# Capacity analysis of an uplink synchronized multicarrier DS-CDMA system

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# Capacity Analysis of an Uplink Synchronized Multicarrier DS-CDMA System

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***Abstract*—An uplink synchronous transmission scheme has been proposed in DS-CDMA systems to increase the uplink capacity through transmission timing control and a proper code usage. Since the scheme suppresses the interference along the first paths of other users, it can achieve a significant capacity increase when combined with multicarrier (MC) technique. This letter mathematically compares the proposed uplink synchronized MC DS-CDMA system with conventional DS-CDMA and MC DS-CDMA systems in terms of the maximum number of users.**

***Index Terms*—Multicarrier DS-CDMA, uplink synchronization.**

## I. Introduction

An UPLINK synchronous transmission scheme has been previously proposed to improve the uplink capacity, which controls the transmission timing of mobile users to guarantee orthogonality among spreading codes at the cell site. Its basic concept and timing control algorithm were proposed and system performance was analyzed in [1]. The capacity was theoretically derived in [2], where the impact of fast transmit power control (TPC) and antenna diversity reception were investigated. In this letter, a combination of the uplink synchronous transmission scheme and multicarrier (MC) technique is proposed. The uplink synchronous transmission scheme can suppress the interference from other users along the first resolvable path [2], which tends to carry most of the signal power in MC DS-CDMA systems since each carrier has a narrowband waveform owing to a low chip rate. Therefore, the uplink synchronous transmission scheme can achieve a significant capacity increase when used in MC DS-CDMA systems. This letter mathematically compares an uplink synchronized MC DS-CDMA system with conventional DS-CDMA and MC DS-CDMA systems in terms of the maximum number of users, assuming fast TPC and antenna diversity reception.

## II. Uplink Synchronized MC DS-CDMA System

### A. MC DS-CDMA System

The input data is first spread with orthogonal spreading codes for channel/user discrimination and scrambled with a common scrambling code for cell discrimination. Then, the stream modulates $M$ band-limited carriers, which enables the frequency diversity whereas the processing gain per carrier is generally $1/M$ times smaller compared to DS-CDMA systems. At the receiver, a matched filter (MF) demodulates the signal of each carrier and then, a combiner performs a maximal-ratio combining for the outputs of $M$ MFs [3].

### B. Uplink Synchronous Transmission Scheme

The scheme consists of transmission timing control and common scrambling code(s). Transmission timing control is required to ensure that received signals are synchronized at the cell site [1]. Users must share a common scrambling code so that only orthogonal spreading codes remain after descrambling. Accordingly, user discrimination is based on spreading codes. Interference among uplink synchronous users can be suppressed owing to orthogonality of spreading codes, similar to the downlink of conventional DS-CDMA systems [2].

### C. Channel Model

In MC DS-CDMA systems, each carrier has a narrowband waveform owing to a low chip rate. Then, the channel can be simplified to have two resolvable multipaths, where the first path tends to carry most signal power. Each path can be modeled as an independent, zero-mean complex Gaussian process [5]. The squared path gains of the first and second paths are exponentially distributed and their means are $1/\alpha$ and $1/\beta$, respectively, where $1/\alpha + 1/\beta = 1$ and $\alpha \ll \beta$. A correlator is used for each carrier because the impact of the second path is quite small. Letting $a_{k,j,l}^{(n)}$ denote the squared path gain of the $l$th path at the $j$th antenna of the $n$th user in the $k$th carrier, $a_{k,j,l}^{(n)}$s $(j, l = 1, 2)$ are mutually independent for different $n, k, j$, and $l$ in the uplink and their means are determined only by path index, i.e., $E[a_{k,p,l}^{(i)}] = E[a_{m,q,l}^{(j)}]$ for any $i, j, k, m, p, q$ and $l$.

In a conventional DS-CDMA system, $L$ mutually independent multipaths with a uniform power delay profile are assumed to exist, i.e., $E[a_{j,l}^{(n)}] = 1/L$, and they are ideally combined with an $L$-finger Rake receiver. An ideal receive antenna diversity with $J$ antennas is assumed for both DS-CDMA and MC DS-CDMA systems. Throughout this letter, a single cell is considered and all users are assumed to be perfectly synchronized in an uplink synchronized system for simplicity.

## III. Performance Evaluation

### A. Capacity for a Conventional DS-CDMA System

Let $E_b/I_o$ be the ratio of signal energy per information bit to power spectrum density of average interference plus back-

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ground noise. Without fast TPC, the received $E_b/I_o$ of the $m$th path at the $j$th antenna of the zeroth user can be defined as [5]

$$\left(\frac{E_b}{I_o}\right)_{j,m} \triangleq \frac{G\frac{S}{J}a_{j,m}^{(0)}}{\frac{2}{3}\frac{S}{J}E\left[\sum_{n=0}^{N-1}\sum_{l=1}^{L}a_{j,l}^{(n)} - a_{j,m}^{(0)}\right]} \approx \frac{\frac{3}{2}Ga_{j,m}^{(0)}}{\sum_{n=0}^{N-1}\sum_{l=1}^{L}E\left[a_{j,l}^{(n)}\right]} = \frac{3}{2}\frac{G}{N}a_{j,m}^{(0)} \tag{1}$$

where $N$ is the number of users and $G$ is the processing gain. We can drop the terms related to the path loss and shadowing by assuming perfect compensation due to open loop power control. The transmit signal power at a mobile user can be denoted as $S/(\sum_{j=1}^{J}\sum_{l=1}^{L}a_{j,l}^{(0)}) = (S/J)$ and the instantaneous received signal power at a cell site antenna becomes $S/J$ times the corresponding squared path gain. $S$ is the target signal power for power control after antenna diversity reception and is the same for all users because only a single type of traffic is considered. Here, the background noise is neglected, and $a_{j,m}^{(0)}$ can be omitted from the denominator in interference calculation since it has a negligible impact for a large number of users [5]. The term 2/3 in the denominator comes from the assumption of rectangular chip pulse shape [6].

The $E_b/I_o$ of the zeroth user after antenna diversity reception can be obtained by

$$E_b/I_o = \sum_{j=1}^{J}\sum_{l=1}^{L}(E_b/I_o)_{j,l} = \frac{3}{2}\frac{G}{N}\sum_{j=1}^{J}\sum_{l=1}^{L}a_{j,l}^{(0)}. \tag{2}$$

Letting $\gamma$ denote the target $E_b/I_o$ level, the outage probability $P_{out}$ is defined as $P_{\text{out}} \triangleq \Pr\{E_b/I_o < \gamma\}$. Then, $P_{\text{out}}$ is given by

$$P_{\text{out}} = \Pr\left\{\sum_{j=1}^{J}\sum_{l=1}^{L}a_{j,l}^{(0)} < \gamma'\right\} \tag{3}$$

where $\gamma' = (2/3)(N/G)\gamma$. The term on the left-hand side, in the braces, is the sum of $JL$ independent, identically distributed random variables with an exponential distribution and accordingly, it follows an Erlang distribution with pdf being

$$f_Y(y) = \frac{L^{JL}}{(JL-1)!}y^{JL-1}e^{-Ly}U(y). \tag{4}$$

If taking into account the ideal fast TPC in a multipath fading environment, we need to consider instantaneous variations in the path gains. $X_n$, the multipath fading experienced after antenna diversity reception, is expressed as $X_n = \sum_{j=1}^{J}\sum_{l=1}^{L}a_{j,l}^{(n)}$. To keep the received signal power at a desired level, the transmit power at mobile users need to be controlled. Then, the instantaneous transmit signal power $\tilde{P}_T$ at a mobile user becomes inversely proportional to $X_n$ so that the received signal power $\tilde{P}_R$ is maintained at the desired level [5], i.e., $\tilde{P}_R = \tilde{P}_T X_n = (S/X_n)X_n = S$ if we drop the terms related to the path loss and shadowing. $E_b/I_o$ is calculated as

$$E_b/I_o = \sum_{j=1}^{J}\sum_{l=1}^{L}\frac{\frac{3}{2}G\frac{a_{j,l}^{(0)}}{X_0}}{\sum_{n=0}^{N-1}E\left[\sum_{l=1}^{L}\frac{a_{j,l}^{(n)}}{X_n}\right]} = \frac{3}{2}\frac{G}{N}J \tag{5}$$

because $E[(\sum_{l=1}^{L}a_{j,l}^{(n)})/X_n] = (1/J)$ in case of uniform power delay profile. The system capacity is given by $N = (3/2)(G/\gamma)J$.

### *B. Capacity for a Conventional MC DS-CDMA System*

Without fast TPC, the received $E_b/I_o$ of the first path at the $j$th antenna of the zeroth user in the $k$th carrier can be expressed as

$$\left(\frac{E_b}{I_o}\right)_{k,j,1} \approx \frac{G_M a_{k,j,1}^{(0)}}{\frac{2}{3}E\left[\sum_{n=0}^{N-1}\sum_{l=1}^{2}a_{k,j,l}^{(n)}\right]} = \frac{3}{2}\frac{G_M}{N}a_{k,j,1}^{(0)} \tag{6}$$

where $G_M$ is the processing gain for MC DS-CDMA systems and there is no interference coming from users in other carriers [3]. The path diversity is not taken into account for MC DS-CDMA systems since only a single correlator is assumed per carrier.

The $E_b/I_o$ of the zeroth user after antenna diversity reception can be obtained by

$$E_b/I_o = \frac{3}{2}\frac{G_M}{N}\sum_{k=1}^{K}\sum_{j=1}^{J}a_{k,j,1}^{(0)}. \tag{7}$$

Then, $P_{\text{out}}$ is calculated from (4) by replacing $JL$ with $KJ$, $L$ with $\alpha$, and $G$ with $G_M$.

With ideal fast TPC, $X_n$ can be modified to

$$X_n = \sum_{k=1}^{K}\sum_{j=1}^{J}a_{k,j,1}^{(n)}. \tag{8}$$

Finally, we can get

$$\frac{E_b}{I_o} = \frac{\frac{3}{2}G_M}{\sum_{n=0}^{N-1}E\left[\frac{a_{k,j,l}^{(n)}+a_{k,j,2}^{(n)}}{X_n}\right]} = \frac{3}{2}\frac{G_M}{N}\left(\frac{1}{KJ}+\frac{\alpha/\beta}{KJ-1}\right)^{-1} \tag{9}$$

because $E[a_{k,j,1}^{(n)}/X_n] = (1/KJ)$ and $E[a_{k,j,2}^{(n)}/X_n] = (\alpha/\beta)/(KJ-1)$. The capacity is expressed as $N = (3/2)(G_M/\gamma)((1/KJ)+((\alpha/\beta)/(KJ-1)))^{-1}$.

Consider a special case of $1/\alpha = 1$ and $1/\beta = 0$. Compared with (5) in a conventional DS-CDMA system, since $G_M$ is $1/K$ times smaller in MC DS-CDMA systems, (9) is simplified to $E_b/I_o = (3/2)(G_M/N)KJ = (3/2)(G/N)J$. Accordingly, both systems have the same capacity, which is consistent with the finding in [4].

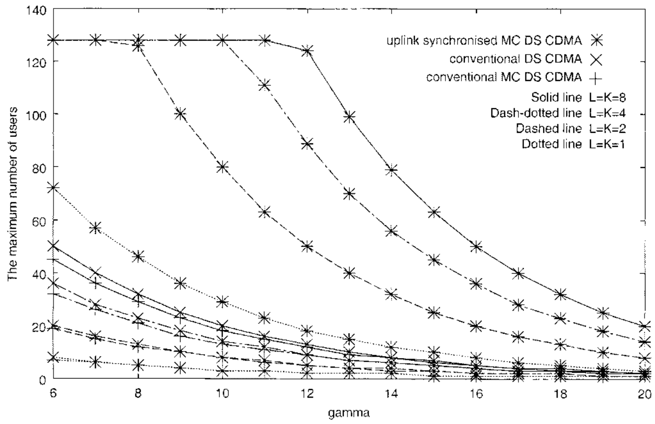


Fig. 1. The maximum number of users for various values of the target $E_b/I_o$, $\gamma$ without fast TPC ($J = 2$ and $P_{\rm out} = 1\%$).

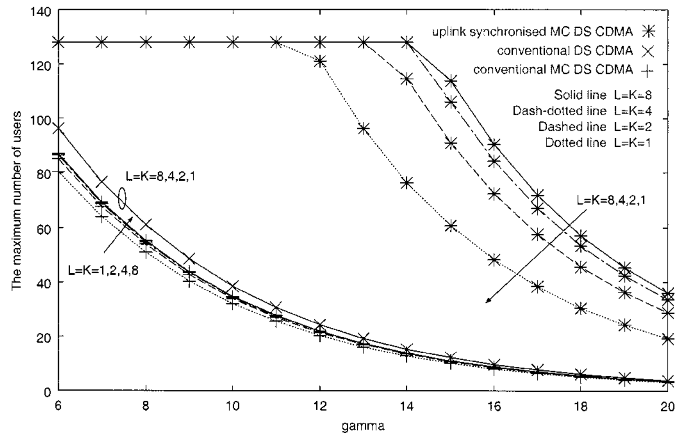


Fig. 2. The maximum number of users for various values of the target $E_b/I_o$, $\gamma$ with fast TPC ($J = 2$).

### C. Capacity for an Uplink Synchronized MC DS-CDMA System

Without fast TPC, the received $E_b/I_o$ of the first path at the $j$th antenna of the zeroth user in the $k$th carrier can be approximated as

$$\left(\frac{E_b}{I_o}\right)_{k,j,1} \approx \frac{G_M a_{k,j,1}^{(0)}}{\frac{2}{3}\sum_{n=0}^{N-1} E\left[a_{k,j,2}^{(n)}\right]} = \frac{3}{2}\frac{\beta G_M}{N} a_{k,j,1}^{(0)} \qquad (10)$$

because the signals along the first paths of other users are not counted as interference.

The $E_b/I_o$ of the zeroth user after antenna diversity reception can be obtained by

$$E_b/I_o = \frac{3}{2}\frac{\beta G_M}{N}\sum_{k=1}^{K}\sum_{j=1}^{J} a_{k,j,1}^{(0)}. \qquad (11)$$

Then, $P_{\rm out}$ can be obtained as for a conventional MC DS-CDMA system except $\gamma'$ is replaced with $(2/3)(N/\beta G_M)\gamma$.

With ideal fast TPC, $X_n$ is given as in (8) and $E_b/I_o$ is calculated as

$$E_b/I_o = \frac{\frac{3}{2}G_M}{\sum_{n=0}^{N-1} E\left[\frac{a_{k,j,2}^{(n)}}{X_n}\right]} = \frac{3}{2}\frac{G_M}{N}\left(\frac{\alpha/\beta}{KJ-1}\right)^{-1}. \qquad (12)$$

The system capacity becomes $N = (3/2)(G_M/\gamma)((\alpha/\beta)/(KJ-1))^{-1}$.

## IV. Numerical Results and Conclusions

For numerical examples, $G$ and $J$ are set to 128 and 2, respectively. The number of multipaths in a conventional DS-CDMA system is assumed to be equal to the number of carriers in MC DS-CDMA systems, i.e., $L = K$, and $\beta/\alpha$ is set to 10 for MC DS-CDMA systems. Fig. 1 plots the maximum number of users satisfying $P_{\rm out} = 1\%$ without fast TPC for varying the target $E_b/I_o$, $\gamma$ when $L = K$ varies 1 to 8. The conventional MC DS-CDMA system is slightly inferior to the conventional DS-CDMA system due to 10% uncaptured signal power with a single correlator per carrier. The uplink synchronized MC DS-CDMA system greatly improves the system capacity, reaching the upper limit of 128 with a low value of $\gamma$. This upper limit is equal to $G$ and comes from the number of possible orthogonal spreading sequences. This limit can be overcome by assigning additional scrambling code(s) to a cell.

Fig. 2 shows the maximum number of users with fast TPC for various values of $\gamma$. Since ideal fast TPC can perfectly compensate the variation due to fading, the conventional DS-CDMA system achieves an equal capacity regardless of $L$. In the case of MC DS-CDMA systems, the uncaptured signal power results in different impact of $L$ on system capacity. However, the uplink synchronized MC DS-CDMA system can still achieve a great capacity improvement.

In conclusion, we have investigated mathematically the attainable capacity improvement when the uplink synchronous transmission scheme is combined with multicarrier technique. Compared with conventional DS-CDMA and MC DS-CDMA systems, the uplink synchronized MC DS-CDMA system can greatly increase the system capacity. The effect of fast TPC and the target $E_b/I_o$ were both considered in view of attainable system capacity. The impact of imperfections in synchronization and fast TPC are left for further study, as is extension to a multiple cell system.

## References


[1] E.-K. Hong, S.-H. Hwang, K.-J. Kim, and K.-C. Whang, "Synchronous transmission technique for the reverse link in DS-CDMA terrestrial mobile systems," *IEEE Trans. Commun.*, vol. 47, pp. 1632–1635, Nov. 1999.

[2] D. K. Kim, S.-H. Hwang, E.-K. Hong, and S. Y. Lee, "Capacity estimation for an uplink synchronised CDMA system with fast TPC and two-antenna diversity reception," *IEICE Trans. Commun.*, vol. E84-B, pp. 2309–2312, Aug. 2001.

[3] S. Kondo and L. B. Milstein, "Performance of multicarrier DS-CDMA systems," *IEEE Trans. Commun.*, vol. 44, pp. 238–246, Feb. 1996.

[4] E. A. Sourour and M. Nakagawa, "Performance of orthogonal multicarrier CDMA in a multipath fading channel," *IEEE Trans. Commun.*, vol. 44, pp. 356–367, Mar. 1996.

[5] D. K. Kim and F. Adachi, "Theoretical analysis of reverse link capacity for an SIR-based power-controlled cellular CDMA system in a multipath fading environment," *IEEE Trans. Veh. Technol.*, vol. 50, pp. 452–464, Mar. 2001.

[6] F. Adachi and D. K. Kim, "Interference suppression factor in DS-CDMA systems," *Electron. Lett.*, vol. 35, pp. 2176–2177, Dec. 1999.