CONSTRAINED NONLINEAR PREDICTIVE CONTROL BASED ON IMC-OPTIMIZATION

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Abstract: A difficulty with constrained nonlinear control is the minimization of the cost function. With complex system representations such as fundamental models, the required optimization algorithm may be complex to implement, setting its parameters may be difficult and the calculation time may be long. To overcome these problems, an innovative optimization algorithm is proposed. The quadratic criterion is written as a function of approximate linear models which makes possible the use of well known and easy to implement optimization techniques. The properties of the algorithm are analyzed and an example illustrates its very good performances.

Keywords: Nonlinear control, predictive control, optimization, physical models.

1. INTRODUCTION

For many nonlinear processes, linear controllers can perform quite adequately. However, nonlinear control can be justified when the plant is highly nonlinear and subject to large and frequent disturbances or when the operating points span a wide range of nonlinear dynamics (Qin and Badgwell, 1997). Nonlinear predictive control has to be considered as a solution if safety and actuator constraints are present, which is always the case for real processes.

To describe the dynamic behavior of a process, two possible ways are physical modeling and empirical modeling, both having attractive characteristics and drawbacks (Söderström and Stoica, 1988). Fundamental models are obtained in an analytical way from basic physical laws while empirical modeling is an experimental approach consisting in adjusting the parameters of an empirical mathematical relation between the variables of interest to fit the recorded data. The main drawback of physical models is that some processes are so complex that it is almost impossible to explain their behavior using only first principles. Empirical models are much easier to obtain and to use but their parameters do not have any physical meaning and *a priori* information is almost completely neglected. Furthermore, unlike fundamental models, they represent adequately the process only for conditions (operating points, types of inputs, etc.) similar to those found in the recorded data. On the other hand, if the underlying assumptions of the fundamental models are respected, they can mimic behaviours outside the range of calibration and less data is required for their development.

Because of their qualities, fundamental models have been used for nonlinear model predictive control. However, the considered plants are almost always a single unit operation with a relatively

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simple dynamic model (Henson, 1998). Writing fundamental models is a difficult task but commercial dynamic simulators are now available. However, using commercial dynamic simulators for nonlinear predictive control does not seem to have been reported yet in the litterature (Henson, 1998). A main reason why complex fundamental nonlinear models are not used to design predictive control is certainly that the complexity of the online solution of the nonlinear programming problem increases with the one of the model, hence leading to computational and reliability difficulties. Another reason why commercial simulators are not used is probably the unavailability of the model equations to the control designer (Henson, 1998).

In recent years, many research works have focused on the nominal stability problem for nonlinear model predictive control. Most proposed solutions consist in insuring nominal stability by imposing penalties or constraints on the terminal state of the prediction horizon (Qin and Badgwell, 1997; Mayne et al., 2000). These solutions are usually computationally quite demanding. Fortunaltely, algorithms to reduce the computational effort are now appearing in the litterature (Fontes, 2001; Magni et al., 2001). However, all main industrial nonlinear model predictive controllers do not use terminal state constraints of any kind (Qin and Badgwell, 1997). They instead allow to set the prediction horizon long enough to go beyond the steady-state hence approximating the infinite horizon solution, which leads to nominal stability (Meadows et al., 1995).

The proposed scheme makes use of two internal model control (IMC) structures, restricting its application to plants that are stable in the operating region. The first IMC acts as a disturbance estimator. The second IMC structure is an innovative way to optimize a static constrained nonlinear problem (Desbiens and Shook, 2003) and it is used to minimize the performance index at each sampling time. The predictive control quadratic criterion is not written as a function of nonlinear models but it is instead based on approximate linear models. Since the cost is then a function of linear models, traditional techniques, such as quadratic programming, can be used to find the global optimum of that convex problem. Obviously, that minimum does not necessarily correspond to an optimum of the nonlinear predictive control problem. Therefore, a correction is calculated by applying the solution to both linear and nonlinear models and evaluating the difference between their two outputs, through an IMC structure. The nonlinear plant models may be complex and commercial fundamental simulators could be used even if the equations are completely unknown.

The proposed controller is presented in its simplest form, without relying on terminal state constraints. This is still the norm in industry where using a long enough prediction horizon seems to lead to successful applications (Qin and Badgwell, 1997).

2. NONLINEAR PREDICTIVE CONTROL

2.1 Notation

The plant inputs and outputs at time $t = k$ are respectively $\boldsymbol{u}(k) \in \Re^{n_u}$ and $\boldsymbol{y_P}(k) \in \Re^{n_y}$ (all vectors in the paper are columns). The set points are $\boldsymbol{r_P}(k) \in \Re^{n_y}$. The best possible plant model M_{Ny}, possibly based on phenomenological relationships and therefore probably highly complex and nonlinear, is described by

$$\boldsymbol{x_{Ny}}(k+1) = \boldsymbol{f_y}\left(\boldsymbol{x_{Ny}}(k), \boldsymbol{u}(k)\right) \tag{1}$$

$$\boldsymbol{y_N}(k) = \boldsymbol{g_y}\left(\boldsymbol{x_{Ny}}(k)\right) \tag{2}$$

A simplified linear plant model is M_{Ly}, whose dynamics are given by

$$\boldsymbol{x_{Ly}}(k+1) = \boldsymbol{A_y}\boldsymbol{x_{Ly}}(k) + \boldsymbol{B_y}\boldsymbol{u}(k) \tag{3}$$

$$\boldsymbol{y_L}(k) = \boldsymbol{C_y}\boldsymbol{x_{Ly}}(k) \tag{4}$$

Other states or secondary outputs of the plant are denoted $\boldsymbol{w_P}(k) \in \Re^{n_w}$ and $\boldsymbol{t_P}(k) \in \Re^{n_t}$. Again these signals can be predicted using either nonlinear (M_{Nw}, M_{Nt}) or simpler linear models (M_{Lw}, M_{Lt})

$$\boldsymbol{x_{Nw}}(k+1) = \boldsymbol{f_w}\left(\boldsymbol{x_{Nw}}(k), \boldsymbol{u}(k)\right) \tag{5}$$

$$\boldsymbol{w_N}(k) = \boldsymbol{g_w}\left(\boldsymbol{x_{Nw}}(k)\right) \tag{6}$$

$$\boldsymbol{x_{Nt}}(k+1) = \boldsymbol{f_t}\left(\boldsymbol{x_{Nt}}(k), \boldsymbol{u}(k)\right) \tag{7}$$

$$\boldsymbol{t_N}(k) = \boldsymbol{g_t}\left(\boldsymbol{x_{Nt}}(k)\right) \tag{8}$$

$$\boldsymbol{x_{Lw}}(k+1) = \boldsymbol{A_w}\boldsymbol{x_{Lw}}(k) + \boldsymbol{B_w}\boldsymbol{u}(k) \tag{9}$$

$$\boldsymbol{w_L}(k) = \boldsymbol{C_w}\boldsymbol{x_{Lw}}(k) \tag{10}$$

$$\boldsymbol{x_{Lt}}(k+1) = \boldsymbol{A_t}\boldsymbol{x_{Lt}}(k) + \boldsymbol{B_t}\boldsymbol{u}(k) \tag{11}$$

$$\boldsymbol{t_L}(k) = \boldsymbol{C_t}\boldsymbol{x_{Lt}}(k) \tag{12}$$

To represent the plant disturbances, the following stochastic model M_S is used

$$\boldsymbol{x_S}(k+1) = \boldsymbol{A_S}\boldsymbol{x_S}(k) + \boldsymbol{B_S}\boldsymbol{\xi}(k) \tag{13}$$

$$\boldsymbol{y_S}(k) = \boldsymbol{C_S}\boldsymbol{x_S}(k) + \boldsymbol{D_S}\boldsymbol{\xi}(k) \tag{14}$$

where $\boldsymbol{\xi}(k) \in \Re^{n_y}$ is a zero mean random signal and $\boldsymbol{y_S}(k) \in \Re^{n_y}$. The model M_S usually contains an integration to represent non-stationary disturbances hence adding an integral action in the proposed control scheme.

In the following, the notation $\widehat{\boldsymbol{S}}(1:H)$ will refer to the vector of predictions of the vector signal $\boldsymbol{s}$ over a future horizon H

$$\begin{aligned}\widehat{\boldsymbol{S}}(1:H) = [\ &\widehat{\boldsymbol{s}}^T(k+1/k) \quad \widehat{\boldsymbol{s}}^T(k+2/k) \\ &\dots \qquad \widehat{\boldsymbol{s}}^T(k+H/k)\ ]^T\end{aligned} \tag{15}$$

The vector $\boldsymbol{U}(0:H-1)$ denotes the present and future values of the plant inputs

$$\begin{aligned}\boldsymbol{U}(0:H-1) = [\ &\boldsymbol{u}^T(k)\ \ \boldsymbol{u}^T(k+1) \\ &\dots \quad \boldsymbol{u}^T(k+H-1)\ ]^T\end{aligned} \tag{16}$$

A similar notation is also used for $\boldsymbol{U'}(0:H-1)$, whose elements are defined by $\boldsymbol{u'}(k) = \boldsymbol{Q}(z^{-1})\boldsymbol{u}(k)$ where usually $\boldsymbol{Q}(z^{-1}) = \boldsymbol{I}\left[1-z^{-1}\right]$. The corresponding state-space representation is

$$\boldsymbol{x_Q}(k+1) = \boldsymbol{A_Q}\boldsymbol{x_Q}(k) + \boldsymbol{B_Q}\boldsymbol{u}(k) \tag{17}$$

$$\boldsymbol{u'}(k) = \boldsymbol{C_Q}\boldsymbol{x_Q}(k) + \boldsymbol{D_Q}\boldsymbol{u}(k) \tag{18}$$

2.2 Controller design

According to a receding horizon procedure, the objective consists in minimizing at each sampling time the following cost function

$$\begin{aligned}J_N = &\tfrac{1}{2}\widehat{\boldsymbol{\Xi}}^T(1:H)\boldsymbol{\Omega}\,\widehat{\boldsymbol{\Xi}}(1:H) \\ &+\tfrac{1}{2}\boldsymbol{U'}^T(0:H-1)\boldsymbol{\Lambda}\boldsymbol{U'}(0:H-1)\end{aligned} \tag{19}$$

where $\widehat{\boldsymbol{\Xi}}(1:H) = \widehat{\boldsymbol{Y}}_{\boldsymbol{N}}(1:H) + \widehat{\boldsymbol{Y}}_{\boldsymbol{S}}(1:H) - \widehat{\boldsymbol{R}}_{\boldsymbol{P}}(1:H)$. The positive definite matrices $\boldsymbol{\Omega}$ and $\boldsymbol{\Lambda}$ are weights to make a trade off between the control actions and the deviations to the set points. The minimization of the cost function is subject to a control horizon (H_c) constraint

$$\begin{aligned}\boldsymbol{u}(k+H-1) &= \boldsymbol{u}(k+H-2) = \dots \\ &= \boldsymbol{u}(k+H_c-1)\end{aligned} \tag{20}$$

and to constraints on $\boldsymbol{w_N}$ and $\boldsymbol{t_N}$ over the prediction horizon H

$$\widehat{\boldsymbol{W}}_{\boldsymbol{N}}(1:H) < \boldsymbol{W_{max}}(1:H) \tag{21}$$

$$\widehat{\boldsymbol{T}}_{\boldsymbol{N}}(1:H) = \boldsymbol{T_{eq}}(1:H) \tag{22}$$

To achieve the above objective, a new predictive control scheme is proposed. The control is calculated by repeating the following receding horizon steps at every sampling time.

Step 1: Measure the plant outputs $\boldsymbol{y_P}(k)$.

Step 2: The plant disturbance is estimated with a first IMC structure: $\widehat{\boldsymbol{y}}_{\boldsymbol{S}}(k) = \boldsymbol{y_P}(k) - \boldsymbol{y_N}(k)$ where $\boldsymbol{y_N}(k)$ is calculated with (2). Using the stochastic model (13) and (14), stochastic predictions $\widehat{\boldsymbol{Y}}_s(1:H)$ can then be calculated as described in Desbiens et al. (2000).

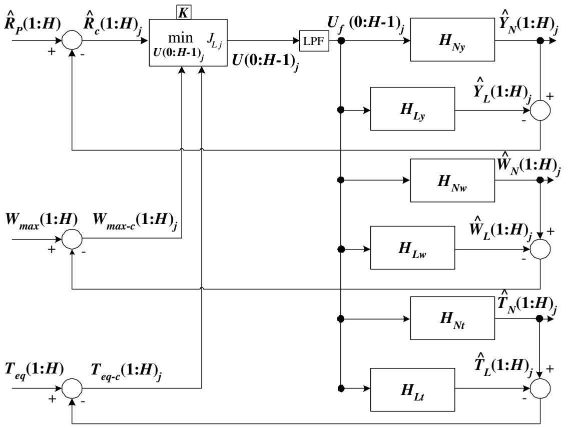

Fig. 1. IMC-optimization

Step 3: This step consists in minimizing the cost function (19) with respects of constraints (20), (21) and (22). The minimization is achieved by simulating the system depicted in Figure 1 until convergence. The subscript j represents the simulation steps (which will be referred to as the optimization steps) for the optimization at time k. The cost function appearing in Figure 1 is defined as follows

$$\begin{aligned}J_{L\,j} = &\tfrac{1}{2}\widehat{\boldsymbol{E}}^T(1:H)_j\boldsymbol{\Omega}\widehat{\boldsymbol{E}}(1:H)_j \\ &+\tfrac{1}{2}\boldsymbol{U'}^T(0:H-1)_j\boldsymbol{\Lambda}\boldsymbol{U'}(0:H-1)_j\end{aligned} \tag{23}$$

where $\widehat{\boldsymbol{E}}(1:H)_j = \widehat{\boldsymbol{Y}}_{\boldsymbol{L}}(1:H)_j + \widehat{\boldsymbol{Y}}_{\boldsymbol{S}}(1:H)_j - \widehat{\boldsymbol{R}}_{\boldsymbol{c}}(1:H)_j$. The minimization of the cost function is subject to the control horizon constraint (20) and to

$$\widehat{\boldsymbol{W}}_{\boldsymbol{L}}(1:H)_j < \boldsymbol{W_{max-c}}(1:H)_j \tag{24}$$

$$\widehat{\boldsymbol{T}}_{\boldsymbol{L}}(1:H)_j = \boldsymbol{T_{eq-c}}(1:H)_j \tag{25}$$

Note that the set point in $J_{L\,j}$ and the above constraints are the ones corrected through the IMC structure at each step j. As with any IMC structure, the addition of a unitary gain low-pass filter (LPF) increases the stability but slows down the convergence. The cost $J_{L\,j}$ is not function of the nonlinear models M_{Ny}, N_{Nw} and M_{Nt}. They only need to be simulated using $\boldsymbol{U_f}(0:H-1)_j$ as a sequence of inputs to generate the predictions $\widehat{\boldsymbol{Y}}_{\boldsymbol{N}}(1:H)_j$, $\widehat{\boldsymbol{W}}_{\boldsymbol{N}}(1:H)_j$ and $\widehat{\boldsymbol{T}}_{\boldsymbol{N}}(1:H)_j$. Those calculations are represented by the static operators or deterministic predictors $\boldsymbol{H_{Ny}}$, $\boldsymbol{H_{Nw}}$ and $\boldsymbol{H_{Nt}}$ in Figure 1. Linear models M_{Ly}, N_{Lw} and M_{Lt} are also simulated in a similar way, represented by the operators $\boldsymbol{H_{Ly}}$, $\boldsymbol{H_{Lw}}$ and $\boldsymbol{H_{Lt}}$. When calculating the outputs of the six models for the sequence of inputs $\boldsymbol{U_f}(0:H-1)_j$, the initial conditions are set to $\boldsymbol{x_{Ny}}(k)$, $\boldsymbol{x_{Ly}}(k)$, $\boldsymbol{x_{Nw}}(k)$, $\boldsymbol{x_{Lw}}(k)$, $\boldsymbol{x_{Nt}}(k)$, $\boldsymbol{x_{Lt}}(k)$ at every new step j. The quadratic objective $J_{L\,j}$ is based on the linear models which greatly facilitates its minimization. An analytical solution even exists if the control horizon is the only constraint. If the constraints (21) and (22) are also taken into account, quadratic programming (Maciejowski, 2002) or alternative methods (Kouvaritakis and Cannon, 2002) exist to solve the constrained minimization. Once Figure 1 has converged ($j = j_{end}$), apply $\boldsymbol{u}(k) = \boldsymbol{u}_f(k)_{j_{end}}$ to the plant and reset $j = 1$.

Step 4: Update the state vectors using (1), (3), (5), (7), (9), (11), (13) and (17). Go back to Step 1 at the next sampling time ($k = k + 1$).

In practice, during step 3, the simulation can be stopped if

$$\|\boldsymbol{U_f}_{(0:H_c-1)_j} - \boldsymbol{U_f}_{(0:H_c-1)_{j-1}}\|_2 < \eta\sqrt{n_u H_c} \quad (26)$$

where $\eta > 0$. The parameter η is therefore the desired precision for the elements of the solution, equivalent to the termination tolerance for usual optimization algorithms. To accelerate the convergence, before starting simulating Figure 1 at time $t = k$, some variables should be initialized equal to the values they had at the steady-state ($j = j_{end}$) of the preceding sampling time ($k-1$). This is the case for the initial state of the low-pass filter and for $\widehat{\boldsymbol{R}}_{\boldsymbol{c}}(1:H)_1$, $\boldsymbol{W_{max-c}}(1:H)_1$ and $\boldsymbol{T_{eq-c}}(1:H)_1$.

2.3 IMC-optimization properties

Constraints on the manipulated variables and constraints such as (21) and (22) are often necessary in practice because of actuators, operation and safety limits. Even in presence of mismatches between nonlinear and linear models, it is obvious that constraints on $\boldsymbol{u}$ could be added and would be respected with the proposed algorithm. Constraints (21) and (22) will also be respected because of the corrections brought by the IMC structure, as demonstrated in Desbiens and Shook (2003). Since the equality constraints are respected, terminal state constraints could then be implemented with the proposed controller.

Because of the nonlinearities, establishing the convergence conditions when simulating Figure 1 is a difficult topic. Since the constraints (21) and (22) are respected, it will be assumed that the control horizon is the only constraint. Figure 1 is an IMC structure and therefore the first stability condition is that $\widehat{\boldsymbol{Y}}_{\boldsymbol{L}}(1:H)_j$ is bounded if there is no feedback. In terms of usual IMC, and referring to the labels of Figure 1, the "controller" $\boldsymbol{K}$ must stabilize the "model" $\boldsymbol{H_{Ny}}$ in open-loop. This is achieved by correctly selecting the predictive control parameters (a long enough prediction horizon, etc.).

The second condition is to preserve the stability when $\widehat{\boldsymbol{Y}}_{\boldsymbol{N}}(1:H)_j - \widehat{\boldsymbol{Y}}_{\boldsymbol{L}}(1:H)_j$ is fed back. A tool to analyze the stability of discrete multivariable nonlinear systems is the Tsypkin criterion, which is the discrete version of the Popov criterion used for analyzing continuous nonlinear systems. Even if $\boldsymbol{H_{Ly}}$ and $\boldsymbol{H_{Ny}}$ can be seen as static operators acting on $\boldsymbol{U_f}(0:H-1)_j$ to respectively generate $\widehat{\boldsymbol{Y}}_{\boldsymbol{L}}(1:H)_j$ and $\widehat{\boldsymbol{Y}}_{\boldsymbol{N}}(1:H)_j$, structural restrictions of even the more recent results (Kapila and Haddad, 1996; Park and Kim, 1998; Larsen and Kokotović, 2001) make impossible the stability analysis of Figure 1. Indeed, to apply the Tsypkin criterion and its extensions, it is required to know the nonlinearities sector bounds which is difficult in practice when $\boldsymbol{H_{Ny}}$ consists of phenomenological equations (which is the ultimate goal). But even worse, $\boldsymbol{H_{Ny}}$ must be diagonal.

Fortunately, it is known that increasing ρ eventually makes the IMC structure stable for linear systems (Morari, 1987) if both $\boldsymbol{H_{Ly}}$ and $\boldsymbol{H_{Ny}}$ have the same 'sign', i.e. if $\det\boldsymbol{H_{Ly}}/\det\boldsymbol{H_{Ny}} > 0$. Theorem 6.7 in Skogestad and Postlethwaite (1996) also confirms that property for any controller with integral action in all channel (which is the case for Figure 1). The parameter ρ of the proposed IMC-optimization is therefore similar to the step length for usual optimization algorithms: a larger value makes it more robust but slows down the convergence.

Even if the IMC-optimization converges, is the steady-state solution optimal? For an easier analysis and understanding, it will be again assumed that the control horizon is the only constraint and that M_{Ny} is linear. As it will be illustrated by the simulation example, the conclusions drawn for linear systems are similar for nonlinear systems.

Theorem 1. (Optimality). If M_{Ny} is linear and $n_u H_c \leq n_y H$, the IMC-optimization solution (for the weights $\boldsymbol{\Lambda}$ and $\boldsymbol{\Omega_{IMC}}$) corresponds to the optimal solution (for the weights $\boldsymbol{\Lambda}$ and $\boldsymbol{\Omega}$), if $\boldsymbol{\Omega_{IMC}}$ is adequately selected.

Proof. In the linear case with a control horizon constraint, the optimal solution can analytically be found. The solution provided by simulating Figure 1 until convergence can also be easily calculated. It can then be demonstrated that both solutions become identical if $\boldsymbol{\Omega_{IMC}}$ is correctly choosen - see Desbiens and Shook (2003) for more details. □

In practice, the limitation $n_u H_c \leq n_y H$ is not restrictive since usually $n_u \approx n_y$ and $H_c << H$. Also, the difference in the weight on the deviations is not critical since it must often be fine tuned by trials and errors. The important point is that $\boldsymbol{\Omega_{IMC}}$ has the expected effect on $\boldsymbol{U}(0:H_c-1)$ (but not with the same magnitude as a direct optimization of the nonlinear problem).

If M_{Ny} is linear and $\boldsymbol{\Lambda} = \boldsymbol{0}$, it could also be shown that the optimal solution and the IMC-optimization solution are identical if M_{Ny} differs from M_{Ly} only by its static gain or if $n_y H = n_u H_c$.

The example presented in Sections 3, dealing with a nonlinear system, illustrates that when $\boldsymbol{\Lambda} = \boldsymbol{0}$, a direct minimization of (19) and the method proposed in this paper both lead to very similar results. Other examples confirming that similarity are detailed in Desbiens and Shook (2003) and Pomerleau et al. (2003a).

Setting $\mathbf{\Lambda} \neq \mathbf{0}$ with nonlinear systems is similar to what was concluded with linear systems: the weight $\mathbf{\Lambda}$ for the proposed method has the expected effect on the manipulated variables but not with the same magnitude as with a direct optimization based on the nonlinear model. However, selecting $\mathbf{\Lambda} = \mathbf{0}$ is not limitative because a similar smoothing can be obtained by low-pass filtering the set points (tracking tuning) and by adding high-pass filters to M_S (regulation tuning). This is even recommended since it allows independent tunings for tracking and regulation, which is not possible to achieve with $\mathbf{\Lambda}$, while leading to a solution close to the true optimum.

3. SIMULATION EXAMPLE

The plant to be controlled is a pellet cooling phenomenological simulator described in Pomerleau et al. (2003b). It simulates the cooling zone of an induration furnace used for the concentration and agglomeration of iron ore oxide pellets. The two manipulated variables are the shutter positions for two fans forcing the air circulation through the moving bed of pellets (one above and one below). The gas temperature and pressure above the pellet bed are the controlled variables. The simulator is based on energy balance equations for the pellets and the gas. The pressure drop in the bed is calculated with the Ergun's model. The fans characteristics, the pressure drops in the shutters and the pressure loss in the outlet resistance are explained by nonlinear empirical relationships. The linear model was obtained by applying a 1 % step to each manipulated variable and by fitting first- or second-order models for each input-output transfer function.

The sampling period is 10 seconds. The initial operating points are $u_1 = 75$ %, $u_2 = 75$ %, $y_1 = -105.401$ Pa and $y_2 = 1344.626$ K. The horizons are $H = 25$ and $H_c = 1$. To give each output approximately the same weight in the cost function, the following are selected: $\mathbf{\Lambda} = \mathbf{0}$ and $\mathbf{\Omega} = \text{diag}\left(1,\ 10^4,\ 1,\ 10^4,\ \ldots,\ 1,\ 10^4\right)$. The settings for the nonlinear controller based on the IMC-optimization (IMNLPC) are $\rho = 0.6$ and $\eta = 1/6$. The stochastic model is

$$\boldsymbol{y_S}(k) = \left[\frac{1 - 0.8z^{-1}}{1 - z^{-1}}\boldsymbol{I}\right]\boldsymbol{\xi}(k) \tag{27}$$

The nonlinear model M_{Ny} is identical to the plant. The set points are 144.599 Pa and 1349.626 K. Figures 2 and 3 compare IMNLPC and a nonlinear predictive controller which directly minimizes (19) (NLPC). IMNLPC uses the same optimization routine (*fmincon* from Matlab) as NLPC to minimize (23) even if an analytical solution exists. Both approaches lead to vey similar results but IMNLPC requires a smaller number of iterations at each sampling period (8 to 45 times smaller).

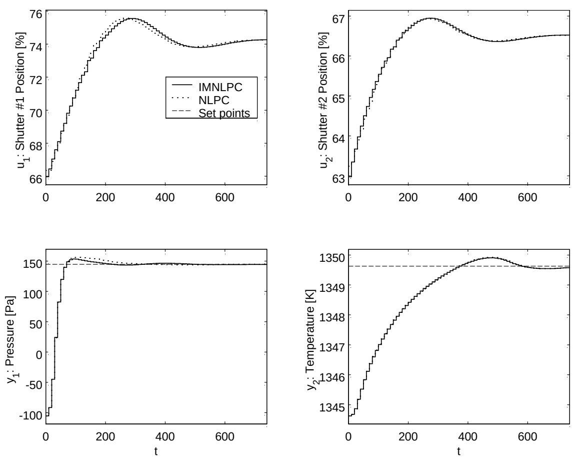

Fig. 2. u and y

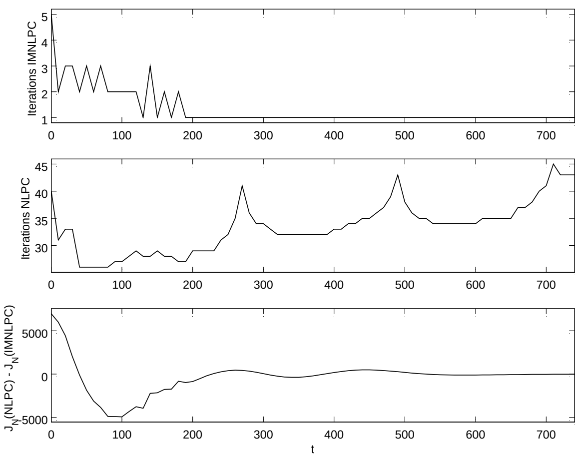

Fig. 3. Iterations and cost fuction

4. CONCLUSION

A constrained nonlinear predictive controller is presented. A simulation example illustrates its very good performances in terms of optimality and rate of convergence.

The novelty of the proposed scheme lies in the minimization of the cost function at each sampling period. The optimization is seen and solved as a control problem by itself, showing a duality between optimization and control (Desbiens and Shook, 2003). The criterion is written as a function of linear transfer functions approximating the plant nonlinear models. The model mismatches are compensated with an IMC structure allowing to find a solution very close to the optimal specially when $\mathbf{\Lambda} = \mathbf{0}$ while respecting the constraints (filters can be used to obtain a similar effect of having $\mathbf{\Lambda} \neq \mathbf{0}$).

The main advantages of using the proposed IMC-optimization to calculate the solution at each sampling time are:

- Use of a simple optimization algorithm: Since the quadratic criterion is a function of linear models, its minimum can be found simply using quadratic programming. An analytical solution even exists if the control horizon is the only constraint.
- Easy to implement: Because of the preceding point, the IMC-optimization is easier to implement in an industrial environment than complex optimization algorithms. Indeed, commercial and simple optimization routines can be used. In Desbiens and Shook (2003), a commercial routine written in C was used to optimize a static nonlinear phenomenological simulator. Divergence occurred frequently when directly solving the nonlinear problem, which has never been the case with the IMC-optimization (using the same commercial routine).
- Easy to tune: Because of the simplicity of the model appearing in the cost function, the default parameters of the optimization routine are usually adequate. The only parameters to tune are η and ρ, both with clear meanings. The proposed algorithm seems less sensitive to the optimization tuning parameters than a direct nonlinear optimization.
- Fast convergence (short calculation time): As illustrated by the example (other examples can be found in Desbiens and Shook (2003) and Pomerleau et al. (2003a)), the IMC-optimization usually converges much faster than a direct nonlinear optimization and leads to a similar final cost. This would also be the case even if the nonlinear optimization was initialized with the solution found with the linear model (Desbiens and Shook, 2003). It seems that the difference of convergence rate becomes even larger for higher dimension problems. The convergence will slow down if the linear models become significantly different from the nonlinear models (for instance, when controlling at various operating points). It is then recommended to switch between different linear models. Unlike most multi-model techniques, there is no need to weight in any way the contribution of the various linear models.

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