COMPARISON OF INTERPOLATION ALGORITHMS IN NETWORK-BASED GPS TECHNIQUES

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ABSTRACT

High precision GPS surveying and navigation applications have been constrained to the short-range case due to the presence of distance-dependent biases in the between-receiver single-differenced observables. Over the past few years, the use of a GPS reference station network approach, to extend the inter-receiver distances (user-to-reference receiver separation), has shown great promise. In order to account for the distance-dependent residual biases, such as the atmospheric biases and orbit errors, several techniques have been developed. They include the Linear Combination Model, Distance-Based Linear Interpolation Method, Linear Interpolation Method, Lower-Order Surface Model, and Least Squares Collocation. All of these methods aim to model (or interpolate) the distance-dependent biases between the reference station(s) and the user receiver with the support of a reference station network.

In this paper the interpolation methods associated with these techniques are compared in detail, and the advantages and disadvantages of each are discussed. On an epoch-by-epoch and satellite-by-satellite basis, all of the abovementioned methods use a n-1 independent error vector generated from a n reference station network to model the distance-dependent biases at the user station. General formulas for all of the methods involve the computation of the n-1 coefficients first, and then the formation of a n-1 linear combination with a n-1 error vector from the reference stations to mitigate the spatially correlated errors for the user station(s). Test data from GPS (and Glonass) reference stations was used to evaluate the performance of the interpolation methods. The numerical results show that all of the proposed implementations of the multiple reference station approach can significantly reduce the distance-dependent biases associated with carrier phase and pseudo-range measurements at the GPS user station. The performance of all the methods is similar.

INTRODUCTION

High precision GPS surveying and navigation techniques have been constrained to 'short-range' due to the presence of distance-dependent biases in the between-receiver single-differenced observables. Over the past few years the concept of using reference station networks for kinematic GPS positioning (including in real-time) has been promoted strongly by several investigator groups. The basic idea is that, with the pre-determined coordinates of reference stations and fixed GPS carrier phase ambiguities, the so-called 'correction terms' for the atmospheric biases and orbit errors can be generated to support 'medium-range' carrier phase-based positioning. See, for example, Gao et al. (1997), Han & Rizos (1996); Raquet (1997); Wanninger (1995, 1997); Wübbena et al. (1996). A detailed review and comparison of the various multi-reference receiver approaches can be found in Fotopoulos & Cannon (2001) and Dai et al. (2001b).

After the double-differenced ambiguities associated with the reference station receivers have been fixed to their correct values (for more details concerning this issue see, e.g., Gao et al., 1997; Colombo et al., 1999; Chen, 2000; Dai et al., 2001a), the double-differenced GPS/Glonass residuals can be generated. The spatially correlated errors to be interpolated could be the pseudo-range and carrier phase residuals for the L1 and/or L2 frequencies, or other linear combinations.

One core issue for multi-reference receiver techniques is how to interpolate the distance-dependent biases generated from the reference station network for the user's location? Over the past few years, in order to interpolate (or model) the distance-dependent residual biases, several interpolation methods have been proposed. They include the Linear Combination Model (Han & Rizos, 1996; 1998), the Distance-Based Linear Interpolation Method (Gao et al., 1997; 1998), the Linear Interpolation Method (Wanniger, 1995; Wübbena et al., 1996), the Low-Order Surface Model (Wübbena et al., 1996; Fotopoulos & Cannon, 2000), and the Least Squares Collocation Method (Raquet, 1997; Marel, 1998). (It should be emphasised that the Virtual Reference Station (VRS) technique promoted by the Trimble GPS Company is merely an implementation of the multiple-reference receiver approach, and all of the aforementioned interpolation methods can be applied.)

In this paper, the aforementioned interpolation methods are compared in detail, and the advantages and disadvantages of each of these techniques are discussed. An underlying common formula for all of the interpolation methods has been identified, and their performance will be demonstrated through case study examples of GPS (and Glonass) reference station networks.

INTERPOLATION METHODS

Linear Combination Model (LCM)

A linear combination of single-differenced observations was proposed by Han & Rizos (1996, 1998) to model the spatially correlated biases (i.e. orbit bias $\Delta\rho_{orb,i}$, residual ionospheric bias $\Delta d_{ion,i}$ and residual tropospheric bias $\Delta d_{trop,i}$), and to mitigate multipath $\Delta d^{\phi}_{mp,i}$ and noise $\varepsilon_{\sum_{i=1}^{n}\alpha_i\cdot\Delta\phi_i}$:

$$\begin{aligned}\sum_{i=1}^{n}\alpha_i\cdot\Delta\phi_i &= \sum_{i=1}^{n}\alpha_i\cdot\Delta\rho_i+\sum_{i=1}^{n}\alpha_i\cdot\Delta d\rho_i - c\cdot\sum_{i=1}^{n}\alpha_i\cdot\Delta dT_i+\lambda\cdot\sum_{i=1}^{n}\alpha_i\cdot\Delta N_i-\sum_{i=1}^{n}\alpha_i\cdot\Delta d_{ion,i}+\sum_{i=1}^{n}\alpha_i\cdot\Delta d_{trop,i}\\ &+\sum_{i=1}^{n}\alpha_i\cdot\Delta d^{\phi}_{mp,i}+\varepsilon_{\sum_{i=1}^{n}\alpha_i\cdot\Delta\phi_i}\end{aligned} \tag{1}$$

where n is the number of reference stations in the network, i indicates the ith reference station, and u the user station. A set of parameters α_i is estimated, satisfying the following conditions:

$$\sum_{i=1}^{n}\alpha_i = 1 \tag{2}$$

$$\sum_{i=1}^{n}\alpha_i(\hat{X}_u-\hat{X}_i)=0 \tag{3}$$

$$\sum_{i=1}^{n}\alpha_i^2 = Min \tag{4}$$

where $\hat{X}_u$ and $\hat{X}_i$ are horizontal coordinate vectors for the user station and the ith reference station respectively.

Based on Equations (1)-(4), the impact of orbit errors can be eliminated, and ionospheric biases, tropospheric biases, multipath and measurement noise can be significantly mitigated. As a result, the double-differenced observables can be formed after ambiguities in the reference station network have been fixed to their correct integer values:

$$\nabla\Delta\phi_{u,n}-[\alpha_1\cdot V_{1,n}+...+\alpha_i\cdot V_{i,n}+...+\alpha_{n-1}\cdot V_{n-1,n}=\nabla\Delta\rho_{u,n}+\lambda\nabla\Delta N_{u,n}+\varepsilon_{\sum_{i=1}^{n}\alpha_i\nabla\Delta\phi_i} \tag{5}$$

where $V_{i,n}$ (referred to her as the 'correction terms') is the residual vector generated from the double-differenced measurements between reference stations n and i:

$$V_{i,n}=\nabla\Delta\phi_{i,n}-\nabla\Delta\rho_{i,n}-\lambda\nabla\Delta N_{i,n}\ (\mathrm{i}=1,...,\mathrm{n}-1) \tag{6}$$

Distance-Based Linear Interpolation Method (DIM)

A distance-based linear interpolation algorithm for ionospheric correction estimation has been suggested by Gao et al. (1997), described by the following equations:

$$\nabla\Delta\hat{I}_u=\sum_{j=1}^{n-1}\frac{w_i}{w}\nabla\Delta\hat{I}_i \tag{7}$$

$$w_i=\frac{1}{d_i} \tag{8}$$

$$w=\sum_{j=1}^{n-1}w_i \tag{9}$$

where n is the number of reference stations in the network, and d_i is the distance between the ith reference station and the user station. $\nabla\Delta\hat{I}_i$ is the double-differenced ionospheric delay at the ith reference station.

In order to improve interpolation accuracy, two modifications were made by Gao & Li (1998). The first modification is to replace the ground distance with a distance defined on a single-layer ionospheric shell at an altitude of 350km. The second modification is to extend the model to take into account the spatial correction with respect to the elevation angle of the ionospheric delay paths on the ionospheric shell.

Linear Interpolation Method (LIM)

Wanninger (1995) first suggested a regional differential ionospheric model derived from dual-frequency phase data from at least three GPS monitor stations surrounding the user station. Unambiguous double-differenced ionospheric biases can be obtained on a satellite-by-satellite and epoch-by-epoch basis after ambiguities in the reference station network have been fixed to their correct integer values. Ionospheric corrections for any station in the area can be interpolated by using the known coordinates of the reference stations and approximate coordinates of the station(s) of interest. Wübbena et al. (1996) extended this method to model the distance-dependent biases such as the residual ionospheric and tropospheric biases, and the orbit errors. Similar methods have been proposed by Wanninger (1999), Schaer (1999), Chen et al. (2000), Vollath et al. (2000), and others.

For a network with three or more stations, the linear model can be described by:

$$\begin{bmatrix} V_{1n} \\ V_{2n} \\ \vdots \\ V_{n-1,n} \end{bmatrix} = \begin{bmatrix} \Delta X_{1n} & \Delta Y_{1n} \\ \Delta X_{2n} & \Delta Y_{2n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} \tag{10}$$

where ΔX and ΔY are the plane coordinate differences referred to the master reference station. Parameters a and b are the coefficients for ΔX and ΔY (the so-called 'network coefficients' according to Wübbena et al., 1996). In the case of more than three reference stations, the coefficients a and b can be estimated by a Least Squares adjustment on an epoch-by-epoch, satellite-by-satellite basis. Then the GPS user within the coverage of the network can apply the following 2D linear model to interpolate the distance-dependent biases:

$$V_{un} = a \cdot \Delta X_{un} + b.\Delta Y_{un} \tag{11}$$

Low-Order Surface Model (LSM)

The distance-dependent biases exhibit a high degree of spatial correlation across a reference station network. Low-order surfaces can be used to 'fit' the distance-dependent biases (Wübbena et al., 1996; Fotopoulos, 2000). The fitted surfaces are known as trend or regression surfaces, and they model the major trend of the distance-dependent biases. The coefficients of the low-order surfaces can be estimated via a Least Squares adjustment using data from the reference station network. The variables of the fitting function could be two (i.e. the horizontal coordinates), or three (horizontal coordinates and height). The fitting orders could be one, two or higher. Some fitting functions are:

$$V = a \cdot \Delta X + b.\Delta Y + c \tag{12}$$

$$V = a \cdot \Delta X + b.\Delta Y + c \cdot \Delta X^2 + d.\Delta Y^2 + e \cdot \Delta X \Delta Y + f \tag{13}$$

$$V = a \cdot \Delta X + b.\Delta Y + c \cdot \Delta H + d \tag{14}$$

$$V = a \cdot \Delta X + b.\Delta Y + c \cdot \Delta H + d.\Delta H^2 + e \tag{15}$$

Schaer et al. (1999) have proposed that Equation (12) be used to model residual ionospheric refraction on a satellite-by-satellite and epoch-by-epoch basis after double-differencing, and that Equation (14) could be used to estimate the tropospheric zenith delay. Equations (14) and (15) can be derived by applying partial derivative principles (Varner & Cannon, 1997; Varner, 2000). After the fitted coefficients are computed, they can be used to predict the biases for the user station(s).

Least Squares Collocation (LSC)

Least Squares Collocation has been used for many years to interpolate gravity at any given location using only measurements at some discrete locations (e.g., Tscherning, 1974; 2001; Schwartz, 1978). The following is the basic interpolation equation:

$$\hat{U} = C_{vu} \cdot C_v^{-1} \cdot V \tag{16}$$

where C_v is the covariance matrix of the measurement vector V, and C_{uv} is the cross-covariance matrix between the interpolated vector $\hat{U}$ and the measurements vector V. If these covariance matrices are computed correctly, and the measurements satisfy the conditions of zero mean and a normal distribution, Equation (16) gives the optimal estimator (Raquet & Lachapelle, 2001). Least Squares Collocation is also well suited to interpolating the distance-dependent biases in a network. Raquet (1997) proposed the NetAdjust method, which in essence is equivalent to Least Squares Collocation.

The challenge for this method is to calculate the covariance matrices C_v and C_{uv}. The following covariance function was proposed (Raquet, 1998):

$$C_{ab}^{x} = \mu^2(\varepsilon)\cdot\left[\delta_{c_z}^2(P_a,P_0)+\delta_{c_z}^2(P_b,P_0)-\delta_{c_z}^2(P_a,P_b)\right] \tag{17}$$

where the computation of the double-differenced covariance matrices can be decomposed into two mathematical functions. First, a correlated variance function which maps the zenith variance of the correlated errors over the network area is computed:

$$\delta_{c_z}^2(P_n,P_m) = k_1 d + k_2 d^2 \tag{18}$$

where $\delta_{c_z}^2(P_n,P_m)$ is the differential zenith variance of the correlated errors for points p_n and p_m in the network. This function is based on the two-dimensional distance d between the reference stations. k_1 and k_2 are constant coefficients (k_1 = 1.1204e-4 and k_2 = 4.8766e-7 for L1 phase in their paper). Secondly, a mapping function is needed to map the zenith correlated and uncorrelated errors to the elevation of the satellite at each epoch:

$$\mu(\varepsilon) = \frac{1}{\sin\varepsilon} + \mu_k(.53-\frac{\varepsilon}{180})^3 \tag{19}$$

where $\mu(\varepsilon)$ is a dimensionless scale factor which, when multiplied by the zenith variance obtained from Equation (18), gives the correlated variance for the specified satellite elevation ε, and μ_k is a constant coefficient (μ_k = 3.9393 for L1 phase in their paper). Tests have shown that the estimated corrections are not sensitive to the choice of the covariance function. However, estimated variances are sensitive to the covariance function used (Raquet & Lachapelle, 2001).

Based on the principles of Least Squares Collocation, a practical interpolator for ionospheric biases (or tropospheric biases) is (Marel, 1998; Odijk et al., 2000):

$$I_{1u}^{1s} = \begin{bmatrix} C_{u1}^s & C_{u2}^s & \cdots & C_{un}^s \end{bmatrix} \cdot \begin{bmatrix} C_0 & C_{12}^s & \cdots & C_{1n}^s \\ C_{21}^s & C_0 & \cdots & C_{2n}^s \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1}^s & C_{n1}^s & \cdots & C_0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \hat{I}_{12}^{1s} \\ \hat{I}_{13}^{1s} \\ \vdots \\ \hat{I}_{1n}^{1s} = 0 \end{bmatrix} \tag{20}$$

The spatial covariance function C_{kl}^s is linearly dependent on the distance between the stations, or rather, the distance between their ionospheric pierce points:

$$C_{kl}^s = l_{\max} - l_{ks}^s \tag{21}$$

In this covariance function C_{kl}^s is the distance between the ionospheric points of stations k and l with respect to satellite s, with $l_{\max} > l_{kl}^s$, where $l_{\max}$ (300km was used in their paper) is a distance which is larger than the longest distance between the ionospheric points of the stations in the network. Therefore, the larger the distance between the points, the smaller the correlation.

COMPARISON OF INTERPOLATION METHODS

General Formula

On an epoch-by-epoch and satellite-by-satellite basis all of the abovementioned methods use a n-1 independent error vector generated from a n reference station network to interpolate (or estimate) the distance-dependent biases for the user station location. One significant characteristic shared by all of the methods is that it is necessary to first compute the n-1 coefficients, and then to form a n-1 linear combination with the n-1 error vector generated by the reference station network:

$$\hat{V}_u = \vec{\alpha} \cdot \vec{V} = \alpha_1 V_{1n} + \alpha_2 V_{2n} + \cdots + \alpha_{n-1} V_{n-1,n} \tag{22}$$

It should be emphasised that all the coefficients can be calculated without using any actual measurements, and are constant if the user receiver is not in motion. The coefficients depend on the geometry between the user station and the reference station network (and the GPS satellite geometry). They refer to one master reference station and one reference satellite.

The formulas for the determination of the coefficients, and a discussion of the advantages and disadvantages of each interpolation method, are presented below.

Coefficient Determination

Linear Combination Model

Equations (2) and (3) can be re-written as:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ \Delta X_{1n} & \Delta X_{2n} & \cdots & \Delta Y_{n-1n} & 0 \\ \Delta Y_{1n} & \Delta Y_{2n} & \cdots & \Delta Y_{n-1n} & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} 1 \\ \Delta X_{un} \\ \Delta Y_{un} \end{bmatrix} \tag{23}$$

If three or more reference stations are used, the n coefficient vector α can be determined using the Least Squares condition adjustment based on Equation (4):

$$\vec{\alpha} = B^T (BB^T)^{-1} W \tag{24}$$

$$\text{where } B = \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ \Delta X_{1n} & \Delta X_{2n} & \cdots & \Delta Y_{n-1n} & 0 \\ \Delta Y_{1n} & \Delta Y_{2n} & \cdots & \Delta Y_{n-1n} & 0 \end{bmatrix}, \vec{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}, W = \begin{bmatrix} 1 \\ \Delta X_{un} \\ \Delta Y_{un} \end{bmatrix} \tag{25}$$

In this method, although a total of n coefficients can be derived from Equation (24), only n-1 coefficients are used to interpolate the distance-dependent biases. Coefficient α_n is related to the master reference station.

The Linear Combination Model is formed from the single-differenced functional equation for baselines from the user receiver to two or more reference stations. The advantage of this model is the elimination of the orbit bias. The residual ionospheric delay and the tropospheric delay can also be reduced to the same degree that the epoch-by-epoch and satellite-by-satellite ionosphere and the troposphere models are able to. Multipath and measurement noises can be reduced if the user receiver is located within the network of reference stations, so that the coefficients are less than one. Otherwise the multipath and noise may be amplified (because the coefficients might be larger than one).

Distance-Based Linear Interpolation Method

From Equation (7), it can be seen that the n-1 coefficients can be determined as follows:

$$\vec{\alpha} = \begin{bmatrix} \frac{w_1}{w} & \frac{w_2}{w} & \cdots & \frac{w_{n-1}}{w} \end{bmatrix} \tag{26}$$

In this method it should emphasised that the coefficients always are less than one, even if the user receiver is located outside the network of reference stations. Although this method was originally proposed by Gao et al. (1997) to interpolate residual ionospheric biases, it can also, to a certain degree, mitigate other distance-dependent biases such as tropospheric bias and orbit errors.

Linear Interpolation Method

If three or more reference stations are available, the parameters $\hat{a}$ and $\hat{b}$ can be estimated using Least Squares based on Equation (10):

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (A^T A)^{-1} A^T V \tag{27}$$

$$\text{where } V = \begin{bmatrix} V_{1n} \\ V_{2n} \\ \vdots \\ V_{n-1,n} \end{bmatrix}, \text{A} = \begin{bmatrix} \Delta X_{1n} & \Delta Y_{1n} \\ \Delta X_{2n} & \Delta Y_{2n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{bmatrix} \tag{28}$$

After the parameters $\hat{a}$ and $\hat{b}$ have been estimated, the biases at the user location within the coverage of the network can be interpolated using Equation (11):

$$\hat{V}_{1u} = \left[\Delta X_{un} \ \Delta Y_{un}\right] \cdot \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left[\Delta X_{un} \ \Delta Y_{un}\right] \cdot (A^T A)^{-1} A^T V \tag{29}$$

From Equation (29) it can be seen that the n-1 coefficient vector α can be written as:

$$\vec{\alpha} = \left[\Delta X_{un} \ \Delta Y_{un}\right] \cdot (A^T A)^{-1} A^T \tag{30}$$

The coefficients can also be derived using the satellite-by-satellite, epoch-by-epoch ionospheric model, to reduce residual ionosphere and troposphere delay. It can be proven that if only three reference station are used, the coefficients α_1 and α_2 are exactly the same for the Linear Combination Model as for the Linear Interpolation Method. However, they are different when the number of reference stations is greater than 3 (see Experiments) because the Linear Combination Model eliminates the orbit bias as well. The advantage of this method for real-time implementation is that the implementation is easier because only two coefficients for each satellite pair are required for transmission to the user.

Low-Order Surface Model

The different variables and orders of the fitting surfaces result in a different n-1 coefficient vector α. However, the computation procedure is the same. Here, an example of a plane-fitting function will be used.

If four or more reference stations are available, the parameters $\hat{a}$, $\hat{b}$ and $\hat{c}$ can be estimated using Least Squares based on Equation (12):

$$\begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{bmatrix} = (A^T A)^{-1} A^T V \tag{31}$$

$$\text{where } V = \begin{bmatrix} V_{1n} \\ V_{2n} \\ \vdots \\ V_{n-1,n} \end{bmatrix}, \text{A} = \begin{bmatrix} \Delta X_{1n} & \Delta Y_{1n} & 1 \\ \Delta X_{2n} & \Delta Y_{2n} & 1 \\ \vdots & \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} & 1 \end{bmatrix} \tag{32}$$

After the parameters $\hat{a}$, $\hat{b}$ and $\hat{c}$ have been estimated, the biases at the user location within the coverage of the network can be interpolated using Equation (12):

$$\hat{V}_{1u} = \begin{bmatrix}\Delta X_{un} & \Delta Y_{un} & 1\end{bmatrix} \cdot \begin{bmatrix}\hat{a} \\ \hat{b} \\ \hat{c}\end{bmatrix} = \begin{bmatrix}\Delta X_{un} & \Delta Y_{un} & 1\end{bmatrix} \cdot (A^T A)^{-1} A^T V \tag{33}$$

From Equation (33) it can be seen that the n-1 coefficient vector α can be written as:

$$\vec{\alpha} = \begin{bmatrix}\Delta X_{un} \; \Delta Y_{un} \; 1\end{bmatrix} \cdot (A^T A)^{-1} A^T \tag{34}$$

For a Low-Order Surface Model the required number of reference stations depends on the fitting variable and the fitting order. In general, the minimum number of reference stations is four if the plane-fitting function is used. It is obvious that the Linear Interpolation Method is a special case of the plane-fitting function.

Least Squares Collocation

For the Least Squares Collocation Method the n-1 coefficients can be determined using Equation (16):

$$\vec{\alpha} = C_{uv} \cdot C_v^{-1} \tag{35}$$

The n coefficients in the interpolator suggested by Marel (1998) can be determined from:

$$\vec{\alpha} = \begin{bmatrix}C_{u1}^s \; C_{u2}^s \cdots C_{un}^s\end{bmatrix} \cdot \begin{bmatrix} C_0 & C_{12}^s & C_{1n}^s \\ C_{21}^s & C_0 & C_{2n}^s \\ \vdots & \vdots & \vdots \\ C_{n1}^s & C_{n2}^s & C_{n1}^s \end{bmatrix}^{-1} \tag{36}$$

It should be emphasised that although there are n coefficients in this interpolator, only the first n-1 coefficients are used for interpolation because the nth coefficient is related to the reference satellite and a zero error value has been assigned to this satellite.

This method explicitly attempts to minimise the differenced phase-code biases between any reference station receiver and the user receiver. Note that the accuracy of the Least Squares Collocation Method is dependent upon the accuracy of the covariance matrix (Raquet, 1998). In practice it is very difficult to calculate precise covariance matrices.

Coefficient Comparison in a Simulated Multiple-Reference Station Network

From the previous discussion it can be seen that all the methods use n-1 coefficients to form a linear combination with the ‘correction terms’ to mitigate spatially correlated biases at user stations. In fact the coefficients can be considered as weighting for the ‘correction terms’. Therefore, the major differences between all the methods are only the coefficients. In order to further analyse the coefficient differences for the different interpolation methods, a simulation study has been carried out. Figure 1 shows the configuration of the reference station network used in the simulation. ‘Ref. 1’-‘Ref. 7’ and ‘Master Ref.’ indicate the seven reference stations and one master reference station respectively.

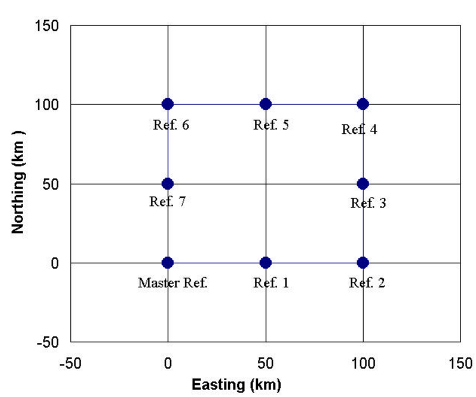

Figure 1. Configuration of the simulated reference station network

Figures 2a to 2g show the distribution of all the coefficients for the user location within (100km x 100km) and outside (50km) the reference station network, using the seven different interpolation methods respectively. Figures 2d and 2e refer to the Low-Order Surface Model using the Equations (12) and (13) respectively. It can be seen from Figures 2a, 2c and 2d that for the Linear Combination Model, the Linear Interpolation Method, and the 1st Order Surface Model, each coefficient distribution lies in one plane whose form is defined by the reference station coordinates. This can be proven using Equations (24), (30) and (34) respectively. Figures 2a and 2c also show that the corresponding coefficients (α_1 to α_7) are quite similar. Therefore, the performance of the two methods should be similar too. Figure 2e shows that each coefficient form is a 2nd order surface defined by the reference station coordinates. Figures 2b, 2f and 2g show that the closer to the reference station the user location is, the larger (up to 1) the corresponding coefficient. It is interesting that every coefficient trend is almost the same for the Least Squares Collocation methods suggested by Raquet (1998) and Marel (1998), even though their derived formulas are quite different.

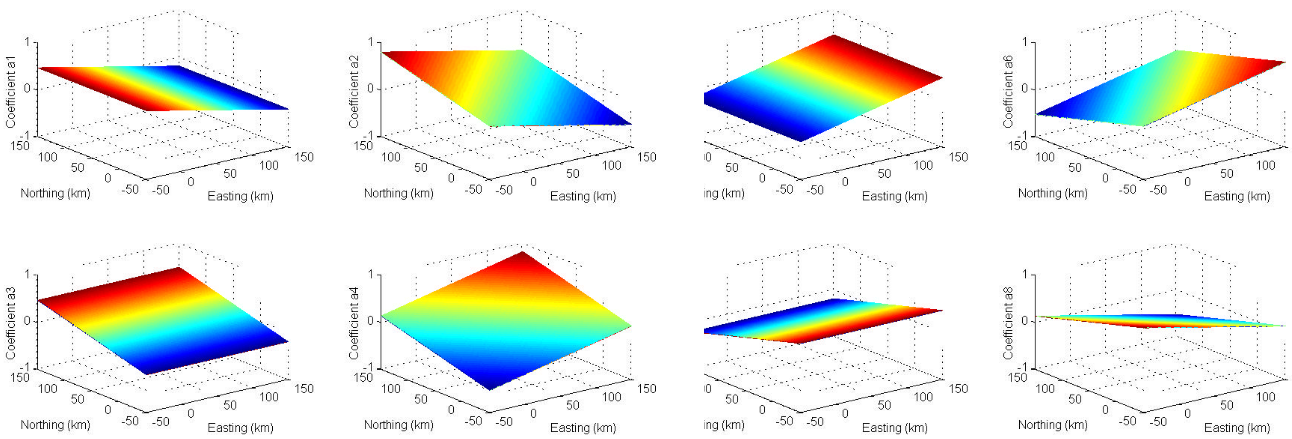

Figure 2a. Coefficients generated by the Linear Combination Model

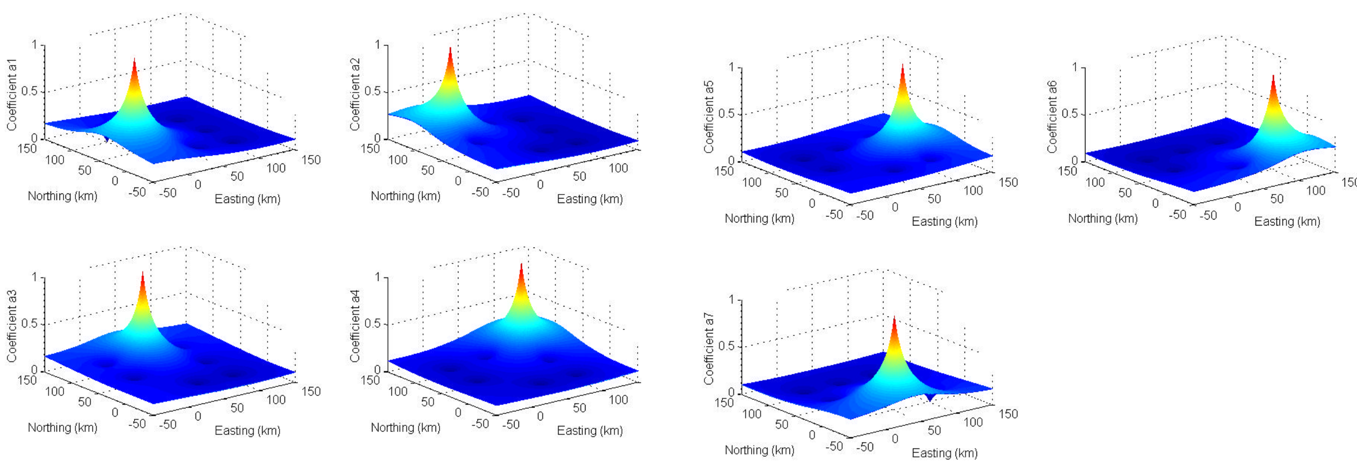

Figure 2b. Coefficients generated by the Distance-Based Linear Interpolation Method

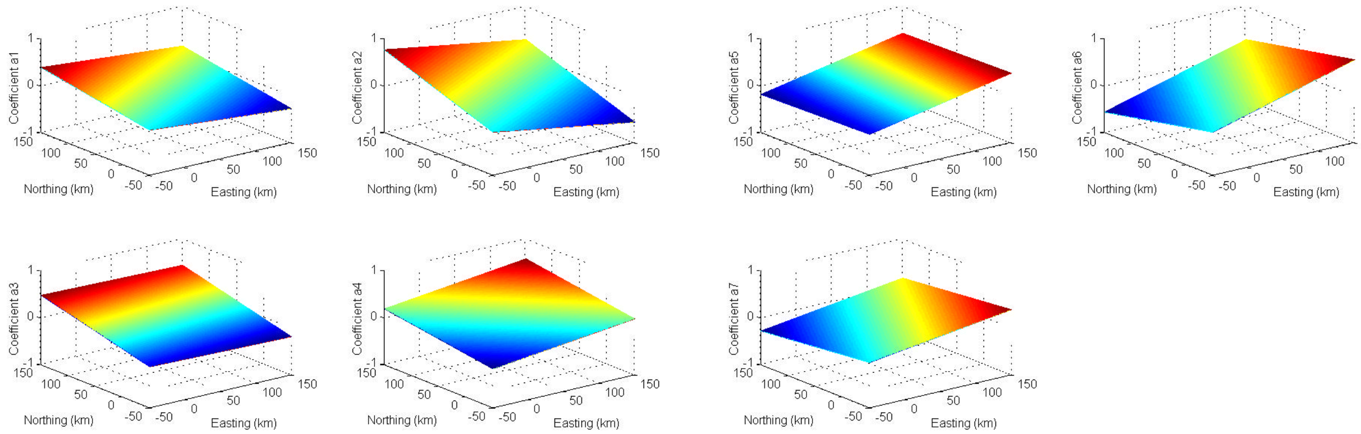

Figure 2c. Coefficients generated by the Linear Interpolation Method

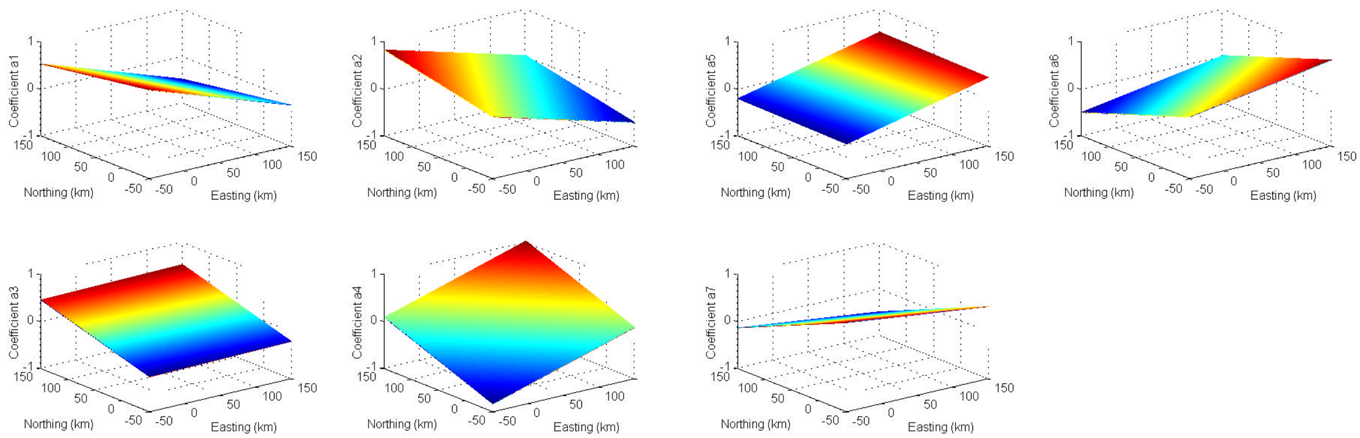

Figure 2d. Coefficients generated by the Low-Order Surface Model (1st order)

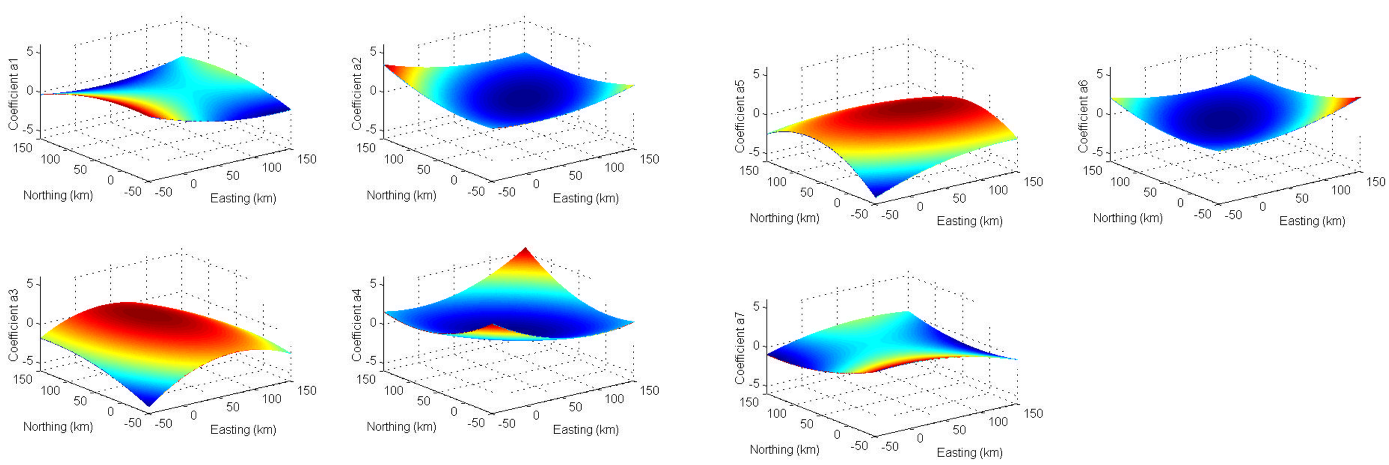

Figure 2e. Coefficients generated by the Low-Order Surface Model (2nd order)

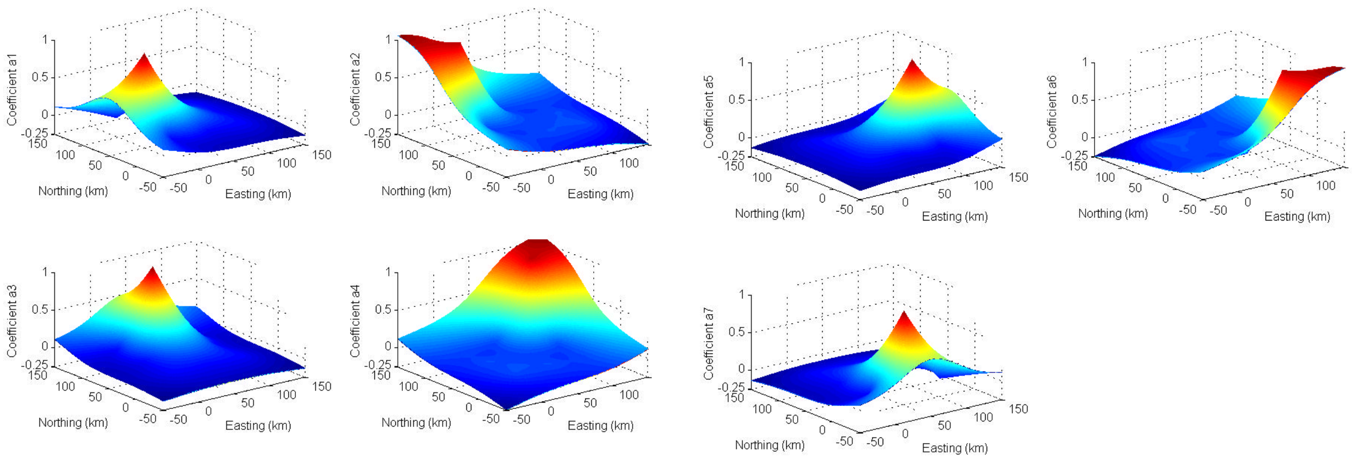

Figure 2f. Coefficients generated by the Least Squares Collocation Method proposed by Raquet (1998)

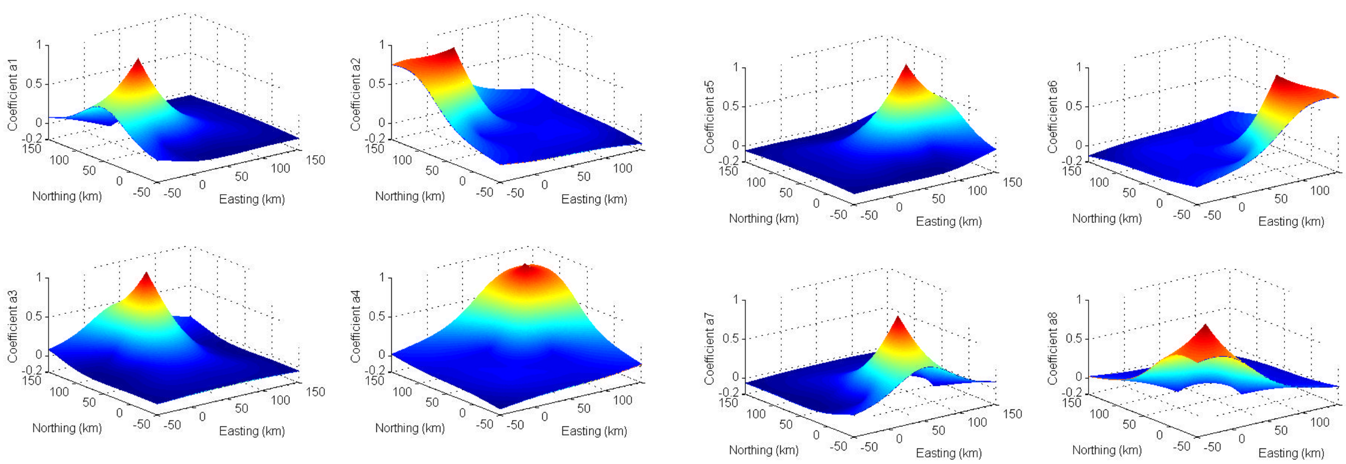

Figure 2g. Coefficients generated by the Least Squares Collocation Method proposed by Marel (1998)

EXPERIMENTS

In order to compare the performance of the different interpolation methods, two experiments were carried out.

Sydney: GPS and Glonass Reference Stations

This experiment was carried out on 15 May 2000, using four dual-frequency integrated GPS/Glonass JPS receivers to simulate a reference station network (Figure 3). One of the reference stations was located on the roof of the Geography and Surveying Building, at The University of New South Wales (UNSW). The other two reference stations were located at Camden and Richmond. The distances between the reference stations were 55.9km, 48.2km and 49.5km. The user receiver was located at the side of Motorway No.4, 31.4km, 26.5km and 32.4km away from the UNSW, Richmond and Camden stations respectively. The station UNSW was selected as the master reference station. The experiment commenced at 8:30AM and concluded at 12:30PM. A total of 3 hours of GPS and Glonass measurements for all the receivers, with one-second sampling rate and a 15^{o} cut-off angle, were collected. During the period, between 5 and 9 GPS, and between 3 and 5 Glonass satellites were tracked.

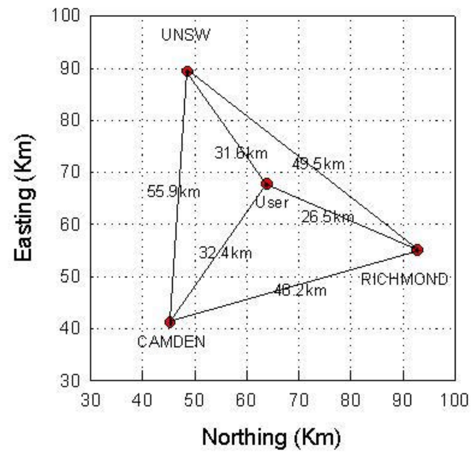

Figure 3. Configuration of the Sydney GPS/Glonass reference receiver network

The reference station ambiguities were correctly determined in the post-processing mode using the recorded GPS and Glonass measurements. Table 1 shows the coefficients for the different interpolation methods. The last two columns denote the sum and square sum of the n-1 coefficients. The square sum factor is an indicator of noise for the 'correction terms', hence the smaller the better. The LSC1 and LSC2 refer to the Least Squares Collocation Method suggested by Raquet (1998) and Marel (1998) respectively. It can be seen that the coefficients for the LCM and LIM are exactly same, and that the coefficients for LSC1 and LSC2 are very close. However, there is a larger difference for the DIM method.

Table 1. Coefficients generated for the different methods (experiment 1)

	Ref. Sta.	LCM	DIM	LIM	LSC1	LSC2
α_1	CAMD	0.193	0.450	0.193	0.249	0.256
α_2	RICH	0.448	0.550	0.448	0.421	0.424
α_3	UNSW	0.360				0.337
$\sum_{i=1}^{2} \alpha_i$		0.640	1.000	0.640	0.670	0.680
$\sqrt{\sum_{i=1}^{2} \alpha_i^2}$		0.487	0.711	0.487	0.489	0.495

Figures 4a to 4d show the L1 and L2 residuals for the baseline UNSW-USER, for satellite pairs PRN39-41 and PRN16-11, with and without the Linear Combination Model. The distance-dependent biases have been reduced significantly after the 'correction terms' from the reference station network were applied. In this experiment there are two data gaps caused by data loss at the user receiver when recording. If the data gap had occurred at the reference station receivers, correction terms can be predicted for up to a few minutes using a Kalman filter or by linear function fitting (see Dai et al., 2002).

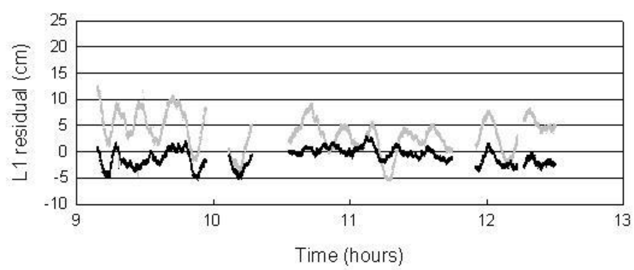

Figure 4a. L1 residuals for Glonass PRN39-41 with (black) and without (grey) the Linear Combination Model

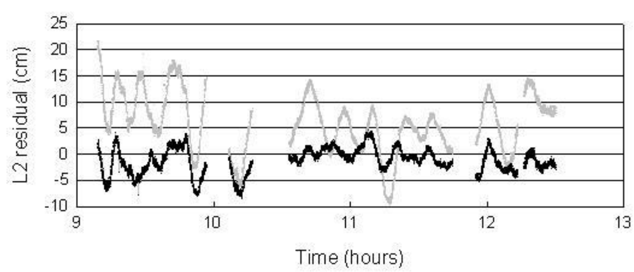

Figure 4b. L2 residuals for Glonass PRN39-41 with (black) and without (grey) the Linear Combination Model

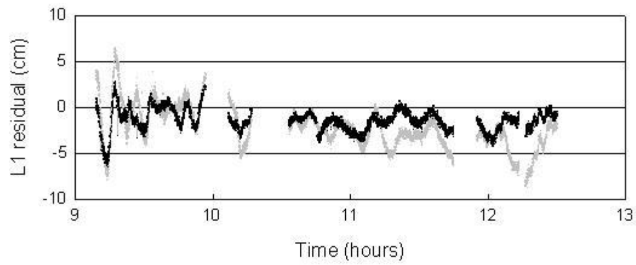

Figure 4c. L1 residuals for GPS satellite pair PRN16-11 with (black) and without (grey) the Linear Combination Model

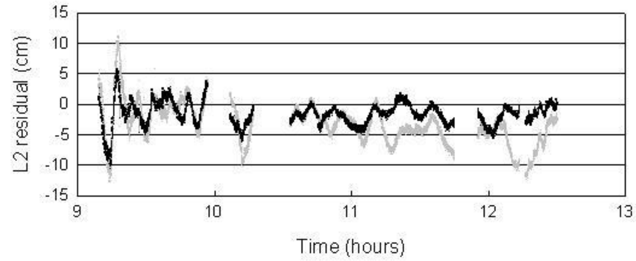

Figure 4d. L2 residuals for GPS satellite pair PRN16-11 with (black) and without (grey) the Linear Combination Model

Figure 5a shows the original L1 resiudals for all the satellite pairs at the baseline UNSW-USER. It can be seen that the residuals can be up to 20cm for the 31.6km baseline. Figures 5b, 5c, and 5d show the L1 residuals after the correction terms from the reference stations are applied using the LIM, DIM and LSC methods respectively. As the coefficients are the same, or very close, for the LCM and LIM, and the LSC1 and LSC2, the results for these are not plotted. It can be seen that the LCM and LSC methods give almost the same results, but the DIM method gives slightly worse results.

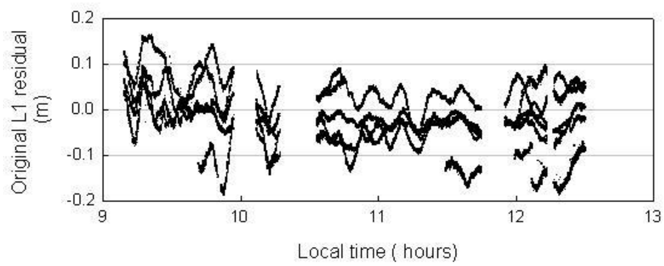

Figure 5a. Original double-differenced L1 residuals for all the satellites pairs

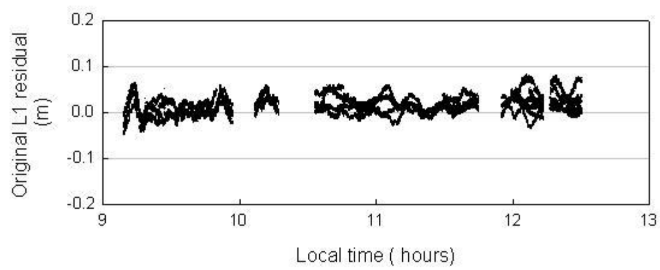

Figure 5b. Double-differenced L1 residuals for all the satellites pairs using the Linear Combination Model

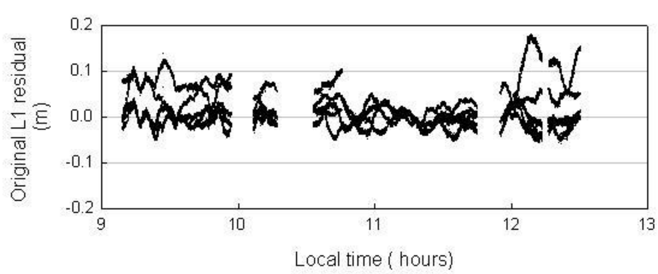

Figure 5c. Double-differenced L1 residuals for all the satellites using the Distance-Based Linear Interpolation Method

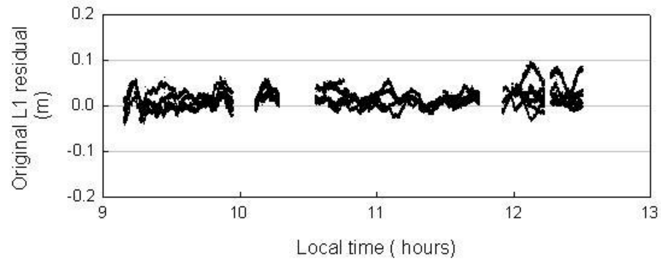

Figure 5d. Double-differenced L1 residuals for all the satellites pairs using the Least Squares Collocation Method

Figure 6 shows the L1, L2, P1 and P2 RMS statistics for the original residuals (ORG), and after the different interpolation methods (LCM, DIM, LIM, SC1 and LSC2) were applied. The conclusion can be made that all the interpolation methods can significantly mitigate the distance-dependent biases in the L1, L2, P1 and P2 double-differenced observables.

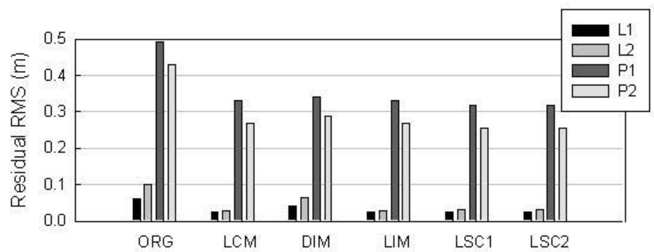

Figure 6. L1, L2, P1 and P2 RMS statistics for the different interpolation methods (experiment 1)

Taiwan: Multiple Reference Receiver Test

In order to further investigate the performance of the different interpolation methods, data from permanent GPS stations established for deformation monitoring purposes in the Taiwan region (Figure 7) have also been analysed. The data was collected on 31 December 2000, logged at a 30-second sampling rate and a cut-off angle of 15^{o}. Of the six reference stations (S011, S104, S058, I007, FCWS and S01R) S011 was selected as the master reference station and I007 as the user. There were two Leica CRS1000 recevers at stations S011 and I007, and four Trimble SSI receivers at stations S01R, FCWS, S058 and S104.

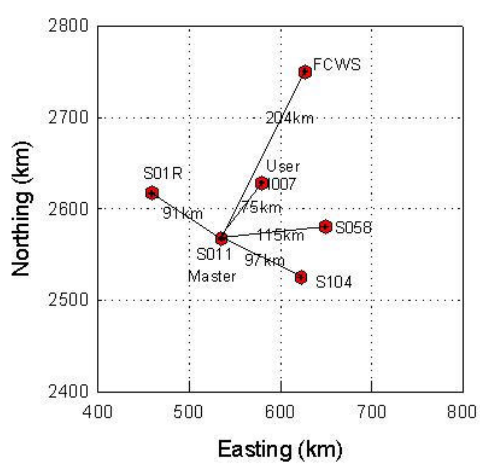

Figure 7. Configuration of the Taiwan reference receiver network

The reference station ambiguities were determined using the Bernese software v4.2 in the standard post-processing mode. Due to the high geomagnetic activity in the Taiwan region over recent years, the ambiguities between the reference stations were very difficult to determine correctly. Therefore, a cut-off angle 25° was used in the data processing. Table 2 shows the coefficients for the LCM, DIM, LIM, LSM, LSC1 and LSC2 interpolation methods. The coefficients for LSC1 and LSC2 are very similar.

Table 2. Coefficients generated for the different methods (experiment 2)

	Ref. Sta.	LCM	DIM	LIM	LSM	LSC1	LSC2
α_1	FCWS	0.297	0.209	0.329	0.283	0.316	0.266
α_2	S01R	0.208	0.225	0.004	0.305	0.024	0.163
α_3	S104	0.142	0.244	0.016	0.202	-0.081	0.004
α_4	S058	0.180	0.322	0.114	0.211	0.282	0.330
α_5	S011	0.173					0.344
$\sum_{i=1}^{4}\alpha_i$		0.827	1.000	0.463	1.000	0.540	0.763
$\sqrt{\sum_{i=1}^{4}\alpha_i^2}$		0.429	0.507	0.349	0.508	0.431	0.454

Figure 8a shows the original L1 residuals for all the satellite pairs. It can be seen that the residuals can be up to 3 metres for the 75km baseline between S011 and I007! It should be emphasised that the distance-dependent biases became quite large and variable between local time 13:00-22:00. This is likely to be due to the high solar activity. Figures 8b to 8g show the L1 residuals after the 'correction terms' from the reference station network have been applied, using the LIM, DIM, LIM, LSM, LSC1 and LSC2 interpolation methods respectively. It can be seen that all six methods can significantly reduce the distance-dependent biases, and demonstrate similar interpolation accuracy. Again, the DIM method does give slightly worse results. It is obvious that during high solar activity the accuracy of the interpolation for all the methods is reduced significantly.

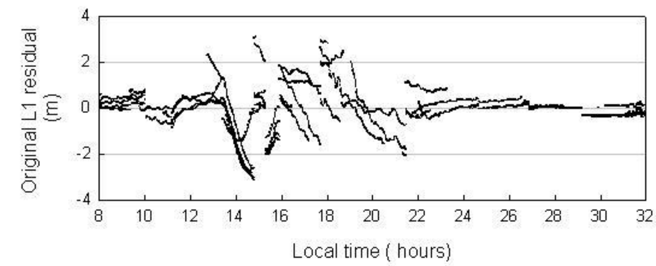

Figure 8a. Original double-differenced L1 residuals for all the satellites pairs

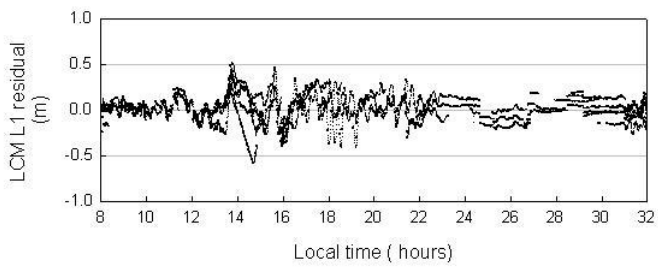

Figure 8b. Double-differenced L1 residuals for all the satellites pairs using the Linear Combination Model

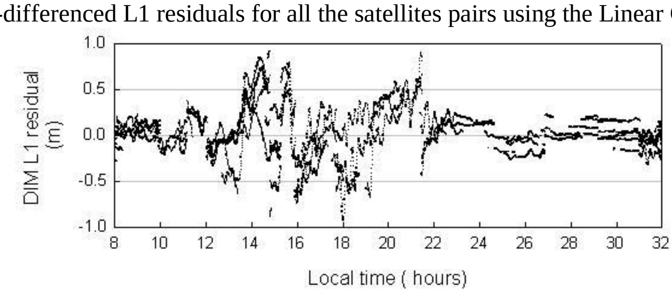

Figure 8c. Double-differenced L1 residuals for all the satellites using the Distance-Based Linear Interpolation Method

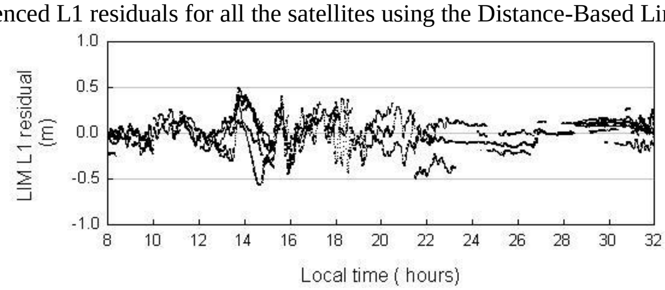

Figure 8d. Double-differenced L1 residuals for all the satellites pairs using the Linear Interpolation Method

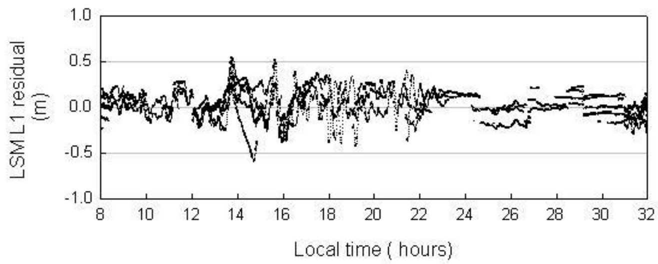

Figure 8e. Double-differenced L1 residuals for all the satellites pairs using the Low-Order Surface Method (bivariate linear function fitting)

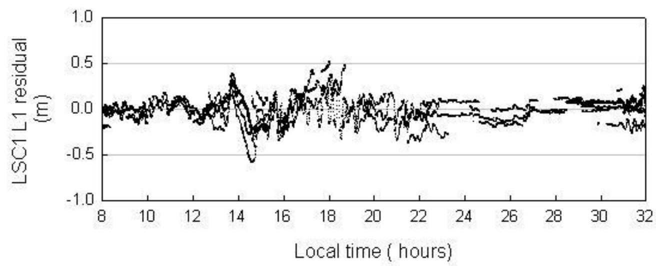

Figure 8f. Double-differenced L1 residuals for all the satellites pairs using the Least Squares Collocation Method proposed by Raquet (1998)

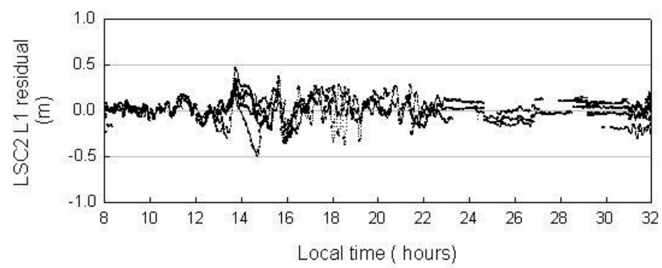

Figure 8g. Double-differenced L1 residuals for all the satellites pairs using the Least Squares Collocation Method proposed by Marel (1998)

Figure 9 shows the L1, L2, P1 and P2 RMS statistics for the original residuals (ORG) and after the different interpolation methods (LCM, DIM, LIM, LSM, SC1 and LSC2) have been applied. It can be seen that in the case of the original residuals (in Figure 9) there are similar RMS values for L1 and P1, and for L2 and P2. This could be due to the dominant ionospheric biases compared to the pseudo-range noise. However, the RMS values for carrier phase are much smaller than for pseudo-ranges after the correction terms are applied. The conclusion can be made again that all the interpolation methods can significantly mitigate the distance-dependent biases in the L1, L2, P1 and P2 double-differenced observables.

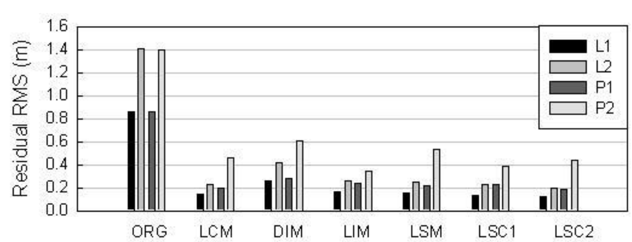

Figure 9. L1, L2, P1 and P2 RMS statistics using the different interpolation methods (experiment 2)

CONCLUDING REMARKS

In this paper several interpolation methods suitable for reference station network techniques, including the Linear Combination Model, the Distance-Based Linear Interpolation Method, the Linear Interpolation Method, the Low-Order Surface Model, and the Least Squares Collocation Method, have been compared in detail. The advantages and disadvantages of each of these techniques have been discussed, and for all of the abovementioned methods, the essential common formula has been identified. All use n-1 coefficients and the n-1 independent 'correction terms' generated from a n reference station network to form a linear combination that mitigates spatially correlated biases at user stations.

Test data from several GPS/Glonass reference station networks were used to evaluate the performance of these methods. The numerical results show that all of the methods for multiple-reference receiver implementations can significantly reduce the distance-dependent biases in the carrier phase and pseudo-range measurements at the user station. The performance of all of the methods is similar, although the distance-dependent Linear Interpolation Method does demonstrate slightly worst results in the two experiments analysed.

REFERENCES

Chen H.Y. (2000), An instantaneous ambiguity resolution procedure suitable for medium-scale GPS reference station networks, 13th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Salt Lake City, Utah, 19-22 September, 1061-1070.

Chen X., Han S., Rizos C. & Goh P.C. (2000), Improving real-time positioning efficiency using the Singapore Integrated Multiple Reference Station Network (SIMRSN), 13th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Salt Lake City, Utah, 19-22 September, 9-18.

Colombo O.L., Hernández-Pajares M., Juan J.M., Sanz J. & Talaya J. (1999), Resolving carrier phase ambiguities on the fly, at more than 100km from nearest reference site, with the help of ionospheric tomography, 12th International Technical Meeting of the Satellite Div. of the U.S. Institute of Navigation, Nashville, Tennessee, 14-17 September, 1635-1642.

Dai L., Wang J., Rizos C. & Han S. (2001a), Real-time carrier phase ambiguity resolution for GPS/Glonass reference station networks. Int. Symp. on Kinematic Systems in Geodesy, Geomatics & Navigation (KIS2001), Banff, Canada, 5-8 June, 475-481.

Dai, L., Han, S., Wang, J., & Rizos, C., (2001b) A study of GPS/Glonass multiple reference station techniques for precise real-time carrier phase-based positioning. 14th Int. Tech. Meeting of the Satellite Division of the U.S. Inst. of Navigation. Salt Lake City, Utah, 11-14 September, 392-403.

Dai L., Wang J., Rizos C. & Han S. (2002), Predicting atmospheric biases for real-time ambiguity resolution in GPS/Glonass reference station networks, accepted for publication by the Journal of Geodesy.

Fotopoulos G. (2000), Parameterization of carrier phase corrections based on a regional network of reference stations, 13th International Technical Meeting of the Satellite Div. of the U.S. Institute of Navigation, Salt Lake City, Utah, 19-22 September, 1091-1102

Fotopoulos G. & Cannon M.E. (2001), An overview of multi-reference station methods for cm-level positioning, GPS Solutions, 4(3), 1-10.

Gao Y., Li Z. & McLellan J.F. (1997), Carrier phase based regional area differential GPS for decimeter-level positioning and navigation, 10th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Kansas City, Missouri, 16-19 September, 1305-1313.

Gao Y. & Li. Z. (1998), Ionosphere effect and modelling for regional area differential GPS network, 11th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Nashville, Tennessee, 15-18 September, 91-97.

Han S. & Rizos C. (1996), GPS network design and error mitigation for real-time continuous array monitoring systems, 9th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Kansas City, Missouri, 17-20 September, 1827-1836.

Han S. & Rizos C. (1998), Instantaneous ambiguity resolution for medium-range GPS kinematic positioning using multiple reference stations, International Association of Geodesy Symposia, vol. 118, Advances in Positioning and Reference Frames, ISBN 3-540-64604-3, 283-288.

Marel H. van der (1998), Virtual GPS reference stations in The Netherlands, 11th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Nashville, Tennessee, 15-18 September, 49-58.

Odijk D., Marel H. van der & Song I. (2000), Precise GPS positioning by applying ionospheric corrections from an active control network, GPS Solutions, 3(3), 49-57.

Raquet J. (1997), Multiple user network carrier-phase ambiguity resolution, Int. Symp. on Kinematic Systems in Geodesy, Geomatics and Navigation (KIS-97), Banff, Canada, 3-6 June, 45-55.

Raquet J., Lachapelle G. & Fortes L. (1998), Use of a covariance analysis technique for predicting performance of regional area differential code and carrier-phase networks, 11th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Nashville, Tennessee, 15-18 September, 1345-1354.

Raquet J. & Lachapelle G. (2001), RTK positioning with multiple reference stations, GPS World, 12(4), 48-53.

Schaer S., Beutler G., Rothacher M., Brockmann E., Wiget A. & Wild U. (1999), The impact of the atmosphere and other systematic errors on permanent GPS networks, Presented at IAG Symposium on Positioning, Birmingham, UK, 21 July.

Schwartz K.P. (1978), On the application of least-squares collocation models to physical geodesy, in Approximation Methods in Geodesy, edited by H. Moritz and H. Sunkel, H. Wichmann-Verlag, Karlsruhe, Germany, 89-116.

Tscherning, C.C. (1974), A FORTRAN IV Program for the Determination of the Anomalous Potential Using Stepwise Least Squares Collocation. Reports of the Department of Geodetic Science No. 212, The Ohio State University, Columbus, Ohio.

Tscherning, C.C. (2001), Computation of spherical harmonic coefficients and their error estimates using least-squares collocation, Journal of Geodesy, 75(1), 12-18.

Varner C. (2000), DGPS carrier phase networks and partial derivative algorithms, Ph.D. Thesis, University of Calgary, Dept. of Geomatics Engineering, Report No. 20129, Calgary, Canada.

Vollath U., Bücherl A., Landau H., Pagels C. & Wager B. (2000), Multi-base RTK positioning using virtual reference stations, 13th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Salt Lake City, Utah, 19-22 September, 121-131.

Wanninger L. (1995), Improved ambiguity resolution by regional differential modelling of the ionosphere, 8th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Palm Springs, California, 12-15 September, 55-62.

Wanninger L. (1999), The performance of virtual reference stations in active geodetic GPS-networks under solar maximum conditions, 12th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Nashville, Tennessee, 14-17 September, 1419-1427.

Wübbena G., Bagge A., Seeber G., Böder V. & Hankemeier P. (1996), Reducing distance dependent errors for real-time precise DGPS applications by establishing reference station networks, 9th Int. Tech. Meeting of the Satellite Div. of the U.S. Institute of Navigation, Kansas City, Missouri, 17-20 September, 1845-1852.