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Life-Cycle Channel Coordination Issues in Launching an Innovative Durable Product

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We analyze the dynamic strategic interactions between a manufacturer and a retailer in a decentralized distribution channel used to launch an innovative durable product (IDP). The underlying retail demand for the IDP is influenced by word-of-mouth from past adopters and follows a Bass-type diffusion process. The word-of-mouth influence creates a trade-off between immediate and future sales and profits, resulting in a multi-period dynamic supply chain coordination problem. Our analysis shows that while in some environments, the manufacturer is better off with a far-sighted retailer, there are also environments in which the manufacturer is better off with a myopic retailer. We characterize equilibrium dynamic pricing strategies and the resulting sales and profit trajectories. We demonstrate that revenue-sharing contracts can coordinate the IDP's supply chain with both far-sighted and myopic retailers throughout the entire planning horizon and arbitrarily allocate the channel profit.

Key words: supply chain coordination; innovative durable products; differential games; revenue-sharing contracts
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1. Introduction

This research addresses the dynamic strategic interactions between manufacturers with innovative durable products (IDPs) and the specialized retailers that sell the IDPs to final users. We consider a stylized model of a supply chain in which a monopolist manufacturer produces an IDP and sells it through an independent retailer who serves the final market. We assume that the window of opportunity to sell the product is exogenously determined (by competition or other factors), and assume there are a finite number of potential adopters for the IDP and each potential adopter purchases at most one unit (i.e., no repeat purchases).

Although this research was motivated by the distribution of computer-aided design hardware and software, the insights obtained apply to the distribution of multiple innovative industrial products. These products are technically very sophisticated, and the buyers for these products require extensive technical information and attention before purchasing a unit. Therefore, these products are often distributed through intermediaries known as value adding resellers (VARs).

There are several motivations for a manufacturer to use the VARs channel. First, the VARs are often already experienced in dealing with the needs and idiosyncrasies of potential adopters as the VARs may be providing the potential adopters with other related products and services. Second, the VARs may already have a business relationship with the IDP target customers; hence, they can reach potential customers faster and more economically. Third, because they are typically located in the same geographical region as their customer base, they are more efficient in providing after-sales field support.

Although the potential efficiencies of distributing an IDP through VARs are significant, introducing an intermediary in the distribution channel complicates the coordination of this supply chain. This problem presents inter-temporal trade-offs between current and future profits, and possible differences in the term of optimization objectives of the retailers (i.e., *myopic* vs. *far-sighted*), which combine to create complex double marginalization issues. Specifically, lowering current period prices may stimulate immediate sales, possibly at the expense of immediate profits, while an increase in the number of current adopters increases future demand through word-of-mouth or network influences, possibly leading to larger future profits. Furthermore, if both the manufacturer and the VAR (i.e., the retailer) make independent pricing decisions, neither of them has full control of their own profitability or the profitability of the supply chain. The VAR, through pricing, affects demand for the IDP, but its profitability is also affected by the wholesale prices charged by the manufacturer. The manufacturer, on the other hand, can affect retail prices and sales only indirectly. Even in cases in which the manufacturer has the ability to set the retail price for the physical IDP, the VAR can affect the total cost of ownership for the customer by varying the prices charged by the after-sales services and hence affect the demand. For simplicity, in the rest of the paper, we will refer by retail price (set by the VAR) to the total cost of ownership of the IDP. Similarly, in the rest of the paper we will refer to the exogenously defined window of opportunity to sell the product simply as the IDP's life cycle; although we must acknowledge this definition is unconventional.

In our analysis, we concentrate on the following two challenges: (a) the effect of differences in the term of the optimization objectives of the VAR and (b) the double marginalization problems that arise in this dynamic multi-period environment. Below, we elaborate on each of these challenges.

(a) *Effect of differences in the VAR's inter-temporal optimization objectives*: We assume throughout the analysis that the manufacturer is interested in maximizing its profits over the IDP's life cycle. The VAR, on the other hand, can either have a *myopic* optimization objective (i.e., to maximize its immediate profits), or it can be committed to maximizing its profits over the IDP's life cycle; in this latter case we will call it a *far-sighted* retailer.

(b) *Multi-period double marginalization*: As an independent decision maker, the VAR will formulate the pricing strategies to maximize its own profits disregarding the profitability of the manufacturer. The manufacturer, on the other hand, will select a wholesale pricing strategy to maximize its own profits over the IDP's selling horizon.

The maximization of life-cycle profits derived from an IDP presents us with a multi-period, dynamic supply chain coordination problem. In this context, it is relevant to ask whether it is possible to fully coordinate the supply chain throughout the entire selling horizon of the IDP. If so, what are the terms of such coordinating contracts and how will the profits be split between the manufacturer and the retailer?

We formulate the life-cycle channel coordination problem in an optimal control framework. We assume the manufacturer takes the leader role and the retailer is the follower in a Stackelberg (sequential) game. Specifically, the manufacturer sets the wholesale price to the retailer, and the retailer sets the retail price that maximizes its profits. The manufacturer takes the retailer's optimal reaction into consideration when it makes its wholesale pricing decision. Our solution for the Stackelberg differential game is an open-loop equilibrium; that is, at the start of the game, the manufacturer and retailer decide on a strategy that depends on time, and we assume that the manufacturer is able to credibly commit to its wholesale pricing strategy.

We find that in some instances the manufacturer will prefer the retailer to react to the wholesale prices by setting retail prices myopically to maximize its immediate (instantaneous) profits rather than its long-term profits over the entire horizon. In our analysis we will elaborate on the circumstances leading the manufacturer to prefer each type of retailer behavior.

We demonstrate that revenue-sharing contracts are in principle capable of coordinating a durable product supply chain with either a far-sighted or a myopic retailer and arbitrarily allocate the channel profit between the manufacturer and the retailer. Moreover, as the coordinating contracts achieve the same profits as an integrated channel, they overcome any sub-optimality induced by the open-loop policy assumed in the optimization of the Stackelberg game.

The rest of the paper is organized as follows. In the next section we review the related literature. In section 3, we introduce the demand model. In section 4, we study the case of a far-sighted retailer. In section 5, we study the case of a myopic retailer. In section 6, we present a numerical study that compares the cases of far-sighted and myopic retailers. In section 7, we use revenue-sharing contracts to fully coordinate the channel. We conclude the paper by summarizing the results and the managerial implications and pointing out future research avenues in section 8. All proofs are in the on-line Appendix.

2. Literature Review

This work is related to multiple streams of literature, but the three most closely related are (1) diffusion models in the marketing literature, (2) revenue-sharing contracts in supply chain management literature, and (3) differential games applications in management science.

In the marketing literature, Bass (1969) and its variants have been widely used to forecast the demand of a new durable product. We refer readers to Mahajan et al. (1990) and Mahajan et al. (2000) for comprehensive reviews on diffusion models. A number of papers have extended the Bass model by incorporating the (competitive) price impact on retail demand of an IDP, including Robinson and Lakhani (1975), Bass (1980), Dolan and Jeuland (1981), Bass and Bultez (1982), Kalish (1983), Kalish and Lilien (1983), Clarke and Dolan (1984), Thompson and Teng (1984), Rao and Bass (1985), Eliashberg and Jeuland (1986), Raman and Chatterjee (1995), and Krishnan et al. (1999). Eliashberg and Jeuland (1986) and Thompson and Teng (1984) analyze oligopoly pricing strategies while the rest analyze the optimal monopolist pricing strategies. Levin et al. (2010) study the optimal dynamic pricing of a perishable product to strategic consumers. Using a Markovian model, Hall et al. (2009) consider a supply chain in which a make-to-order manufacturer sells a product to the core customers at a fixed price and to ''fill-in'' customers at a current price. They examine both state-independent (static) and state-dependent pricing policies in a make-to-order environment. Their results show that constant pricing up to a cutoff state can significantly outperform a static pricing policy.

In order to derive the dynamic pricing strategies, we need to make a key assumption about the firm's profit-maximizing strategy: does the firm maximize the short-term or long-term profits? Bass (1980) and Bass and Bultez (1982) assume the firm maximizes the current period (instantaneous) profits. The corresponding pricing strategies in these two papers are called *myopic* pricing strategies as compared with (global) optimal pricing strategies which maximize the firm's aggregated profits over the product's life cycle. Robinson and Lakhani (1975) compared the results of optimal pricing with those of myopic pricing strategies. Their numerical results show that the differences are significant while Bass and Bultez (1982) report small differences.

It is critical to properly incorporate the impact of pricing in the demand model. Several papers, including Robinson and Lakhani (1975), Dolan and Jeuland (1981), and Thompson and Teng (1984), assume the demand is an exponential function of price. Other researchers, like Eliashberg and Jeuland (1986) and Raman and Chatterjee (1995), assume that demand is a linearly decreasing function of retail price. In this paper, we select a linear demand model to explore contracting and coordination issues in a decentralized dynamic supply chain.

All of the above papers assume an integrated distribution channel in which all pricing and production decisions are centralized on a single decision maker. Therefore, they are unable to examine the pricing implications of having an independent retailer as part of the distribution channel of the IDP.

In the supply chain management literature, a number of supply contracts have been designed to mitigate or eliminate the double marginalization and incentive misalignment problems due to the independent decisions of a retailer in a decentralized channel. We refer the readers to Krishnan et al. (2004) and Cachon (2003) for excellent reviews on the supply chain contracting literature. The papers focusing on revenue-sharing contracts are more closely related to this research. Gerchak and Wang (2004) study revenue-sharing contracts between an assembler/retailer and its component suppliers. Cachon and Lariviere (2005) study revenue-sharing contracts using a single-period model of a decentralized two-stage supply chain facing stochastic demand. Gerchak et al. (2006) study revenue-sharing contracts in the video rental industry; in their model, the movie studios and the video rental retailers play a Stackelberg game and make independent decisions. All of the above papers focus on a single interaction between an upstream supplier/manufacturer and a downstream retailer. By contrast, we study the channel coordination between a manufacturer and a retailer over a finite-time selling horizon, in which both channel members interact multiple times making dynamic retail and wholesale pricing decisions.

In our model, we assume the manufacturer and the retailer play a Stackelberg differential game. This approach is very popular to study the problems involving dynamic environments (see He et al. 2007 for a comprehensive review of differential game models in supply chain and marketing channels). Jorgensen et al. (2003) study the dynamic advertising strategies of a manufacturer and a retailer in a decentralized setup. In their model, the retailer can be myopic (maximizes the instantaneous payoff) or far-sighted (maximizes the long-term payoff). He et al. (2009) apply a Stackelberg differential game to model the pricing and co-op advertising policy in a decentralized supply chain. Eliashberg and Steinberg (1987) formulate a Stackelberg differential game to study joint production, distribution, and pricing strategies in a decentralized supply chain consisting of a manufacturer who sets a constant transfer price and a downstream distributor who can dynamically optimize retail prices. By contrast, in this research both the manufacturer and the retailer dynamically optimize their wholesale and retail prices, respectively, and, unlike the above models, we focus on an IDP where demand is generated by a diffusion model.

3. The Demand Model

We assume the retail demand for the IDP follows a modified Bass model. Let $x(t)$ be the instantaneous sales rate at time t. The demand dynamics are described by the following differential equation:

$$x(t) = \dot{X}(t) = \frac{dX(t)}{dt} = (M - X(t))(\alpha + \beta X(t))(1 - \gamma r(t)),$$
$$X(0) = X_0,$$

where $X(t)$ is the cumulative product sales at time t, M is the total market potential for the product, the term $(M - X(t))$ is the unsaturated market size, and $\alpha \geq 0$, $\beta \geq 0$ are the coefficients of innovation and imitation, respectively. The positive parameter γ measures the customer's sensitivity to the retail price $r(t)$.

Note that we use a multiplicatively separable function to model the impact of price and cumulative sales (past sales) on the instantaneous demand rate $x(t)$, and the demand rate is a linearly decreasing function of the retail price $r(t)$.

We use the superscripts "*L*" and "*S*" to denote the long-term (far-sighted) and short-term (myopic) retailer profitability strategy, respectively. Subscripts "*M*," "*R*," and "*SC*" denote the manufacturer, the retailer, and the supply chain, respectively.

4. Far-Sighted Retailer Model

Consider the case of a far-sighted retailer who maximizes its profits over the entire life cycle T of the IDP. We use a Stackelberg game framework to formulate the manufacturer's and the retailer's problems. The manufacturer acts as the game leader and the retailer acts as the follower. The sequence of the events is as follows. The manufacturer announces the wholesale price path $\{w^L(t)\colon t \in [0, T]\}$ at time 0. Then after observing $w^L(t)$, the retailer decides the retail price path $\{r^L(t)\colon t \in [0, T]\}$. We look for a Stackelberg equilibrium of the differential game. The optimal pricing strategies are obtained as open-loop strategies. Using open-loop strategies means that the manufacturer (retailer) commits to using wholesale (retail) pricing strategies based on time only.

The retailer's instantaneous profit rate function is given by $[r^L(t) - w^L(t) - s]x^L(t)$, where s is the retailer's cost associated with selling the product. This cost should include not only the variable costs associated with closing the sale of the physical IDP, but it should also include the variable cost of the additional services provided. The far-sighted retailer's optimization problem is given by

$$\Pi_R^{L^*}(T) = \max_{r^L(t)} \int_0^T [r^L(t) - w^L(t) - s]x^L(t)dt, \quad (1)$$

$$\text{s.t.} \quad x^L(t) = (M - X^L(t))(\alpha + \beta X^L(t))(1 - \gamma r^L(t)), \quad (2)$$

$$X^L(0) = X_0^L, \quad (3)$$

where $X^L(0)^1$ is the initial sales. Note that Equations (1)–(3) formulate an optimal control problem with the retail price $r^L(t)$ and the cumulative sales $X^L(t)$ as control and state variables, respectively. The differential equation (2) along with the initial condition (3) explicitly describe how the cumulative sales and retail price jointly determine the instantaneous demand rate $x^L(t)$. Note that Equation (1) has been constructed under the assumption that the far-sighted retailer does not discount its profit between time 0 and the end of the finite selling horizon T. This assumption is made for mathematical tractability; however, this assumption does not change the qualitative nature of our results. We make a similar assumption to construct the manufacturer's objective function.

We first solve the retailer's problem and use the retailer's best response to formulate the manufacturer's problem. From now on, for notational simplicity, we may omit the time argument in some equations. Let $F(X^L) = (M - X^L)(\alpha + \beta X^L)$ and $f(X^L) = dF(X^L)/dX^L = -\alpha + M\beta - 2\beta X^L$. The retailer's Hamiltonian H_R^L is given by

$$H_R^L = F(X^L)(1 - \gamma r^L)[r^L - w^L - s + \lambda_R^L], \quad (4)$$

where λ_R^L, the shadow price associated with the state variable X^L, satisfies the adjoint equation:

$$\dot{\lambda}_R^L = -\frac{\partial H_R^L}{\partial X^L} = -f(X^L)(1 - \gamma r^L) \times [r^L - w^L - s + \lambda_R^L],\ \lambda_R^L(T) = 0. \quad (5)$$

Let r^{L^*} be the retailer's best response retail price. The first-order condition $\partial H_R^L / \partial r^L = 0$ gives us the best response r^{L^*}

$$r^{L^*} = \frac{1 + \gamma(w^L + s - \lambda_R^L)}{2\gamma}. \quad (6)$$

The economic interpretation of $\lambda_R^L(t)$ is the value of additional unit of sales. For given $w^L(t)$, $\lambda_R^L(t) > 0$ implies that the retailer benefits from current sales (see Sethi and Thompson 2000 for a detailed discussion of the economic interpretation of the shadow price). Accordingly, the retailer sets $r^L(t)$ below the myopic retail response, which is defined as the price that would result if we set $\lambda_R^L = 0$. With a myopic response, the retailer does not take into account the impact of current sales on future sales. On the other hand, when $\lambda_R^L(t) < 0$, the retailer has no incentive to sacrifice current profits for future profits, and the retailer will increase $r^L(t)$ above the myopic price level.

Let $H_R^{L^*}$ be the retailer's maximized Hamiltonian, which is given by

$$H_R^{L^*} = \frac{F(X^L)[1 - \gamma(w^L + s - \lambda_R^L)]^2}{4\gamma}.$$

It is easy to verify that $H_R^{L^*}$ is concave and continuously differentiable with respect to X^L for all $t \in [0, T]$. Therefore r^{L^*} is an optimal path.

Note that for each wholesale price path $\{w^L(t)\colon t \in [0, T]\}$ the manufacturer announces, there is a corresponding optimal retail price path $\{r^{L^*}(t)\colon t \in [0, T]\}$. The manufacturer takes the retailer's best response into consideration when solving the optimization problem. Assume that the manufacturer incurs a constant per unit production cost c_0. The manufacturer's optimization problem is given by

$$\Pi_M^{L^*}(T) = \max_{w^L} \int_0^T [w^L - c_0]x^{L^*}dt$$

$$\text{s.t.} \quad x^{L^*} = \frac{F(X^{L^*})\{1 - \gamma[w^L + s - \lambda_R^L]\}}{2}, \quad (7)$$

$$\dot{\lambda}_R^L = -\frac{f(X^{L^*})\{1 - \gamma[w^L + s - \lambda_R^L]\}^2}{4k}, \quad (8)$$

$$X^L(0) = X_0^L,\ \lambda_R^L(T) = 0, \quad (9)$$

where Equations (7) and (8) are obtained by substituting (6) into (2) and (5), respectively. Note that the manufacturer has two state variables: X^{L^*} and λ_R^L. Dockner et al. (2000) used a similar approach. By now, we have defined a two-player differential game with two control variables w^L and r^L.

The manufacturer's Hamiltonian equation H_M^L is given by

$$H_M^L = (w^L - c_0 + \lambda_M^L)x^{L^*} + \mu\dot{\lambda}_R^L, \qquad (10)$$

where λ_M^L and μ are the shadow prices associated with x^{L^*} and $\dot{\lambda}_R^L$, respectively. λ_M^L and μ satisfy the following conditions:

$$\dot{\lambda}_M^L = -\frac{\partial H_M^L}{\partial X^L} = -\frac{f(X^L)(w^L - c_0 + \lambda_M^L)[1 - \gamma(w^L + s - \lambda_R^L)]}{2}, \qquad (11)$$

$$\dot{\mu} = -\frac{\partial H_M^L}{\partial \lambda_R^L} = -\frac{\gamma F(X^L)(w^L - c_0 + \lambda_M^L)}{2} + \frac{\mu f(X^L)[1 - \gamma(w^L + s - \lambda_R^L)]}{2}, \qquad (12)$$

with boundary conditions $\lambda_M^L(T) = 0$ and $\mu(0) = 0$. We impose $\lambda_R^L(T) = 0$ because $X^L(T)$ is free to move and impose $\mu(0) = 0$ because our problem is controllable, i.e., the associated initial state $\lambda_R^L(0)$ is dependent on w^L. Substituting (7) into (10), we obtain

$$H_M^L = \frac{F(X^L)(w^L - c_0 + \lambda_M^L)\{1 - \gamma[w^L + s - \lambda_R^L]\}}{2} - \frac{\mu f(X^L)\{1 - \gamma[w^L + s - \lambda_R^L]\}^2}{4\gamma}. \qquad (13)$$

The necessary first-order condition to maximize H_M^L, $\partial H_M^L/\partial w^L = 0$, gives us

$$w^{L^*} = \frac{1 + \gamma(\lambda_R^L - s)}{\gamma} - \frac{F(X^{L^*})\psi^L}{\gamma[2F(X^{L^*}) + \mu f(X^{L^*})]}, \qquad (14)$$

where $\psi^L = 1 + \gamma(\lambda_M^L + \lambda_R^L - s - c_0)$. Substituting (14) into (6), we have

$$r^{L^*} = \frac{1}{\gamma} - \frac{F(X^{L^*})\psi^L}{2\gamma[2F(X^{L^*}) + \mu f(X^{L^*})]}. \qquad (15)$$

Substituting (14) into (13), after simplification, we obtain the maximized Hamiltonian equation $H_M^{L^*}$:

$$H_M^{L^*} = \frac{[\psi^L F(X^{L^*})]^2}{4\gamma[2F(X^{L^*}) + \mu f(X^{L^*})]}.$$

Substituting (14) into (7), (8), (11), and (12), respectively, we have

$$\dot{X}^{L^*} = \frac{\psi^L[F(X^{L^*})]^2}{2[2F(X^{L^*}) + \mu f(X^{L^*})]}, \quad X^L(0) = X0^L, \qquad (16)$$

$$\dot{\lambda}_R^L = \frac{-f(X^{L^*})[\psi^L F(X^{L^*})]^2}{4\gamma[2F(X^{L^*}) + \mu f(X^{L^*})]^2}, \quad \lambda_R^L(T) = 0, \qquad (17)$$

$$\dot{\lambda}_M^L = -\frac{F(X^{L^*})(\psi^L)^2\left\{f(X^{L^*})F(X^{L^*}) + \mu\left[(f(X^{L^*}))^2 + \beta F(X^{L^*})\right]\right\}}{2\gamma[2F(X^{L^*}) + \mu f(X^{L^*})]^2},$$

$$\lambda_M^L(T) = 0, \qquad (18)$$

$$\dot{\mu} = -\frac{\psi^L[F(X^{L^*})]^2}{2[2F(X^{L^*}) + \mu f(X^{L^*})]}, \quad u(0) = 0. \qquad (19)$$

We assume that ψ^L has the same sign as $2F(X^{L^*}) + \mu f(X^{L^*})$ so that the instantaneous sales rate is positive in (16). Under this assumption, we find that $\dot{\mu} = -\dot{X}^{L^*} < 0$, $\forall t \in [0, T]$. This implies that $\mu(t) < 0$, $\forall t \in [0, T]$, because $\mu(0) = 0$. We observe, from (17), that the sign of $\dot{\lambda}_R^L$ depends on the sign of $f(X^{L^*})$: if $f(X^{L^*}) > 0$, $\dot{\lambda}_R^L < 0$, i.e., λ_R^L is decreasing; otherwise, $\dot{\lambda}_R^L \geq 0$.

Equations (16)–(19) consist of a system of four differential equations with four unknowns, which, along with the boundary conditions, imply a solution; however, it is very difficult to derive analytical solutions for all the variables as functions of time and system parameters. (See Eliashberg and Jeuland 1986 for a discussion of the complexity of the solutions to a similar system of non-linear differential equations.)

5. Myopic Retailer Model

Our analysis assumes the manufacturer owns the rights to produce and distribute the IDP. Therefore, the manufacturer will set wholesale prices to maximize its own life-cycle profits. However, it is not clear if it is in the VAR's best interest to have a myopic optimization objective (i.e., to maximize its profit rate) or if the VAR should set retail prices to maximize its profits over the IDP's life cycle. Its preferred optimization objectives will be determined by multiple factors including features in the contract the VAR signs with the manufacturer and the characteristics of the IDP itself. For example, the VAR would take a myopic approach to pricing the IDP if its contract with the manufacturer does not give the VAR certainty about longer-term distribution of the IDP.

The model in this section assumes that the retailer sets its prices myopically to maximize its profit rate; the manufacturer knows the retailer has a myopic optimization objective, and using this knowledge it sets the wholesale prices to maximize its life-cycle profits. This model will be used in section 6 to compare the relative desirability, from the manufacturer's perspective, of having a myopic or a far-sighted retailer.

At any time t, the myopic retailer's instantaneous profit rate is $\pi_R^S(t) = [r^S(t) - w^S(t) - s]x^S(t)$ at time t. Its objective is to maximize the instantaneous profit rate:

$$\pi_R^{S^*}(t) = \max_{r^S(t)}[r^S(t) - w^S(t) - s]x^S(t), \tag{20}$$

$$\text{s.t.} \quad x^S(t) = F(X^S(t))[1 - \gamma r^S(t)], \quad X^S(0) = X_0^S, \tag{21}$$

where $F(X^S(t)) = (M - X^S(t))(\alpha + \beta X^S(t))$. Solving the first-order condition $\partial \pi_R^S(t)/\partial r^S(t) = 0$ gives us the retailer's best response:

$$r^{S^*} = \frac{1}{2\gamma}[1 + \gamma(w^S + s)]. \tag{22}$$

The manufacturer takes the retailer's best response into consideration when solving the optimization problem. The manufacturer maximizes its life-cycle profits, and its optimization problem is given by

$$\Pi_M^{S^*}(T) = \max_{w^S(t)} \int_0^T [w^S(t) - c_0]x^S(t)dt,$$

$$\text{s.t.} \quad x^S = \frac{1}{2}F(X^S)[1 - \gamma(w^S + s)], \quad X^S(0) = X_0^S. \tag{23}$$

where the expression for x^S in (23) is obtained by substituting (22) into (21). The manufacturer has a control variable w^S and a state variable X^S. The manufacturer's Hamiltonian equation H^S is given by

$$H^S = \frac{1}{2}F(X^S)[1 - \gamma(w^S + s)][w^S - c_0 + \lambda^S],$$

where λ^S, the shadow price associated with the state variable X^S, satisfies the adjoint equation:

$$\dot{\lambda}^S = -\frac{\partial H^S}{\partial X^S} = -\frac{1}{2}f(X^S)[1 - \gamma(w^S + s)][w^S - c_0 + \lambda^S],$$
$$\lambda^S(T) = 0, \tag{24}$$

where $f(X^S) = dF^S/dX^S = -\alpha + M\beta - 2\beta X^S$. We can determine the optimal control w^{S^*} from the first-order condition of $\partial H^S/\partial w^S = 0$:

$$w^{S^*} = \frac{1}{2\gamma}[1 + \gamma(c_0 - s - \lambda^S)]. \tag{25}$$

The maximized Hamiltonian H^{S^*} is given by

$$H^{S^*} = \frac{F(X^S)[1 + \gamma(\lambda^S - c_0 - s)]^2}{8\gamma}.$$

We can easily verify that H^{S^*} is concave in X^S and continuously differentiable with respect to X^S for all $t \in [0, T]$. Therefore, w^{S^*} is an optimal path.

Substituting (25) into Equations (22)–(24), we derive the optimal retail price, instantaneous sales rate, and shadow price as follows:

$$r^{S^*} = \frac{1}{4\gamma}[3 - \gamma(\lambda^S - s - c_0)], \tag{26}$$

$$x^{S^*} = \frac{F(X^{S^*})}{4}[1 + \gamma(\lambda^S - s - c_0)], \tag{27}$$

$$\dot{\lambda}^S = -\frac{f(X^{S^*})}{8\gamma}[1 + \gamma(\lambda^S - s - c_0)]^2. \tag{28}$$

The solution to the problem is determined by solving (27) and (28) simultaneously together with boundary conditions: $X^S(0) = X_0^S$ and $\lambda^S(T) = 0$. In the rest of this section, we characterize the equilibrium of this game in terms of cumulative sales.

The following lemma derives the relationship between λ^S and X^{S^*}. We assume that $1 - \gamma(c_0 + s) > 0$.

Lemma 1. *The shadow price trajectory $\lambda^S(t)$ is given by*

$$\lambda^S(t) = \frac{1 - \gamma(c_0 + s)}{\gamma}\left[\sqrt{\frac{F(X^{S^*}(T))}{F(X^{S^*}(t))}} - 1\right], \tag{29}$$
$$t \in [0, T].$$

Remarks. We note that the sign of $\lambda^S(t)$ is determined by the sign of $\left(\sqrt{F(X^{S^*}(T))/F(X^{S^*}(t))} - 1\right)$. If $F(X^{S^*}(T)) > F(X^{S^*}(t))$, $\lambda^S(t) > 0$. If $F(X^{S^*}(T)) < F(X^{S^*}(t))$, $\lambda^S(t) < 0$. Lemma 1 enables us to eliminate the shadow price from the optimality conditions and characterize the important variables in terms of the cumulative sales $X^{S^*}(t)$.

Lemma 2. *With a short-term retailer profitability strategy, for $t \in [0, T]$,*

(i) The instantaneous shadow price $\dot{\lambda}^S(t)$ is given by

$$\dot{\lambda}^S(t) = -\frac{[1 - \gamma(c_0 + s)]^2 f(X^{S^*}(t))}{8\gamma}\frac{F(X^{S^*}(T))}{F(X^{S^*}(t))}.$$

(ii) The equilibrium retail price trajectory $r^{S^*}(t)$ and wholesale price trajectory $w^{S^*}(t)$ are

$$r^{S^*}(t) = \frac{1}{4\gamma}\left[4 - (1 - \gamma(c_0 + s))\sqrt{\frac{F(X^{S^*}(T))}{F(X^{S^*}(t))}}\right],$$
$$w^{S^*}(t) = \frac{1}{2\gamma}\left[2(1 - \gamma s) - (1 - \gamma(c_0 + s))\sqrt{\frac{F(X^{S^*}(T))}{F(X^{S^*}(t))}}\right].$$

(iii) The equilibrium instantaneous sales rate $x^{S^*}(t)$ is given by

$$x^{S^*}(t) = \frac{1-\gamma(c_0+s)}{4}\sqrt{F(X^{S^*}(t))F(X^{S^*}(T))}.$$

(iv) The retailer's and manufacturer's equilibrium instantaneous profit rates are given by

$$\pi_R^{S^*}(t) = \frac{[1-\gamma(c_0+s)]^2 F\big(X^{S^*}(T)\big)}{16\gamma},$$

$$\pi_M^{S^*}(t) = \frac{[1-\gamma(c_0+s)]^2}{8\gamma}\Big[2\sqrt{F(X^{S^*}(t))F(X^{S^*}(T))} - F\big(X^{S^*}(T)\big)\Big].$$

Remarks. According to part (i), the sign of $\dot{\lambda}^S(t)$ is determined by the sign of $f\big(X^{S^*}(t)\big)$. We have $\dot{\lambda}^S(t)>0$ if $f\big(X^{S^*}(t)\big) = -\alpha + M\beta - 2\beta X^{S^*}(t) < 0$, i.e., $X^{S^*}(t) > (-\alpha + M\beta)/(2\beta)$, and $\dot{\lambda}^S(t)<0$, if $X^{S^*}(t) < (-\alpha+M\beta)/(2\beta)$. This result is consistent with that in the case of a far-sighted retailer. In part (ii), in order to obtain a meaningful wholesale price trajectory, i.e., $w^{S^*}(t)>0$, the parameters should satisfy the condition $\sqrt{F(X^{S^*}(T))/F(X^{S^*}(t))} < 2(1-\gamma s)/(1-\gamma(c_0+s))$. According to part (iv), the myopic retailer achieves a constant instantaneous profit rate over time. This result is surprising because both the instantaneous sales volume and the retail margin vary over time. Our assumption of multiplicatively separable demand function partially contributes to this result. On the other hand, the manufacturer's instantaneous profit rate varies over time.[2]

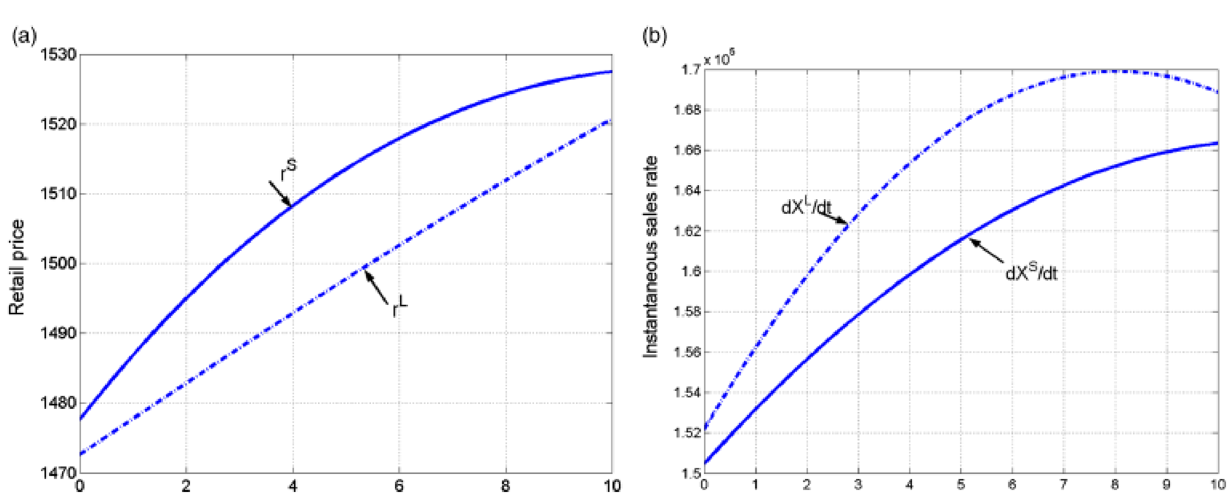
Proposition 1. *The optimal retail price r^{S^*}, wholesale price w^{S^*}, and instantaneous sales rate x^{S^*} peak at the same time. We can observe three retail/wholesale pricing patterns, depending on the system parameters.*

(i) If $X^{S^*}(T) < \frac{M}{2} - \frac{\alpha}{2\beta}$, $r^{S^*}(t)$, and $w^{S^*}(t)$ are both monotonically increasing over the entire selling horizon.
(ii) If $X^{S^*}(T) > \frac{M}{2} - \frac{\alpha}{2\beta} > X^S(0)$, $r^{S^*}(t)$, and $w^{S^*}(t)$ are increasing up to the peak sales and decreasing thereon.
(iii) If $X^S(0) > \frac{M}{2} - \frac{\alpha}{2\beta}$, $r^{S^*}(t)$, and $w^{S^*}(t)$ are monotonically decreasing over the entire selling horizon.

Proposition 1 states that we may observe three different patterns of retail and wholesale price trajectories: monotonically increasing, increasing then declining, and monotonically declining. The ultimate determinant of pricing patterns is the interaction between the different effects driving the demand dynamics: the diffusion effect (word-of-mouth) and the saturation effect. When the market saturation level is low, the diffusion effect outweighs the saturation effect, and the retailer (manufacturer) will start with relatively low price to stimulate early sales. As the cumulative sales grow, the saturation effect dominates the diffusion effect; in a saturated market, every additional sale will have the effect of decreasing the future sales rate, and the retailer (manufacturer) will price high to capture the immediate profit rather than sacrificing current profits for future profits. This result is a generalization of Kalish (1983), which examined the optimal dynamic pricing strategy within a centralized channel setting. Here, we show that with a short-term retailer profitability strategy, the optimal retail price and wholesale price patterns should follow the sales curves. In contrast, in the decentralized channel with a far-sighted retailer, neither the retail price nor the wholesale price pattern mimics the sales curves as illustrated by Figure 1.

From Lemma 2, we have demonstrated the importance of obtaining the cumulative sales trajectory

Figure 1 Comparisons with Far-Sighted or Myopic Retailers. (a) Retail Price and (b) Instantaneous Sales Rate



Note: The parameters are $M=4\times10^7$, $X_0=1\times10^7$, $\alpha=0.016$, $\beta=8\times10^{-9}$, $\gamma=5\times10^{-4}$, $c_0=100$, $s=10$, $T=10$.

$X^{S^*}(t)$. The following lemma provides a method to calculate $X^{S^*}(t)$.

LEMMA 3. *The equilibrium (optimal) cumulative sales trajectory $X^{S^*}(t)$ can be determined by the unique solution to the following equation for $\forall t \in [0, T]$:*

$$\tan^{-1}\left[\frac{f(X^{S^*}(t))}{2\sqrt{\beta F(X^{S^*}(t))}}\right] = \tan^{-1}\left[\frac{f(X^{S}(0))}{2\sqrt{\beta F(X^{S}(0))}}\right] - \frac{t[1-\gamma(c_0+s)]\sqrt{\beta F(X^{S^*}(T))}}{4}. \tag{30}$$

Managerially, Lemma 3 is very useful for two major reasons. First, it is useful for forecasting purposes. The manufacturer can assess the life-cycle cumulative sales $X^{S^*}(T)$ using (30) to solve for the unique solution. Note that $X^{S^*}(T)$ is a function of initial sales $X^S(0)$ and parameters of the problem. Once the life-time cumulative sales is obtained, the cumulative sales trajectory $X^{S^*}(t)$ for any given time t is determined as well as the wholesale and retail price trajectories. Second, the managers can plan for operations decisions such as production rate and production capacity at any instant t.

Let $\Pi^{S^*}_{SC}(T)$ denote the channel profit up to time t, which is the sum of the manufacturer's and retailer's profits over the selling horizon. The following result characterizes the equilibrium (optimal) profits up to time t, $\Pi^{S^*}_{R}(t)$ and $\Pi^{S^*}_{M}(t)$ and $\pi^{S^*}_{SC}(T)$, in terms of the cumulative sales $X^{S^*}(t)$.

LEMMA 4. *Let $\Delta X^{S^*}(t) = X^{S^*}(t) - X^{S}(0)$.*

(i) The manufacturer's optimal cumulative profit function up to time t, $\Pi^{S^*}_{M}(t)$, and retailer's optimal cumulative profit up to time t, $\Pi^{S^*}_{R}(t)$, are given by

$$\Pi^{S^*}_{M}(t) = \frac{[1-\gamma(c_0+s)]\Delta X^{S^*}(t)}{\gamma} - \frac{t[1-\gamma(c_0+s)]^2 F(X^{S^*}(T))}{8\gamma},$$

$$\Pi^{S^*}_{R}(t) = \frac{t[1-\gamma(c_0+s)]^2 F^{S^*}(T)}{16\gamma}.$$

(ii) The life-cycle supply chain profit function $\Pi^{S^*}_{SC}(T)$ is given by

$$\Pi^{S^*}_{SC}(T) = \frac{[1-\gamma(c_0+s)]\Delta X^{S^*}(T)}{\gamma} - \frac{T[1-\gamma(c_0+s)]^2 F(X^{S^*}(T))}{16\gamma}.$$

We conclude this section by pointing out that it is not clear whether a retailer with a myopic optimization objective is desirable or undesirable from the manufacturer's perspective; this will be addressed in the next section.

6. Numerical Analysis: Myopic vs. Far-Sighted Retailers

In the previous sections, we analyzed the models with both far-sighted and myopic retailers. We now compare the manufacturer's profits under these two retailer optimization strategies. Our results show that the manufacturer will not always be better off with a far-sighted retailer instead of a retailer who follows a short-term (myopic) optimization strategy.

We conducted a numerical study with different values of T and β (recall that β captures the word-of-mouth effect, or the network effect). Table 1 reports for each (T, β) combination (a) the manufacturer's preferred retailer profitability strategy denoted as P with possible values of either L denoting a life-cycle focus or S denoting a short-term or myopic optimization focus, (b) the market saturation level at the end of selling horizon denoted as SL and expressed as a percentage of total market captured throughout the life cycle, and (c) the manufacturer's percentage gain in profits, G, with its preferred retailer's optimization strategy. We have a few observations. First, in some cases, the manufacturer is better off with a far-sighted, L, retailer while in other cases the manufacturer is better off with a myopic, S, retailer. Second, for a fixed value of β, the manufacturer's preferences shift from the far-sighted, L, to the myopic, S, retailer as T increases. Similarly, for a fixed value of T, preferences shift in the same order as β increases. Third, the manufacturer's profit gain, G, can be as high as 12% with its preferred retailer optimization strategy (i.e., a myopic retailer) when the market is highly saturated at the end of the selling horizon, which occurs when either T or β or both are very large. However, when the market is not highly saturated, the profit gain of the manufacturer with its preferred retailer's optimization focus (i.e., a far-sighted retailer) is smaller.

Table 1 Preferred Retailer Strategy, Market Saturation, and Profit Gain

	$\beta = 2.5\times10^{-9}$			$\beta = 8\times10^{-9}$			$\beta = 2\times10^{-8}$			$\beta = 5\times10^{-8}$		
T	P	SL (%)	G (%)	P	SL (%)	G (%)	P	SL (%)	G (%)	P	SL (%)	G (%)
1	—	—	—	L	27	<1	L	29	2	L	36	5
2	—	—	—	L	29	2	L	34	4	L	49	5
5	L	28	3	L	35	3	L	49	4	S	74	5
10	L	32	5	L	45	4	S	68	2	S	90	12

Problem parameters are $M = 4\times10^7$, $X_0 = 1\times10^7$, $\alpha = 0.016$, $\gamma = 5\times10^{-4}$, $c_0 = 100$, $s = 10$.

Let P denote the manufacturer preferred retailer optimization strategy, $P \in \{S, L\}$.

SL denotes the market saturation level at the end of the selling horizon, defined as $SL = \frac{X(T)}{M} \times 100\%$.

G is the percentage of the manufacturer's profit gain with its preferred retailer profitability strategy.

These results can be explained as follows. A myopic retailer ignores the impact of current sales on the future profits while a far-sighted retailer takes the future profit into consideration when it sets the retail price. For both the manufacturer and the far-sighted retailer, the shadow prices represent the future value (profit) of an additional sale. The magnitude of the shadow prices depends on the market saturation level which is itself a function of the problem parameters (i.e., T, α, β, c_0, and s). When the market saturation is low, the retailer shadow price is positive. However, when the market saturation is high enough, the retailer's shadow price becomes negative.

A myopic retailer ignores its shadow price when making pricing decisions. A positive retailer's shadow price leads the far-sighted retailer to lower its profit margin below the myopic level in order to stimulate the current sales, while a negative shadow price leads a far-sighted retailer to increase its profit margins above the myopic level. Therefore, the manufacturer will benefit from the retailer's far-sighted behavior as long as the market saturation is low and the retailer's shadow price is positive, and it would prefer a myopic retailer if the retailer's shadow price is negative for a large enough portion of the life cycle.

We emphasize that the variable driving the manufacturer's preference of the retailer's optimization objective is the level of market saturation reached by the diffusion process. In our numerical analysis, we tested the sensitivity of the manufacturer's preferences for different values of T and β because the diffusion process is very sensitive to these parameters. However, we emphasize that the other parameters, c_o and α and s, also affect the diffusion process thereby the market saturation level. Accordingly, varying those parameters, we can observe a similar pattern of the change of the manufacturer's preferences.

7. Revenue-Sharing Contracts

Cachon and Lariviere (2005) show in a single-period setting that revenue-sharing contracts are very effective in a wide range of static supply chains. Specifically, revenue-sharing contracts coordinate a supply chain with a single retailer and arbitrarily allocate the supply chain's profit. In this section, we examine whether the revenue-sharing contracts can be effective in coordinating a dynamic supply chain. In section 7.1, we obtain the optimal price, sales, and profit trajectories for an integrated supply chain; these results will be used as benchmarks to evaluate the performance of a decentralized supply chain. In sections 7.2 and 7.3, we analyze channel coordination mechanisms for the far-sighted and myopic retailers, respectively.

7.1. Integrated Channel

We now consider an integrated channel in which the manufacturer makes centralized decisions to maximize the supply chain profits over the life cycle of the IDP. The channel maximizes the life-cycle profit obtained from selling the IDP. The channel incurs a constant per unit production cost c_0 and a selling cost s. This problem corresponds to a specialization of the demand function in Kalish (1983). For this special case, we obtain an implicit expression for the optimal sales trajectory (Lemma 7) and are able to express the optimal retail price trajectory (Lemma 6) and the optimal profit trajectory (Lemma 8) as functions of the cumulative sales. The integrated channel's profit-maximization problem is given by

$$\begin{aligned} \Pi^{I^*}(T) &= \max_{r^I(t)} \int_0^T [r^I(t) - c_0 - s]x^I(t)dt, \\ \text{s.t.} \quad x^I &= (M - X^I)(\alpha + \beta X^I)(1 - \gamma r^I), \quad X^I(0) = X_0^I. \end{aligned}$$

Let $\lambda^I(t)$ denote the shadow price associated with the state variable X^I. Using a similar approach to the one applied in the previous sections, we establish the relationship between the optimal cumulative sales X^{I^*} and shadow price λ^I in the following lemma.

Lemma 5. *For the integrated channel, $\lambda^I(t)$ is given by*

$$\lambda^I(t) = \frac{1 - \gamma(c_0 + s)}{\gamma}\left(\sqrt{\frac{F(X^{I^*}(T))}{F(X^{I^*}(t))}} - 1\right), \quad \lambda^I(T) = 0.$$

Lemma 6. *For the integrated channel, the optimal retail price, the instantaneous sales rate, and the instantaneous profit rate are given by*

$$\begin{aligned} r^{I^*}(t) &= \frac{1}{2\gamma}\left[2 - (1 - \gamma(c_0 + s))\sqrt{\frac{F(X^{I^*}(T))}{F(X^{I^*}(t))}}\right], \\ x^{I^*}(t) &= \frac{1 - \gamma(c_0 + s)}{2}\sqrt{F(X^{I^*}(T))F(X^{I^*}(t))}, \\ \pi^{I^*}(t) &= \frac{(1 - \gamma(c_0 + s))^2}{4\gamma}\Big[2\sqrt{F(X^{I^*}(T))F(X^{I^*}(t))} \\ &\quad - F\big(X^{I^*}(T)\big)\Big]. \end{aligned}$$

Lemma 7. *The optimal cumulative sales trajectory $X^{I^*}(t)$ is determined by the unique solution to the following equation for $\forall t \in [0, T]$:*

$$\begin{aligned} \tan^{-1}\left[\frac{f\big(X^{I^*}(t)\big)}{2\sqrt{\beta F(X^{I^*}(t))}}\right] &= \tan^{-1}\left[\frac{f\big(X^I(0)\big)}{2\sqrt{\beta F(X^I(0))}}\right] \\ &\quad - \frac{t(1 - \gamma(c_0 + s))\sqrt{\beta F(X^{I^*}(T))}}{2}. \end{aligned}$$

Using the results from Lemmas 6 and 7, we can now establish the profit trajectory for the integrated channel.

LEMMA 8. *Let* $\Delta X^{I^*}(t) = X^{I^*}(t) - X^I(0)$. *The optimal integrated channel's cumulative profit function* $\Pi^{I^*}(t)$, $\forall t \in [0, T]$ *is given by*

$$\Pi^{I^*}(t) = \frac{(1-\gamma(c_0+s))\Delta X^{I^*}(t)}{\gamma} - \frac{t(1-\gamma(c_0+s))^2 F\left(X^{I^*}(T)\right)}{4\gamma}.$$

In the next two sections, we use these results to elicit revenue-sharing contracts leading to the coordination of a decentralized distribution channel.

7.2. Revenue Sharing with a Far-Sighted Retailer

We consider a revenue-sharing contract with two parameters $\{q^L, \hat{w}^L(t)\}$, where $q^L \in [0,1]$ is the manufacturer's share of revenue per unit sold by the retailer, and $\hat{w}^L(t)$ is the wholesale price that the manufacturer charges the retailer per unit at time t. The manufacturer's revenue share, q^L, is assumed to be constant over time. Note that we use the hat-accent "^" to indicate that the variable is associated with a revenue-sharing contract. The manufacturer's objective is to set $\hat{w}^L(t)$ such that the supply chain profit (sales) is the same as that achieved by an integrated channel.

The far-sighted retailer's problem is the following:

$$\hat{\Pi}_R^{L^*}(T) = \max_{\hat{r}^L(t)} \int_0^T [(1-q^L)\hat{r}^L(t) - \hat{w}^L(t) - s]\hat{x}^L(t)dt,$$

$$\text{s.t.} \quad \hat{x}^L(t) = F\left(\hat{X}^L(t)\right)[1-\gamma\hat{r}^L(t)], \quad \hat{X}^L(0) = \hat{X}_0^L.$$

Let $\hat{\lambda}_R^L(t)$ be the shadow price associated with the state variable $\hat{X}^L(t)$. Let $\hat{\Pi}_{SC}^{S^*}(T)$ be the channel's optimal life-cycle profit function under the revenue sharing with a far-sighted retailer.

THEOREM 1. *Consider a revenue-sharing contract with* $q^L \in [0,1]$ *and the wholesale price trajectory* $\hat{w}^{L^*}(t)$ *set as follows:* $\hat{w}^{L^*}(t) = \hat{w}^{L^*} = (1-q^L)c_0 - q^L s$.

(i) The retailer's instantaneous profit rate and its life-cycle profit function are

$$\hat{\pi}_R^{L^*}(t) = (1-q^L)\pi^{I^*}(t),$$
$$\hat{\Pi}_R^{L^*}(T) = (1-q^L)\Pi^{I^*}(T).$$

(ii) The manufacturer's instantaneous profit rate and its life-cycle profit function are

$$\hat{\pi}_M^{L^*}(t) = q^L\pi^{I^*}(t),$$
$$\hat{\Pi}_M^{L^*}(T) = q^L\Pi^{I^*}(T).$$

(iii) The above revenue-sharing contract coordinates the channel, i.e., $\hat{\Pi}_{SC}^{L^*}(T) = \Pi^{I^*}(T)$. *The retailer's instantaneous sales rate is* $x^{I^*}(t)$ *and the retail price is set at* $r^{I^*}(t)$.

This theorem shows that, although we allow the wholesale price to change over time, the coordinating wholesale price is constant over time and it is below the manufacturer's cost of production. Therefore, the manufacturer loses money in the wholesale transaction, but it can make a profit after receiving its share of the sales revenue; Cachon and Lariviere (2005) obtain a similar result in their setting. The theorem also shows that, at any instant, q^L is also the manufacturer's share of the supply chain's profit rate in addition to its share of revenue. Furthermore, since $q^L \in [0,1]$, the type of revenue-sharing contract specified above is able to coordinate the supply chain and arbitrarily allocate the supply chain profits between the manufacturer and the retailer through the entire selling horizon.

7.3. Revenue Sharing with a Myopic Retailer

In this section, we consider a revenue-sharing contract specified by $\{q^S, \hat{w}^S(t)\}$, and signed by a manufacturer and a myopic retailer; under this contract, the retailer will pay the manufacturer a constant share, q^S, of the IDP's sales revenue, and the manufacturer commits to a wholesale price trajectory specified by $\hat{w}^S(t)$. In our analysis, we assume the manufacturer devises the proposed wholesale price trajectory $\hat{w}^S(t)$ so that the instantaneous sales rate $\hat{x}^S(t) = x^I(t)$. Under this contract, the myopic retailer's problem is to maximize its instantaneous profit rate:

$$\hat{\pi}_R^{S^*}(t) = \max_{\hat{r}^S(t)} [(1-q^S)\hat{r}^S(t) - \hat{w}^S(t) - s]\hat{x}^S(t),$$

$$\text{s.t.} \quad \hat{x}^S(t) = F\left(\hat{X}^S(t)\right)[1-\gamma\hat{r}^S(t)], \quad \hat{X}^S(0) = \hat{X}_0.$$

Let $\hat{\Pi}_{SC}^{S^*}(T)$ be the channel's optimal life-cycle profit function under the revenue sharing with a myopic retailer.

THEOREM 2. *Consider a revenue-sharing contract with* $q^S \in [0,1]$ *and the wholesale price trajectory* $\hat{w}^{S^*}(t)$ *set as*

$$\hat{w}^{S^*}(t) = \frac{1}{\gamma}(1-q^S)\left[1-(1-\gamma(c_0+s))\sqrt{\frac{F(X^{I^*}(T))}{F(X^{I^*}(t))}}\right] - s.$$

(i) The resulting retailer's instantaneous profit rate and its cumulative life-cycle profit function are

$$\hat{\pi}_R^{S^*}(t) = \frac{(1-q^S)(1-\gamma(c_0+s))^2}{4\gamma}F(X^{I^*}(T)),$$

$$\hat{\Pi}_R^{S^*}(T) = \frac{(1-q^S)(1-\gamma(c_0+s))^2}{4\gamma}TF\left(X^{I^*}(T)\right).$$

(ii) The manufacturer's instantaneous profit rate and its cumulative life-cycle profit function are

$$\hat{\pi}_M^{S^*}(t) = \frac{(1-\gamma(c_0+s))^2}{4\gamma}\Big[2\sqrt{F(X^{I^*}(t))F(X^{I^*}(T))} - (2-q^S)F(X^{I^*}(T))\Big],$$

$$\hat{\Pi}_M^{S^*}(T) = \frac{(1-\gamma(c_0+s))\Delta X^{I^*}(T)}{\gamma} - \frac{T(1-\gamma(c_0+s))^2(2-q^S)F\big(X^{I^*}(T)\big)}{4\gamma}.$$

(iii) The above revenue-sharing contract coordinates the channel, i.e., $\hat{\Pi}_{SC}^{S^*}(T) = \Pi^{I^*}(T)$. *At any time t, the retailer's instantaneous sales rate is* $x^{I^*}(t)$ *and the retail price* $r^{I^*}(t)$.

As in the case of the far-sighted retailer, our model allows the wholesale price to change over time; however, with a myopic retailer, the wholesale price that coordinates the supply chain is not constant. Note that at the end of selling horizon, $\sqrt{F(X^{I^*}(T))/F(X^{I^*}(t))} = 1$; hence if $q^L = q^s = q$, Theorem 2 implies that at the end of the IDP's life cycle, the coordinating contract's wholesale price of the myopic retailer converges to the coordinating contract's (constant) wholesale price of the far-sighted retailer, $\hat{w}^{S^*}(T) = \hat{w}^{L^*} = (1-q)c_0 - qs$. Furthermore, if $F\big(X^{I^*}(t)\big)$ is increasing over time, then $\hat{w}^{S^*}(t)$ decreases in t. Initially, $\hat{w}^{S^*}(t)$ may be greater than c_0, which means that the manufacturer may earn a profit both by having a positive profit margin on the transfer of the product and also by sharing the sales revenue with the retailer. As $F\big(X^{I^*}(t)\big)$ further increases, $\hat{w}^{S^*}(t) < c_0$, and the manufacturer faces a negative profit margin on the transfer of the IDP to the retailer, but it earns a profit by sharing the sales revenue with the retailer. The theorem also demonstrates that under the coordinating revenue-sharing contract, the myopic retailer's optimal instantaneous profit rate is constant rather than varying over time as with the far-sighted retailer. As in the case of the far-sighted retailer, a revenue-sharing contract can arbitrarily split the supply chain profits.

8. Summary and Conclusions

This study addresses channel coordination issues in a dynamic decentralized supply chain which produces and sells an IDP over a finite selling horizon. We derive optimal dynamic pricing strategies with two retailer profitability strategies: far-sighted and myopic, and examine the manufacturer's preference over the retailer's profitability strategies. We find that the manufacturer is better off with a far-sighted retailer when the market saturation level is low and is better off with a myopic retailer when the market saturation level is high.

We further explore whether and how the revenue-sharing contracts coordinate the dynamic supply chain. The results show that revenue-sharing contracts can coordinate the dynamic supply chain with both myopic and far-sighted retailers. Sales commission agreements have the properties of a revenue-sharing agreement, and we can point out that they are actually common in the VARs distribution channel. However, to guarantee that a sales commission agreement actually coordinates the channel, we need the price of the product to be inclusive of the service component of the IDP paid by the customer.

Our dynamic diffusion model of a decentralized supply chain opens up several avenues for future research. One direction is to study the dynamic channel with retailers competing in prices. It is of interest to investigate how the manufacturer's pricing strategy and its preference over the retailer profitability strategy will change in competitive environments. Future research may consider different demand situations, such as products with repeat purchases. Our model assumes that the consumers do not postpone purchase on purpose in anticipation of future product price. Modeling such strategic consumer behavior is a natural extension to this work.

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Notes

[1]In this paper, we allow $X(0)$ to be a non-negative number. A positive value of $X(0)$ corresponds to the case in which the IDP faces the sales of an earlier generation when the diffusion process starts.

[2]We note that the retailer's equilibrium profit rate is obviously positive for any given set of parameters. The manufacturer's profit rate may be positive or negative within the selling horizon, depending on the parameter values. If the parameters are such that $(\alpha+M\beta)^2/(4\alpha\beta M)<4$, the manufacturer's instantaneous profit will be positive at any instant.

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Supporting Information

Additional supporting information may be found in the online version of this article:

Appendix SA1: Life-cycle Channel Coordination Issues in Launching an Innovative Durable Product.