DOWNLINK BEAMFORMING WITH PER-ANTENNA POWER CONSTRAINTS

Wei Yu and Tian Lan

Electrical and Computer Engineering Department
University of Toronto, Toronto, Ontario M5S 3G4, Canada
emails: {weiyu, lantian}@comm.utoronto.ca

ABSTRACT

This paper addresses the problem of linear beamformer design for a multiuser downlink channel in which the base-station is equipped with multiple antennas and the remote users are equipped with a single-antenna each. The design objective is to minimize the maximum transmit power on each transmit antenna while satisfying an SINR constraint for each user. This per-antenna power optimization is more realistic than the usual sum-power criterion because in practice each transmit antenna has its own power amplifier. The main contribution of this paper is a uplink-downlink duality result for this new setting. We prove that the optimal downlink transmit beamformers with per-antenna power constraints are identical to the optimal receive beamformers of a dual uplink channel *with a uncertain noise*. This mirrors our previous result on the capacity region duality with per-antenna power constraints, and it extends the previously known beamforming duality for sum-power constrained channels. The per-antenna beamforming duality derived in this paper is based on a new interpretation of beamforming duality based on Lagrangian duality. Duality is useful in practice because the uplink problem is typically easier to solve. In the final part of this paper, we propose efficient numerical algorithms to solve the per-antenna problem.

1. INTRODUCTION

Duality is a powerful concept for system design in wireless communications. The optimal design for a multi-antenna downlink channel can often be derived from the optimal design of an uplink channel in which the transmit and receive antennas are reversed and the channel matrix is transposed. This so-called uplink-downlink duality has been proved in two very different contexts. From an information theoretical point of view, [1] and [2] showed that the capacity region of a downlink broadcast channel, achieved by the so-called dirty-paper precoding, is exactly the same as the capacity region of a dual uplink multiple-access channel under the same power constraint. On the other hand, from a signal processing perspective, a similar beamforming duality result also holds [5]-[7]. For a multi-antenna downlink channel with a single antenna in each remote user, under the same power constraint, the best achievable SINR region with transmit beamforming is exactly the same as that of an uplink channel with receiver beamforming. Although the proofs of the two results are quite different, in both cases, duality is extremely useful because the uplink problem is much easier to solve than the downlink problem.

In this paper, we focus on the beamforming problem and give a new interpretation of SINR duality. We establish the equivalence of uplink-downlink duality and Lagrangian duality in optimization and extend the beamforming duality beyond the sum-power constrained channels. In particular, we show that the uplink and downlink beamforming problems are related to each other in the sense that their Lagrangian duals are the same. Further, with a per-antenna power constraint, the dual of the optimal downlink beamforming problem becomes an uplink problem with an uncertain channel noise. This result is akin to our previous work on an optimization viewpoint of capacity region duality [9], where the uplink and downlink capacity region optimization problems are shown to be the Lagrangian duals of each other and a similar extension for the per-antenna power constrained problem is established. It is interesting to note that despite the similarity in the two final results, the proofs in these two settings are again very different.

Our Lagrangian approach is different from previous treatment of SINR duality [5]-[7] which is mainly based on the manipulation of the optimality conditions and SINR constraints. It has long been recognized that the uplink beamforming problem has an analytical structure that is much easier to handle [3]. Thus, the main motivation for duality is to transform the downlink problem into the uplink problem. Toward this end, [5] identified an iterative algorithm that solves the downlink beamforming optimization problem in the dual uplink domain. Later, [6] offered an optimality proof for uplink-downlink duality based on an examination of the Karush-Kuhn-Tucker (KKT) conditions for the optimization problem. An alternative approach is provided in [7] where the iterative algorithm is generalized to take into account fairness among the users by maximizing the smallest individual SINR under a total power constraint.

In a subsequent work, [8] introduced a convex optimization framework for this problem. They showed that the downlink beamforming problem can be formulated as a semidefinite programming (SDP) problem and proposed a simple and fast fixed-point iteration algorithm for its solution. All of these above approaches essentially solve the same problem: the downlink beamforming problem with a sum-power constraint.

This paper advocates yet another approach to the downlink beamforming problem based on Lagrangian duality in optimization. This viewpoint not only illustrates the previously known duality result in a new perspective, it also allows a new downlink beamforming problem with per-antenna power constraint to be solved.

2. BEAMFORMING DUALITY WITH PER-ANTENNA POWER CONSTRAINTS

In practical multi-antenna implementations, each transmit antenna is usually equipped with its own power amplifier. Thus, an individual power constraint on each antenna individually is more realistic than a sum power constraint across all the antennas. Consider a downlink channel with N transmit antennas and K users, each equipped with a single antenna only:

$$\mathbf{y} = H\mathbf{x} + \mathbf{z} \tag{1}$$

where $\mathbf{x} = [x_1, \cdots, x_N]$ is the transmitted signal, $\mathbf{y} = [y_1, \cdots, y_K]$ is the received signal, H is an $K \times N$ channel matrix (which is assumed to be known both at the transmitter and at the receivers), and $\mathbf{z}$ is an i.i.d additive Gaussian noise with variance σ^2 on each component. We focus on a scenario where an individual power constraint P_i needs to be satisfied at each transmit antenna:

$$\mathbb{E}[x_i^2] \leq P_i, \quad \forall i = 1 \cdots n. \tag{2}$$

One way to formulate a single optimization problem for this setting is to uniformly minimize the margin of $\mathbb{E}[x_i^2]/P_i$ over all possible beamformers and transmit powers. i.e.

$$\min_{\alpha, \mathbf{w_i}} \alpha \quad \text{s.t. } \mathbb{E}[x_i^2] \leq \alpha P_i. \tag{3}$$

A set of SINR targets are feasible if and only if the optimum α is less than or equal to one. This formulation therefore provides a single measure that reflects the individual transmit power on each antenna.

We consider a beamforming problem in which the transmitted signal is of the form $\mathbf{x} = \sum_{i=1}^{K} \mathbf{w_i} u_i$, where $\mathbf{w_i}$ is a $K \times 1$ beamformer for u_i, user i's information signal. Without loss of generality, let $\mathbb{E}[u_i^2] = 1$. The received signal for user k is

$$y_i = \sum_{j=1}^{K} u_j \mathbf{w_j}^H \mathbf{h_i} + z_i, \tag{4}$$

where $\mathbf{h_i}$ denotes the $K \times 1$ channel vector for user i.

Given a set of SINR targets $\gamma_1, \ldots \gamma_K$, the optimal downlink beamforming problem is to minimize the per-antenna power, i.e. to solve (3) subject to the SINR constraints:

$$\min_{\alpha, \mathbf{w_i}} \quad \alpha \tag{5}$$

$$\text{s.t.} \quad \left[\sum_{j=1}^{K} \mathbf{w_j} \mathbf{w_j}^H\right]_{i,i} \leq \alpha P_i, \quad \forall i = 1 \cdots N \tag{6}$$

$$\frac{|\mathbf{w_i}^H \mathbf{h_i}|^2}{\sum_{j \neq i} |\mathbf{w_j}^H \mathbf{h_i}|^2 + \sigma^2} \geq \gamma_i, \quad \forall i = 1 \cdots K \tag{7}$$

where $[\cdot]_{i,i}$ denotes the (i,i)-entry of a matrix and (6) is the per-antenna power constraint. It is easy to see that, at the optimum point of (5), all SINR constraints (7) must be active.

If the design objective were to minimize the sum total transmit power, the downlink beamforming problem could have been easily solved via a dual uplink channel with the same SINR constraints [5]-[7]. The main result of this paper is a generalization of these results in the sense that the downlink beamforming problem with individual power constraints can still be solved via a dual uplink channel with the same SINR constraints, but the uplink channel must be modified. The modified dual uplink channel has an unusual noise covariance which is not fixed but constrained within a convex set. The SINR constraint must be satisfied with the worst noise in the set. This new result is established using a Lagrangian duality approach as shown in the proof of the following theorem.

Theorem 1 *The optimal downlink beamforming problem (5) with per-antenna power constraints $(P_1, \ldots, P_N)$ can be solved via a dual uplink channel in which the noise is uncertain. The SINR constraints remain the same, but the SINR's have to be satisfied for all diagonal noise covariance matrices Q in the convex constraint set (10) below. More precisely, the Lagrangian dual of the downlink beamforming problem (5) is identical to the Lagrangian dual of the following minimax problem:*

$$\max_{Q} \min_{\lambda_i, \hat{\mathbf{w}}_\mathbf{i}} \quad \sum_{i=1}^{K} \lambda_i \sigma^2 \tag{8}$$

$$\text{s.t.} \quad \frac{\lambda_i |\hat{\mathbf{w}}_\mathbf{i}^H \mathbf{h_i}|^2}{\sum_{j \neq i} \lambda_j |\hat{\mathbf{w}}_\mathbf{i}^H \mathbf{h_j}|^2 + \hat{\mathbf{w}}_\mathbf{i}^H Q \hat{\mathbf{w}}_\mathbf{i}} \geq \gamma_i \tag{9}$$

$$\mathrm{tr}(Q\Phi) \leq 1, \;\; Q \text{ diagonal}, \;\; Q \geq 0 \tag{10}$$

where λ_i is the dual uplink power and the diagonal matrix $\Phi = \mathrm{diag}(P_1, \ldots P_N)$ is the per-antenna power budget in the downlink problem.

Proof: To show that the Lagrangian duals of the uplink and the downlink problems are identical, we first derive the dual of the downlink beamforming problem (5). With a

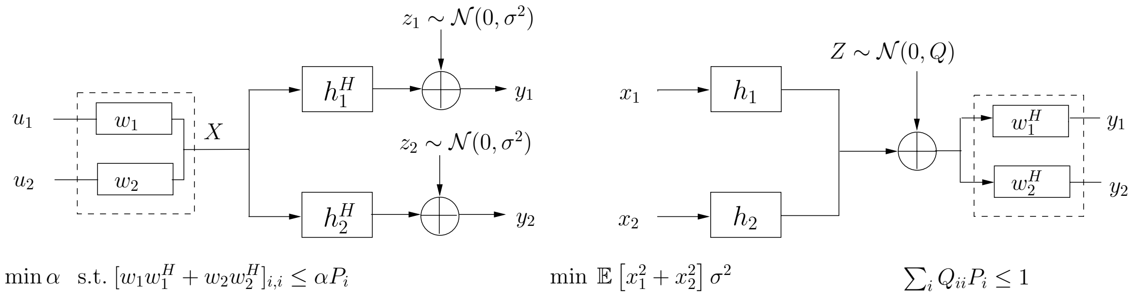

Fig. 1. Uplink-downlink beamforming duality with per-antenna power constraints

simple manipulation of the SINR constraints (7), the Lagrangian for downlink optimization problem (5) is given by

$$L^{DL} = \alpha + \sum_{i=1}^{N} q_i \left\{ \left[\sum_{j=1}^{K} \mathbf{w_j} \mathbf{w_j}^H \right]_{i,i} - \alpha P_i \right\} \qquad (11)$$
$$- \sum_{i=1}^{K} \lambda_i \left\{ \frac{1}{\gamma_i} |\mathbf{w_i}^H \mathbf{h_i}|^2 - \sum_{j \ne i} |\mathbf{w_j}^H \mathbf{h_i}|^2 - \sigma^2 \right\}.$$

where q_i's are the Lagrange multipliers corresponding to per-antenna power constraints (6) and λ_i's are the Lagrange multipliers corresponding to SINR constraints (7).

Let $Q = \mathrm{diag}(q_1, \cdots, q_N)$ and $\Phi = \mathrm{diag}(P_1, \cdots, P_N)$. Rearranging the terms of (11), we obtain

$$L^{DL} = \sum_{i=1}^{K} \lambda_i \sigma^2 - \alpha \left\{ \mathrm{tr}(Q\Phi) - 1 \right\} \qquad (12)$$
$$+ \sum_{i=1}^{K} \mathbf{w_i}^H \left\{ Q + \sum_{j \ne i} \lambda_j \mathbf{h_j} \mathbf{h_j}^H - \frac{\lambda_i}{\gamma_i} \mathbf{h_i} \mathbf{h_i}^H \right\} \mathbf{w_i}$$

The dual objective is therefore

$$g(Q, \lambda_i) = \min_{\mathbf{w_i}} \min_{\alpha} L^{DL}(\alpha, \mathbf{w_i}, Q, \lambda_i). \qquad (13)$$

Since α must be positive and there is no constraint on the beamformer $\mathbf{w_i}$, it is easy to see that $g(Q, \lambda_i) = -\infty$ if $\mathrm{tr}(Q\Phi) \ge 1$ or $Q + \sum_{j \ne i} \lambda_j \mathbf{h_j} \mathbf{h_j}^H - \frac{\lambda_i}{\gamma_i} \mathbf{h_i} \mathbf{h_i}^H \le 0$. As Q and λ_i should be chosen such that the Lagrangian dual $g(Q, \lambda_i)$ exists, the above two inequalities impose constraints on the dual objective function. Formally, the Lagrangian dual problem can be written down as follows:

$$\max_{Q} \max_{\lambda_i} \quad \sum_{i=1}^{K} \lambda_i \sigma^2 \qquad (14)$$
$$\text{s.t.} \quad Q + \sum_{j=1}^{K} \lambda_j \mathbf{h_j} \mathbf{h_j}^H \ge \left(1 + \frac{1}{\gamma_i}\right) \lambda_i \mathbf{h_i} \mathbf{h_i}^H$$
$$\mathrm{tr}(Q\Phi) \le 1, \ \ Q \text{ diagonal}, \ \ Q \ge 0$$

Next, we derive the Lagrangian dual for the uplink optimization problem (8). The key observation is that (8) is equivalent to the following optimization problem

$$\max_{Q} \max_{\lambda_i} \min_{\hat{\mathbf{w}}_\mathbf{i}} \quad \sum_{i=1}^{K} \lambda_i \sigma^2 \qquad (15)$$
$$\text{s.t.} \quad \frac{\lambda_i |\hat{\mathbf{w}}_\mathbf{i}^H \mathbf{h_i}|^2}{\sum_{j \ne i} \lambda_j |\hat{\mathbf{w}}_\mathbf{i}^H \mathbf{h_j}|^2 + \hat{\mathbf{w}}_\mathbf{i}^H Q \hat{\mathbf{w}}_\mathbf{i}} \le \gamma_i$$
$$\mathrm{tr}(Q\Phi) \le 1, \ \ Q \text{ diagonal}, \ \ Q \ge 0$$

where the SINR constraints are reversed and the minimization on λ_i is replaced with a maximization. The reason is that at the optimal solutions of (8) and (15), the SINR constraints must be satisfied with equality. Thus, the optimal set of power allocations $(\lambda_1, \ldots, \lambda_K)$ must be the same for both problems.

The Lagrangian of this new uplink optimization problem (15) is now:

$$L^{UP} = \sum_{i=1}^{K} \lambda_i \sigma^2 - \alpha \left\{ \mathrm{tr}(Q\Phi) - 1 \right\} \qquad (16)$$
$$+ \sum_{i=1}^{K} \delta_i \hat{\mathbf{w}}_\mathbf{i}^H \left\{ Q + \sum_{j \ne i} \lambda_j \mathbf{h_j} \mathbf{h_j}^H - \frac{\lambda_i}{\gamma_i} \mathbf{h_i} \mathbf{h_i}^H \right\} \hat{\mathbf{w}}_\mathbf{i}.$$

where α is the Lagrange multiplier for uncertain noise constraint and δ_i's are Lagrange multipliers for uplink SINR constraints. Setting $\mathbf{w_i} = \sqrt{\delta_i} \hat{\mathbf{w}}_\mathbf{i}$, we observe that L^{DL} is exactly L^{UP}. $\square$

Note that we have not yet established that either the uplink or downlink beamforming problems is convex. However, the fact that the Lagrangians of the uplink and downlink optimization problems are identical is sufficient to ensure that local optima for the uplink and downlink problems must correspond to each other. Thus, the global optimum of the two problems must also be the same.

In fact, it can be shown that for both problems, a local optimum must also be a global optimum. The downlink beamforming problem can be converted into a convex optimization problem using a technique due to Wiesel, Eldar and Shamai [8]. Thus, the duality gap of the downlink problem is zero. A derivation of this fact is included in the appendix. For the uplink problem, Visotsky and Madhow [6] proved that the global optimum solution can be obtained using an iterative power update algorithm. Thus, a local optimum in the uplink problem is a global optimum as well.

It is interesting to compare the structures of the uplink optimization problem (8) and the downlink problem (5). It is clear that the Lagrange multipliers λ_i corresponding to the SINR constraints in the downlink problem play the role of transmit power in the uplink problem. The Lagrange multiplier Q corresponding to the per-antenna power constraints in the downlink problem plays the role of noise covariance matrix in the uplink. This correspondence between the primal and dual variables enhances and generalizes the previous sum-power duality as in [5]-[7].

3. OPTIMIZATION ALGORITHMS VIA DUALITY

The main motivation for establishing the uplink-downlink duality is that the uplink beamforming problem is typically more amendable to numerical computation. Thus, duality leads to an efficient numerical solution for the original downlink problem.

To solve the dual uplink beamforming problem for the per-antenna power constrained downlink channel, a simultaneous maximization of the noise covariance Q and minimization on transmit power and beamformer $(\lambda_i, \hat{\mathbf{w}}_{\mathbf{i}})$ must be done. We propose the following numerical algorithm that computes the minimization of $(\lambda_i, \hat{\mathbf{w}}_{\mathbf{i}})$ and the maximization of Q iteratively.

Under a fixed noise covariance Q, the optimal power and beamformer $(\lambda_i, \hat{\mathbf{w}}_{\mathbf{i}})$ can be found via a fixed-point iteration algorithm as previously proposed for the sum-power problem. In our implementation, we choose the method of [8] as shown in Steps 1 and 2 below. At a fixed $(\lambda_i, \hat{\mathbf{w}}_{\mathbf{i}})$, the update of the worst-noise covariance Q is more difficult. However, as Q is also the Lagrange multiplier for the per-antenna power constraints in the downlink, a subgradient update of Q can easily be implemented in the downlink: Q_{ii} should increase if the transmit power on antenna i exceeds its power budget and decrease otherwise. This is implemented in Step 3. Here, $\mathcal{S}_Q = \{Q : \mathrm{tr}(Q\Phi) \leq 1, Q \geq 0\}$ denotes the constraint set and $\mathcal{P}_{\mathcal{S}_Q}$ is the Euclidean projection on $\mathcal{S}_Q$. The proposed algorithm is as follows:

1. Solve for optimal power allocation λ_i for the uplink using a fixed point iteration:
$\lambda_i^{n+1} = \dfrac{1}{(1+\frac{1}{\gamma_i})\mathbf{h}_{\mathbf{i}}^H(\sum_{j=1}^K \lambda_i^n \mathbf{h}_{\mathbf{j}}\mathbf{h}_{\mathbf{j}}^H + Q^n)^{-1}\mathbf{h}_{\mathbf{i}}}$
2. Update the beamformers for the downlink problem:
$\hat{\mathbf{w}}_{\mathbf{i}}^{n+1} = \sqrt{\delta_i}(\sum_{j=1}^K \lambda_j^{n+1}\mathbf{h}_{\mathbf{j}}\mathbf{h}_{\mathbf{j}}^H + Q^n)^{-1}\mathbf{h}_{\mathbf{i}}$
3. Update Q using a subgradient method with step size t_n: $Q^{n+1} = \mathcal{P}_{\mathcal{S}_Q}\{Q^n + t_n \mathrm{diag}(\sum_{i=1}^K \mathbf{w}_{\mathbf{i}}\mathbf{w}_{\mathbf{i}}^H)\}$.

It is also possible to solve the dual optimization (14) directly using an interior-point method. A numerical example of downlink systems with 10 and 20 mobile users is simulated. Fig. 2 shows the norm distance of the current power vector to the optimal solution as a function of the number of iterations. Algorithm One in the plot is the iterative update described above with a square summable step size $t_n = \frac{1}{n}$. Algorithm Two in the plot is the interior-point method. As one would expect, the iterative update is more effective when the gap is large. However, the interior-point method performs better as the power vector approaches the optimum.

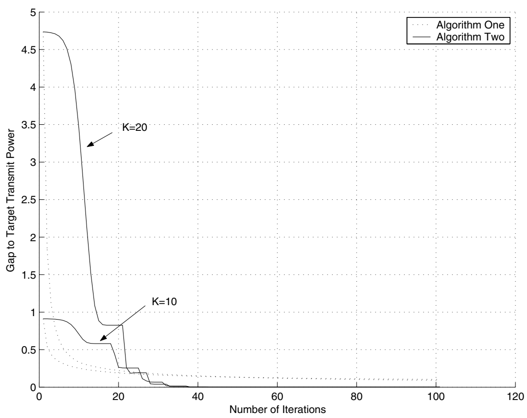

Fig. 2. Convergence behavior of proposed algorithm for downlink systems with 10 and 20 mobile users.

4. CONCLUSION

In this paper, we solve the downlink beamforming problem with per-antenna power constraints using a generalized uplink-downlink duality. Our proof is based on a new relation between beamforming duality and Lagrangian duality in optimization. It turns out that the dual of a downlink beamforming problem with per-antenna power constraint is an uplink problem with uncertain noise. This allows the downlink problem to be solved effectively in the dual domain. Finally, we proposed an iterative algorithm for the computation of the dual uplink beamforming problem followed by a comparison with a general-purpose interior-point implementation.

5. APPENDIX

We prove in this appendix that strong duality holds for the downlink beamforming problem (5) and its Lagrangian dual

(14). This is also an alternative proof of Theorem 1. The same proof also works for the optimization problem (15).

The main idea is to use the technique in [8] to transform the problem into a semi-definite programming problem. First, we re-write the downlink beamforming problem (5) into a equivalent convex form:

$$\min_{\alpha,\mathbf{w_i}} \quad \alpha \tag{17}$$

$$\text{s.t.} \quad \left[\sum_{j=1}^{K} \mathbf{w_j}\mathbf{w_j}^H\right]_{i,i} \leq \alpha P_i, \ \forall i. \tag{18}$$

$$\sqrt{1+\frac{1}{\gamma_i}}\mathbf{w_i}^H\mathbf{h_i} \geq \left\| \begin{array}{c} \mathbf{h_i^H} W \\ \sigma \end{array} \right\|, \ \forall i. \tag{19}$$

where $||\cdot||$ denotes the $\mathcal{L}_2$ Euclidean vector norm and $W = [\mathbf{w_1}, \ldots, \mathbf{w_K}]$. Its Lagrangian is given by

$$\begin{aligned} L = \quad & \alpha + \sum_{i=1}^{K} q_i \left\{ \left[\sum_{j=1}^{K} \mathbf{w_j}\mathbf{w_j}^H\right]_{i,i} - \alpha P_i \right\} \qquad (20) \\ & - \sum_{i=1}^{K} \mu_i \left\{ \sqrt{1+\frac{1}{\gamma_i}}\mathbf{w_i}^H\mathbf{h_i} - \left\| \begin{array}{c} \mathbf{h_i^H} W \\ \sigma \end{array} \right\| \right\}. \end{aligned}$$

The dual objective is therefore

$$g(Q,\mu_i) = \min_{\mathbf{w_i}} \min_{\alpha} L(\alpha, \mathbf{w_i}, Q, \mu_i). \tag{21}$$

Since the optimization objective and constraints in (17) are convex, strong duality holds. In other words, $g(Q,\mu_i)$ maximized over Q and μ_i reaches a maximum at the optimal value of the primal problem (17).

To compute $g(Q,\mu_i)$, let

$$t_i = \left\{ \sqrt{1+\frac{1}{\gamma_i}}\mathbf{w_i}^H\mathbf{h_i} + \left\| \begin{array}{c} \mathbf{h_i^H} W \\ \sigma \end{array} \right\| \right\}. \tag{22}$$

Then, the last term in (20) can be rewritten as

$$\begin{aligned} & \mu_i \left\{ \sqrt{1+\frac{1}{\gamma_i}}\mathbf{w_i}^H\mathbf{h_i} - \left\| \begin{array}{c} \mathbf{h_i}^H W \\ \sigma \end{array} \right\| \right\} \\ & = \frac{\mu_i}{t_i} \left\{ \left(1+\frac{1}{\gamma_i}\right)(\mathbf{w_i}^H\mathbf{h_i})^2 - \left\| \begin{array}{c} \mathbf{h_i}^H W \\ \sigma \end{array} \right\|^2 \right\} \\ & = \frac{\mu_i}{t_i} \left\{ \left(1+\frac{1}{\gamma_i}\right)(\mathbf{w_i}^H\mathbf{h_i})^2 - \mathbf{h_i}^H \left[\sum_{j=1}^{K} \mathbf{w_j}\mathbf{w_j}^H\right] \mathbf{h_i} - \sigma^2 \right\} \end{aligned}$$

Substituting this into the Lagrangian, we obtain

$$\begin{aligned} L = \quad & \sum_{i=1}^{K} \frac{\mu_i}{t_i}\sigma^2 - \alpha\left\{ \mathrm{tr}(Q\Phi) - 1 \right\} \qquad (23) \\ & + \sum_{i=1}^{K} \mathbf{w_i}^H \left\{ Q + \sum_{j\neq i} \frac{\mu_j}{t_j}\mathbf{h_j}\mathbf{h_j}^H - \frac{\mu_i}{t_i\gamma_i}\mathbf{h_i}\mathbf{h_i}^H \right\} \mathbf{w_i}. \end{aligned}$$

Note that t is lower bounded by σ and is strictly positive. Since the only constraint for the maximization of μ_i is $\mu_i \geq 0$, we can change the optimization variable to $\lambda_i = \mu_i/t_i$. Notice that (23) is now exactly (12). Thus, the dual of the SDP (17) is exactly the dual problem (14) derived in Theorem 1. The convexity of (17) guarantees strong duality and zero duality gap between (5) and (14).

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