2002-01-0371

An Analytic Model for Cylinder Pressure in a Four Stroke SI Engine

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ABSTRACT

An analytic model for cylinder pressures in spark ignited engines is developed and validated. The main result is a model expressed in closed form that describe the in-cylinder pressure development of an SI engine. The method is based on a parameterization of the ideal Otto cycle and takes variations in spark advance and air-to-fuel ratio into account. The model consists of a set of tuning parameters that all have a physical meaning. Experimental validation on two engines show that it is possible to describe the in-cylinder pressure of a spark ignited combustion engine operating close to stoichiometric conditions, as a function of crank angle, manifold pressure, manifold temperature and spark timing.

INTRODUCTION

More complex engine designs are continously being considered and developed. To manage the increased complexity the traditional engine control designs that are based on calibrated maps will require much development time. Here the model based techniques can play an important role since the models provide the couplings between inputs, outputs, and parameters and have the potential to reduce calibration time. Cylinder pressure traces contain information about the work and emission producing process which is valuable for the engine management system. For some diagnosis and control problems it would be beneficial to have information about the cylinder pressure available, some examples are spark advance control, estimation of torque generation, and misfire detection.

Current cylinder pressure models are computationally demanding and it is not yet possible to simulate or to have an observer for the cylinder pressure online in conventional engine control units. Therefore computationally simple models for cylinder pressure in combustion engines are tractable for control purposes. Here an analytical model is developed and validated for the cylinder pressure of a spark ignited (SI) engine, which is computationally tractable since it does not require a numerical solution of the ordinary differential equations, and can thus be used on-line. Work in this direction has already been made for diesel engines in [1] where the differential equation for the cylinder temperature is reduced to an Riccati differential equation that is solved analytically.

One of the key ideas behind the model here is based on the observation that the ideal Otto cycle provides valuable information about the compression and expansion processes. These two processes are seen in the real measured pressure traces under normal operating conditions as the asymptotes before the ignition and after the combustion is finished. The real cycle is similar to the ideal Otto cycle and the similarities are largest early in the compression and late in the expansion. It is characterized by the compression and expansion phases which are well defined by the states of the fluid. The second key idea is that the heat release analysis procedure based on pressure ratio management, developed by Matekunas [2, 3, 4], gives a good approximation of the heat release trace. This method can easily be inverted and used to interpolate between the compression and expansion trace.

The model is described using measurements readily available in production engines and using a set of tuning parameters that have physical interpretations and are closely connected to the ideal Otto cycle. This work focuses on investigating the accuracy of the simple model, not on how parameters can be determined or predicted. The interesting question is, how well the simple model describes the in-cylinder pressure during the high pressure and combustion phase. The burn rate of the combustion has a big influence on the pressure and in the validation the burn rate is considered to be known. However, there are several approaches presented for estimating the burn rate which could be used directly in this model. Two slightly different approaches are described in [12, 5] that present ways of predicting the variations of the burn angles over the full engine operating range by utilizing a reference measurement

in a central area of the operating point. Another approach is described in [6] which is based on the ionization current measurements, this approach is also applicable since the ion current has been used in production cars since 1994, see e.g. [7].

MODEL DESCRIPTION

Figure 1 outlines the ideas behind and the structure within the model. The modeled pressure trace $p(\theta)$ is built up by two asymptotic traces and an interpolation between these. The cylinder pressure model is divided into four parts:

- The compression process is well described by a polytropic process. The polytropic process also encapsulates the heat transfer, so that there is no need to explicitly include the heat transfer in the model.
- The expansion asymptote is also well described by a polytropic process. The reference point for expansion temperature and pressure is calculated using a constant-volume combustion process.
- The concept of pressure ratio management provides a convenient way to interpolate from compression to expansion. Its appearance is very close to the mass fraction burned profile and the Vibe function is used to describe the pressure ratio.
- Gas exchange phase. During the period IVO-IVC the pressure is approximated by the intake manifold pressure. During the period EVO-EVC the pressure is approximated by the exhaust manifold pressure. Between the phases the pressure can be determined through an interpolation using for example a cosine function.

Compression Part in the Cycle

It is a well known fact that the compression process can be modeled with good accuracy by a polytropic process. Such a process is described by a polytropic exponent k_c and a value at one reference point. One point that can be used as reference is the intake valve closing (IVC) which gives the following expressions for the compression pressure and temperature

$$p_c(\theta) = p_{ivc} \left(\frac{V_{ivc}}{V(\theta)} \right)^{k_c} \tag{1}$$

$$T_c(\theta) = T_{ivc} \left(\frac{V_{ivc}}{V(\theta)} \right)^{k_c - 1} \tag{2}$$

These traces describe the cylinder pressure and temperature up to the point of ignition. The temperature model is also necessary to have in this approach since it has a direct impact on the second pressure asymptote.

Determination of initial pressure The manifold pressure gives a good indication of the initial pressure for the compression stroke. However pressure drops over valves as well tuning effects in the intake runners also have an influence. Here a reference condition just before IVC is used to determine the initial pressure

$$p_{ivc} = p_{im}(\theta_{ivc})$$

The crank angle for intake valve closing θ_{ivc} is not exactly known due to production tolerances, it is also used as tuning parameter to compensate for pressure drops over the valves etc. Additionally an affine correction in engine speed is tested which improves the accuracy of the compression pressure model slightly,

$$p_{ivc} = p_{im}(\theta_{ivc}) + c_1 + c_2 * N$$

Here c_1 and c_2 are parameters that have to be determined which increases the flexibility but also the model complexity. To maintain simplicity the evaluation is concentrated on the first model.

Determination of initial temperature It is more difficult to determine the fluid temperature at intake valve closing compared to the pressure, since it is influenced by heat transfer and residual gases that are difficult to measure and determine. The air in the intake manifold is heated from T_{im} to T_a by the hot valves and the locally high heat transfer coefficients in the cylinder. Fuel is added in the ports and undergoes an evaporation which also influences the temperature. By considering the energy equation with a lumped process for heating, evaporation, and mixing, the initial air/fuel mixture temperature can be stated as

$$T_{af} = \frac{m_a\, c_{p,a}\, T_a + m_f\, c_{p,f}\, T_f - m_f\, h_{v,f} + Q}{m_a\, c_{p,a} + m_f\, c_{p,f}}$$

where $h_{v,f}$ is the vaporization enthalpy for the fuel and Q is the heat added to the fresh mixture. Both these are difficult to determine. In the cylinder the residual gases the fresh charge is mixed with the residual gases and the mixture temperature is

$$T_{ivc} = \frac{m_{af}\, c_{p,af}\, T_{af} - m_r\, c_{p,r}\, T_r}{m_{af}\, c_{p,af} - m_r\, c_{p,r}}$$

Prior to mixing the residual gases are cooled down by heat transfer to the walls.

Simplifying the temperature model The outlined models for heating, evaporation, and mixing processes are complex and contain several variables that have to be determined. The central question here is to see how well a simple model can capture the process and therefore some simplifications are made. First it is assumed that the specific heats c_p are the same for the residual gas and the fresh air and fuel mixture yields

$$T_{ivc} = T_{af}\,(1 - x_r) + x_r\, T_r \tag{3}$$

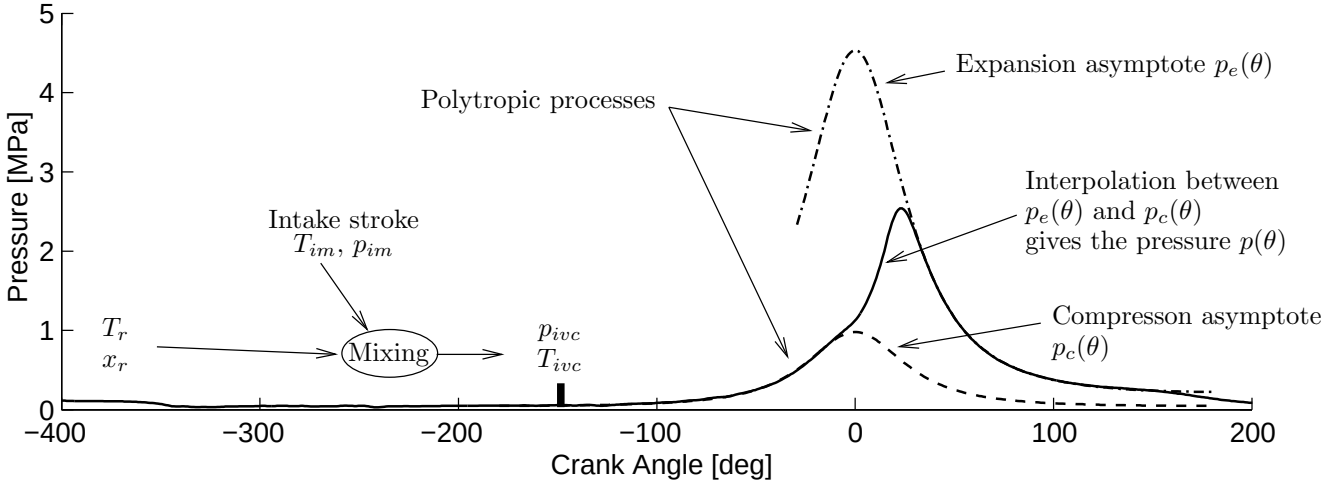

Figure 1: The model is based on the compression pressure, the expansion asymptote, and an interpolation between these. Initial conditions are determined from the intake conditions and the residual gases.

where the residual gas fraction is defined as

$$x_r = \frac{m_r}{m_a + m_f + m_r}$$

The heat transfer to the fresh fluid is also neglected and the fresh fluid is set equal to the temperature in the intake manifold,

$$T_{af} = T_{im}$$

Finally the heat transfer from the residual gas is neglected and the residual gas temperature T_r is set equal to the temperature at the end of the cycle. This approach is mainly justified by its simplicity, but there are some effects that cancels out, e.g. the heat transfer to the fresh mixture and the heat transfer from the residual gases are both neglected and cancels some of the effects of each other. The residual gas fraction x_r is maintained constant but better estimates can be received from an ideal Otto cycle, using for example one of the procedures outlined in [8] or [9].

Asymptotic Final Pressure

The asymptotic expansion process is also modeled as polytropic, with polytropic exponent k_e

$$p_e(\theta) = p_3 \left(\frac{V_3}{V(\theta)} \right)^{k_e} \tag{4}$$

$$T_e(\theta) = T_3 \left(\frac{V_3}{V(\theta)} \right)^{k_e - 1} \tag{5}$$

The determination of V_3, p_3, and T_3, that refer to state three in the ideal Otto cycle, will be discussed below, see Figure 2. The pressure p_3 can be determined experimentally by inverting the pressure ratio analysis [2, 3, 4]. Here a constructive approach is presented it is based on the ideal Otto cycle that accounts for the physical properties in the system. Air-to-fuel ratio and ignition timing both have an influence on the final pressure and these are covered by this approach

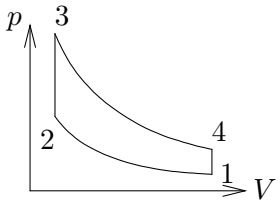

Figure 2: Sketch of the ideal Otto cycle that defines the states 2 and 3.

From state 2 to state 3 in the pV diagram, see Figure 2, the temperature increase is determined by,

$$\Delta T_{comb} = \frac{m_f \, q_{HV} \, \eta_f(\lambda)}{c_v \, m_{tot}} = \frac{(1 - x_r) \, q_{HV} \, \eta_f(\lambda)}{(\lambda (A/F)_s + 1) \, c_v} \tag{6}$$

the fuel conversion efficiency $\eta_f(\lambda)$ comes from figure 3.9 in Heywood [8] and the following expression is used

$$\eta_f(\lambda) = 0,95 \, min(1;\, 1,2\lambda - 0,2)$$

For a thorough discussion of the ideal cycle see e.g. Chapter 5 in [8]. The equation takes the effect of varying air-to-fuel ratios by considering the effect that the fuel mass has on the temperature increase. Exhaust gas recirculation EGR can also be included, this enters the equations in the same way as the residual gas and influences both the initial temperature T_{ivc} and the dilution x_r. In the evaluation the thermodynamic properties of the fluids (i.e. c_v, k_c and k_e of burned and unburned gases) are considered to be independent of λ which is a simplification.

The temperature after the combustion becomes

$$T_3 = T_2 + \Delta T_{comb}$$

Finally the pressure after the combustion is determined from the ideal gas law

$$p_3 = p_2 \frac{T_3}{T_2} \tag{7}$$

where p_2 and T_2 are determined from Equations (1) and (2).

Method to account for combustion phasing Ignition timing and combustion phasing influence the final pressure. This is taken into account by phasing the ideal Otto combustion in a special way using the mass fraction burned trace. The position for the combustion θ_c is chosen to be at TDC if the calculated position for 50% mass fraction burned, mfb_{50}, is at its optimal value, $MFB_{50,OPT}$. If the mfb_{50} position deviates from its optimum, the angle θ_c is set to that deviation in CAD.

$$\begin{aligned} \theta_c &= mfb_{50} - MFB_{50,OPT} \\ mfb_{50} &= \Delta\theta_d + \frac{1}{2}\Delta\theta_b \\ MFB_{50,OPT} &= 8^o\text{ATDC} \end{aligned}$$

where the optimal value for the 50% mass fraction burned is set to 8°. The model above is motivated by the following observations:

- The cycle with the best combustion phasing has best efficiency and lowest exhaust temperature.
- The best phased real cycles have their 50% mass fraction burned position around 8° ATDC.
- The Otto cycle has the best efficiency and lowest exhaust temperature if the combustion is at TDC.

These statements couple the mass fraction burned trace to θ_c in the ideal Otto cycle that defines the volumes at states 2 and 3 to $V_2 = V_3 = V(\theta_c)$.

Combustion part

The pressure ratio management has been investigated in great detail by Matekunas [2, 3, 4]. The pressure ratio is defined using the ratio between the pressure from a firing cycle, $p(\theta)$, and the pressure from a motored cycle, $p_c(\theta)$,

$$\overline{PR}(\theta) = \frac{p(\theta)}{p_c(\theta)} - 1 \tag{8}$$

Traces produced by the pressure ratio are similar to the mass fraction burned profiles, for example the position for $\frac{PR(\theta)}{\max(PR(\theta))} = 0.5$ differs only around $1-2°$ from the position for 50% mass fraction burned [10]. This implies that similar techniques for representing the mass fraction burned profile can be used to simulate the pressure, and this is the method that we have used.

	definition
$\Delta\theta_d$	0-10% burned mass fraction
$\Delta\theta_b$	10-85% burned mass fraction

Table 1 Definition of burn angles.

Interpolation method The combustion part is produced by interpolating between the two assymptotic pressure traces p_c and p_e. The interpolation function is the well known Vibe function [11]

$$PR(\theta) = 1 - e^{-a\left(\frac{\theta-\theta_{SOC}}{\Delta\theta}\right)^{m+1}} \tag{9}$$

which gives the following expression for the pressure

$$p(\theta) = (1 - PR(\theta)) \cdot p_c(\theta) + PR(\theta) \cdot p_e(\theta)$$

From the end of combustion to EVO the pressure follows the expansion asymptote, Eq. (4).

The burn duration $\Delta\theta$ and shape factors a and m in the Vibe function depend on engine speed, air-fuel ratio and EGR ratio. The work presented in [12, 5] develop models that addresses these issues. Here these variations are not considered, instead the parameters are specified in terms of the burn angles $\Delta\theta_d$, $\Delta\theta_b$. The burn angles are defined in Table 1 Given the burn angles the shape parameters a and m can be calculated the approach here is the same as the one used in [13]

$$m = \frac{\ln\left(\frac{ln(1-0.1)}{ln(1-0.85)}\right)}{\ln(\Delta\theta_d) - \ln(\Delta\theta_d + \Delta\theta_b)} - 1 \tag{10}$$

$$a = -\ln(1-0.1)\left(\frac{\Delta\theta}{\Delta\theta_d}\right)^{m+1} \tag{11}$$

Burn duration can be calculated using flame development angle, θ_d, and fast burn angle, θ_b.

$$\Delta\theta \approx 2\theta_d + \theta_b \tag{12}$$

The remaining parts of the cycle

After the exhaust valve has opened the blow down phase begins and the pressure approaches the pressure in the exhaust system. For this phase an interpolation scheme can also be used. The same goes for the phase after the intake valve has opened and the cylinder pressure approaches the intake manifold pressure. For the transitions between these two phases a cosine function can be used for the interpolation. These phases are not validated.

Model parameters and inputs

The model is described by a set of inputs and a set of parameters that have physical interpretation. These inputs

and parameters are summarized in this section The tuning parameters are

c_p	specific heat at the combustion
k_c	polytropic coefficient for the compression pressure
k_e	polytropic coefficient for the expansion pressure
q_{HV}	heating value for the fuel
T_r	residual gas temperature
x_r	residual gas fraction
θ_{ivc}	intake valve closing angle

The inputs are

p_{im}	intake manifold pressure
T_{im}	intake manifold temperature
$\Delta\theta_d$	flame development angle
$\Delta\theta_b$	fast burn angle
θ_{SOC}	start of combustion angle
λ	normalized air to fuel ratio

T_r and the residual gas fraction x_r are used as tuning parameter here but they can also be modeled, for example based on cycle simulations. They are important for the model since they directly influence the initial temperature T_{ivc} Equation 3 and the expansion pressure through Equations 2 and 7. *An illustrative example*: $x_r = 0.07$, $T_{im} = 298$ [K], $T_r = 1000$ [K] yield an initial temperature of $T_{ivc} = 347$ [K], which is a 16% increase. This effect is directly visible in the expansion pressure asymptote.

MODEL EVALUATION

The model consists of a set of parts of which the following will be validated

1. selection of compression pressure
2. selection of compression and expansion polytropic coefficient
3. selection of expansion pressure
4. interpolation between firing and expansion

Finally the influence from air-fuel ratio and ignition timing is studied.

Validation is done in two steps. A model calibration is found to fit the data set best possible. First a general performance is investigated for all data and second a sensitivity analysis is performed on changes in ignition timing, air-fuel ratio and manifold pressure respectively.

Data collection

Two different engines were used for data collection. Engine A is a Daimler Chrysler 3.2L 6 cylinder SI engine and engine B is a Saab 2.3L 4 cylinder SI engine.

The data set for Engine A is a speed-load map of original calibration of ignition and fuel. Engine speed range from 1000 to 5000 rpm and the inlet manifold pressure range from 20 to 100 kPa. The data set from Engine B is taken at 2000 rpm, 100 Nm and with shifted ignition timing and air-fuel ratio.

Compression Pressure

The intake manifold pressure gives a good indication of the starting pressure, p_{ivc} but due to wave effects the initial pressure in the combustion chamber may differ from what is inferred from the intake manifold pressure. There is a possibility to map the initial cylinder pressure as a function of engine speed and load to capture such deviations. Another possibility is to use the air mass flow and volumetric efficiency to support the estimate given by the intake manifold.

Engine A is equipped with an intake manifold pressure sensor positioned close to the intake valve of the cylinder where the cylinder pressure sensor is located. This setup is used to validate how well the intake manifold pressure describes the combustion pressure. The relative error

$$\frac{p_{meas} - p_c}{p_c}$$

at the end of compression was calculated for 89 engine operating conditions. The relative error for all cycles is less than 12%. Adding an affine correction factor for the speed the error was reduced to the range -7–10% , see Figure 3. The compression traces are plotted for a big crank interval to show that there is no systematic deviation during the compression. This indicates that the model assumptions for compression holds and deviations comes from in-accuracy in initial data. The variation -7-10% is not large compared to the general variations in the actual cylinder pressure. To put in perspective to how well the agreement can be for this setup one can use the measured cylinder pressure to determine the initial pressure, p_{ivc} by using the pressure at some place after the intake valves have closed. This has been done on the data and gives a relative error of the same size but with a higher maximum error.

Polytropic exponent for compression and expansion

The data set from Engine B was used for polytropic exponent evaluation. For the compression the polytropic exponent was selected to be $k_c = 1.25$. The expansion phase showed a need for a slightly higher exponent, $k_e = 1.30$ was used. Figure 4 views the choice of polytropic coefficients for one cycle at MBT timing and $\lambda = 1$. The important property of the plot is the slope of the compression and expansion parts. Ideally, both the actual and the simulated pressure traces shall have the same slope in the straight

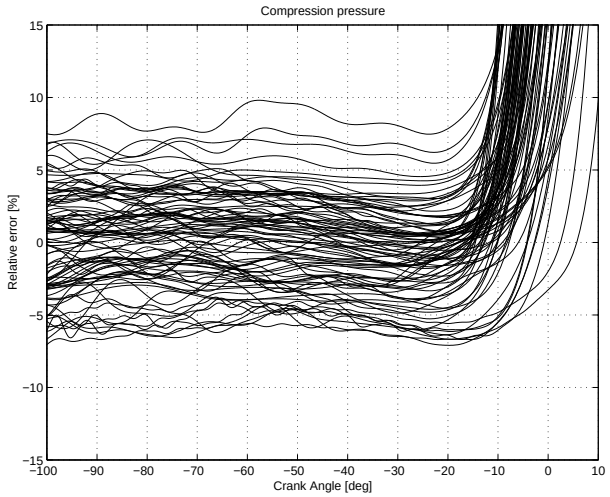

Figure 3: Relative error for compression part, cycle by cycle.

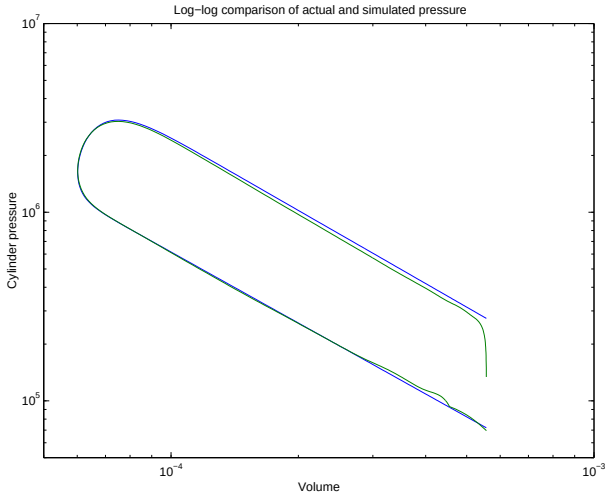

Figure 4: Validation of polytropic coefficient, $k_c = 1.25$ and $k_e = 1.30$

parts. In the log-log scale, the slope is the polytropic coefficient. The polytropic coefficient for the compression part is excellent and for the expansion part it is slightly too small. However, in the simulations it was seen that the expansion coefficient has a dependency to engine operating conditions and for engine B $k_e = 1.30$ was the best choice of a constant parameter.

The polytropic coefficients can be modeled to be dependant on engine operating point and it also differs between engines.

Expansion pressure

The expansion pressure level is calculated according to equations 3 to 7. Using $T_r = 1000$K and $x_r = 0.07$ produces an overall model performance as in Figure 5. Figure 5 is based on the full data set from engine B. The highest deviation is seen around peak pressure where the mean error is less than 4% and the standard deviation is 1%. In the expansion part the mean relative difference is less than 2% but the cycle variations have increased to 4%.

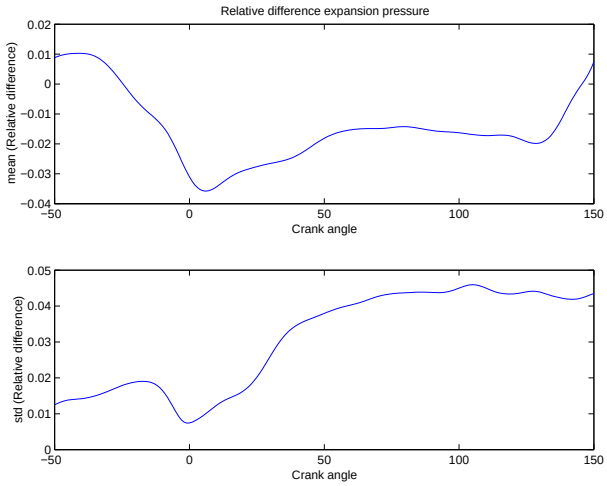

Figure 5: Upper: mean relative error in expansion pressure. Lower: standard deviation in relative error. 2000 rpm, 100 Nm.

The model handles a shift in mfb_{50} position according to figure 6. By shifting ignition timing the timing for mfb_{50} was shifted. The upper graph shows mean values of the pressure point p_{90}, the cylinder pressure at 90° CAD which is a point where the combustion is finished but before EVO. The main property of the mfb_{50} influence is captured even though there is a difference in slope between operating points. The lower graph shows the relative error between the two p_{90} calculations for each combustion cycle.

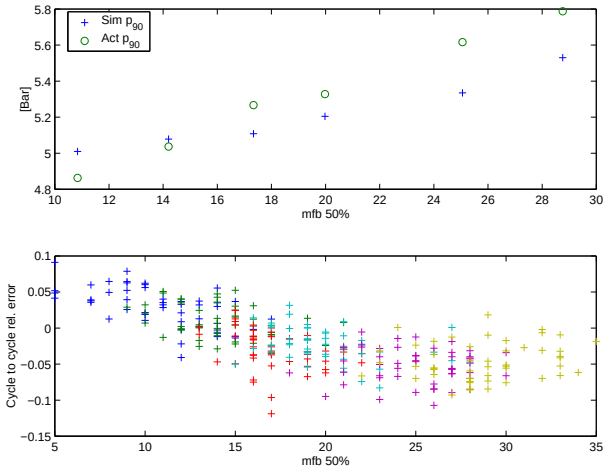

Figure 6: Expansion pressure and combustion timing, modeled and actual. 2000 rpm, 100 Nm.

Interpolation between compression and expansion

One measure for the model quality of the combustion part is the matching of the peak pressure location (PPL) between simulated and actual pressure. The interpolation is done as in equation (8) with PR defined as in (9). The shape factors a and m are calculated from burn angles as in equations (10) and (11). To validate the pressure model assumptions the burn angles for each cycle were used in the simulation of cylinder pressure. The burn angles were calculated from the pressure trace by first calculating the heat release. The most simple form of heat release analysis was used. It is similar to the method *net heat release* described by Kriger and Borman in [14]:

$$dQ = \frac{\gamma}{\gamma - 1} p\, dV + \frac{1}{\gamma - 1} V\, dp$$

Mass fraction burned, mfb, is calculated as the integral of heat release

$$mfb(\theta) = \frac{1}{Q_{tot}} \int_{ivc}^{\theta} dQ$$

The burn angles are extracted from the mfb curve according to table 1. Figure 7 shows the histogram of the difference in PPL between actual and simulated pressure for each cycle in data set B. The mean value of PPL difference is 0.006 CAD and the standard deviation is 1.1 CAD. The resolution in the sampled pressure data was 1 CAD.

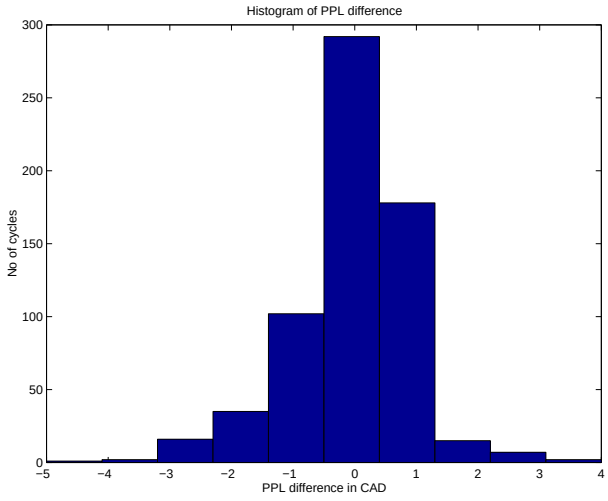

Figure 7: Histogram of difference in PPL, actual and simulated, engine B

Changes in λ and manifold pressure

Figure 8 views the influence from changed air-fuel ratio. In this case there was also a change in mfb_{50} position due to the changed burn speed. The upper graph shows the mean value of relative error. The model captures the behaviour for rich mixtures, where p_{90} decreases when λ decreases from 1. The down slope in the modeled p_{90} for lean mixtures comes from the impact of equation 6 which is obviously not the only effect to consider. Cycle to cycle deviations from the mean error stay at $\pm 7\%$, as seen in the lower graph. Figure 9 views the influence on p_{90} when the

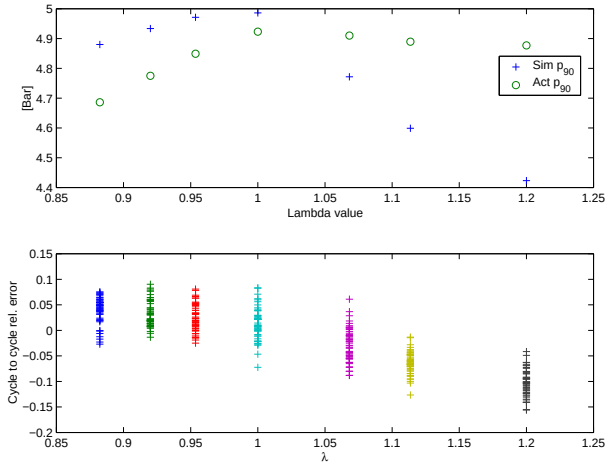

Figure 8: Expansion pressure air-fuel ratio influence, modeled and actual. 2000 rpm, 100 Nm.

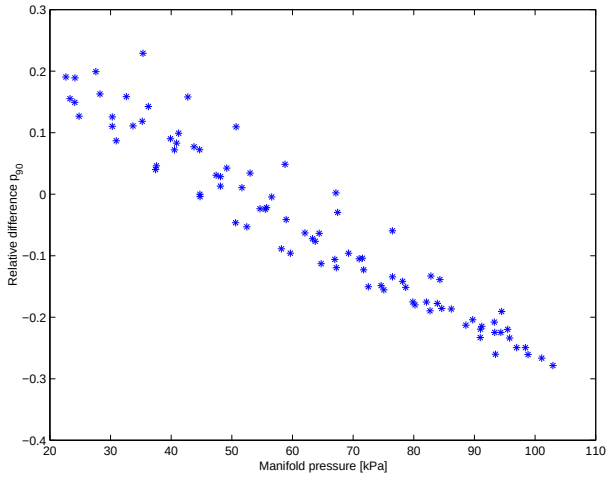

Figure 9: Expansion pressure error, influence from manifold pressure. Engine A.

inlet manifold pressure is changed. Data from engine A is used. The model captures the behaviour of the cylinder pressure at different manifold pressures but a noticeable systematic error is seen. There is a trend where the relative error moves from +20% for low pressure to -30% for high. In the simulations it was noted that the polytropic coefficients k_c and k_e change value significantly. By only changing k_c and k_e it was possible to position the slope in figure 9 at any level and have the error equal to zero for

any manifold pressure. This implies a possible enhanced model performance by modeling the polytropic coefficients as a function of engine operating condition.

SUMMARY AND CONCLUSIONS

An analytic model for the pressure has been developed and evaluated. The pressure model is given in closed form and there is no need to numerically solve the ordinary differential equations. The model is based on physical relations with components that are easy to measure and tune. The closed expression is

$$p(\theta) = \begin{cases} p_c(\theta) = p_{ivc} \left(\frac{V_{ivc}}{V(\theta)} \right)_c^k & IVC < \theta < SOC \\ PR(\theta)p_e(\theta) + (1 - PR(\theta))p_c(\theta) & SOC < \theta < EOC \\ p_e(\theta) = p_3 \left(\frac{V(\theta_c)}{V(\theta)} \right)_e^k & EOC < \theta < EVO \end{cases}$$

and

$$EOC = SOC + \Delta\theta$$

The model contains a number of tuning parameters that effect the accuracy of the pressure model. With a proper tuning of this it is shown that:

- Compression and expansion slopes can be captured well with an accuracy of -7 - 10%.
- When the burn angles are available for the model the peak pressure location stayed within $\pm 1^\circ$ standard deviation from the actual peak pressure location.
- The model can capture large variations in ignition timing.
- The model can capture variations in air to fuel ratio in the rich region.
- Variations in manifold pressure are captured but a significant error trend is seen.

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