Relativistic Effects in Heavy-Ion Collisions at SIS Energies[1]

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Abstract:

The covariant and non-covariant Quantum Molecular Dynamics models are applied to investigate possible relativistic effects in heavy ion collisions at SIS energies. These relativistic effects which arise due to the full covariant treatment of the dynamics are studied at bombarding energies $E_{lab.}$ = 50, 250, 500, 750, 1000, 1250, 1500, 1750 and 2000 MeV/nucl.. A wide range of the impact parameter from b = 0 fm to b = 10 fm is also considered. In the present study, five systems ^{12}C-^{12}C, ^{16}O-^{16}O, ^{20}Ne-^{20}Ne, ^{28}Si-^{28}Si and ^{40}Ca-^{40}Ca are investigated. The full covariant treatment at low energies shows quite good agreement with the corresponding non-covariant approach whereas at higher energies it shows less stopping and hence less thermal equilibrium as compared to the non-covariant approach. The collisions dynamics is less affected. The density using RQMD rises and drops faster than with QMD. The relativistic effects show some influence on the resonance matter production. Overall, the relativistic effects at SIS energies ($\leq$ 2000 MeV/nucl.) are less significant.

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1. Introduction :

In the beginning of the last decade it was thought that by comparing the predictions of the theoretical models with existing experimental data, one may be able to conclude something about the **E**quation **o**f **S**tate (EOS). Unfortunately, it turned out that the EOS is drastically affected by the ingredients of these models. Therefore, up to now the question of the equation of state is still open. In the intermediate energy region [$\leq$ 2000 MeV/nucl.], both main inputs of the nuclear dynamics, namely, the mean field (or mutual two- and three body interactions) and the Nucleon-Nucleon cross-section are found to influence the nuclear dynamics to a larger extent [1-6].

Apart of the choice of possible different interactions and cross-sections, one has to remember that even at these intermediate energies, the velocities of particles are not at all negligible as compared to the velocity of light. For example, at a bombarding energy of 1 GeV/nucl., the boost velocity is about 85% of the speed of light. Thus, even at these energies, the Lorentz-invariance of the theory has to be respected. Once the bombarding energy of a nucleon becomes comparable with its rest mass, the relativistic effects are expected to influence the dynamics.

To study the nuclear dynamics at intermediate energies where no local or global equilibrium is reached, one needs a transport theory which is not based on the assumption of local or global equilibrium. In this energy domain, the Boltzmann-Uehling-Uhlenbeck (BUU) and Quantum Molecular Dynamics (QMD) models with their relativistic versions i.e. RBUU and RQMD are used with big success [1-23].

Recently, Faessler and collaborators have reported their new covariant generalization of the QMD model [i.e. Relativistic Quantum Molecular Dynamics (RQMD)][20]. The first RQMD was made by the Frankfurt group [22]. One of the main advantages of our RQMD is that we have shown for practically the first time that this numerical implementation of a covariant theory gives in the non-relativistic limit the same result(s) as that of the corresponding non-covariant approach. At higher energies, clear relativistic effects in the flow were observed. One should note that the transverse flow is a quite sensitive quantity. Therefore, to study relativistic effects, one has to look for quantities which are not very much sensitive to model inputs or even to different EOS's. Thus, quantities like resonance production, density, rapidity distribution are better candidate for checking the validity of relativistic effects at SIS energies. Hence, in this paper we concentrate fully on these quantities and search for relativistic effects.

We would like to remark here that the nature of relativistic effects can show up very differently because there exits several origins of these effects. Some of them stem from the relativistic kinematics, some from relativistic forces, retardation or meson radiation effects etc. Parts of the relativistic kinematics are trivial and are dealt even in the non-covariant models by using the relativistic energy-momentum relations. But the dynamics in these models is still treated in a non-covariant way and hence it breaks the Lorentz-invariance of the theory. The relativistic effects which we are going to discuss are the one which are originating due to the full covariant treatment of the dynamics.

The paper is organized as follows: Section 2 deals with a brief introduction to the formalisms used in QMD and RQMD. The results are presented in section 3 and finally we summarize our findings in section 4.

2. The Formalism:

The detailed formalism and the numerical realization of QMD and RQMD are given in refs. [13] and [20] , [22], [23], respectively. Here we discuss briefly the main important points of the formalism used in the QMD and RQMD:

2.1 Quantum Molecular Dynamics [QMD]:

In QMD, the nuclei under consideration are chosen by a procedure which is based on the random choice of the coordinate and momentum space. This is done with the help of a standard Monte-Carlo procedure. The nucleons are distributed in a sphere of radius R = 1.14 $A^{1/3}$ which is consistent with the liquid drop model. If the centers of the Gaussians of two nucleons are closer than a distance R_{min} = 1.5 fm, the choice of this coordinate is rejected and other coordinates are chosen. The momenta of the nucleons are chosen randomly between zero and the local Fermi momentum. The successfully initialized nuclei are boosted towards each other with proper center-of-mass velocity using relativistic kinematics [13].

In our approach each nucleon is described by a Gaussian wave packet with a width $\sqrt{L}$ centered around the mean position $\vec{r}_i$(t) and the mean momentum $\vec{p}_i$(t):

$$\psi_i(\vec{r},\vec{p},t) = \frac{1}{(2\pi L)^{3/4}} \ \exp\left\{-\frac{(\vec{r}-\vec{r}_i(t))^2}{4L} + i\vec{p}_i(t)\cdot\vec{r}\right\}, \tag{1}$$

with L = 1.08 fm^2. This choice corresponds to a root mean square radius of the nucleon wave-packet of 1.8 fm. As the width of the Gaussians is kept fixed, the centeriods of the wave -packets are propagated using the classical equation of motion.

$$\frac{d\vec{r}_i}{dt} = \frac{\partial H}{\partial \vec{p}_i}, \tag{2}$$

$$\frac{d\vec{p}_i}{dt} = -\frac{\partial H}{\partial \vec{r}_i}, \tag{3}$$

where the Hamiltonian is given by the classical N-body Hamiltonian

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} + V, \tag{4}$$

with V as the potential, which in present study, is simple Skyrme force

$$V = \sum_{i=1}^{N}\left[\frac{\alpha}{2}\left\{\sum_{j\neq i}\frac{\tilde{\rho}_{ij}}{\rho_0}\right\} + \frac{\beta}{\sigma+1}\left\{\sum_{j\neq i}\frac{\tilde{\rho}_{ij}}{\rho_0}\right\}^{\sigma}\right]. \tag{5}$$

Here $\tilde{\rho}_{ij}$ is the interaction density which is given by

$$\tilde{\rho}_{ij} = \int \rho_i(\vec{r}(t))\rho_j(\vec{r}(t))d\vec{r} = \frac{1}{(4\pi L)^{3/2}} \exp \left[-\frac{(\vec{r}_i - \vec{r}_j)^2}{4L} \right]. \quad (6)$$

The coefficients α, β and σ appearing in eq. (5) are determined by the condition that the bulk properties of infinite nuclear matter has to be reproduced. Different sets of parameters lead to different incompressibilities K which generate different EOS's. Usually two incompressibilities K are chosen: (i) K= 200 MeV corresponds to a soft EOS and (ii) K = 380 MeV corresponds to a hard EOS [13].

During the propagation, two nucleons are assumed to collide if they come closer than a distance A_{min}=$\sqrt{\sigma(\sqrt{s})/\pi}$ where $\sigma(\sqrt{s})$ is the total cross-section depending on the invariant mass $\sqrt{s}$. Whenever a collision occurs, the phase space around the final state of the two scatterers is checked. From the overlap one is able to check the probability whether a collision is Pauli blocked or not. Here we use the cross-sections parametrized by Cugnon [24]. In these new cross-sections both elastic and inelastic channels are considered and different isospins of baryons are also taken into account. Some of the processes considered here are of the type: $NN \rightarrow NN$, $N\Delta \rightarrow N\Delta, NN \leftrightarrow N\Delta, \Delta\Delta \rightarrow \Delta\Delta$ etc. In the following, we demonstrate the main points of the covariant QMD i.e of RQMD.

2.2 Relativistic Quantum Molecular Dynamics [RQMD]:

The RQMD model describes the propagation of all kinds of baryons and mesons in a Lorentz-invariant fashion. The Hamiltonian for an N-particle system is expressed in terms of 8N variables (4N position coordinates $q_{i\mu}$ and 4N momentum coordinates $p_{i\mu}$). This means that here each particle carries its own energy and time. Since the physical events are described as world lines in a 6N dimensional phase-space, extra 2N-1 degrees of freedom have to be eliminated and a global evolution parameter τ has to be defined. This can be achieved with the help of 2N constraints. In our approach, the first N constraints are chosen as Poincaré invariant mass-on shell constraints [20], [22].

$$\xi_i = p_i^{\mu} p_{i\mu} - m_i^2 - \tilde{V}_i = 0 \qquad ; \qquad i = 1, ..., N. \quad (7)$$

This choice of Poincaré invariant constraints requires that the potential part $\tilde{V}_i$ should be a Lorentz scalar and therefore function of Lorentz scalars only. Since in the RQMD, a system with mutual two- and three-body interactions (like in QMD) has to be defined, $\tilde{V}_i$ should be given by the sum of these two-body interactions. Further, as we want to look for relativistic effects in the dynamics, we have to generalize the non-relativistic Skyrme force in such a way that the force is covariant and reduces also to the usual Skyrme force in the non-relativistic limit. This can be done as [20]

$$\tilde{V}_i = \sum_{j \neq i}^{N} \tilde{V}_{ij}(q_{Tij}^2). \tag{8}$$

This shows that the two-body interactions depend only on the Lorentz invariant squared transverse distance

$$q_{Tij}^2 = q_{ij}^2 - \frac{(q_{ij}^{\mu} p_{ij\mu})^2}{p_{ij}^2}, \tag{9}$$

with $q_{ij}^{\mu} = q_i^{\mu} - q_j^{\mu}$ being the simple four dimensional distance and $p_{ij}^{\mu} = p_i^{\mu} + p_j^{\mu}$ the sum of the momenta of the two interacting particles i and j.

The next set of constraints (which fix the relative times of all particles) should be chosen in such a way that these constraints must respect the principle of causality and N-1 of these constraints should be Poincaré invariant so that the world line invariance can also be fulfilled. Another feature which these constraints has to fulfill is the cluster separability. This means that the system can be divided into single particles or clusters as soon as their Minkowski distances are space-like. Furthermore, a global evolution parameter should also be defined. These features can be fulfilled by choosing the following set of time constraints:

$$\chi_i = \sum_{j(\neq i)} \frac{1}{q_{ij}^2/L_C} \exp(q_{ij}^2/L_C)\ p_{ij}^{\mu} q_{ij\mu} = 0 \qquad ; \qquad i = 1, ..., N-1, \tag{10}$$

$$\chi_{2N} = \hat{P}^{\mu} Q_{\mu} - \tau = 0. \tag{11}$$

with $\hat{P}^{\mu} = P^{\mu}/\sqrt{P^2}$, $P^{\mu} = \sum_i p_i^{\mu}$, $Q^{\mu} = \frac{1}{N}\sum_i q_i^{\mu}$.

These time fixations take care that the time coordinates of interacting particles are not too much dispersed in the center of mass system of two particles. The Hamiltonian is a linear combination of the Poincaré invariant constraints:

$$H = \sum_{i=1}^{2N-1} \lambda_i \Psi_i, \tag{12}$$

with

$$\Psi_i = \begin{cases} \xi_i & ; \quad i \leq N \\ \chi_{i-N} & ; \quad N < i \leq 2N-1. \end{cases} \tag{13}$$

This Hamiltonian then generates the equations of motion

$$\frac{dq_i^{\mu}}{d\tau} = [H, q_i^{\mu}], \tag{14}$$

$$\frac{dp_i^\mu}{d\tau} = [H, p_i^\mu]. \tag{15}$$

Here square brackets represent the Poisson brackets. The unknown Lagrange multipliers λ_i in eq. (12) are determined by the condition that all constraints must be fulfilled for all times during the simulations. These equations of motion are used to propagate the baryon during the reaction.

The propagation and the "soft interaction" between baryons is combined with the quantum effects like stochastic scattering and the Pauli-blocking etc.. In RQMD, the collision part is treated in a covariant fashion. Therefore all quantities which determine the collision must be Lorentz invariant. In RQMD two baryons are allowed to collide if their distance $\sqrt{-q_{Tij}^2} \leq \sqrt{\sigma(\sqrt{s})/\pi}$ where q_{Tij}^2 is the Lorentz-invariant squared transversal distance (eq.9) and $\sigma(\sqrt{s})$ is the cross-section depending on the available invariant mass $\sqrt{s}$.

3. Results and Discussion:

All results presented here are calculated using a new simulation package which integrates the RQMD and QMD approaches under one shell and hence this code has been named as UNISCO which stands for **UNI**fied **S**imulation **CO**de [20]. In this paper, five systems ^{12}C-^{12}C, ^{16}O-^{16}O, ^{20}Ne-^{20}Ne, ^{28}Si-^{28}Si and ^{40}Ca-^{40}Ca are considered. For a detailed investigation of the relativistic effects, we simulate these systems at bombarding energies E_{lab} = 50, 250, 500, 750, 1000, 1250, 1500, 1750 and 2000 MeV/nucl.. In addition, a wide range of the impact parameters between b = 0 (central collisions) and b = 10 fm (peripheral collisions) is also considered.

We start with the time evolution of nuclear density. Here the density is calculated in a sphere with a radius of 2 fm. The center of this sphere is located at the point where the two nuclei touch each other in their center of mass system. Fig. 1 shows the time evolution of the maximum density reached in this sphere. Here a semi-central collision of ^{40}Ca-^{40}Ca is considered at a bombarding energy 1.5 GeV/nucl. with both hard and soft EOS's. It is quite interesting to note that RQMD and QMD simulations show a quite different evolution of the maximum density. Compared to QMD, the coordinate space in RQMD is Lorentz-contracted and this leads to more repulsion in the RQMD and the density decreases faster. This rapid decrease in the density shows that the particles are kicked out from the compressed zone and one should get less thermalization using RQMD than QMD. Though the maximal values are not much different, the full shape of evolution of the density is quite different.

What makes this difference? Is it only a Lorentz contraction of the initial phase space distribution? To answer these questions we give the main differences between RQMD and QMD which are clearly visible in addition to the covariant feature of the RQMD model: (i) In RQMD, we have an initial Lorentz-contracted distribution in coordinate space and an elongated distribution in the momentum space. We will come to this point later when QMD simulations with this initial Lorentz-contracted distribution will be presented. (ii) In

RQMD, a multi-time formalism is used. In other words, in RQMD all baryons carry their own time coordinates. (iii) RQMD has a full covariant treatment of the collision part which also includes Pauli-blocking covariantly [22]. (iv) In RQMD, the mean field is a Lorentz-scalar whereas in QMD it is a zero component of the Lorentz vector. Due to the covariant feature, the interactions in RQMD are defined as a function of the distance between the particles in the rest frame of their common center of mass. Therefore, in a moving frame (e.g. in the reference frame corresponding to the center of mass of the two nuclei) these interactions are not spherical but are Lorentz contracted in the direction of the motion of the two particles. Therefore, the strength of the interaction depends strongly on the direction of the center of mass motion of the two nucleons in the rest frame of the two nuclei. When the initial phase space distribution is Lorentz contracted then, naturally, the density of a fast moving nucleus is increased in the CM system. If one includes Lorentz contraction in normal QMD then one finds that it can lead to a tremendous enhancement in the transverse flow [20]. When we use the feature in covariant RQMD, this artificial repulsion due to the initial contraction of the phase space is partially counterbalanced.

To understand these effects we follow in fig.2 the time evolution of the collisions rate for the same reaction as in fig.1 but at an impact parameter of 1 fm. One sees that the collision evolution reflects the density behaviour. The QMD simulations show the first collisions around 4 fm/c whereas RQMD only around 8 fm/c. The collision rate in RQMD falls more rapidly than in QMD. We further note that this behaviour is the same at all higher energies and for all impact parameters. In fig.3, we show the time evolution of the number of collisions for the systems ^{12}C-^{12}C, ^{16}O-^{16}O, ^{20}Ne-^{20}Ne, ^{28}Si-^{28}Si at a bombarding energy of 1.5 GeV/nucl. and for an impact parameter = 0.25 b^{max} (b^{max} = the radius of target + the radius of projectile). Here the hard EOS is used. It is interesting that all reactions show a similar behaviour for the time evolution of the collision rate. The size of the relativistic effects is similar in all reactions.

A further decomposition of the collision rate into elastic and inelastic channels is shown for the case of ^{40}Ca-^{40}Ca in fig.4. The elastic channel in RQMD seems to dominate over the QMD whereas, in the case of inelastic channel, a careful look shows that the situation is not clear. The inelastic channel which contains the formation of resonance matter will be considered later on. In order to look how much collisions one can get when one includes the Lorentz contraction in coordinate space in a normal QMD and keeps the interaction still spherical, we show in fig. 5 the reaction ^{40}Ca-^{40}Ca at 1.5 GeV/nucl. In this figure, the time evolution of the total number of collisions per nucleon is shown using the usual RQMD and QMD and a special version of QMD where the initial phase space is Lorentz boosted (it is labeled as QMD(cont.)). It is interesting to see that RQMD shows more collisions than the normal QMD but this collision number is less than what we get using QMD(cont.). This justify our earlier claim that the covariant treatment of the interactions counterbalances partially the initial contraction. In fig. 6, we switch off the self-consistent field in RQMD, QMD and QMD(cont.) i.e. we use a cascade model. One sees that now RQMD shows more collisions than QMD and QMD(cont.). In a cascade-mode, all particles are allowed to move freely and hence there is no more counterbalancing force. This shows that the covariant treatment of the interaction keeps the nuclei not only stable, but counterbalances also the artificial collisions which can happen due to the higher contracted density.

One of the important quantum feature in all semi-classical models like BUU /QMD /RBUU /RQMD etc. is the inclusion of the Pauli-principle. Therefore, in fig. 7, we compare the percentage of the collisions which are blocked due to the lack of free phase space using RQMD and QMD. In this figure, the ratio of the Pauli-blocked collisions to all attempted collisions is shown for both the soft and the hard EOS's. The general feature i.e. the decrease in the Pauli-blocked collisions with increase of the bombarding energy is well reproduced using QMD and RQMD. Remember that in RQMD, a Lorentz-invariant Pauli-blocking procedure is implemented. It is clear from fig. 7 that at 50 MeV/nucl. RQMD and QMD show good agreement as expected whereas at higher energies (up to 2 GeV/nucl.), RQMD shows more Pauli-blocking than QMD. This result which is valid for all higher energies and masses shows that the covariant treatment of the theory results in more blocked collisions.

In a very recent paper, the name *resonance matter* has been put forward [11], [25] [26]. The name *resonance matter* is based on the fact that especially in central collisions, an appreciable portion of nuclear matter is converted into excited resonances. This resonance matter contains the Δ's and higher resonances. This idea is , however, questioned recently by Frankfurt group [27]. These studies of resonance matter are carried out with non- covariant BUU and QMD. Thus, it is important to look for the effect of a covariant treatment on resonance production. This is done in fig.8. Here the number of delta's which are obtained at the final stage of the reaction (i.e. at 60 fm/c) are plotted as a function of the impact parameter. We note that in central collisions, about 30 % nuclear matter is converted into delta matter. We also note that up to semi-central collisions, the relativistic effects show some reduction in the number of delta's. This reduction in the number of deltas using a covariant theory can be important for the discussion of resonance matter at relativistic energies.

Fig. 9 shows the mass dependence of the relativistic effect on the delta production. Here five systems ^{12}C-^{12}C, ^{16}O-^{16}O, ^{20}Ne-^{20}Ne, ^{28}Si-^{28}Si, ^{40}Ca-^{40}Ca are considered. The straight lines in this figure are the extrapolation of the delta population per nucleon found in ^{40}Ca-^{40}Ca to the whole mass region. In other words, e.g. in case of QMD (hard EOS), we see that the delta population for ^{40}Ca-^{40}Ca is 23 % (about 18 deltas for 80 baryons). Therefore the straight line indicates the delta population of 23 % for all mass region. It is interesting to note that the percentage of the resonance matter is nearly independent of the mass of the colliding nuclei. This result is in agreement with the finding of Ref. [11]. Further, it is also clear that the relativistic effects in delta production are similar for all masses considered here. In case of the ^{40}Ca-^{40}Ca collision, RQMD simulations show about 12 % ($(\Delta(RQMD) - \Delta(QMD))/\Delta(QMD)$ x 100) reduction in the delta population. This reduction can have some influence for the production of kaons and other particles created through mainly resonances [31], [28], [29], [30]. The influence of the bombarding energy on relativistic effects in the delta production is shown in fig.10 for the semi-central collision of ^{40}Ca-^{40}Ca. We note that the relativistic effects increase with the increase in the bombarding energy. For all energies, one can see an unique behaviour i.e. the reduction in the resonance matter when one uses a covariant formalism.

It is also interesting to look for the thermalization in heavy ion collisions. To study this, we analyse the rapidity distribution which is a measure of the stopping of nuclear matter in heavy ion collisions. The rapidity distribution is defined as:

$$Y_i = \frac{1}{2} \ln \frac{E(i) + p_z(i)}{E(i) - p_z(i)}, \tag{16}$$

where E(i) and $p_z(i)$ are the energy and the longitudinal momentum of the ith particle. For full equilibrium, one should get a Gaussian shape peaked at mid-rapidity. In fig. 11, we show the rapidity distribution at an impact parameter of 2 fm using the hard EOS. To establish the relativistic effects over a wide range of energy, we show here the rapidity distribution at four different incident energies i.e. at 50 MeV/nucl., 500 MeV/nucl., 1 GeV/nucl. and 1.5 GeV/nucl.. We assumed that the final rapidity distribution is reached for 50 MeV/nucl. after 100 fm/c and for higher energies after 60 fm/c. The Rapidity distribution at 50 MeV/nucl. is the same for RQMD and QMD. At all higher energies (from 500 MeV/nucl. to 1.5 GeV/nucl.), the simulations using a covariant approach show less stopping than the non-covariant. This is true for both the hard and the soft EOS's. One should note that different forces and in-medium effects modify the transverse momentum and other quantities, but the rapidity distribution is not affected [1], [2], [4], [5] [13],[18]. This reduction of the stopping power using RQMD was earlier predicted when the time evolution of the density was discussed (see fig.1). In this figure 1 one sees a rapid decrease of the density in RQMD which shows that the particles are stopped less in the hot and dense zone. These results are in nice agreement with the available calculations of the Frankfurt group [16].

In fig. 12, we show the final rapidity distributions for the reaction ^{40}Ca-^{40}Ca at the bombarding energy of 1.5 GeV using the hard EOS. Here we take two extreme cases of the impact parameters i.e. a central collision b = 0 fm and a peripheral collision b = 6 fm. It is clear that in central reactions, the collision rate is very high and thus we obtain a complete stopping. As one goes to semi-central collisions (see fig. 11), the stopping starts to decrease and for peripheral collisions (fig.12), one sees nearly no stopping and hence one can still see two peaks at the target and projectile rapidities, respectively. Although the initial distributions in fig. 14 (at 6 fm/c) are higher for QMD than for RQMD, the final rapidity distribution at impact parameter b=6 fm in fig. 12 shows the opposite. It is also clear that at all impact parameters one has less stopping using a full covariant approach. To see this further the rapidity distribution of ^{40}Ca-^{40}Ca at 1 GeV/nucl. is shown in fig. 13 calculated in a pure cascade-mode and in a pure Vlasov-mode. The cascade-mode is obtained by switching off the mutual "soft interactions" (self consistent field) between baryons and the Vlasov-mode is obtained by switching off all collisions i.e. we assume that all collisions are Pauli-blocked in a Vlasov-mode. The cascade simulations show the interesting result that relativistic effects vanish in a cascade mode. Further, when one compares the 1 GeV/nucl. simulations using RQMD and QMD, in fig.11 with the cascade-mode in present fig., one sees that the absence of the mean field produces far more stopping. The cascade mode does not have any kind of repulsion even when nucleons are in the hot and compressed central zone and thus it results in more stopping. Due to the lack of any collisions, the Vlasov-mode shows no stopping. Here one can see peaks at target and projectile rapidities.

In fig. 14, we follow the full time evolution of a rapidity distribution for a central collision for the same reaction, energy and impact parameter as in fig.12. Here we show the result using QMD and RQMD simulations at time t = 6 fm/c, 9 fm/c, 12 fm/c and 18 fm/c. After 18 fm/c, the reaction at this high energy is practically in its asymptotic state [see fig.1]. We also note that the soft and the hard EOS's show similar behaviour at all times. At 6 fm/c, no collisions have occurred and hence it reflects the situation of an initial rapidity distribution . One sees that due to the Lorentz- elongation of the momentum distribution, the RQMD simulations have less high peaks at target and projectile rapidities but the Gaussians are broader in RQMD than in the QMD. Between 6 fm/c and 9 fm/c, the first collisions happen and thus particles start to accumulate in the mid-rapidity region. When one sees fig. 2, one expects that due to more collisions, QMD should show more stopping at 9 fm than RQMD. Interestingly enough at 12 fm/c, one finds just the reverse situation than at 9 fm/c. i.e. RQMD at 12 fm/c shows more stopping. This is quite understandable when we remember that in the time span between 9 fm/c and 15 fm/c, RQMD shows a faster rise in the number of collisions as compared to the QMD and hence more particles are stopped in RQMD than in QMD. After 12 fm/c the reaction in RQMD is already nearly asymptotic, whereas the reaction in the QMD still goes on. As a result, the rapidity distribution in RQMD at 12 fm/c and 18 fm/c is nearly the same whereas in the QMD particles are still interacting and thus more and more particles are stopped and hence at last QMD simulations dominate the mid-rapidity zone after 18 fm/c. Finally the rapidity distribution is shown in fig.15 for different collisions involving ^{12}C to ^{28}Si for hard EOS. It is evident from fig. 15 that though the degree of stopping varies with the mass of the colliding nuclei but, the influence of the relativistic effects (i.e. less stopping using RQMD than in the QMD) is the same.

4. Summary and Outlook :

In this paper, we have investigated the dynamical relativistic effects which are originating from a full covariant treatment of the dynamics of heavy ion reactions. For this purpose, the Quantum Molecular Dynamics with its covariant extension (Relativistic Quantum Molecular Dynamics) was used. This generalization of QMD to a full covariant RQMD is based on the Constraint Hamiltonian Dynamics. For a complete understanding of the relativistic effects, five different systems ^{12}C-^{12}C, ^{16}O-^{16}O, ^{20}Ne-^{20}Ne, ^{28}Si-^{28}Si and ^{40}Ca-^{40}Ca were considered at bombarding energies of 50, 250, 500, 750, 1000, 1250, 1500, 1750 and 2000 MeV/nucl.. In addition, a wide range of impact parameters between 0 fm(central collisions) to 10 fm (peripheral collisions) was also investigated. In this study, we have concentrated fully on the observables which are least affected by the different model inputs. Some of these quantities are: The nucleon density , collision history, formation of the resonance matter(which stands for resonance production), the rapidity distribution which shows the thermalization in heavy-ion collisions and also reflects the stopping of the nuclear matter in heavy ion collisions.

We have shown that the final state rapidity distribution using RQMD and QMD at 50 MeV/nucl. gives very close agreement. At higher energies, the covariant treatment of the dynamics affects not only the maximal values of the nucleon density, but it also affects the shape and size of the hot and compressed zone. The time evolution of density using RQMD shows a faster rise and also a faster decline as compared to QMD. The rapid fall of the density in RQMD gives us a hint that the covariant treatment produces less thermalization and less stopping of nuclear matter at relativistic energies. It shows that in RQMD, the particles feel some kind of repulsion which can be due to the Lorentz-contraction of the initial coordinate space, the covariant formulation of the interactions and also due to the mutli-times of the particles. In addition, the covariant treatment of the collisions can also give different results. The number of collisions using RQMD are found to be larger than for QMD. These number of collisions in RQMD are less than in QMD with an initial Lorentz-contracted distribution. This Lorentz-contraction in the initial distribution in RQMD is counterbalanced by the covariant treatment of the interactions. But when one switches off the interactions i.e. when one simulates heavy-ion collisions in a cascade-mode, one finds that the RQMD simulations show far more collisions than what one gets with a Lorentz-contracted QMD. The ratio of collisions which are blocked (Pauli-blocking) relative to all attempted collisions at 50 MeV/nucl. is the same using RQMD and QMD whereas at higher energies, the covariant treatment of the collisions and of the Pauli-blocking (RQMD) shows more collisions which are blocked as compared to QMD.

Resonance matter especially delta production is found to be affected also by the covariant treatment of the dynamics. RQMD shows less delta production than QMD. The relativistic effect in delta production increases with the increase in the bombarding energy. This indicates that the relativistic effects can influence the subthreshold production of K^+ 's, $\bar{p}$'s etc. where the largest part of the production comes from resonance matter.

A detailed investigation of the thermalization i.e. the rapidity distribution shows that RQMD gives less stopping than QMD. This is quite understandable . The rapid fall of the density indicates that the particles in RQMD are less stopped and thus it results in less stopping of nuclear matter.

The main result of our present investigation is that relativistic effects are not strongly dependent on the parameters of the model. This means that both soft and hard nuclear equation of states show the same results. Further the relativistic effects can be understood when we look to the density evolution. The rapid decrease of the density indicates that RQMD produce less stopping than QMD. In other words, the particles are kicked out from the hot and dense zone which immediately indicates that RQMD should give less resonance production, thermalization and nuclear stopping. These results are the same for all energies and masses considered here. In conclusion, our investigation of the relativistic effects in heavy-ion collisions at SIS energies shows that the influence of the relativistic effects at SIS energies is for the observables considered here of a not too large importance . This may change if subthreshold production of heavier particles like K^+ 's and antiprotons are considered.

The package UNISCO contains fully integrated the QMD code based on the latest code of Jörg Aichelin and coworkers and our RQMD code. The authors are thankful to Prof. Jörg Aichelin for providing us with his new QMD code.

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Figure Captions :

Fig. 1 The time evolution of the maximum density ρ^{max}. Here the density is calculated in a central sphere with radius of 2 fm and the maximum value of density reached anywhere in this sphere is taken. The reaction under consideration is ^{40}Ca-^{40}Ca at the bombarding energy E_{lab} = 1.5 GeV/nucl. and at the impact parameter b = 2 fm. The upper and lower parts of the figure show the results with hard and soft EOS's, respectively.

Fig. 2 The average collision rate (dN_{coll}/dt) for the reaction ^{40}Ca-^{40}Ca at impact parameter b = 1 fm and incident energy 1.5 GeV/nucl.. This collision rate includes the Pauli-blocking for each scattered pair of baryons. RQMD and QMD are shown by solid and dashed histograms.

Fig. 3 The same as in fig.2, but for ^{12}C-^{12}C, ^{16}O-^{16}O, ^{20}Ne-^{20}Ne, ^{28}Si-^{28}Si at incident energy = 1.5 GeV/nucl. and at impact parameter b = 0.25 b^{max}. Here a hard EOS is used.

Fig. 4 The same as in fig. 2 but with further decomposition of the total collision rate into the elastic channel (d$N_{elas.}$/dt) and the inelastic channel d$N_{inel.}$/dt. Left and right parts of the figure are calculated with the hard and the soft EOS's, respectively. The upper part represents the elastic collisions whereas the lower part is for inelastic collisions.

Fig. 5 The evolution of the total number of collisions per nucleon as a function of the reaction time. Here the results are shown for RQMD, QMD and QMD with a Lorentz-contracted initial distribution [labeled as QMD(cont.)] . The reaction under consideration is ^{40}Ca-^{40}Ca at 1.5 GeV/nucl. and using both hard and soft EOS's. Here impact parameter is b = 2 fm.

Fig. 6 The time evolution of the total number of collisions per nucleon for the same reaction in fig.5 but in the cascade-mode. The upper part shows the calculation at incident energy

1.5 GeV/nucl., the lower part at 2 GeV/nucl..

Fig. 7 The percentage of the collisions which are Pauli-blocked due to lack of available free phase space as a function of the bombarding energy. Here we simulate ^{40}Ca-^{40}Ca at an impact parameter b = 2 fm. The displayed results are at 60 fm/c.

Fig. 8 The number of delta's obtained at 60 fm/c as a function of the impact parameter in the simulation of ^{40}Ca-^{40}Ca at an incident energy of 1.5 GeV/nucl.. The upper and lower part represent the results using the hard and the soft EOS's, respectively.

Fig. 9 The same as in fig. 8, but for the number of delta's as a function of the total mass of the colliding nuclei. The upper and lower part of the figure are calculated with the hard and the soft EOS's. For the explanation of the straight lines, see text.

Fig. 10 The same as in fig. 8, but for the delta's obtained at final stage as a function of the bombarding energy. The impact parameter is 2 fm.

Fig. 11 The rapidity distribution of the final stage of the reaction of ^{40}Ca-^{40}Ca at impact parameter b = 2 fm. Note that the rapidity is given in units of the beam rapidity. The upper left and right parts are at energies 50 MeV/nucl. and 500 MeV/nucl., respectively. The lower left and right parts represent the results for 1 GeV/nucl. and 1.5 GeV/nucl., respectively. Here hard EOS was used.

Fig.12 The rapidity distribution dN/dY as a function of Y_{cm}. The reaction is ^{40}Ca-^{40}Ca at 1.5 GeV/nucl. The solid and dashed curves are the results of RQMD and QMD at an impact parameter b = 0 fm. The solid and dashed histograms represent the results of RQMD and QMD at an impact parameter b = 6 fm.

Fig. 13 The rapidity distribution dN/dY as a function of Y_{cm}. The reaction is ^{40}Ca-^{40}Ca at the impact parameter b = 2 fm and at an incident energy of 1.0 GeV/nucl. The upper part of the figure is calculated with a cascade-mode whereas the lower part represents the results in a Vlasov approach.

Fig. 14 The rapidity distribution dN/dY as a function of Y_{cm} for the reaction of ^{40}Ca-^{40}Ca at an incident energy 1.5 GeV/nucl. and at an impact parameter b = 0 fm. Here four different times t = 6, 9, 12 and 18 fm/c are chosen. These results are calculated using the hard EOS.

Fig. 15 The final rapidity distribution as a function of Y_{cm}. Here four different systems ^{12}C-^{12}C, ^{16}O-^{16}O, ^{20}Ne-^{20}Ne, ^{28}Si-^{28}Si are considered at the incident energy = 1.5 GeV/nucl. and the impact parameter b = 0.25 b^{max}. The hard EOS is used.