

A Scheme for Throughput Maximization in a Dual-Class CDMA System

Sudhir Ramakrishna & Jack M. Holtzman,
Wireless Information & Networking Laboratory (WINLAB),
Dept. of Elec. & Comp. Engg., Rutgers University,
PO Box 909, Piscataway, NJ 08855-0909.
Email : {rsudhir, holtzman}@winlab.rutgers.edu

Abstract

This work focuses on the problem of efficient exploitation of the available bandwidth in order to provide high bit rates on the wireless link, as will be required in future wireless systems interfacing to broadband fixed networks. In particular, the uplink of a CDMA system with two user classes is considered. One of the classes consists of delay intolerant users requiring support for a constant information bit rate, while the other consists of delay tolerant users needing support for an information bit rate larger than a given value. It is assumed that when not transmitting information, both classes maintain synchronization contact with the base station at a given rate. The objective is to maximize the throughput of the delay tolerant users, while ensuring that the interference to other cells is as low as possible by minimizing the sum of all the transmit powers used by the mobiles. Two transmission modes for the delay tolerant users are considered. In the first mode, all the users are allowed to transmit information when they wish. In the second mode, the transmissions of the delay tolerant users are scheduled, so that only a limited number of them are transmitting information at any given time instant. It is shown that the second transmission mode, which tends towards a hybrid CDMA/TDMA scheme for the delay tolerant users, affords a better throughput while imposing the same average power requirements as conventional transmission. The results in this paper can be interpreted using results from previous work based on information theory.

1 Introduction

Present CDMA-based cellular systems have primarily been optimized for voice transmission. In order to interface to the broadband networks of the future, wireless systems will be required to support sources with a variety of rates and quality-of-service requirements. Since very high bit rates will be required to be supported, efficient use of the available bandwidth resource will be necessary. In this paper, we consider the problem of making present CDMA systems more efficient with respect to the throughputs they provide, with an emphasis on non-voice sources. The basic approach is to exploit the property of delay tolerance that is a characteristic of many non-voice sources. A similar approach was followed in [1] in order to increase the capacity of an integrated voice/data CDMA system. The basic idea therein (other related references in [1]) was to increase the data rates during periods of low voice activity, thus increasing the data throughput. The approach explored in this paper is fundamentally different. We propose a scheme by which transmissions of the delay tolerant users are arranged so as to reduce, at any time instant, the interference seen by the transmitting users; as a consequence, transmitting users can transfer information at higher rates, leading to an overall increase in throughput. The total transmit power is constrained to be

minimal; this, in addition to imposing low power budgets on the mobiles, also tends to minimize the inter-cell interference.

In [2], a comparison was made between single channel chip rate and multiple chip rate CDMA systems. It was indicated therein that a multiple chip rate system would necessitate more complex receivers as well as additional frequency planning. In this work, a common chip rate on the wireless link is assumed; hence, different users get various spreading gains, depending on their bit rates. Such a system has been proposed in other contexts in [3, 4, 5]. Though our interest is in a multi-class CDMA system, for clarity of exposition, we focus on the simplest special case, namely, a CDMA system with two classes of users. In this paper, we focus on the uplink of such a system. Similar considerations apply on the downlink, and will be explored in later expositions.

The CDMA system under consideration uses a total spreading bandwidth of W Hz, and is required to support two classes of users having the following properties :

- Class 1 : The users in this class are delay intolerant. When transmitting information, they require support for a constant bit rate of R bits/sec; they can tolerate a bit error rate of at most P_b .
- Class 2 : The users in this class are delay tolerant. When transmitting information, they require support for a bit rate of at least R_m bits/sec; they can tolerate a bit error rate of at most P_{b1} .
- Both classes : When not transmitting information, it is assumed that the users still communicate with the base for synchronization purposes. The bit rate used in this synchronization mode is denoted as R_0 bits/sec for both classes, and is referred to as the “idle rate”. One would expect that $R_0 < R$, $R_0 < R_m$; more detailed constraints on R_0 are derived later.

In an actual system, classes 1 & 2 could represent voice and data users, respectively. In the following exposition, an activity factor of unity is assumed for all the users; this model can be readily extended. This corresponds to the worst case situation of all the users wanting to transmit information all the time.

Our objectives are threefold :

- (1) To assign the transmit powers to the mobiles in a cell such that the sum of their powers is minimized; this criterion minimizes the interference caused to other cells.
- (2) At the same time, to assign the class 2 rates and arrange the transmissions so as to maximize each class 2 user’s average throughput.
- (3) Perform (1) and (2) above such that the quality of service requirements are met.

The problem of minimizing the sum of powers and maximizing throughputs has been studied in [5]. This paper focuses on Objective (2), using the results in [5] as a starting point. In particular, two transmission modes for the class 2 users are considered and compared. In the first mode, which is the mode used in conventional CDMA systems, a class 2 user is allowed to transmit information whenever it wishes; this transmission mode serves as the baseline for comparison. In the second mode, the transmissions of the class 2 users are time-scheduled such that at any time instant, only a limited number of them are transmitting information, while the remaining are simply maintaining synchronization contact with the base station. Each class 2 user now has a duty cycle, in that it is allowed to transmit information only a fraction of the time. The expectation is that the reduced

interference seen by an information-transmitting class 2 user will allow operation at a higher rate, high enough to offset the negative effect of the duty cycle, leading to gains in average throughput. The main result of this paper is that under many conditions, this is indeed true, i.e, the time-scheduled transmission mode leads to significant per-user throughput gains for the class 2 users, while requiring the same per-user average power as conventional transmission. The situations in which the scheduled transmissions lead to throughput gains are also identified.

It may be noted that the second transmission mode tends towards hybrid CDMA/TDMA for the class 2 users. A scheme resembling TDMA is used to distinguish between the class 2 users within a cell; CDMA is used to separate class 1 and class 2 users within a cell, and to suppress the effect of the users in one cell on another. An interesting comparison can be made between this approach and some recent information theoretic results on optimal cellular capacity in [6, 7]. Although the contexts of these works are each somewhat different from that of this work, a simplified interpretation gives their essence as saying that to achieve capacity, a necessary condition is to separate the users in a cell and share the available bandwidth between cells ¹. Thus, the time scheduled transmission mode for the class 2 users presented here may be viewed as a constructive approach tending toward what is indicated as optimal by information theory. Further, we show how additional constraints, such as synchronization and peak power requirements, place limits on the achievable gains.

The organization of this paper is as follows. Section (2) reviews previous work to establish conditions for the existence of a solution to our set of objectives. The presence or absence of a constraint on the peak transmit power affects the analyses significantly; specializations to these two cases are considered in Sections (3) and (4) respectively. The problem of maximizing the class 2 throughputs in these two cases is considered in Sections (3.2) and (4.2) respectively. Numerical results are presented and discussed in Sections (3.3) and (4.3), followed by a discussion of implementation issues in Section (6). Section (5) summarizes the work in this paper, and future research directions are indicated in Section (7).

2 Existence of a minimum total transmit power solution

We consider a general situation, where one has N_1 class 1 users and N_2 class 2 users present in the system. Our aim is to assign transmit powers to all the mobiles such that their sum is as small as possible, and such that the SIR requirements of all the users are met. In previous work [5], it is shown that at any given time instant, a unique solution to this problem exists if and only if

$$\sum_{i=1}^{N_2} \frac{1}{\left(\frac{W}{R_i^{(v)}} \cdot \frac{1}{\text{SIR}_i^{(v)}} + 1\right)} + \sum_{i=1}^{N_1} \frac{1}{\left(\frac{W}{R_i^{(c)}} \cdot \frac{1}{\text{SIR}_i^{(c)}} + 1\right)} < 1 - \frac{IW}{\min_i \left\{ \left[p_i h_i \left(\frac{W}{R_i \cdot \text{SIR}_i} + 1 \right) \right] \right\}_{i=1}^{N_1+N_2}}, \quad (1)$$

where

– $\text{SIR}_i^{(v)}$, $\text{SIR}_i^{(c)}$: (instantaneous) signal-to-interference ratios (SIR's) required to be received, for the i^{th} variable bit rate (VBR, denoted by superscript “v”, referring to class 2) and constant bit rate (CBR, superscript “c”, referring to class 1) user respectively. In this work, SIR will refer to the energy-per-bit to total interference ratio.

– $R_i^{(v)}$, $R_i^{(c)}$: (instantaneous) rates of the i^{th} VBR and CBR user respectively.

¹Joint decoding or interference cancellation, which are not considered in this work, are also indicated by the information theoretic results.

- SIR_i, R_i : (instantaneous) SIR required to be received, and the instantaneous rate, of the i^{th} generic user.
- p_i : peak power permitted to be used by the i^{th} generic user.
- h_i : (mobile to base) gain of the i^{th} generic user; $h_i^{(c)}$ or $h_i^{(v)}$ will be used to refer specifically to the gains of a class 1 or a class 2 user, respectively.
- $I = I_o + n_0$ = (spreading bandwidth normalized) other cell interference + background noise spectral density, respectively. The background noise is assumed to be white and Gaussian.

It is assumed that the SIR variables translate at the physical layer to the required BERs P_b and P_{b1} respectively. Also, we assume that I is a slowly changing variable.

A unique, and minimum sum set of transmit powers can always be assigned if the constraint (1) is satisfied. Hence, in our attempt to find user populations, rates and transmission modes to maximize the class 2 per-user average throughput, we always ensure that those solutions lie in the region defined by the constraint (1). In this way, the property of the assigned transmit powers having the smallest possible sum is always maintained.

The presence or (lack of it) of an upper limit on the mobiles peak transmit power has a significant effect on subsequent analyses and results. Hence, we study these two cases separately. The case with perfect power control, and with unconstrained peak transmit powers, is considered first.

3 Case 1 : Unconstrained peak transmit powers

Here, the mobiles are allowed to use any transmit power instantaneously; one would, however, expect some constraint on the average power permitted to be used. To start with, we assume the presence of perfect power control. Extensions to the case with imperfect power control are treated in a later section. The received SIR's can therefore be described as :

- $SIR_i^{(c)} = \gamma = \text{constant} \forall$ CBR (class 1) users i . It is assumed that $SIR \gamma \iff BER P_b$ for the CBR users.
- $SIR_i^{(v)} = \gamma_1 = \text{constant} \forall$ VBR (class 2) users i . By assumption, $SIR \gamma_1 \iff BER P_{b1}$ for the VBR users.

We now specialize the basic existence constraint of the previous section to account for the lack of constraints on the peak transmit power.

3.1 Existence of a minimum total power solution

We consider a situation where we have k_c class 1 users present. Let k_v denote the maximum number of class 2 users supportable along with the k_c class 1 users. The set $\{k_c, k_v\}$ is maximal, in the sense that if one more user of either class enters the system, it will not be possible to meet the quality-of-service requirements of all the users. Clearly, this corresponds to a situation where each class 2 user gets exactly its minimum required rate R_m .

□ Note : We remind the reader that the subscript “ v ” refers to variable bit rate, not voice.

Using the notation of the previous section, and noting that $p_i = \infty \forall i$, the constraint (1) now becomes

$$\frac{k_v}{\left(\frac{W}{R_m \gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} < 1. \quad (2)$$

We note here that similar constraints have been derived in [8, 9]. Hence, given that k_c class 1 users are present, one may support at most

$$k_v = \left\lfloor 1 + \frac{W}{R_m \gamma_1} - k_c \cdot \frac{\left(\frac{W}{R_m \gamma_1} + 1\right)}{\left(\frac{W}{R\gamma} + 1\right)} \right\rfloor \text{ class 2 users.} \quad (3)$$

Given that k_c class 1 and upto k_v class 2 users are present, the transmit powers assigned to the users will then be such that

- Each class 1 user will achieve an SIR of exactly γ ; similarly, each class 2 user will have a received SIR of exactly γ_1 . The bit error rate requirements of users of both classes will therefore also be met with equality.
- The transmit powers will be such that their sum is as small as possible; hence, the interference to other cells is minimized.

It is assumed that admission control will handle the task of ensuring that the number of users of each class satisfy the constraints in (2) and (3). We now consider the other objective, which is to maximize the throughput of the admitted class 2 users.

3.2 Maximization of Class 2 throughput

We consider a situation in which one has k_c class 1 users and $k_1 \leq k_v$ class 2 users present in the system. We consider the following two transmission modes for the class 2 users:

- Mode 1 : All k_1 of them are allowed to transmit information, each at a rate R_1 . The rate R_1 is chosen to be the largest possible so as to satisfy the constraint (1). It may be noted that this is the transmission mode followed in present systems. A more efficient (from the point of view of throughput) version of this scheme would allow each class 2 user to transmit at an appropriate, different rate. We, however, constrain all the class 2 rates to be identical for simplicity.
- Mode 2 : The class 2 transmissions are scheduled in such a way that at any given time instant, only $k_2 (< k_1)$ of them are transmitting information, while the remaining $(k_1 - k_2)$ are in contact with the base at the idle/synchronization rate R_0 bits/sec. When transmitting information, a class 2 user is allowed to transmit at a rate R_2 , which, again, is chosen so as to be the maximum value satisfying constraint (1). Thus, assuming a fair division of time, each class 2 user has a ‘‘duty cycle’’, or fraction of time when it is transmitting information, given by $\frac{\binom{k_1-1}{k_2-1}}{\binom{k_1}{k_2}} = \frac{k_2}{k_1}$; the remaining fraction of time is spent in maintaining synchronization with the base at a rate R_0 .

Intuitively, we see that in mode 2, each information-transmitting class 2 user has to contend with lesser interference than a corresponding situation in mode 1; hence, the rate R_2 is expected to be higher than R_1 , which has a positive effect on the per-user throughput. However, a class 2 user can no longer transmit all the time in mode 2, as it could in mode 1, which affects the achievable throughput negatively. Hence, the question one is interested in is : Does the gain in throughput due to R_2 being greater than R_1 offset the loss in throughput imposed by the duty cycle? This question is answered in the following sections, where we compute the average throughput achieved by a class 2 user in each transmission mode.

3.2.1 Mode 1 : Unscheduled class 2 transmissions

Following constraint (1), let the rate R_1 be chosen such that

$$\frac{k_1}{\left(\frac{W}{R_1\gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} = \frac{k_v}{\left(\frac{W}{R_m\gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} < 1 . \quad (4)$$

This corresponds to increasing the transmission rate of each class 2 user to the maximum possible value; at this point, the system is equivalent, from the point of view of constraint 1, to the fully loaded case (which has k_c class 1 and k_v class 2 users). From (4), it can be shown that the corresponding rate R_1 of the class 2 users is given by

$$R_1 = R_m \cdot \frac{\frac{W}{\gamma_1} k_v}{\left\{ \frac{W}{\gamma_1} k_1 - R_m (k_v - k_1) \right\}} . \quad (5)$$

Since an activity factor of unity has been assumed, R_1 represents both the instantaneous as well as the average rate of the class 2 users. Clearly, assuming a negligible number of retransmissions, R_1 is also the average throughput of each class 2 user.

Using the expressions derived in [5], the transmit powers assigned to the various users, in order to achieve their quality-of-service requirements, are given as follows :

- $P_{1,i}^{(c)}$, the power assigned to the i^{th} class 1 user (denoted by the superscript “c”) in mode 1 (the subscript), is

$$P_{1,i}^{(c)} = \frac{IW}{\left(\frac{W}{R\gamma} + 1\right) h_i^{(c)} Y} , \quad (6)$$

where Y is given by

$$Y = 1 - \frac{k_1}{\left(\frac{W}{R_1\gamma_1} + 1\right)} - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} .$$

- $P_{1,i}^{(v)}$, the power assigned to the i^{th} class 2 user (superscript “v”) in mode 1, is

$$P_{1,i}^{(v)} = \frac{IW}{\left(\frac{W}{R_1\gamma_1} + 1\right) h_i^{(v)} Y} . \quad (7)$$

We note that in this mode, the average and peak transmit powers are the same (for both user classes). This is a consequence of the assumption of an activity factor of unity for all the users, along with the fact that users transmit at a constant rate in this mode.

3.2.2 Mode 2 : Scheduled class 2 transmissions

Now, the class 2 transmissions are time-shared so that at any time instant, only k_2 of them are transmitting information, while the remaining $(k_1 - k_2)$ are simply maintaining synchronization with the base. The “active” users, which are transmitting information, transmit at a rate R_2 , while the other class 2 users transmit at the idle rate R_0 . Each user transmits information a fraction $\frac{k_2}{k_1}$ of the time; the remaining fraction of time is devoted to transmission for synchronization purposes. Following constraint (4), let the class 2 rate R_2 be chosen such that

$$\frac{k_2}{\left(\frac{W}{R_2\gamma_1} + 1\right)} + \frac{k_1 - k_2}{\left(\frac{W}{R_0\gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} = \frac{k_1}{\left(\frac{W}{R_1\gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} = \frac{k_v}{\left(\frac{W}{R_m\gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} < 1. \quad (8)$$

As mentioned before, this constraint physically corresponds to increasing the rate R_2 to the maximum possible value, at which point one has a fully loaded system. The rate R_2 can be derived as

$$R_2 = \frac{k_2 R_0 \left(\frac{W}{\gamma_1} + R_1\right) + k_1 \frac{W}{\gamma_1} (R_1 - R_0)}{k_2 \left(\frac{W}{\gamma_1} + R_1\right) - k_1 (R_1 - R_0)}. \quad (9)$$

Before expanding further on the average throughput being achieved, we digress to take note of the transmit powers being used in this mode. We note here that unlike in mode 1, the average and peak transmit powers used by the class 2 mobiles are not the same; this is due to the duty cycle associated with the class 2 transmissions in this mode. Proceeding as in the previous section, we have :

- $P_{2,i}^{(c)}$, the power assigned to the i^{th} class 1 user in mode 2, is

$$P_{2,i}^{(c)} = \frac{IW}{\left(\frac{W}{R\gamma} + 1\right) h_i^{(c)} Z}, \quad (10)$$

where Z is given as

$$Z = 1 - \frac{k_2}{\left(\frac{W}{R_2\gamma_1} + 1\right)} - \frac{k_1 - k_2}{\left(\frac{W}{R_0\gamma_1} + 1\right)} - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}.$$

Using Equation (8), it is apparent that one has $Z = Y$, where the parameter Y appears in Equations (6) and (7). Consequently, one has $P_{2,i}^{(c)} = P_{1,i}^{(c)}$, where $P_{1,i}^{(c)}$ is given as in Equation (6). Hence, for the class 2 users, transmission modes 1 and 2 impose identical average transmit power requirements.

- $P_{2,i}^{(v)}$, the average power expended by the i^{th} class 2 user in mode 2, is

$$P_{2,i}^{(v)} = \frac{k_2}{k_1} \cdot \frac{IW}{\left(\frac{W}{R_2\gamma_1} + 1\right) h_i^{(v)} Z} + \frac{(k_1 - k_2)}{k_1} \cdot \frac{IW}{\left(\frac{W}{R_0\gamma_1} + 1\right) h_i^{(v)} Z},$$

where the factors $\frac{k_2}{k_1}$ and $\frac{(k_1 - k_2)}{k_1}$ account for the fractions of information transmission and plain synchronization times respectively. Using $Z = Y$, and Equation (8), the above can be written as

$$P_{2,i}^{(v)} = \frac{IW}{\left(\frac{W}{R_1\gamma_1} + 1\right) h_i^{(v)} Y} = P_{1,i}^{(v)}, \quad (11)$$

where $P_{1,i}^{(v)}$ is given as in Equation (7). Hence, one concludes that each class 2 mobile expends the same average power in either of the transmission modes (independent of the value of k_2 or R_0 used in mode 2 transmission).

- $P_{2,i,peak}^{(v)}$, the peak transmit power expended by the i^{th} class 2 user in mode 2, is simply the power used when transmitting at the rate R_2 . It is given as

$$P_{2,i,peak}^{(v)} = \frac{IW}{\left(\frac{W}{R_2\gamma_1} + 1\right) h_i^{(v)} Y} ; \quad (12)$$

it can be shown that $P_{2,i,peak}^{(v)} \approx \frac{k_1}{k_2} \cdot P_{1,i}^{(v)}$. Hence, for the same average power, mode 2 requires a peak power which is approximately $\frac{k_1}{k_2}$ times that expended in mode 1.

In summary, mode 2 transmission imposes the same average transmit power requirements as mode 1; the class 2 users, however, are required to expend a peak transmit power which is about $\frac{k_1}{k_2}$ that in mode 1.

We now consider the average throughput achieved by the class 2 users. Clearly, the fraction of time spent in synchronization does not contribute to the useful throughput; only the information transmission at rate R_2 need be considered. Hence, using Equation (9) and assuming a negligible retransmission rate, the average throughput is given as

$$T = \frac{k_2}{k_1} \cdot R_2 = \frac{k_2^2 R_0 \left(\frac{W}{\gamma_1} + R_1\right) + k_1 k_2 \frac{W}{\gamma_1} (R_1 - R_0)}{k_1 k_2 \left(\frac{W}{\gamma_1} + R_1\right) - k_1^2 (R_1 - R_0)} . \quad (13)$$

Clearly, one would like to maximize T by selecting the variable k_2 (number of class 2 users allowed to transmit information simultaneously) appropriately. The conditions under which T exceeds the mode 1 throughput R_1 are of special interest. We make the following easily proven observations with regard to this maximization.

Observation 1 *For any admissible value of k_2 , T is a decreasing function of the idle rate R_0 .*

Observation 2 *Given a non-trivial R_0 , one has the following possibilities :*

Case 1 $R_0 \leq \frac{R_1^2}{\left(\frac{W}{\gamma_1} + 2R_1\right)} :$

– In this case, one has $T \geq R_1$ for any admissible value of k_2 , i.e., $\forall k_2 \in [1, \dots, k_1 - 1]$.

Case 2 $\frac{R_1^2}{\left(\frac{W}{\gamma_1} + 2R_1\right)} < R_0 \leq \frac{k_1 R_1^2}{\left(\frac{W}{\gamma_1} + [k_1 + 1] R_1\right)} :$

– In this case, one has $T \geq R_1$ for the set of k_2 values $k_2 \in \left\{1, \dots, \left\lfloor \frac{k_1 R_1 (R_1 - R_0)}{R_0 \left(\frac{W}{\gamma_1} + R_1\right)} \right\rfloor\right\}$.

Also, given that $k_1 \geq 1$, this set is always non-empty, i.e., always includes the point $k_2 = 1$.

$$\underline{\text{Case 3}} \ R_0 > \frac{k_1 R_1^2}{\left(\frac{W}{\gamma_1} + [k_1 + 1] R_1\right)} :$$

– In this case, one has $T < R_1$ for any admissible k_2 .

In either Case 1 or Case 2, T is a decreasing function of $k_2 \ \forall \ k_2 \ \text{s.t.} \ T(k_2) \geq R_1$; hence, T is maximized by choosing $k_2 = 1$.

Observation 1 states an intuitively obvious fact; a lower synchronization rate would imply reduced interference seen by the information-transmitting users, and thus a higher per-user throughput. The actual value of R_0 , however, will be determined purely by the system set up. Hence, it is Observation 2 which is of utility in a real system. Cases 1 and 2 put limits on the synchronization rate R_0 for throughput gains in mode 2; the upper and lower limits in case 2 will henceforth be referred to as $R_{0,upper,1}$ and $R_{0,upper,2}$ respectively. This observation then states that given $R_0 < R_{0,upper,2}$, the throughput T achieved by the class 2 users in mode 2 is greater than that in mode 1, for a set of k_2 values which always includes $k_2 = 1$. Also, the choice $k_2 = 1$ maximizes T . From the previous discussion on the powers used, this increase in throughput is achieved by using the same average power per user as in mode 1 transmission. We will focus on the maximum throughput increase case $k_2 = 1$, which physically corresponds to allowing only one class 2 user to transmit information at any given time instant. We denote the throughput T for $k_2 = 1$ by T_2 . One then has the throughput gain G measured by the ratio of mode 2 to mode 1 throughputs as

$$G = \frac{T_2}{R_1} = \frac{\frac{R_0}{R_1} \left(\frac{W}{R_1 \gamma_1} + 1\right) + k_1 \frac{W}{R_1 \gamma_1} \left(1 - \frac{R_0}{R_1}\right)}{k_1 \left(\frac{W}{R_1 \gamma_1} + 1\right) - k_1^2 \left(1 - \frac{R_0}{R_1}\right)}. \quad (14)$$

3.3 Numerical results and discussions

In order to compute the throughput gain G for various situations, the system parameters were chosen to be as follows :

- Spreading bandwidth $W = 1.23 \text{ MHz}$.
- Class 1 bit rate $R = 9.6 \text{ Kbps}$, with a minimum SIR of $\gamma = 7 \text{ dB}$ (5) required to be received.
- Minimum class 2 bit rate, $R_m = 14.4 \text{ Kbps}$, with a minimum SIR of $\gamma_1 = 8.5 \text{ dB}$ (7.0795) required to be received.
- Idle bit rate $R_0 = 1.2 \text{ Kbps}$.

The class 2 SIR requirement γ_1 is chosen under the conservative assumption that power control at higher rates might involve higher overheads. In an actual system, one might have $\gamma = \gamma_1$. The number of class 1 users k_c was taken to be the primary variable; based on this, the maximum number k_v of class 2 users permitted was computed according to Equation (3). The number of class 2 users in the system was then varied from 1 to k_v , and the corresponding gain G was computed from Equation (14). The results are plotted in Figures (1a, 1b, 1c), for three values of k_c , varying from a predominantly class 1 system to a predominantly class 2 system.

The throughput gains offered are seen to be quite substantial; this is especially true as the fraction of class 2 users in the system increases, as can be seen by comparing the Figures (1a), (1b)

and (1c). This can be explained by noting that a reduction in the number k_c of class 1 users in the system leads to the class 2 users experiencing lesser interference as well. This allows increases in the information transmission rate R_2 , thus making scheduling more efficient.

The values assumed by the parameters $R_{0,upper,1}$ and $R_{0,upper,2}$ are also of interest. For example, a value of $R_{0,upper,2}$ lower than that dictated by synchronization requirements might imply that scheduling is not practical. For the case in Figure (1b), we plot the variation of $R_{0,upper,1}$ and $R_{0,upper,2}$ in Figure (1d). It is seen in this example that the most stringent value of $R_{0,upper,2}$ is about 5 kbps, which would translate to a rather weak requirement on R_0 . Hence, we may conclude that the requirements imposed on R_0 by the parameters $R_{0,upper,1}$ and $R_{0,upper,2}$ are not unreasonably tight.

The modifications to these results in the presence of imperfect power control are studied next.

3.4 Extensions to the case of imperfect power control

In this section, we consider the changes to the results of the previous sections when one has imperfections in the power control mechanism. We assume the presence of “bang-bang” power control, which is elaborated upon in [10]. The imperfections in such a power control scheme can be shown to lead to the received SIR’s being log-normal variables; this observation has been validated in field trials as well, as reported in [11]. Assuming the presence of k_c class 1 and k_v class 2 users, where, as before, the set $\{k_c, k_v\}$ is maximal, the constraint (1) now becomes

$$\sum_{i=1}^{k_v} \frac{1}{\left(\frac{W}{R_m} \cdot \frac{1}{\text{SIR}_i^{(v)}} + 1\right)} + \sum_{i=1}^{k_c} \frac{1}{\left(\frac{W}{R} \cdot \frac{1}{\text{SIR}_i^{(c)}} + 1\right)} < 1, \quad (15)$$

where

- $\{\text{SIR}_i^{(c)}\}_i$ are iid according to $e^{Z^{(c)}}$, where $Z^{(c)} \sim \mathcal{N}(\gamma_c, \sigma_c^2)$. This distribution on the received SIR is assumed to lead to a bit error rate better than or equal to P_b , the maximum allowed.
- $\{\text{SIR}_i^{(v)}\}_i$ are iid according to $e^{Z^{(v)}}$, where $Z^{(v)} \sim \mathcal{N}(\gamma_v, \sigma_v^2)$. By assumption, this distribution on the received SIR leads to a bit error rate better than or equal to P_{b1} , the maximum allowed.

Since the SIR’s are now random variables, requiring the constraint (15) to be valid at every time instant would be too stringent. Hence, we demand that the instantaneous constraint (15) be valid a certain fraction of the time. This would correspond to the existence of a unique solution to the minimum total transmit power problem, and hence, the SIR targets being met, a certain fraction of the time. The event wherein the SIR targets are not met is referred to as an SIR outage; the new constraint, therefore, defines a certain acceptable “outage probability”. Hence, the new condition of interest is

$$P(V < 1) = 1 - \eta, \quad (16)$$

where

- $V = \sum_{i=1}^{k_v} \frac{1}{\left(\frac{W}{R_m} \cdot \frac{1}{\text{SIR}_i^{(v)}} + 1\right)} + \sum_{i=1}^{k_c} \frac{1}{\left(\frac{W}{R} \cdot \frac{1}{\text{SIR}_i^{(c)}} + 1\right)}$
- η is the outage probability (a typical value of which could be 1%).

In order to compute the maximal set $\{k_v, k_c\}$ given the other parameters, $F_V(v)$, the CDF of the random variable V , needs to be available. The following observations are used to compute $F_V(v)$.

- If X is log-normal, then $\frac{1}{X}$ is also a log-normal random variable.
- If X is log-normal, then $a \cdot X$, where a is a constant, is also a log-normal random variable.
- For log-normal $X \sim e^A$, where $A \sim \mathcal{N}(\gamma_A, \sigma_A^2)$, the random variable $(X + 1)$ is, to a very good approximation, also log-normal. This is particularly true when $|\gamma_A| \gg 1$.
- Given the N independent (not necessarily identically distributed) log-normal random variables $\{X_i\}$, consider the sum $Y = \sum_{i=1}^N X_i$. Extensive work has been done on characterizing such a sum (a summary and comparison of the various characterizations can be found in [12]). We use the method in [13], by which Y can be approximated, to a high degree of accuracy, as another log-normal random variable.

From the above observations, we conclude that V is very well approximated as a log-normal random variable. Thus, $V \sim e^Y$, where $Y \sim \mathcal{N}(\gamma_Y, \sigma_Y^2)$; the parameters γ_Y and σ_Y^2 can be computed in terms of the parameters of the individual log-normal variables $\text{SIR}_i^{(c)}$ and $\text{SIR}_i^{(v)}$ using the method outlined in [13]. The cdf $F_V(v)$, and consequently, an explicit constraint for the existence of a minimum total power solution, can now be found.

In a similar way, we can replace the constraints (4) in Section (3.2.1) and (8) in Section (3.2.2), referring to transmission modes 1 and 2 respectively, by their probabilistic equivalents as in (16). The rates R_1 and R_2 , which are the rates assigned to the class 2 users in modes 1 and 2 respectively, can then be computed in closed form. From R_2 , one can compute T_2 , the class 2 per-user throughput when only one class 2 user transmits information at a time, in closed form. One would expect it to be possible, as in the set of Observations (2), to derive conditions where T_2 exceeds R_1 . However, the computations involved in deriving such conditions in closed form are very tedious. We prefer, therefore, to numerically investigate the variation of the throughput gain $G = \frac{T_2}{R_1}$ with varying degrees of power control imperfections.

In order that comparisons may be made to the previous results, the parameters were chosen to be the same as those used to generate Figures (1a), (1b) and (1c), i.e, as

- Spreading bandwidth $W = 1.23$ MHz.
- Class 1 bit rate $R = 9.6$ Kbps, with the power controlled received SIR parameter $\gamma_c = 7$ dB (5).
- Minimum class 2 bit rate, $R_m = 14.4$ Kbps, with the power controlled received SIR parameter $\gamma_v = 8.5$ dB (7.0975).
- Idle bit rate $R_0 = 1.2$ Kbps.

The standard deviation parameter of the power controlled SIR was taken to be the same for both classes, i.e, we take $\sigma_c = \sigma_v = \sigma$. This corresponds physically to assuming that the power control mechanism works in an identical fashion for users of both classes. As before, the number of class 1 users k_c is taken to be the primary variable; the number of class 2 users supportable, k_v , is then

computed in accordance with the constraint (16). The number of class 2 users is then varied from 1 to k_v , and the throughput gain G is computed. This is done for various values of the standard deviation parameter σ , varying from $\sigma = 0$ dB (perfect power control) to $\sigma = 2.5$ dB. The results are presented in Figures (2a), (2b) and (2c).

It is seen that power control imperfections significantly affect the achievable throughput gains. However, the gains remain substantial, especially when the class 2 users predominate, as in Figure (2c).

This concludes our study of the case where the peak transmit powers of the class 2 mobiles were unconstrained. The other possible case is studied in the next section.

4 Case 2 : Constrained peak transmit powers

We go on now to consider the case where there are constraints on the peak transmit power that a class 2 mobile may use. In order to motivate the following discussion, consider a situation where an upper limit is placed on the peak interference a particular cell can create in another. Clearly, such a constraint would translate to peak transmit power limits on the mobiles in that cell; also, mobiles located close to the boundary between the cells would have more stringent peak transmit power limits than those in the interior. Considering the application of the scheduled transmission mode described earlier to such a situation, we note that the presence of constraints on the peak transmit powers translates to constraints on the peak transmission rate, which limits the throughput gains due to scheduling. Thus, in order to better exploit the looser constraints on the class 2 users in the cell interior, it might be advantageous in such situations to schedule the transmissions of only a certain subset of the class 2 users in the cell. In this paper, we develop analytical tools by which such situations, and in particular, the subdivision of the class 2 users into subsets, can be studied; the detailed analysis will be left for future expositions.

We proceed here along the same lines as in Section (3), which dealt with the case with unconstrained mobile transmit powers. We assume the presence of perfect power control; extensions to cases with imperfect power control, in the fashion of the previous section, will be handled in later expositions. The basic constraint of Section (2) is now specialized to the case in question.

4.1 Existence of a minimum total power solution

As before, we first consider the situation where we have k_c class 1 and k_v class 2 users in the system, where $\{k_c, k_v\}$ is a maximal set. Using the notation of Section (2), Equation (1) now becomes

$$\frac{k_v}{\left(\frac{W}{R_m \gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} < 1 - \frac{IW}{\min_i \left\{ \left[p_i h_i \left(\frac{W}{R_i \cdot \text{SIR}_i} + 1 \right) \right] \right\}_{i=1}^{k_v+k_c}}, \quad (17)$$

where $\text{SIR}_i = \gamma$ or γ_1 , depending on whether the i^{th} generic user belongs to class 1 or 2, respectively. The class 2 users can be expected to have a higher minimum rate requirement than the class 1 users. Hence, other things being equal, one would expect the uplink of a class 2 to form the “weakest link”. Accordingly, we assume that this particular user is user j , belonging to class 2, i.e, we assume

$$p_j h_j \left(\frac{W}{R_j \cdot \gamma_1} + 1 \right) = \min_i \left\{ \left[p_i h_i \left(\frac{W}{R_i \cdot \text{SIR}_i} + 1 \right) \right] \right\}_{i=1}^{k_v+k_c}.$$

The constraint (17) then becomes

$$\frac{k_v}{\left(\frac{W}{R_m \gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} < 1 - \frac{IW}{\left[p_j h_j \left(\frac{W}{R_m \cdot \gamma_1} + 1\right)\right]} . \quad (18)$$

Given that k_c class 1 users are present, Equation (18) gives the number k_v of class 2 users supportable. Given a situation where one has k_c class 1 and upto k_v class 2 users in the system, the transmit powers assigned will be such that

- The class 1 and class 2 users achieve exactly their required SIR's of γ and γ_1 , respectively.
- The sum of the powers will be as small as possible. In addition, at any instant, the transmit power assigned to any user will not exceed its maximum permitted value.

We now consider the problem of maximizing the throughput of the class 2 users.

4.2 Maximization of Class 2 throughput

As in Section (3.2), we consider a situation in which one has k_c class 1 users and $k_1 \leq k_v$ class 2 users present, and the two transmission modes defined therein. We note that due to the presence of an upper limit on its maximum transmit power, an information-transmitting class 2 user cannot exploit the reduced interference due to scheduling as efficiently as it could before. Hence, it is expected that the throughput gains in this case will be lower than before; however, it is yet not clear if mode 2 transmission yields any gains over mode 1 in this case. This question is investigated in the following sections, where the average throughput achieved by the class 2 users in the transmission modes 1 and 2 (as defined in Section (3.2)) are computed.

4.2.1 Mode 1 : Unscheduled class 2 transmissions

In this mode, all the class 2 users are allowed to transmit information simultaneously, at a rate $R_1 (\geq R_m)$. We note here that if a particular uplink forms the “weakest link”, it will continue to do so when the corresponding mobile transmits at a higher rate. Hence, the class 2 user j , which formed the weakest link at a rate R_m , continues to do so when transmitting information at rate R_1 . Accordingly, let the rate R_1 be chosen to be its maximum possible value, so that²

$$\frac{k_1}{\left(\frac{W}{R_1 \gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} = 1 - \frac{IW}{\left[p_j h_j \left(\frac{W}{R_1 \cdot \gamma_1} + 1\right)\right]} . \quad (19)$$

Upon comparing with the constraint (18), it can be shown that

$$R_1 = R_m \cdot \frac{\frac{W}{\gamma_1} \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right]}{\left\{ \frac{W}{\gamma_1} \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right] - R_m (k_v - k_1) \right\}} .$$

Assuming a negligible retransmission rate, R_1 is also the per-user average throughput of the class 2 users in this mode.

²Strictly speaking, one should have an inequality, l.h.s < r.h.s at this point; we are, however, considering maximum values

Using the expressions derived in [5], the transmit powers assigned to the various users can be written as follows. We note that the peak and average transmit powers are the same in this mode for both user classes.

- $P_{1,i}^{(c)}$, the power assigned to the i^{th} class 1 user in mode 1 is

$$P_{1,i}^{(c)} = \frac{IW}{\left(\frac{W}{R\gamma} + 1\right) h_i^{(c)} Y}, \quad (20)$$

where

$$Y = 1 - \frac{k_1}{\left(\frac{W}{R_1\gamma_1} + 1\right)} - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}.$$

- $P_{1,i}^{(v)}$, the power assigned to the i^{th} class 2 user in mode 1 is

$$P_{1,i}^{(v)} = \frac{IW}{\left(\frac{W}{R_1\gamma_1} + 1\right) h_i^{(v)} Y}, \quad (21)$$

the quantity Y being as defined above.

□ Note : The transmit power assigned to user j of class 2 can be seen to be $P_{1,j}^{(v)} = p_j$. Thus, the transmit powers are assigned such that the user corresponding to the weakest link is transmitting at its allowed maximum power.

4.2.2 Mode 2 : Scheduled class 2 transmissions

As in Section (3.2.2), the class 2 transmissions are now time-shared so that at any time instant, k_2 of them are transmitting information at a rate R_2 , while the remaining $(k_1 - k_2)$ are simply maintaining synchronization with the base at a rate R_0 . As noted before, the class 2 user j continues to form the weakest link. Let the rate R_2 be chosen to be its maximum allowed value, such that

$$\frac{k_2}{\left(\frac{W}{R_2\gamma_1} + 1\right)} + \frac{k_1 - k_2}{\left(\frac{W}{R_0\gamma_1} + 1\right)} + \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} = 1 - \frac{IW}{\left[p_j h_j \left(\frac{W}{R_2\gamma_1} + 1\right)\right]}.$$

Upon comparing with the constraint (19), it can be shown that R_2 is given by

$$R_2 = R_0 \cdot R_1 \cdot \frac{\left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} - \frac{(k_1 - k_2)}{\left(\frac{W}{R_0\gamma_1} + 1\right)}\right]}{\left\{R_0 \cdot \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right] - R_1 \cdot \frac{(k_1 - k_2)}{\left(\frac{W}{R_0\gamma_1} + 1\right)}\right\}}. \quad (22)$$

Before going on to compute the throughputs being achieved, we pause to compare the assigned transmit powers in this mode to those of mode 1. We note that in this mode, the peak and average powers are the same for the class 1 users, but are different for the class 2 users. Using the expressions in [5], we have

- $P_{2,i}^{(c)}$, the transmit power assigned to the i^{th} class 1 user in mode 2 is

$$P_{2,i}^{(c)} = \frac{IW}{\left(\frac{W}{R\gamma} + 1\right) h_i^{(c)} Z}, \quad (23)$$

where

$$Z = 1 - \frac{k_2}{\left(\frac{W}{R_2\gamma_1} + 1\right)} - \frac{k_1 - k_2}{\left(\frac{W}{R_0\gamma_1} + 1\right)} - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} = Y \cdot \frac{\left(\frac{W}{R_1\gamma_1} + 1\right)}{\left(\frac{W}{R_2\gamma_1} + 1\right)},$$

Y being as in Equations (20, 21). It can be shown that $P_{2,i}^{(c)} \leq P_{1,i}^{(c)}$, where $P_{1,i}^{(c)}$ is as given in Equation (20). Hence, a class 1 users expends lesser average (and peak) transmit power in mode 2 than it does in mode 1.

- $P_{2,i}^{(v)}$, the average transmit power expended by the i^{th} class 2 user in mode 2, is

$$P_{2,i}^{(v)} = \frac{k_2}{k_1} \cdot \frac{IW}{\left(\frac{W}{R_2\gamma_1} + 1\right) h_i^{(v)} Z} + \frac{(k_1 - k_2)}{k_1} \cdot \frac{IW}{\left(\frac{W}{R_0\gamma_1} + 1\right) h_i^{(v)} Z}, \quad (24)$$

where, as before, the factors $\frac{k_2}{k_1}$ and $\frac{(k_1 - k_2)}{k_1}$ account for the fractions of information transmission and synchronization times respectively, and the quantity Z is as defined above. It can be shown that $P_{2,i}^{(v)} \leq P_{1,i}^{(v)}$, where $P_{1,i}^{(v)}$ is as in Equation (21). Hence, a class 2 mobile uses lesser average transmit power in mode 2 as compared to mode 1.

- $P_{2,i,peak}^{(v)}$, the peak transmit power expended by this user, is simply the power used when transmitting at the peak rate of R_2 . This is given as

$$P_{2,i,peak}^{(v)} = \frac{IW}{\left(\frac{W}{R_2\gamma_1} + 1\right) h_i^{(v)} Z}; \quad (25)$$

it can be shown easily that $P_{2,i,peak}^{(v)} = P_{1,i}^{(v)}$ = peak power used by the same user in mode 1 transmission. Thus, the class 2 users transmit at the same peak power in either mode.

□ Note : As in mode 1, the peak transmit power assigned to user j of class 2 can be seen to be $P_{2,j,peak}^{(v)} = p_j$, the maximum allowed.

In summary, mode 2 transmission imposes lower average transmit power requirements than mode 1 transmission for users of both classes. In addition, the peak power requirements for the class 2 users are identical in both the transmission modes. From this, one can conclude that at any time instant, the sum of the transmit powers is lesser in mode 2 than in mode 1 transmission; hence, mode 2 transmission leads to lesser instantaneous other-cell interference.

We go on now to consider the average throughput being achieved by the class 2 users. As before, only the fraction of time when information is being transmitted at rate R_2 contributes to the useful throughput. Assuming a negligible retransmission rate and using Equation (22), we get the average class 2 throughput T as

$$T = \frac{k_2}{k_1} \cdot R_2 = \frac{k_2}{k_1} \cdot R_1 \cdot \frac{\left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} - \frac{(k_1 - k_2)}{\left(\frac{W}{R_0\gamma_1} + 1\right)}\right]}{\left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} - \frac{R_1}{R_0} \cdot \frac{(k_1 - k_2)}{\left(\frac{W}{R_0\gamma_1} + 1\right)}\right]}. \quad (26)$$

We are interested, as before, in maximizing T ; the conditions under which $T > R_1$, the mode 1 throughput, are of particular interest. We make the following observations about this maximization.

Observation 3 For any admissible value of k_2 , T is a decreasing function of the idle rate R_0 .

Observation 4 Given a non-trivial R_0 , one has the following possibilities :

$$\begin{aligned} \text{Case 1 } R_0 &\leq \frac{\frac{W}{\gamma_1} \cdot \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right]^2}{\left\{k_1 + (k_1 - 1) \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right] - \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right]^2\right\}} : \\ &- \text{ In this case, one has } T \geq R_1 \text{ for the set of } k_2 \text{ values} \\ &k_2 \in \left\{1, \dots, \left\lceil k_c \cdot \frac{\left(\frac{W}{R_0\gamma_1} + 1\right)}{\left(\frac{W}{R\gamma} + 1\right)} + k_1 \cdot \frac{R_1}{R_0} - \left(\frac{W}{R_0\gamma_1} + 1\right) \right\rceil\right\}; \text{ this set is non-empty iff the con-} \\ &\text{ dition} \\ &\frac{IW}{h_j p_j} \leq 1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} - \frac{k_1 \cdot \left[2 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right]}{\left\{\left(\frac{W}{R_0\gamma_1} + 1\right) \cdot \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right] + 1\right\}} \end{aligned} \quad (27)$$

is satisfied. In the above condition, it can be shown that r.h.s $\in [0, 1)$.

$$\text{Case 2 } R_0 > \frac{\frac{W}{\gamma_1} \cdot \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right]^2}{\left\{k_1 + (k_1 - 1) \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right] - \left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)}\right]^2\right\}} :$$

- In this case, one has $T < R_1$ for any admissible k_2 , i.e., $\forall k_2 \in [1, \dots, k_1 - 1]$.

In Case 1, T is a decreasing function of $k_2 \forall k_2$ s.t $T(k_2) \geq R_1$; hence, T is maximized by choosing $k_2 = 1$. In addition, T is a decreasing function of the parameter $\frac{IW}{h_j p_j}$.

As before, Observation 3 states a fact that is intuitively obvious; it is Observation 4 which has more practical value. It sets an upper limit on the synchronization rate R_0 , beyond which mode 2 transmission leads to throughput losses as compared to mode 1; the upper limit is henceforth referred to as $R_{0,upper}$. In addition, a condition is set up, in terms of the parameter $\frac{IW}{h_j p_j}$, as to when

scheduling leads to throughput gains. The quantity $\frac{IW}{h_j p_j}$ is the ratio of the power received from all sources outside the cell to that of the “weakest link” user at the base station, assuming that that user is transmitting at its maximum power; it will henceforth be referred to as the “received power ratio” (RPR). Hence, given a certain system setup, the RPR constraint (27) can be used (along with the constraint on synchronization rate) to make the decision as to whether effort should be expended in scheduling the class 2 users. The constraint (27) can also be used as a pointer to what maximum transmit power limits a mobile should possess. As such, the fact that T is a decreasing function of the RPR indicates that a higher maximum transmit power limit leads to greater throughput gains; this is intuitively apparent, since the system then tends towards the one studied earlier, where one had no constraints on the peak transmit power.

In what follows, we assume that all the parameters are such that case 1 in Observation 4 is valid. We focus on the maximum throughput case $k_2 = 1$, which corresponds to only one class 2 user transmitting information at a time, and denote the corresponding class 2 throughput as T_2 . One then has the throughput gain afforded by mode 2 as compared to mode 1 transmission as

$$G = \frac{T_2}{R_1} = \frac{1}{k_1} \cdot \frac{\left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} - \frac{(k_1 - 1)}{\left(\frac{W}{R_0\gamma_1} + 1\right)} \right]}{\left[1 - \frac{k_c}{\left(\frac{W}{R\gamma} + 1\right)} - \frac{R_1}{R_0} \cdot \frac{(k_1 - 1)}{\left(\frac{W}{R_0\gamma_1} + 1\right)} \right]}. \quad (28)$$

4.3 Numerical results and discussions

As before, the following system parameters were chosen in order to compute the throughput gain G for various situations :

- Spreading bandwidth $W = 1.23 MHz$.
- Class 1 bit rate $R = 9.6$ Kbps, with a minimum SIR of $\gamma = 7$ dB (5) required to be received.
- Minimum class 2 bit rate, $R_m = 14.4$ Kbps, with a minimum SIR of $\gamma_1 = 8.5$ dB (7.0795) required to be received.
- Idle bit rate $R_0 = 1.2$ Kbps.

In addition, the RPR was chosen as 0.1; the variation of G with the RPR is also studied later in this section. The number of class 1 users k_c was taken to be the primary variable; based on this, the maximum number k_v of class 2 users permitted was computed according to Constraint (18). The number of class 2 users in the system was then varied from 1 to k_v , and the corresponding gain G was computed from Equation (28). The results are plotted in Figures (3a, 3b, 3c), for three values of k_c , varying from a predominantly class 1 system to an almost entirely class 2 system. As before, it is seen that when class 1 users predominate in the system, the gains offered are small (Figure (3a)). However, as the fraction of class 2 users in the system increases, the per-user throughput gain offered by mode 2 transmission are quite substantial as compared to mode 1 transmission. A comparison can be made between Figures (3a, 3b, 3c) and the results for the corresponding cases with unconstrained maximum power in Figures (1a, 1b, 1c). As expected, it is seen that the presence of the peak transmit power constraints leads to a reduction in the throughput gains; however, the losses are not significant.

The variability of the RPR constraint, as well as $R_{0,upper}$, are also of interest. For example, a very low RPR requirement would indicate a very high peak power requirement; similarly, as mentioned before, a value of $R_{0,upper}$ lower than that dictated by synchronization requirements might indicate that scheduling is not practical. With respect to Figure (3b), the RPR constraint and $R_{0,upper}$ are plotted as functions of the class 2 population in Figures (4a) and (4b) respectively. It is seen that in this particular example, neither of these parameters puts stringent requirements on the system. For example, from Figure (4b), we see that the most stringent value of $R_{0,upper}$ is about 6 kbps, which would imply a rather loose requirement on the synchronization rate R_0 .

Finally, the variation of the throughput gain G with the RPR for the same case as above is plotted in Figure (4c). As indicated earlier, it is seen that G is a decreasing function of the RPR.

A comparison of Figures (4a) and (4c) validates the RPR constraint (27); when the RPR is above that prescribed by the RPR constraint in Figure (4a), G falls below unity.

This concludes the comparison between the two transmission modes in the case where the class 2 mobiles's peak powers are constrained. The main results of this paper are now summarized.

5 Summary of Results

In this paper, two transmission modes for the delay tolerant class 2 users were compared; in mode 1, the baseline, a class 2 user could transmit information at any time, while in the proposed mode 2, the class 2 users were scheduled so that at any time instant, only a limited number transmit information, while the remaining simply maintain synchronization. It was seen that in many situations, a special case of mode 2 transmission, in which only one class 2 user is allowed to transmit information at a time, offers significant per user throughput gains as compared to mode 1 transmission. The transmit powers required to achieve these gains were also investigated, and we have two sets of results, corresponding to the following cases :

- Case 1 - No constraints on the class 2 peak transmit power : In this case, we have
 - Each user of both user classes expends the same average power in either transmission mode.
 - In mode 2 transmission, each class 2 user expends a peak power which is about $\frac{k_1}{k_2}$ times that required in mode 1.
- Case 2 - Constrained peak transmit powers for the class 2 users : In this case
 - Each user of either class expends lesser average power in mode 2 transmission than in mode 1 transmission.
 - Each class 2 user expends identical peak powers in both transmission modes.

We can therefore conclude that the throughput gains do not come at the expense of increased average power; they are simply a consequence of the transmission scheme.

We go on now to discuss the additional complexity that mode 2 transmission entails.

6 Implementation issues

It is clear that since mode 2 transmission requires some control to be imposed on the class 2 transmission, some additional complexity would have to be added to the system. Some of the requirements of mode 2 transmission, and an outline of possible solutions can be enumerated as follows :

- The class 2 mobiles must be capable of variable rate transmission, and the base station capable of the corresponding reception. Clearly, the information transmission rate to be used by the class 2 users (which was denoted as R_2) will be decided by the base station, using information about the population distribution in the system, and, in the constrained transmit power case, knowledge of the RPR parameter. In practice, the user population distribution may be a slowly changing variable. This would imply that class 2 information rate changes do not

have to be effected too often. The class 2 mobiles transmission rate alternates between the information rate R_2 and the synchronization rate R_0 ; hence, there must also be a mechanism to coordinate these rate changes with the base station.

- There must be a mechanism to schedule the class 2 transmissions. This can be achieved very simply by having the class 2 mobiles transmit in a “round-robin” fashion, with each mobile getting a fixed information transmission time of τ units within each cycle time of C units. The actual value of C will be decided based on practical considerations such as, for example, the amount of buffer space to be provided at the mobile, the average number of class 2 users expected, etc.; once C is known, τ is simply given by $\tau = \frac{C}{k_1}$, where as before, k_1 is the number of class 2 users.

Clearly, this form of scheduling will also have to be centrally controlled by the base station, using information about the user population distribution. There has to be a mechanism by which the base station informs a particular class 2 mobile about the cycle time C , its information transmission time τ , as well as its “place” within the cycle; we refer to this as the slotting mechanism. We note that though it is desirable to have a fine slotting of the class 2 users, i.e, an arrangement of the slots such that their transmissions do not overlap, it may be difficult to achieve in practice without a significant increase in complexity. In that case, one could resort to coarse slotting, in which some (as small as possible) part of the slot assigned to a user overlaps with that assigned to another user. The overlapping portions would then correspond to the case $k_2 = 2$ rather than the desired best case $k_2 = 1$. This would lead to a certain reduction in the throughput gains, but would simplify the implementation of the slotting mechanism.

It may be noted that more sophisticated scheduling schemes, which exploit the traffic characteristics of the class 2 mobiles, can be designed above the basic scheme. For example, it may happen that a class 2 mobile may have no information to transmit in its assigned slot, in which case that particular slot could be reassigned to some other user. Clearly, such schemes would lead to additional throughput gains; however, the base station would need additional knowledge about the state of the data in the mobiles, and would add some additional scheduling load to the system.

- A class 2 mobile would need to use significantly higher transmit power in its information transmission slot time τ as compared to the rest of the time in a cycle in order to maintain the same SIR. Hence, some changes would have to be made to the conventional power control algorithm. During its information transmission slot of time τ , a class 2 mobile must increase its transmit power to correspond to the rate R_2 , and lower it the rest of the cycle to correspond to the rate R_0 . However, since the transmission cycles are repetitive, the computations as to what power to use in what part of the cycle may not be a major problem; one could make use of the knowledge about the transmit powers used in the corresponding parts of previous cycles.

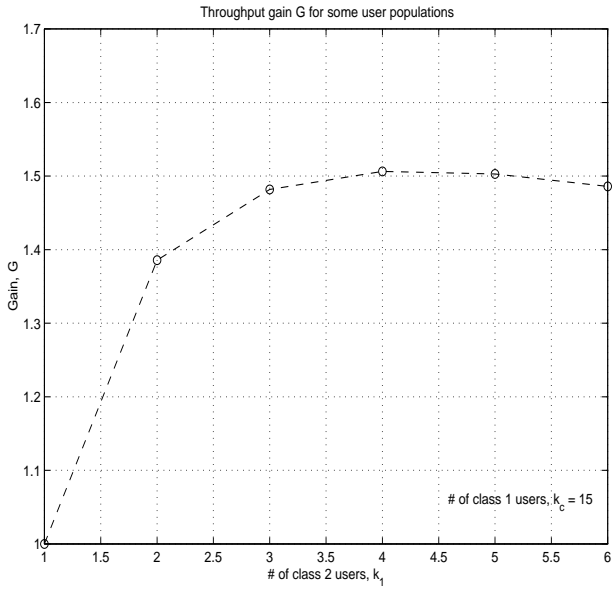
It may be noted that the base station retains the flexibility to decide on the transmission mode. For example, in a particular situation, the gains afforded by mode 2 transmission may not be significant in comparison to the effort. In this situation, the base station could order the class 2 mobiles to revert to mode 1 transmission.

The future work proposed to be done is now indicated.

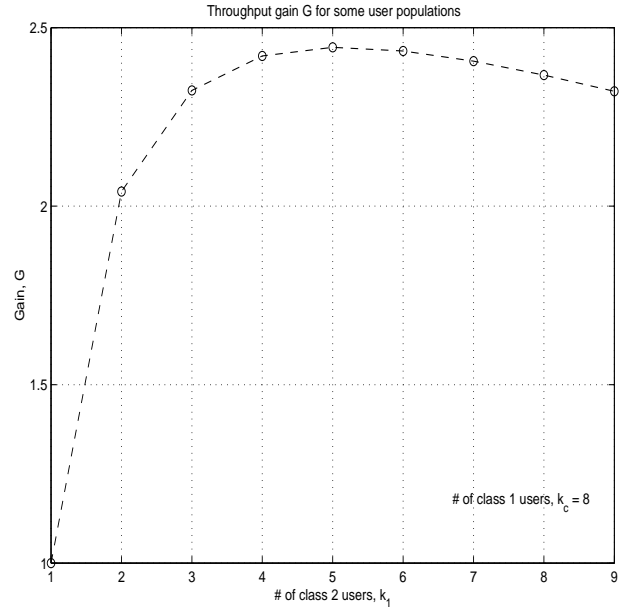
7 Future Work

As indicated Section (4), future work will consider scenarios in which one has constraints on the peak power that a mobile is permitted to use, and in which an upper limit is placed on the maximum instantaneous interference a particular cell can create in another. In this case, mobiles located close to the boundary between the cells will have more stringent maximum instantaneous transmit power limits than those in the interior. For the transmission scheme proposed in this paper, such constraints will reduce the potential throughput gains if the users were apriori assigned to classes 1 and 2. In order to better exploit the looser constraints on the class 2 users in the cell interior, it might be advantageous in such situations to schedule the transmissions of only a certain subset of the class 2 users in the cell, and re-assign the remaining class 2 users to the class 1 set. Using the analysis in Section (4) as a starting point, we propose to study strategies by which such a classification of the class 2 users may be achieved, and compare the throughputs achieved to corresponding cases with unscheduled transmissions.

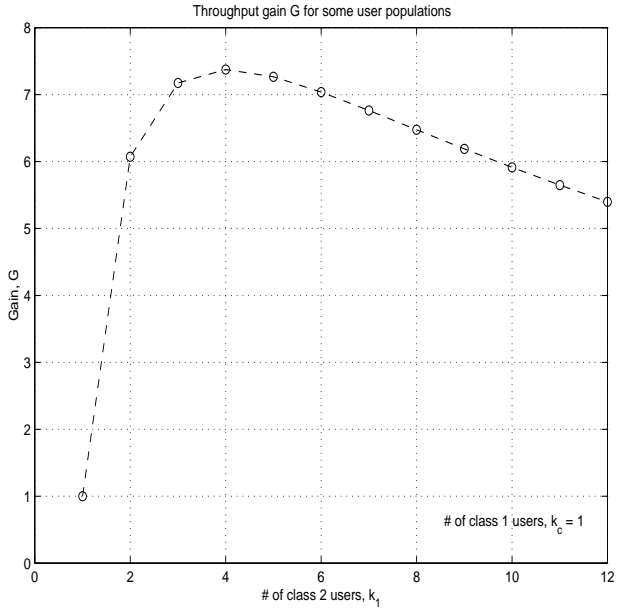
We also propose to investigate the suitability of the transmission scheme proposed here for the downlink of the CDMA system. The straightforward generalizations to a CDMA system consisting of several user classes could also be performed.



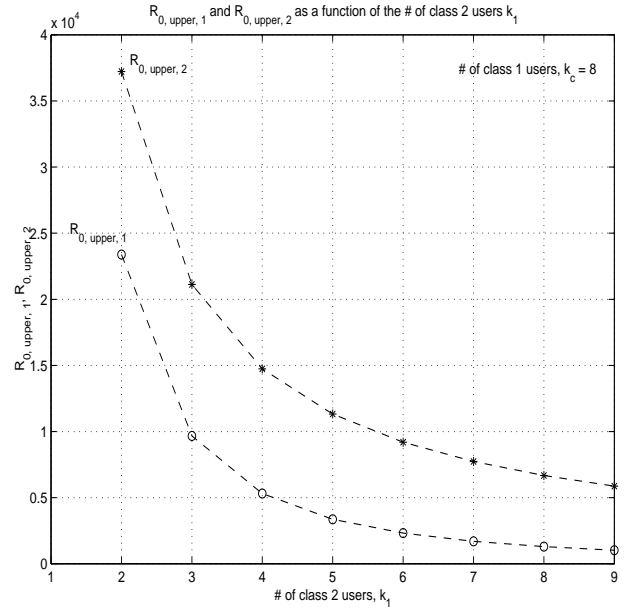
(a)



(b)

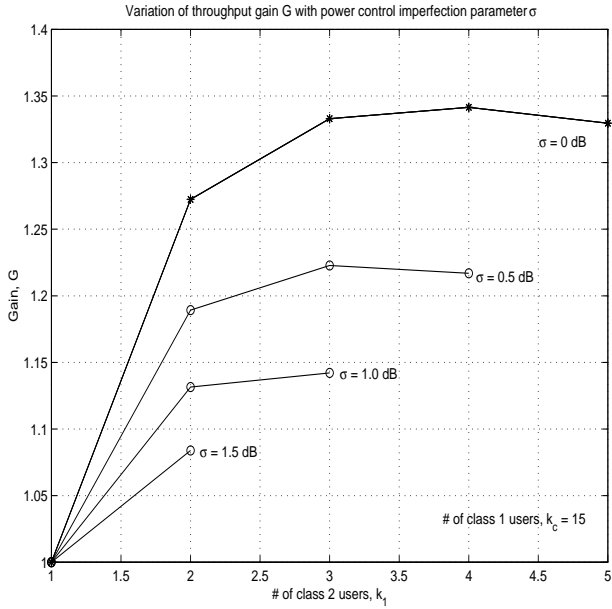


(c)

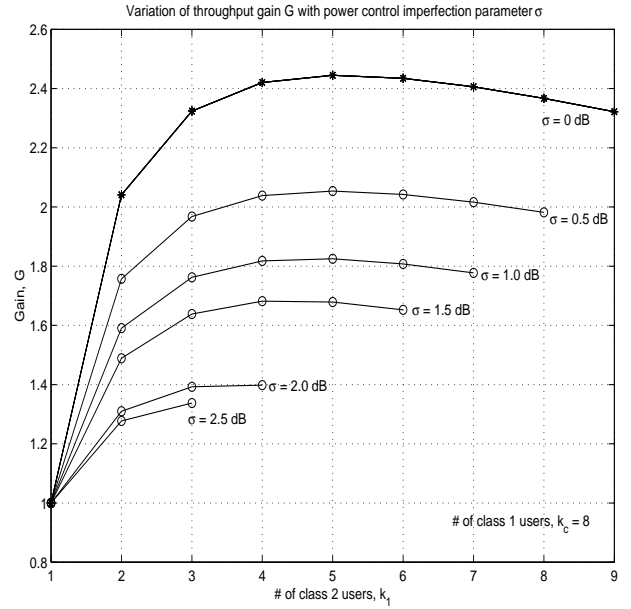


(d)

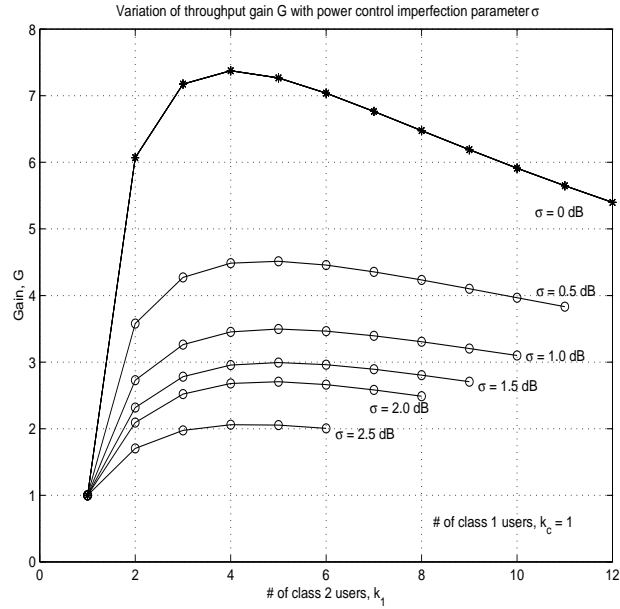
Figure 1: The gain $G = \frac{T_2}{R_1}$ as a function of class 2 population k_1 , for (a) $k_c = 15$, $k_v = 6$ (b) $k_c = 8$, $k_v = 9$ and (c) $k_c = 1$, $k_v = 12$. (d) shows the variation of the synchronization rate limits $R_{0,upper,1}$ and $R_{0,upper,2}$ as a function of k_1 , for the $k_c = 8$ case.



(a)

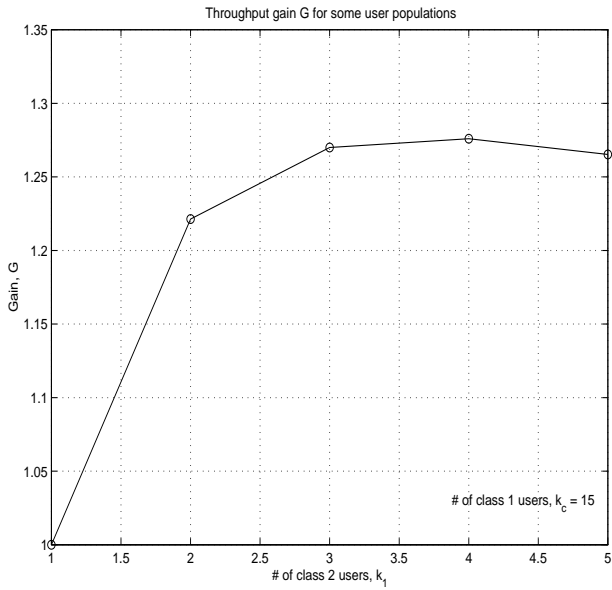


(b)

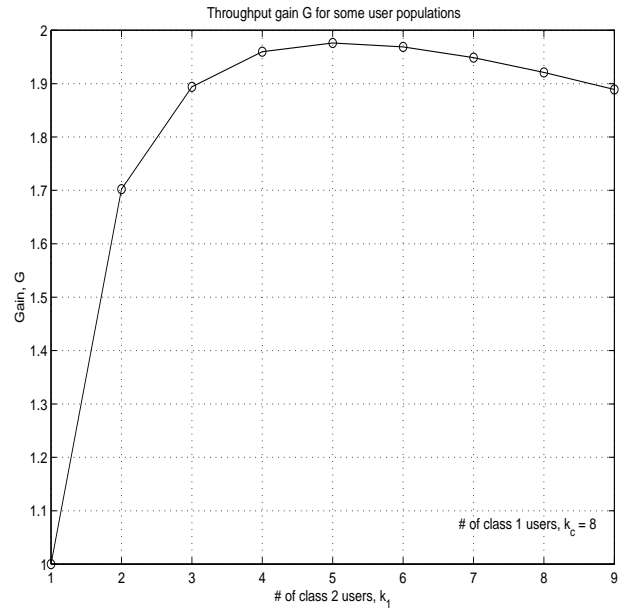


(c)

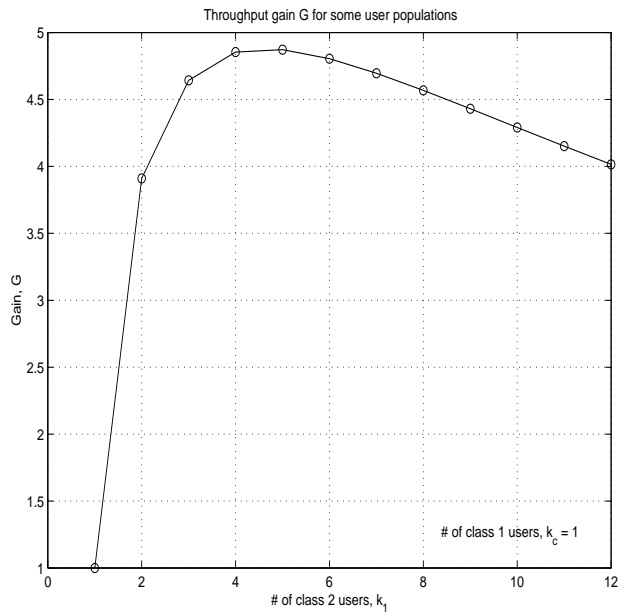
Figure 2: The gain $G = \frac{T_2}{R_1}$ as a function of the standard deviation σ of the power controlled SIR, for various class 2 populations k_1 , for (a) $k_c = 15$ (b) $k_c = 8$ and (c) $k_c = 1$.



(a)

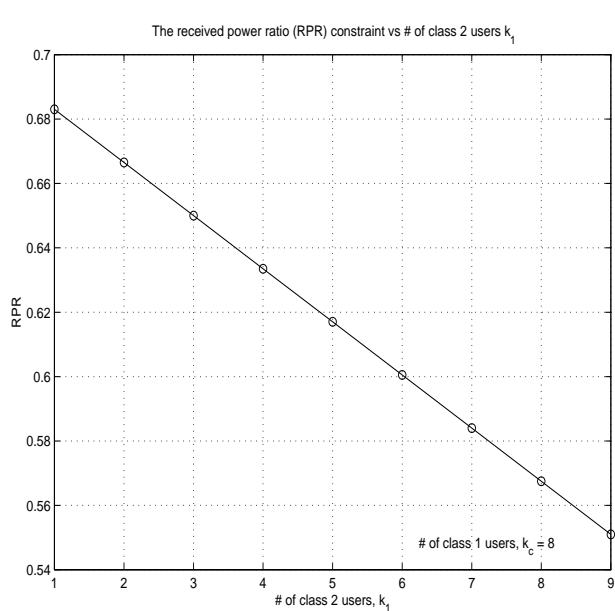


(b)

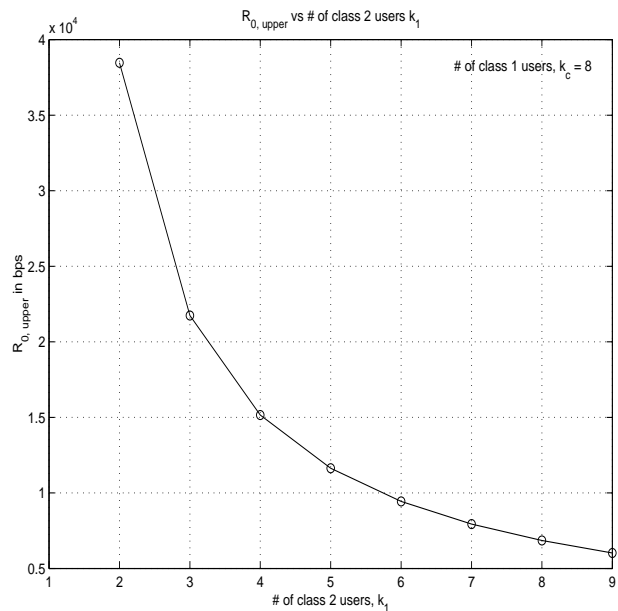


(c)

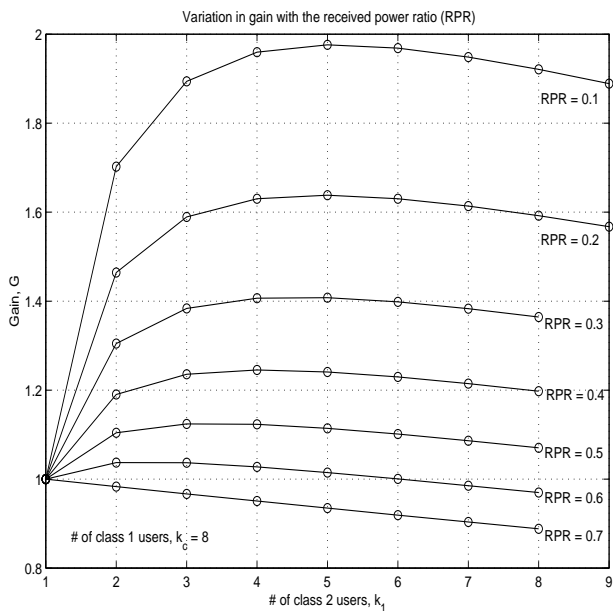
Figure 3: The gain $G = \frac{T_2}{R_1}$ as a function of class 2 population k_1 , with RPR = 0.1, for (a) $k_c = 15$, $k_v = 5$ (b) $k_c = 8$, $k_v = 9$ and (c) $k_c = 1$, $k_v = 12$.



(a)



(b)



(c)

Figure 4: For the $k_c = 8$ case in Figure (3b), (a) shows the variation of the RPR as a function of k_1 , (b) shows the variation of the synchronization rate limit $R_{0, upper}$ as a function of k_1 and (c) shows the variation of the throughput gain G as a function of the RPR and k_1 .

References

- [1] A. Sampath and J. M. Holtzman. “Access Control of Data in Integrated Voice/Data CDMA Systems : Benefits and Tradeoffs”. In *ICC'97*.
- [2] R. Wyrwas, M. J. Miller, R. Anjaria and W. Zhang. “Multiple Access Options for Multi-Media Wireless Systems”. In *Proceedings of the Third WINLAB Workshop on Third Generation Wireless Information Networks*, pages 289–294, April 1992.
- [3] Chih-Lin I and K. K. Sabnani. “Variable Spreading Gain CDMA with Adaptive Control for true Packet Switching Wireless Network”. In *Proceedings of ICC'95*, pages 725–730, June 1995.
- [4] A. Baier, U. C. Fiebig, W. Granzow, W. Koch, P. Teder and J. Thielecke. “Design Study for a CDMA-Based Third-Generation Mobile Radio System”. *IEEE Journal on Special Areas in Communications*, 12(4), May 1994.
- [5] A. Sampath, P. S. Kumar and J. M. Holtzman. “Power Control and Resource Management for a Multimedia Wireless CDMA System”. In *PIMRC'95*, September 1995.
- [6] S. V. Hanly and P. Whiting. “Information-Theoretic Capacity of Multi-receiver Networks”. *Telecommunication Systems*, 1:1–42, 1993.
- [7] A. D. Wyner. “Shannon-Theoretic Approach to a Gaussian Cellular Multiple Access Channel”. *IEEE Transactions on Information Theory*, 40(6):1713–1727, November 1994.
- [8] S. V. Hanly. “*Information Capacity of Radio Networks*”. PhD thesis, University of Cambridge, 1993.
- [9] L. Yun and D. Messerschmitt. “Power Control for Variable QOS on a CDMA Channel”. In *Proceedings of MILCOM'94*, June 1994.
- [10] A. J. Viterbi. “*Principles of Spread Spectrum Communication*”. Addison-Wesley, 1995.
- [11] A. M. Viterbi and A. J. Viterbi. “Erlang Capacity of a Power Controlled CDMA System”. *IEEE Journal on Selected Areas in Communications*, 11(6):892–900, August 1993.
- [12] N. C. Beaulieu, A. A. Abu-Dayya and P. J. McLane. “Estimating the Distribution of a Sum of Independent Lognormal Random Variables”. *IEEE Transactions on Communications*, 43(12):2869–2873, December 1995.
- [13] S. C. Schwartz and Y. S. Yeh. “On the Distribution Function and Moments of Power Sums with Lognormal Components”. In *Bell Systems Tech. Journal*, volume 61, pages 1441–1462. Bell Laboratories, September 1982.