Exploitation of Common Property Resources when Happiness Depends on Relative Consumption

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Abstract

We introduce a dynamic model of resource-grabbing by status-conscious agents, i.e., agents value not only their absolute consumption levels, but also the relative status within his/her reference group. The purpose of this paper is to explore the effect of the "positional externalities" on the urge to seek rent and to connect the "tragedy of the commons" problem with relative consumption. Our model shows that the greater is agents' concern about their relative status, the more aggressively they tend to behave. Consequently, the social welfare is lower because the growth rate of the public asset is reduced due to higher extraction rate. After introducing heterogeneity, we show that the social welfare decreases as the distribution of status-consciousness among agents widens.

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1 Introduction

In discussing the pattern of economic development in the past thirty years, many economists point out that the most successful economies, such as the Asian tigers, are not well endowed with natural resources, while many resource-rich countries, such as Nigeria, seem to be stagnant. This observation has led to the notion of “resource curse”: being well endowed with natural resources may be a burden (see Sachs and Warner 2001). Some economists have refined this view by adding factors that they consider necessary for the resource curse to take place: imperfect property rights, rent-seeking and poor governance (see Baland and Patrick 2000, Torvik 2002, Mehlum, Moene and Torvik 2006).

Many economists attribute poor growth to rent-seeking activities. Some have modelled a dynamic rent-seeking game, where agents extract from a common-property resource (either in the literal sense of a natural resource stock, or in the figurative sense, as in Tornell and Lane, 1999). These models are based on the assumption that rent-seekers’ utility is dependent only on their absolute consumption level. On the other hand, there is mounting empirical evidence that supports the view that individuals care a great deal about their relative consumption, i.e., a person’s happiness depends on the comparison of her consumption level with that of other members of her peer group. An individual is happier the more her consumption (or income) level exceeds the per-capita consumption (or income) of her reference group, as shown in the empirical studies by Clark and Oswald (1996), Neumark and Postlewaite (1998), Luttmer (2005), Dynan and Ravina (2007), and others.

The purpose of this paper is to explore the effect of this “positional externalities” on the urge to seek rent and to connect the “tragedy of the commons” problem with relative consumption. We model rent-seeking as exploitation from a common-property resource stock, as in Tornell and Velasco 1992, and Tornell and Lane 1996, 1999. Their models are however different from ours in two important respects: first, their agents care only about absolute consumption, and second, they assume that rent-seekers are homogeneous[1]. In contrast, we assume that agents gain utility from both absolute consumption and relative consumption, and we also consider the case where agents differ with respect to some characteristics.

The literature of relative consumption can be traced back to Smith (1759) and Veblen (1899). Duesenberry (1949) and Pollak (1976) were among the first to formalize the theory

[1] Furthermore, agents in their models can transfer revenue from a public capital stock to personal accounts, in which property rights are perfectly secured. Long and Sorger (2006) extend the model to the case of heterogeneous agents, and explicitly introduce effort costs.

of relative consumption. In the more recent literature, the interdependence in consumption has been subjected to rigourous refinements, and has been variously described as "keeping up with the Joneses" (Gali 1994), "status" (Fisher and Hof 2000), "jealousy" (Dupor and Liu 2003), or "envy" (Eaton and Eswaran 2003). These authors maintain the assumption that each person is the owner of his capital stock, and therefore the problem of rent-seeking does not arise in their models of status-seeking[2].

In our paper, we combine rent-seeking with status-seeking, and analyse the "status-seeking effect" on the "tragedy of the commons" problem. We show that an increase in the status-seeking parameter (e.g., an increase in the degree of envy) worsens the problem of over-exploitation of resources. Agents tend to behave more aggressively if they are more concerned about their relative status. Consequently, the social welfare is lower. In addition, the growth rate of the public asset is reduced due to higher extraction rates. We also show that with rent-seeking, an exogenous technical progress in the resource-extraction sector can reduce welfare, and the magnitude of this welfare-worsening effect is an increasing function of the status-seeking parameter. In a final section, we introduce heterogeneity, and show that the social welfare decreases if agents become more heterogeneous in terms of status-seeking, but it increases if they become more heterogeneous in terms of appropriation costs.

The rest of the paper is organized as follows. Section 2 presents the model and discuss the key assumptions. Section 3 characterizes the solution to a cooperative equilibrium (or the solution of a social planner's problem). Section 4 characterizes the Markov Perfect Nash Equilibrium and offer welfare comparisons. This is followed by introducing heterogeneity among agents, and studying the implications of increases in heterogeneity. Some concluding remarks and some discussion on policy implications are offered in Section 5.

2 A Simple Model

There are n agents. Let $c_i(t)$ denote the absolute consumption level of agent i at time t. Let $C_{-i}(t)$ denote the average consumption level of agent i's peers:

$$C_{-i}(t) \equiv \frac{1}{n-1} \sum_{j \neq i} c_j(t)$$

[2] Another study related to ours is Alvarez-Cuadrado and Long (2007), who assume, however, that property rights are perfectly enforced and that there is no renk-seeking. Our paper is different from theirs in that we deal with a common-property resource stock, and we explore the impact of the "status-consciousness" on the "tragedy of the commons" problem.

We define $z_i(t)$ to be agent i's relative consumption level:

$$z_i(t) \equiv \frac{c_i(t)}{C_{-i}(t)}$$

Let $E_i(t)$ denote agent i's extraction rate from a common-property resource. We assume that the consumption rate $c_i(t)$ is a fraction of the extraction rate $E_i(t)$. Specifically, $E_i(t) = (1+\theta_i)c_i(t)$. Here θ_i is a non-negative number that represents agent i's "wastage rate", which may be interpreted as reflecting his degree of inefficiency in transforming the extracted resource into the consumption good, or perhaps as the bribes or penalties that he must pay to third parties in his illicit resource-appropriation process.

Let $X(t)$ denote the stock level of the common-property resource. We assume that the rate of growth of X is given by the differential equation

$$\dot{X}(t) = AX(t) - \sum_{i=1}^{n} E_i(t)$$

where $A \geq 0$ is a constant. In what follows, we will omit the time index for simplicity of notation.

The *net-utility function* of agent i is denoted by $V(z_i, c_i, X, E_i)$ where

$$V = U(z_i, c_i, X) - \kappa_i E_i$$

The variable X appears in the utility function, because the stock X provides a flow of amenities (e.g. recreational uses) that each agent values. The non-negative parameter κ_i represents "the effort cost" of extracting the resource. This parameter may represent (a) a technological coefficient between effort and harvest level, so that a fall in κ_i represents a technological progress in resource extraction, or (b) the difficulty with which the agent hides his illegal activities. Note that we have introduced two separate parameters, θ_i and κ_i, that represent different types of cost of appropriation: κ_i is the "effort cost" which is measured in utility units, while θ_i is the "wastage cost", which acts like an income tax.

We assume that each individual's *gross-utility function* $U(z_i, c_i, X)$ is non-decreasing in her relative consumption, z_i, and increasing in her absolute consumption, c_i, and in the amenities provided by the stock, X:

$$\frac{\partial U}{\partial z_i} \geq 0, \ \frac{\partial U}{\partial c_i} > 0, \ \frac{\partial U}{\partial X} > 0$$

Furthermore, for any given C_{-i}, we denote by U_{c_i} the total derivative of U with respect to c_i :

$$U_{c_i} \equiv \frac{\partial U}{\partial z_i}\frac{dz_i}{dc_i} + \frac{\partial U}{\partial c_i} = \frac{\partial U}{\partial z_i}\frac{1}{C_{-i}} + \frac{\partial U}{\partial c_i}$$

and we assume that $U_{c_i} > 0$ and $U_{c_i c_i} < 0$. This means that, for any given C_{-i}, the individual's utility is strictly increasing and strictly concave in his own consumption level, c_i. Strict concavity is assumed so that the second order condition for individual maximization is satisfied. To proceed further, we make the following specific assumptions:

Assumption A.1: The gross-utility function takes the form

$$U(z_i, c_i, X) = G(z_i)F(c_i, X)$$

where $F(c_i, X)$ is homogeneous of degree one[3], strictly-quasi-concave, and increasing in (c_i, X), with $F_{c_i}(0, X) = \infty$, and $G(z_i)$ is positive and non-decreasing in z_i.

Without loss of generality, we set $G(1) = 1$. If $G'(.) > 0$, we say that the agents are envious (concerned about relative consumption), while if $G'(.) = 0$ identically, we say that the agents are non-envious.

For given z_i, the marginal rate of substitution of consumption c_i for X is

$$MRS_{c_i X} \equiv \frac{F_{c_i}}{F_X}$$

It is useful to define the ratio of consumption to amenity services by $\beta_i = c_i/X$ .Since $F(c_i, X)$ is homogeneous of degree 1, we obtain

$$F(c_i, X) = XF(\beta_i, 1) \equiv Xf(\beta_i)$$

Under Assumption A1, it follows that $f'(\beta_i) = F_c > 0$, $f''(\beta_i) < 0$, $r(\beta_i) \equiv f(\beta_i) - \beta_i f'(\beta_i) = F_X > 0$ and $r'(\beta_i) = -\beta_i f''(\beta_i) > 0$.Hence

$$MRS_{c_i X} \equiv \frac{F_{c_i}}{F_X} = \frac{f'(\beta_i)}{f(\beta_i) - \beta_i f'(\beta_i)} \equiv \omega(\beta_i)$$

Clearly the marginal rate of substitution is diminishing in β_i :

$$\omega'(\beta_i) = \frac{f(\beta_i) f''(\beta_i)}{[f(\beta_i) - \beta_i f'(\beta_i)]^2} < 0$$

Assumption A.2: The function f satisfies the following Inada conditions:

$$\lim_{\beta \to 0} f'(\beta) = \infty, \ \lim_{\beta \to \infty} f'(\beta) = 0$$

Our analysis at a general level does not rely on a specific functional form for F nor G, however at places it will be convenient to specialize in the following Cobb-Douglas case:

$$U(z_i, c_i, X) = z_i^{\lambda} c_i^{\mu} X^{1-\mu} \text{ where } \lambda > 0 \text{ and } 0 < \mu < 1 \text{ and } \lambda + \mu < 1$$

[3]The assumption of homogeneity of degree one in (c_i, X) is borrowed from Long and Sorger (2006). It greatly simplifies the analysis.

Here, the parameter λ is an indicator of the strength of the status-consciousness. Note that U is strictly concave and increasing in c_i for given C_{-i}:

$$U_{c_i} = \lambda z_i^{\lambda-1} c_i^{\mu} X^{1-\mu}\left(\frac{1}{C_{-i}}\right) + \mu z_i^{\lambda} c_i^{\mu-1} X^{1-\mu} = X^{1-\mu} c_i^{\mu+\lambda-1} C_{-i}^{-\lambda}(\lambda+\mu) > 0$$

$$U_{c_i c_i} = (\mu+\lambda-1) X^{1-\mu} c_i^{\mu+\lambda-2} C_{-i}^{-\lambda}(\lambda+\mu) < 0$$

3 The Cooperative Equilibrium

It is useful to begin with the following benchmark scenario. All agents are identical, and they cooperate by agreeing on a common rate of resource extraction: $E_i(t) = E(t)$. It follows that $c_i(t) = c(t)$ and $z_i(t) = 1$. It is as if there were a social planner seeking to solve the following optimization problem. Choose $c(t)$ to maximize

$$\int_0^{\infty} e^{-\rho t}\left[G(1)F(c, X) - \kappa(1+\theta)c\right] dt \tag{1}$$

subject to

$$\dot{X} = AX - n(1+\theta)c$$

with $X(0) = X_0$ and

$$\lim_{t\to\infty} X(t) \geq 0$$

To ensure convergence of the integral, we will assume:

Assumption A.3: The rate of discount exceeds the natural growth rate of the stock: $\rho > A$.

Recall that $G(1) = 1$.The social planner's problem reduces to finding the time path of the control variable $\beta(t)$ that maximizes the welfare of the representative agent:

$$W^p = \int_0^{\infty} e^{-\rho t}\left[f(\beta) - \kappa(1+\theta)\beta\right] X dt$$

subject to

$$\dot{X} = X\left[A - n(1+\theta)\beta\right]$$

with $X(0) = X_0$ and

$$\lim_{t\to\infty} X(t) \geq 0$$

Let ψ denote the shadow price of the stock X. The Hamiltonian function is

$$H = \left[f(\beta) - \kappa(1+\theta)\beta\right] X + \psi X\left[A - n(1+\theta)\beta\right]$$

The necessary conditions include

$$\frac{\partial H}{\partial \beta} = X\left\{f'(\beta) - \kappa(1+\theta) - n\psi(1+\theta)\right\} = 0$$

$$\dot{\psi} = (\rho - A)\psi - [f(\beta) - (1+\theta)(\kappa + n\psi)\beta]$$

and the transversality condition is

$$\lim_{t\to\infty} \psi(t)e^{-\rho t} \geq 0,\ \lim_{t\to\infty} X(t) \geq 0,\ \lim_{t\to\infty} \psi(t)e^{-\rho t}X(t) = 0 \tag{2}$$

Let us consider a candidate solution where $\beta(t) = \overline{\beta}$ (a constant). This yields a corresponding constant $\overline{\psi}$ where

$$f'(\overline{\beta}) = (1+\theta)(\kappa + n\overline{\psi}) \tag{3}$$

or

$$\overline{\psi} = \frac{1}{n}\left[\frac{f'(\overline{\beta})}{(1+\theta)} - \kappa\right] \tag{4}$$

which implies that $\dot{\psi} = 0$, hence

$$(\rho - A)\overline{\psi} = f(\overline{\beta}) - (1+\theta)(\kappa + n\overline{\psi})\overline{\beta} \tag{5}$$

Using (3) and (5),

$$(\rho - A)\overline{\psi} = f(\overline{\beta}) - \overline{\beta}f'(\overline{\beta}) > 0 \tag{6}$$

Substituting (4) into (6), we get the following equation which determines the optimal $\overline{\beta}$, say $\overline{\beta}^*$

$$\left[\frac{f'(\overline{\beta})}{(1+\theta)} - \kappa\right] = \frac{n\left[f(\overline{\beta}) - \overline{\beta}f'(\overline{\beta})\right]}{\rho - A} \tag{7}$$

Proposition 1: *Under Assumptions A1, A2 and A3, the cooperative solution consists of following the consumption strategy* $c = \overline{\beta}^* X$*, where* $\overline{\beta}^*$ *is the unique positive solution of equation (7).*

Proof:

First, let us show that $\overline{\beta}^*$ is unique. As shown in Fig. 1, the left-hand side (LHS) of equation (7) is decreasing in $\overline{\beta}$, and as $\overline{\beta}$ varies from zero to infinity, the LHS varies from infinity to $-\kappa$. The RHS is positive for all positive $\overline{\beta}$, and increases as $\overline{\beta}$ increases. Thus the curve that represents the LHS must intersect the curve that represents the RHS exactly at one value, say $\overline{\beta}^*$. At $\overline{\beta}^*$, we have

$$\frac{f'(\overline{\beta}^*)}{(1+\theta)} - \kappa > 0 \tag{8}$$

(This is because the numerator of the right-hand side of (7) is positive for all $\beta > 0$, and the denominator is positive because $\rho > A$).

At the constant ratio $\overline{\beta}^*$ of consumption to stock, the growth rate of the stock is

$$g \equiv \frac{\dot{X}}{X} = A - n(1+\theta)\overline{\beta}^* < A < \rho$$

(which may be positive or negative) and thus

$$X(t) = X_0 e^{gt}$$

Next, to show that the strategy $c = \overline{\beta}^* X$ is optimal, we can verify that all the necessary and sufficient conditions are satisfied. The transversality condition (2) is met, because $\psi(t) = \overline{\psi}^* > 0$ by (4) and (8), and because

$$\lim_{t\to\infty} \psi(t) e^{-\rho t} X(t) = 0 = \overline{\psi}^* X_0 \lim_{t\to\infty} e^{-\rho t} e^{gt} = 0$$

Since the objective function (1) is concave in (c, X), and the constraints are linear, the necessary conditions are also sufficient. ■

Remark 1: (Interpretation) Condition (7) has a straightforward interpretation. Given any $\overline{\beta}$, consider a small decrease in per-capita extraction, say dE at time zero. This will lead to a small decrease in consumption by $dc = dE/(1+\theta)$. The marginal utility loss from reduced consumption (net of reduced extraction cost κ) is thus $\left[f'(\overline{\beta})(1+\theta)^{-1} - \kappa\right] dE$. On the other hand, the impact effect on the stock is an increase by ndE, which leads to a stream of gain in marginal utility of amenities:

$$\int_0^\infty e^{-\rho t} \left\{ \left[f(\overline{\beta}) - \overline{\beta} f'(\overline{\beta}) \right] (ndE) e^{At} \right\} dt = \frac{n\left[f(\overline{\beta}) - \overline{\beta} f'(\overline{\beta}) \right]}{\rho - A} dE$$

At the optimal $\overline{\beta}^*$, the marginal utility loss from reduced consumption must equal the marginal utility gain from increased amenity services.

Remark 2: In the Cobb-Douglas case, assuming $\kappa = 0$, it can be verified that

$$\overline{\beta}^* = \frac{\mu(\rho - A)}{n(1-\mu)(1+\theta)}$$

and thus the growth rate of the public asset is

$$g = A - \frac{\mu(\rho - A)}{1-\mu}$$

which can be negative or positive.

Proposition 2: *The welfare of the representative agent under cooperation is*

$$W^{coop} = \overline{\psi}^* X_0$$

where

$$\overline{\psi}^* = \frac{1}{n}\left[\frac{f'(\overline{\beta}^*)}{(1+\theta)} - \kappa\right]$$

An increase in κ or in θ will reduce both $\overline{\beta}^*$ and welfare.

Proof: Since $X(t) = X_0 e^{gt}$

$$W^{coop} = \int_0^\infty e^{-\rho t}\left[f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*\right] X_0 e^{gt} dt$$

$$W^{coop}(X_0) = \left[f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*\right] X_0 \frac{1}{\rho - g} = X_0 \frac{f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*}{\rho - A + n(1+\theta)\overline{\beta}^*}$$

where, since $\rho - A > 0$, $\rho - g > 0$.

Now, from (5) and (6),

$$(\rho - A)\overline{\psi} = f(\overline{\beta}) - \overline{\beta} f'(\overline{\beta}) = f(\overline{\beta}) - \overline{\beta}(1+\theta)(\kappa + n\overline{\psi}) \tag{9}$$

we obtain

$$\left(\rho - A + n(1+\theta)\overline{\beta}^*\right)\overline{\psi}^* = f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*$$

It follows that

$$\frac{f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*}{\rho - A + n(1+\theta)\overline{\beta}^*} = \overline{\psi}^* = \frac{1}{n}\left[\frac{f'(\overline{\beta}^*)}{(1+\theta)} - \kappa\right] \tag{10}$$

where the last inequality comes from (4). Therefore

$$W^{coop}(X_0) = \overline{\psi}^* X_0 \tag{11}$$

Thus welfare (per person) is the product of the shadow price $\overline{\psi}^*$ and the stock X_0.

An increase in κ or θ will shift down the curve representing the left-hand side (LHS) of equation (7), so the intersection $\overline{\beta}^*$ is moved to the left. Direct computation shows that

$$\frac{\partial \overline{\beta}^*}{\partial \kappa} = \frac{(\rho - A)(1+\theta)}{\left[\rho - A + n(1+\theta)\overline{\beta}^*\right] f''(\overline{\beta}^*)} < 0 \tag{12}$$

Thus

$$\begin{aligned}\frac{\partial W^{coop}}{\partial \kappa} &= \frac{\partial \overline{\psi}^*}{\partial \kappa} X_0 = \frac{1}{(1+\theta)n}\left[f''(\overline{\beta}^*)\frac{\partial \overline{\beta}^*}{\partial \kappa} - (1+\theta)\right] \\ &= \frac{1}{n}\left[\frac{-n(1+\theta)\overline{\beta}^*}{\rho - A + n(1+\theta)\overline{\beta}^*}\right] < 0\end{aligned}$$

A similar calculation shows that welfare falls if θ increases. ■

4 Non-cooperative resource extraction by envious agents

In this section, we study a differential game involving n identical players. Consider individual i. She faces $n-1$ rival rent-seekers. Suppose she thinks that each rival j adopts a consumption strategy having a stationary feedback (i.e., stationary Markovian) form

$$c_j(t) = \phi_j(X(t)) \text{ where } \phi_j'(X) > 0 \text{ and } \phi_j(0) = 0$$

That is, at any moment of time, individual j's consumption depends only on the currently observed stock level $X(t)$. The restriction that $\phi_j(0) = 0$ makes sense: when the resource stock is zero, it is impossible to extract any resource.

Then

$$C_{-i}(t) = \frac{1}{n-1} \sum_{j \neq i} \phi_j(X(t)) \equiv \Phi(X(t))$$

The optimization problem for individual i is then to choose a time path of consumption $c_i(t) \geq 0$ that maximizes her life-time utility

$$\int_0^\infty e^{-\rho t} \left\{ U\left(\frac{c_i(t)}{\Phi(X(t))}, c_i(t), X(t) \right) - \kappa(1+\theta)c_i \right\} dt$$

subject to

$$\dot{X}(t) = AX(t) - (n-1)(1+\theta)\Phi(X(t)) - (1+\theta)c_i(t)$$

and

$$\lim_{t \to \infty} X(t) \geq 0$$

This problem is a standard optimal control problem. Suppose the problem has a solution: a pair of time paths $(c_i(t), X(t))$ that maximizes the objective function. Then one can express the optimal control $c_i(t)$ as a function of the stock $X(t)$. Denote this function by $g_i(X)$:

$$c_i(t) = g_i(X(t))$$

Such a function $g_i(X)$ is player i's "optimal Markovian strategy", given $\Phi(X)$.More formally, we say that the function $g_i(.)$ is player i's Markovian best reply to the $(n-1)$ tuple of Markovian strategies of her rivals, $(\phi_1(.), \phi_2(.), ..., \phi_{i-1}(.), \phi_{i+1}(.), ..., \phi_n(.))$.

We are interested in the scenario where all players are facing similar optimization problems. This is a differential game among n players.

Definition: A Markov-perfect Nash equilibrium of the game described above is a $n-$tuple of Markovian strategies $(\phi_1^*(.), \phi_2^*(.), ..., \phi_n^*(.))$ such that, for each player i $(i = 1, 2, ..., n)$, the function $\phi_i^*(.)$ is player i's Markovian best reply to the $(n-1)$tuple of Markovian strategies of her rivals, $(\phi_1^*(.), \phi_2^*(.), ..., \phi_{i-1}^*(.), \phi_{i+1}^*(.), ..., \phi_n^*(.))$.(For a more precise and more general definition, see Dockner et al., 2000, or Long and Sorger, 2006.)

4.1 Finding a Markov-perfect Nash equilibrium: the case of identical agents

In this subsection, we will show that, when agents are identical, the game described above has a symmetric Markov-perfect Nash equilibrium, in which all players adopt the same linear Markovian strategy

$$c_j(t) = \beta X(t)$$

where β is a positive constant.

Suppose player i knows that all other players use the strategy $c_j(t) = \beta X(t)$.The optimization problem of agent i is to choose a time path of $c_i \geq 0$ that maximizes

$$\int_0^\infty e^{-\rho t} \left\{ G\left(\frac{c_i}{\beta X}\right) F(c_i, X) - \kappa(1+\theta)c_i \right\} dt$$

subject to

$$\dot{X} = AX - (n-1)(1+\theta)\beta X - (1+\theta)c_i$$

$$\lim_{t\to\infty} X(t) \geq 0$$

We may interpret $A - (n-1)(1+\theta)\beta$ as player i's net rate of return on holding the asset.

Let ψ_i be the co-state variable. The Hamiltonian is

$$H_i = G\left(\frac{c_i}{\beta X}\right) F(c_i, X) - \kappa(1+\theta)c_i + \psi_i \left[AX - (n-1)(1+\theta)\beta X - (1+\theta)c_i\right]$$

The optimality conditions are

$$\frac{\partial H_i}{\partial c_i} = G'\left(\frac{c_i}{\beta X}\right)\left(\frac{1}{\beta X}\right) F(c_i, X) + G\left(\frac{c_i}{\beta X}\right) F_{c_i}(c_i, X) - (\kappa + \psi_i)(1+\theta) = 0 \tag{13}$$

$$\dot{\psi}_i = \psi_i \left[\rho - A + (n-1)(1+\theta)\beta\right] + G'\left(\frac{c_i}{\beta X}\right)\left(\frac{c_i}{\beta}\right) X^{-2} F - G F_X \tag{14}$$

$$\dot{X} = \frac{\partial H_i}{\partial \psi_i} = AX - (n-1)(1+\theta)\beta X - (1+\theta)c_i \tag{15}$$

$$\lim_{t\to\infty} e^{-\rho t}\psi_i(t) \geq 0 \text{ and } \lim_{t\to\infty} e^{-\rho t}\psi_i(t)X(t) = 0 \tag{16}$$

Let us try a symmetric equilibrium, with

$$\frac{c_i(t)}{X(t)} = \frac{c_j(t)}{X(t)} = \beta \tag{17}$$

We must verify that the optimality conditions (13) to (16) are satisfied when the strategies described by equation (17) are used, for some suitable constant $\beta > 0$.

Using symmetry, equation (13) becomes

$$G'(1)\left(\frac{1}{\beta}\right) f(\beta) + G(1) f'(\beta) - \kappa(1+\theta) - (1+\theta)\psi_i(t) = 0 \tag{18}$$

This equation implies that $\psi_i(t)$ is a constant, i.e. $\dot{\psi}_i = 0$ along the equilibrium play. Hence we must have

$$\begin{aligned} &\psi_i \left[\rho - A + (n-1)(1+\theta)\beta\right] = \\ &-G'(1) f(\beta) + G(1) \left[f(\beta) - f'(\beta)\beta\right] \end{aligned} \tag{19}$$

These two equations are satisfied iff there exists some $\widehat{\beta} > 0$ which satisfies the following condition

$$\begin{aligned} &\left[\frac{G'(1)f(\beta)\frac{1}{\beta} + G(1)f'(\beta)}{1+\theta} - \kappa\right]\left[(\rho - A) + (n-1)(1+\theta)\beta\right] + G'(1) f(\beta) = \\ &G(1)\left[f(\beta) - f'(\beta)\beta\right] \end{aligned} \tag{20}$$

Proposition 3: *A Markov-perfect Nash equilibrium, where all players play a linear feedback strategy of the form $c = \beta X$, exists iff the equation (20) has a solution $\widehat{\beta} > 0$.*

Example: The Cobb-Douglas Case

$$U = z_i^{\lambda} c_i^{\mu} X^{1-\mu}$$

Here, $G(z) = z^{\lambda}$, $G'(z) = \lambda z^{\lambda - 1}$, $G(1) = 1$, $G'(1) = \lambda$, $f(\beta) = \beta^{\mu}$, $f'(\beta) = \mu\beta^{\mu-1}$, $f(\beta) - f'(\beta)\beta = (1-\mu)\beta^{\mu}$

Eq (20) becomes

$$\begin{aligned} &\left[\frac{\lambda\beta^{\mu-1} + \mu\beta^{\mu-1}}{1+\theta} - \kappa\right]\left[(\rho - A) + (n-1)(1+\theta)\beta\right] = \\ &-\lambda\beta^{\mu} + (1-\mu)\beta^{\mu} \end{aligned}$$

i.e.

$$\left[\frac{\lambda+\mu}{1+\theta} - \kappa\beta^{1-\mu}\right] = \frac{(1-\lambda-\mu)}{(\rho - A)\frac{1}{\beta} + (n-1)(1+\theta)} \tag{21}$$

The LHS of equation (21) is decreasing in β. As β varies from zero to infinity, the LHS falls from $(\lambda+\mu)/(1+\theta)$ to minus infinity if $\kappa > 0$. The RHS is increasing in β, varying from zero to $(1-\lambda-\mu)/\left[(n-1)(1+\theta)\right]$ as β varies from zero to infinity. It follows that if $\kappa > 0$, there exists a unique positive $\widehat{\beta}$ that equates the LHS with the RHS. Furthermore, an increase in κ will lower the curve representing the RHS, resulting in a smaller value of $\widehat{\beta}$. An increase in λ will shift the curve representing the RHS down, and shift the curve representing the LHS up, resulting in a higher value of $\widehat{\beta}$. (If $\kappa = 0$ then a positive $\widehat{\beta}$ exists if and only if $n(\lambda + \mu) < 1$.)

Do these results apply to the general case? The answer is yes, provided the equation (20) has a unique solution $\widehat{\beta} > 0$. Without loss of generality, we set $G(1) = 1$ and treat $G'(1)$ as a parameter: the higher is $G'(1)$, the higher is the degree of status-consciousness of the players. To simplify notation, denote the status consciousness parameter by $\lambda \equiv G'(1)$.

Proposition 4: (*The general case*) *Assume $\widehat{\beta}$ is unique. Then*

(a) A higher degree of status-consciousness will result in a higher equilibrium rate of extraction and a lower public asset growth rate.

(b) An increase in κ or A will reduce the equilibrium rate of extraction, $\widehat{\beta}$ and thus increase the growth rate of the public asset.

Proof: An increase in $G'(1)$ will shift upwards the curve representing the LHS of (20). Hence the intersection point $\widehat{\beta}$ must move to the right. Similarly, an increase in κ or A shift downwards the curve representing the LHS of (20), thus moving $\widehat{\beta}$ to the left. The growth rate of the public asset in the Markov-perfect equilibrium (MPE) is

$$\frac{\dot{X}}{X} = g^{MPE} = A - n(1+\theta)\widehat{\beta}$$

It follows that an increase in κ or A will increase the growth rate of the public asset. ■

Remark: The result (b) above is in sharp contrast to that of Long and Sorger (2006), where an increase in κ (interpreted as an increase in the cost of money laundering) will increase extraction, and reduce the growth rate of the public asset. The reason for the difference is that in Long and Sorger (2006), agents can “store” the amount they extract from the common-property resources by investing it in a private asset. In our model, the quantity extracted must be consumed. Also, for the same reason, our result is different from Tornell and Lane (1999), in that in our model an increase in A, the return of the public asset, will not result in greater appropriation rates.

Proposition 5: *(comparing the cooperative solution with the non-cooperative equilibrium) The cooperative rate of extraction, $\overline{\beta}^*$, is lower than the non-cooperative rate of extraction $\widehat{\beta}$.*

Proof: Re-write eq (7) as follows

$$\left[\frac{f'(\beta)}{(1+\theta)} - \kappa\right]\left(\frac{\rho - A}{n}\right) = f(\beta) - \beta f'(\beta) \tag{22}$$

and compare with

$$\left[\frac{\lambda f(\beta)\frac{1}{\beta}+f'(\beta)}{1+\theta}-\kappa\right][\rho-A+(n-1)(1+\theta)\beta]+\lambda f(\beta)=f(\beta)-\beta f'(\beta) \tag{23}$$

We first prove that when $\lambda = 0$, $\widehat{\beta}$ must exceed $\overline{\beta}^*$. Both equations have the same right-hand side, which is an increasing function of β; as β varies from 0 to infinity, $f(\beta) - \beta f'(\beta)$ rises continuously. The left-hand side of equation (22) is downward sloping, and is positive for all $\beta < \beta_H$ where by definition $f'(\beta_H) = (1+\theta)\kappa$. For all $\beta < \beta_H$, the value of the LHS of eq (23) is greater than that of equation (22). It follows that $\widehat{\beta}$ exceeds $\overline{\beta}^*$. Now, if $\lambda > 0$, this will make $\widehat{\beta}$ even greater. ■

Proposition 6: *(comparing welfare levels) The cooperative solution yields a higher welfare level than that of the Markov perfect equilibrium.*

Proof:

Recall from the cooperative solution that

$$W^{coop} = \overline{\psi}^* X_0$$

$$\overline{\psi}^* = \frac{f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*}{\rho - A + n(1+\theta)\overline{\beta}^*} = \frac{1}{(1+\theta)n}\left[f'(\overline{\beta}^*) - \kappa(1+\theta)\right]$$

The welfare of the representative agent in the Markov-perfect equilibrium is

$$W^{MPE} = \int_0^\infty e^{-\rho t}\left[f(\widehat{\beta}) - \kappa(1+\theta)\widehat{\beta}\right] X_0 e^{gt} dt$$

$$= \left[f(\widehat{\beta}) - \kappa(1+\theta)\widehat{\beta}\right] X_0 \frac{1}{\rho - g} = X_0 \frac{f(\widehat{\beta}) - \kappa(1+\theta)\widehat{\beta}}{\left(\rho - A + n(1+\theta)\widehat{\beta}\right)}$$

Now,

$$(\rho - A + (1+\theta)(n-1)\widehat{\beta})\widehat{\psi} = f(\widehat{\beta}) - \widehat{\beta} f'(\widehat{\beta}) - G'(1) f(\widehat{\beta})$$

$$= f(\widehat{\beta}) - \widehat{\beta}\left[f'(\overline{\beta}) + G'(1)\frac{f(\widehat{\beta})}{\widehat{\beta}}\right]$$

$$= f(\widehat{\beta}) - \widehat{\beta}(1+\theta)(\kappa + \widehat{\psi})$$

where the first equality comes from (19) and the third one comes from (18). Therefore

$$\widehat{\psi} = \frac{f(\widehat{\beta}) - \kappa(1+\theta)\widehat{\beta}}{\rho - A + n(1+\theta)\widehat{\beta}}$$

$$W^{MPE} = \widehat{\psi} X_0 \tag{24}$$

Let's denote

$$\psi = \psi(\beta) = \frac{f(\beta) - \kappa(1+\theta)\beta}{\rho - A + n(1+\theta)\beta} \tag{25}$$

We want to show that

$$\overline{\psi}^* > \widehat{\psi}$$

The cooperative equilibrium can be transformed to an equivalent problem:

$$\underset{\beta}{Max}\ W^{coop} = \psi(\beta)X_0$$

Therefore, the first-order condition of the problem above must yield

$$\psi'(\beta) = 0$$

which gives

$$\frac{\partial\psi(\beta)}{\partial\beta} = \frac{[f'(\beta) - \kappa(1+\theta)]\,[\rho - A + n(1+\theta)\beta] - n(1+\theta)\,[f(\beta) - \kappa(1+\theta)\beta]}{(\rho - A + n(1+\theta)\beta)^2} = 0$$

Rearrange terms in the numerator, we have

$$\left[\frac{f'(\beta)}{(1+\theta)} - \kappa\right] = \frac{n\,[f(\beta) - \beta f'(\beta)]}{\rho - A} \tag{26}$$

which is identical to (7) used to determine the cooperative equilibrium strategy $\overline{\beta}^*$ in Section 3. The second order condition is satisfied. This implies that the curve $\psi(\beta)$ defined by (25) reaches its maximum at $\beta = \overline{\beta}^*$. Therefore the MPE solution $\widehat{\beta}$ must yields a smaller ψ, hence a lower welfare. Figure 2 depicts the curve $\psi(\beta)$ ■.

Remark: Since $\widehat{\beta} > \overline{\beta}^*$ as shown in Proposition 5, we must have $\psi'(\widehat{\beta}) < 0$, which indicates the welfare in the MPE case is decreasing in β, i.e. $\widehat{\beta}$ always lies to the right of $\overline{\beta}^*$(Fig 2 illustrate this situation).

Combining Propositions 5 and 6, it is interesting to note that the cooperative equilibrium has both higher welfare level and greater resource growth rate. Let's explore some intuition behind these results. In the cooperative equilibrium or the social planner's problem, the agents know ex ante that their consumption levels will be equal thus the status-conscisous parameter λ doesn't play a role in the equilibrium. In the MPE case, however, the agents will observe the resource stock at the beginning of each period and make her own decision about the extraction rate, each trying not to be behind, even though they know that in the symmetric equilibrium their consumption levels will be equal ex post. The "positional externalities" imposed by the status-consciousness can only be eliminated by cooperation.

We have shown in Proposition 2 that a fall in κ leads a a higher welfare in the cooperative equilibrium. We now show that, in contrast, in the case of a non-cooperative equilibrium, a fall in κ can decrease the non-copperative welfare, i.e., technological progress in resource extraction can be welfare-worsening when agents are non-cooperative. Furthermore, the absolute magnitude of the negative impact of technological progress on welfare is an increasing function of the degree of status-consciousness. The next proposition is a formalization of this result.

Proposition 7: *A technological progress in resource extraction can reduce welfare in the non-cooperative case. This fall in welfare is an increasing function of the degree of status-consciousness.*

Proof: By (18), and recall that $G(1) = 1$,

$$\widehat{\psi} = \frac{1}{1+\theta}\left[G'(1)\frac{f(\widehat{\beta})}{\widehat{\beta}} + f'(\widehat{\beta}) - \kappa(1+\theta)\right] \tag{27}$$

Thus, using (27) and (24),

$$\frac{dW^{MPE}}{d\kappa} = X_0\frac{d\widehat{\psi}}{d\kappa} = \frac{X_0}{1+\theta}\left\{\left[G'(1)\left(\frac{\widehat{\beta}f'(\widehat{\beta}) - f(\widehat{\beta})}{\widehat{\beta}^2}\right) + f''(\widehat{\beta})\right]\frac{d\widehat{\beta}}{d\kappa} - (1+\theta)\right\} \tag{28}$$

Now, since the term inside the square brackets is negative, and $\frac{d\widehat{\beta}}{d\kappa}$ is also negative, the sign of the expression inside the curly brackets is ambiguous. Let us explore the special Cobb-Douglas case.

Implicit differentiation of equation (21) shows that, if $\theta = 0$,

$$\frac{d\widehat{\beta}}{d\kappa} = \frac{-\beta^{1-\mu}\left[\rho - A + (n-1)\beta\right]}{1 - n(\lambda+\mu) + (n-1)\kappa\beta^{1-\mu} + \kappa(1-\mu)\beta^{-\mu}\left[\rho - A + (n-1)\beta\right]} < 0$$

We evaluate this derivative at $\kappa = 0$:

$$\frac{\partial\widehat{\beta}}{\partial\kappa} = \frac{-\widehat{\beta}^{1-\mu}\left[\rho - A + (n-1)\widehat{\beta}\right]}{1 - n(\lambda+\mu)} < 0$$

Now, from (21), at $\kappa = 0 = \theta$,

$$\rho - A + (n-1)\widehat{\beta} = \frac{(1-\lambda-\mu)\widehat{\beta}}{\lambda+\mu}$$

So, at $\kappa = 0$

$$\frac{\partial\widehat{\beta}}{\partial\kappa} = -\widehat{\beta}^{2-\mu}\left[\frac{(1-\lambda-\mu)}{(\lambda+\mu)\left(1 - n\left(\lambda+\mu\right)\right)}\right] \tag{29}$$

Substituting (29) into (28), we see that the effect of an increase in κ on the equilibrium welfare level is positive if and only if

$$(1-\mu)(1-\mu-\lambda) > [1-n(\mu+\lambda)](1+\theta)$$

For $\theta = 0$, this inequality is equivalent to

$$n > \frac{\mu}{\mu+\lambda} + (1-\mu)$$

Since the right-hand side is smaller than 2, it follows that the condition is satisfied if $n \geq 2$. We conclude that for the Cobb-Douglas case, with $\theta = 0$, a marginal increase in κ from a sufficiently small initial value κ_0 will increase the Markov-perfect equilibrium welfare level. The greater is λ, the greater is the magnitude of the increase in welfare, because

$$\frac{d}{d\lambda}\left[\frac{(1-\mu)(1-\mu-\lambda)}{[1-n(\mu+\lambda)]} - (1+\theta)\right] > 0$$

■.

Remark: This result represents the situation that a small increase in κ may be welfare-improving because the benefits from resouce stock preserving outweight the utility losts from less extraction and consumption (see the case in Figure 2, $\hat{\beta}$ reduces to $\hat{\beta}^{'}$ but the welfare is higher than before). However, it won't happen in the cooperative equilibrium since the cooperative equilibrium extraction rate $\overline{\beta}^{*}$ is always the welfare-maxmizing extraction rate.

4.2 Heterogeneous agents

So far we have focused the case of homogeneous players. This section examines the effects of heterogeneity among agents on the properties of Markov-perfect Nash equilibria. To simplify the analysis, we focus on the case where there are only two groups of players. More specifically, let us assume that there are $n_1 \geq 2$ players described by the parameters $(\rho_1, \theta_1, \kappa_1)$ with the utility function G_1 and f_1, and $n_2 \geq 2$ players described by the parameters $(\rho_2, \theta_2, \kappa_2)$ with the utility function G_2 and f_2. The total number of players is $n = n_1 + n_2$. We assume that assumptions A1-A3 hold for both group of players, and the agents in each group compare her consumption with other members in the same group only.

4.2.1 Analysis

Following the method used in section 4.1, we can set up the maximization problem for each group and solve the Hamiltonians. It is worth to note that the transition equations for each group are now different, i.e., for agent i in group 1:

$$\dot{X} = AX - (n_1-1)(1+\theta_1)\beta_1 X - (1+\theta_1)c_{i1} - n_2(1+\theta_2)\beta_2 X$$

For agent i in group 2:

$$\dot{X} = AX - (n_2 - 1)(1 + \theta_2)\beta_2 X - (1 + \theta_2)c_{i2} - n_1(1 + \theta_1)\beta_1 X$$

The Hamiltonians become

$$\begin{aligned} H_{i1} &= G_1\left(\frac{c_{i1}}{\beta_1 X}\right) F_1(c_{i1}, X) - \kappa_1(1 + \theta_1)c_{i1} + \\ &\quad \psi_{i1}\left[AX - (n_1 - 1)(1 + \theta_1)\beta_1 X - (1 + \theta_1)c_{i1} - n_2(1 + \theta_2)\beta_2 X\right] \end{aligned} \tag{30}$$

$$\begin{aligned} H_{i2} &= G_2\left(\frac{c_{i2}}{\beta_2 X}\right) F_2(c_{i2}, X) - \kappa_2(1 + \theta_2)c_{i2} + \\ &\quad \psi_{i2}\left[AX - (n_2 - 1)(1 + \theta_2)\beta_2 X - (1 + \theta_2)c_{i2} - n_1(1 + \theta_1)\beta_1 X\right] \end{aligned} \tag{31}$$

The optimality conditions are

$$\begin{aligned} \frac{\partial H_{i1}}{\partial c_{i1}} &= G_1'\left(\frac{c_{i1}}{\beta_1 X}\right)\left(\frac{1}{\beta_1 X}\right) F_1(c_{i1}, X) + G_1\left(\frac{c_{i1}}{\beta_1 X}\right) F_{c_i 1}(c_{i1}, X) \\ &\quad -\kappa_1(1 + \theta_1) - (1 + \theta_1)\psi_{i1} \\ &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial H_{i2}}{\partial c_{i2}} &= G_2'\left(\frac{c_{i2}}{\beta_2 X}\right)\left(\frac{1}{\beta_2 X}\right) F_2(c_{i2}, X) + G_2\left(\frac{c_{i2}}{\beta_2 X}\right) F_{c_i 2}(c_{i2}, X) \\ &\quad -\kappa_2(1 + \theta_2) - (1 + \theta_2)\psi_{i2} \\ &= 0 \end{aligned}$$

Each type of agents has the corresponding necessary conditions, for example, for n_1 type of agents:

$$\begin{aligned} \dot{\psi}_{i1} &= \psi_{i1}\left[\rho_1 - A + (n_1 - 1)(1 + \theta_1)\beta_1 + n_2(1 + \theta_2)\beta_2\right] \\ &\quad + G'\left(\frac{c_i}{\beta X}\right)\left(\frac{c_i}{\beta}\right) X^{-2} F - G F_X \end{aligned} \tag{32}$$

$$\lim_{t\to\infty} e^{-\rho t}\psi_{i1}(t) \geq 0 \text{ and } \lim_{t\to\infty} e^{-\rho t}\psi_{i1}(t)X(t) = 0 \tag{33}$$

Again we assume that there exist two symmetric linear solutions for these two groups:

$$\frac{c_{i1}(t)}{X(t)} = \beta_1, \frac{c_{i2}(t)}{X(t)} = \beta_2 \text{ where } \beta_1 \text{ and } \beta_2 \text{ are constants} \tag{34}$$

Substitution yields

$$G_1'(1)\left(\frac{1}{\beta_1}\right) f_1(\beta_1) + G_1(1) f_1'(\beta_1) - \kappa_1(1 + \theta_1) - (1 + \theta_1)\psi_{i1} = 0$$

$$G_2'(1)\left(\frac{1}{\beta_2}\right)f_2(\beta_2)+G_2(1)\,f_2'(\beta_2)-\kappa_2(1+\theta_2)-(1+\theta_2)\psi_{i2}=0$$

These two equations imply that ψ_1 and ψ_2 are also constants, i.e. $\dot{\psi}=0$ along the equilibrium path. Hence we must have

$$\left[\frac{G_1'(1)f_1(\beta_1)\frac{1}{\beta_1}+G_1(1)f_1'(\beta_1)}{1+\theta_1}-\kappa_1\right][\rho_1-A+(n_1-1)(1+\theta_1)\beta_1+n_2(1+\theta_2)\beta_2]=$$
$$-G_1'(1)\,f_1(\beta_1)+G_1(1)\left[f_1(\beta_1)-f_1'(\beta_1)\,\beta_1\right] \tag{35}$$
$$\left[\frac{G_2'(1)f_2(\beta_2)\frac{1}{\beta_2}+G_2(1)f_2'(\beta_2)}{1+\theta_2}-\kappa_2\right][\rho_2-A+(n_2-1)(1+\theta_2)\beta_2+n_1(1+\theta_1)\beta_1]=$$
$$-G_2'(1)\,f_2(\beta_2)+G_2(1)\left[f_2(\beta_2)-f_2'(\beta_2)\,\beta_2\right] \tag{36}$$

The growth rate of the public asset is therefore given by

$$g=A-n_1(1+\theta_1)\beta_1-n_2(1+\theta_2)\beta_2 \tag{37}$$

We use the previous Cobb-Douglas example to show some analytical results. The equations analog to (35) and (36) are

$$\left[\frac{\lambda_1\beta_1^{\mu_1-1}+\mu_1\beta_1^{\mu_1-1}}{1+\theta_1}-\kappa_1\right][\rho_1-A+(n_1-1)(1+\theta_1)\beta_1+n_2(1+\theta_2)\beta_2]=$$
$$-\lambda_1\beta_1^{\mu_1}+(1-\mu_1)\beta_1^{\mu_1} \tag{38}$$
$$\left[\frac{\lambda_2\beta_2^{\mu_2-1}+\mu_2\beta_2^{\mu_2-1}}{1+\theta_2}-\kappa_2\right][\rho_2-A+(n_2-1)(1+\theta_2)\beta_2+n_1(1+\theta_1)\beta_1]=$$
$$-\lambda_2\beta_2^{\mu_2}+(1-\mu_2)\beta_2^{\mu_2} \tag{39}$$

To solve the system of two equations analytically, we assume that $\kappa_1=\kappa_2=0$.There are two equations for two unknowns, the solutions are:

$$\hat{\beta}_1=\frac{1}{1+\theta_1}\frac{(\lambda_1+\mu_1)[\rho_1-n_2(\rho_1-\rho_2)(\lambda_2+\mu_2)-A]}{1-n_1(\lambda_1+\mu_1)-n_2(\lambda_2+\mu_2)} \tag{40}$$

$$\hat{\beta}_2=\frac{1}{1+\theta_2}\frac{(\lambda_2+\mu_2)[\rho_2-n_1(\rho_2-\rho_1)(\lambda_1+\mu_1)-A]}{1-n_1(\lambda_1+\mu_1)-n_2(\lambda_2+\mu_2)} \tag{41}$$

(Note that if $\theta_1=\theta_2$, $\lambda_1+\mu_1=\lambda_2+\mu_2<1/n$ and $n_1=n_2=n/2$, then $\hat{\beta}_1>\hat{\beta}_2$ if and only if $\rho_1>\rho_2$, i.e., the more impatient group extracts the resource stock at a faster rate.)

Since this model is featured by relative consumption appearing in the agents' utility function, we are especially interested in the effect of heterogeneity in the status-conscious parameter λ on the equilibrium outcome. For example, if we assume there is a mean-preserving spread of λ among agents, i.e., $\lambda_1 = \lambda + \frac{\eta}{n_1}$, $\lambda_2 = \lambda - \frac{\eta}{n_2}$ with $\eta > 0$, how are the growth rate of public assets and welfare affected by an increase in η? The following proposition explains this effect.

Proposition 8 *In the Cobb-Douglas case,*

(a) A mean-preserving spread in the distribution of the status-conscious parameter λ leads to an increase of the public asset growth rate iff $\rho_2 > \rho_1$, i.e., iff the members of the group with stronger status-consciousness are more patient.

(b) If the status-conscious parameter λ is the only source of heterogeneity, a mean-preserving spread in the distribution of λ across agents leads to an decrease of the social welfare.

Proof:

(a) Substitute $\hat{\beta}_1$ and $\hat{\beta}_2$ into (37) and take derivative with respect to η will yield

$$\frac{\partial g}{\partial \eta} = \frac{\rho_2 - \rho_1}{1 - n_1(\lambda_1 + \mu_1) - n_2(\lambda_2 + \mu_2)}$$

by definition, $1 - n_1(\lambda_1 + \mu_1) - n_2(\lambda_2 + \mu_2) > 0$, therefore $\frac{\partial g}{\partial \eta} > 0$ iff $\rho_2 > \rho_1$.

(b) The social welfare is the total sum of individual welfare and is given by

$$SW = n_1 W_1 + n_2 W_2 = \frac{n_1 \beta_1^{\mu_1} X_0}{\rho_1 - g} + \frac{n_2 \beta_2^{\mu_2} X_0}{\rho_2 - g}$$

If $\lambda_1 = \lambda + \frac{\eta}{n_1}$, $\lambda_2 = \lambda - \frac{\eta}{n_2}$ and all other parameters are equal across two groups, we have

$$\begin{aligned} \frac{\partial SW}{\partial \eta} &= 0 \Rightarrow \left(\frac{n\lambda - \eta + n\mu}{n(1+\theta)}\right)^{\mu-1} = \left(\frac{\eta + n\lambda + n\mu}{n(1+\theta)}\right)^{\mu-1} \\ &\Rightarrow \eta^* = 0 \end{aligned}$$

$$\frac{\partial^2 SW}{\partial \eta^2} < 0 \text{ at } \eta^* = 0$$

■.

The above proposition shows that if λ differs across the two groups, the social welfare will be lower than the case of homogeneous agents. If the policy maker observes this and looks for some policy to improve this situation, the government could impose two different costs θ_1 and θ_2 to each group. In fact, this policy can achieve a second-best outcome and it will not affect the public asset growth. The next section illustrates this and the proof is given in the Appendix (See Appendix, Proposition A.1).

4.2.2 Simulation results: the joint effects of λ and θ on social welfare

In this section the joint effects of λ and θ on social welfare are given by simulation. Again, suppose $\theta_1 = \theta + \frac{\varepsilon}{n_1}$, $\theta_2 = \theta - \frac{\varepsilon}{n_2}$ and $\lambda_1 = \lambda + \frac{\eta}{n_1}$, $\lambda_2 = \lambda - \frac{\eta}{n_2}$. Substituting them into the social welfare function in 4.2, we can express social welfare as a function of ε and η. The plot of social welfare is given in Fig. 3 (assuming $X_0 = 1$, $\rho = 0.2$, $A = 0.1$, $\lambda = 0.2$, $n_1 = 10$, $n_2 = 10$, $\mu = 0.2$, $\theta = 0.1$).

The saddle-shape diagram allows us to confirm our findings in 4.2 that a mean-preserving spread in the distribution of λ across agents leads to an decrease of the social welfare, while a mean-preserving spread in the appropriation cost θ will increase the social welfare, ceteris paribus. Therefore, if the agents are different in the degree of status consciousness, which reduces the social welfare, the policy maker can apply two tax rates to these agents and can still achieve a second-best outcome.

5 Concluding remarks

This paper explores the role of status-consciousness in rent-seeking in a dynamic setting. The agents in the economy are concerned with not only their absolute level of consumption, but also the relative consumption level within their groups. In the cooperative equilibrium, or equivalently the social planner's problem, the outcome is not affected by the concern for relative consumption. If agents behave non-cooperatively, we show that the status-consciousness parameter λ indeed plays an important role in the model. A higher degree of λ leads to more aggressive extraction efforts, therefore the social welfare and the growth rate of the public resource are lower. This effect has not been explored in the previous literature on rent-seeking models. We have therefore shown that "positional externalities" worsen the "tragedy of the commons" problem.

Another feature of our model is that we introduce two types of cost within the rent-seeking process, a "wastage-cost" θ and an "effort-cost" κ. In contrast with Long and Sorger (2006), we show that an increase in κ will reduce the equilibrium rate of extraction and increase the growth rate of the public asset. Thus if the policy maker's primary objective is to protect the public asset from over-extraction, imposing a higher effort-cost (stricter policing of money-laundering) is preferred. We also show that a technological progress, i.e., a smaller κ, can worsen welfare in a rent-seeking equilibrium. The magnitude of this welfare-worsening effect is an increasing function of the degree of status-consciousness. In the analysis for heterogeneous agents, we show that the heterogeneity in the status-conscious parameter λ will reduce social welfare. However, if the agents are different in both θ and λ, we show that positional externalities caused by λ can be mitigated by different wastage-costs, which can be achieved by discriminational tax rates.

There are several ways our model can be extended. First, one may suppose there exist some external limits for the extraction of the public asset. Thus the agents will optimize their extraction in a constrained problem. Second, with the use of a Cobb-Douglas utility function, one can derive all results in closed form and obtain linear or log-linear equations that are readily adaptable for empirical tests. These extensions are parts of our future research plans.

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APPENDIX

The effect of heterogeneity in θ on the public asset growth and welfare.

Proposition A.1: *In the Cobb-Douglas case*

(a) The growth rate of the public asset is not related to the production costs, θ_1, θ_2.

(b) If the appropriation cost θ is the only source of heterogeneity, a mean-preserving spread in the distribution of this cost across agents leads to an increase of the social welfare.

Proof:

(a) Denote

$$B_1 = \frac{(\lambda_1+\mu_1)\left(\rho_1 - n_2\left(\rho_1-\rho_2\right)\left(\lambda_2+\mu_2\right) - A\right)}{1-n_1\left(\lambda_1+\mu_1\right)-n_2\left(\lambda_2+\mu_2\right)}$$

$$B_2 = \frac{(\lambda_2+\mu_2)\left(\rho_2 - n_1\left(\rho_2-\rho_1\right)\left(\lambda_1+\mu_1\right) - A\right)}{1-n_1\left(\lambda_1+\mu_1\right)-n_2\left(\lambda_2+\mu_2\right)}$$

Substitution yields

$$g = A - n_1(1+\theta_1)\hat{\beta}_1 - n_2(1+\theta_2)\hat{\beta}_2 = A - n_1 B_1 - n_2 B_2$$

where it is clear that g is not affected by θ_1 and θ_2.

(b) Let's consider the social welfare under heterogeneity,

$$SW = n_1 W_1 + n_2 W_2 = \frac{n_1 \beta_1^{\mu_1} X_0}{\rho_1 - g} + \frac{n_2 \beta_2^{\mu_2} X_0}{\rho_2 - g}$$

Suppose $\theta_1 = \theta + \frac{\varepsilon}{n_1}$, $\theta_2 = \theta - \frac{\varepsilon}{n_2}$,

Let's assume that $\mu_1 = \mu_2 = \mu$ and denote $f(\varepsilon) = (\rho_2 - g)n_1\beta_1^{\mu} + (\rho_1 - g)n_2\beta_2^{\mu}$,

We have,

$$f'(\varepsilon) = -\frac{(\rho_2 - g)\mu\beta_1^{\mu-1}}{(1+\theta_1)^2}B_1 + \frac{(\rho_1 - g)\mu\beta_2^{\mu-1}}{(1+\theta_2)^2}B_2 = 0$$

$$\Rightarrow \varepsilon^* = \frac{(\theta+1)\left(1-C\right)}{\frac{1}{n_1}C + \frac{1}{n_2}}$$

Where

$$C = \left(\frac{B_2^{\mu}}{B_1^{\mu}}\frac{g-\rho_1}{g-\rho_2}\right)^{\frac{1}{\mu+1}}$$

and

$$f''(\varepsilon^*) = \frac{(\mu+1)}{n_1 n_2}\left(\frac{(\rho_1 - g)B_2^{\mu}n_1}{(1+\theta_2)^{\mu+2}} + \frac{(\rho_2 - g)B_1^{\mu}n_2}{(1+\theta_1)^{\mu+2}}\right) > 0$$

If $\rho_1 = \rho_2, \lambda_1 = \lambda_2$

$$\varepsilon^* = \frac{(\theta+1)\left(1-1\right)}{\frac{1}{n_1} + \frac{1}{n_2}} = 0$$

■.

Figure 1: The determination of the equilibrium extraction rate

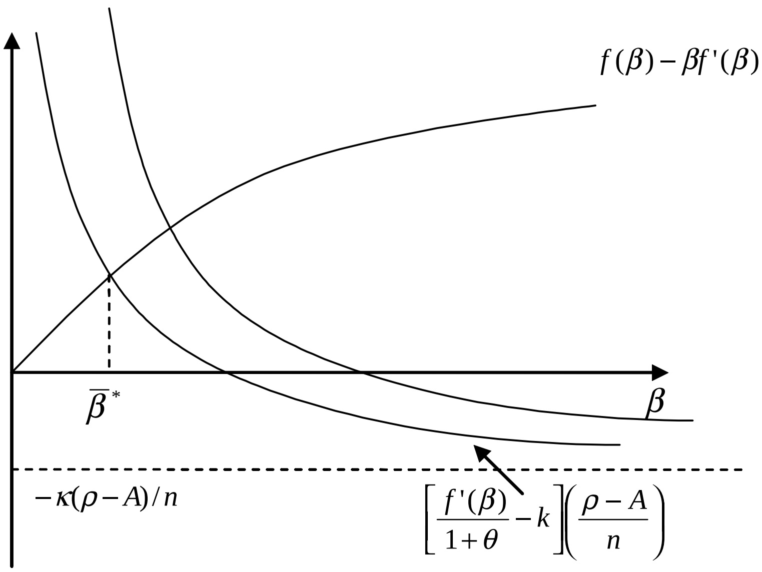

Figure 2: The effect of an increase in k on welfare and extraction rates

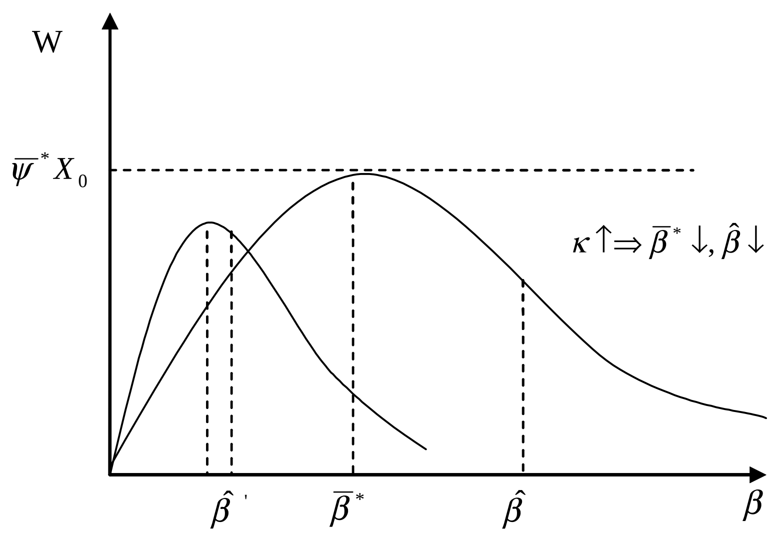

Figure 3: The joint effect of heterogeneity in λ and θ on social welfare