Scaling Laws for Distance Limited Communications in Vehicular Ad Hoc Networks

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***Abstract*—In this paper we propose a framework to study the asymptotical capacity of Vehicular Ad Hoc Networks (VANET)s when nodes are expected to communicate only when they reside in a certain distance of each other. This is quite a favorable scenario for VANETs when they are utilized for accident avoidance and safety applications. Moreover we develop formulations to predict the behavior of VANETs with specific geometrical shapes like the single road and grid topologies. Also, the capacity scaling behavior of VANETs when a node needs to transmit to *all* its neighbors within a certain range, is studied. Results are obtained by combining geometrical analysis, network flow arguments, and probabilistic study of VANETs.**

***Index Terms*—VANET, DSRC, Vehicle Infrastructure Integration (VII).**

I. Introduction

The emergence of the evolving field of VANETs has attracted much attention in recent years. The main driving force behind the ever increasing desire towards VANET deployment is their direct influence on road accident downfall. This is of great importance as road accidents are believed to be one of the main reasons of mortality, worldwide. Along with safety purposes, traffic monitoring and route-finding and wireless on board Internet access are other examples of VANET utilization. The fundamental structure of a VANET consists of On Board Units (OBU)s mounted on vehicles and also base stations known as Road Side Units (RSU)s which are deployed at predetermined locations along the roadside (see Figure 1). Therefore in its most general case we have a hybrid network of mobile nodes within a rigid backbone of RSU infrastructure. Each vehicle, gathers data on its current location through a GPS and then updates nearby RSUs or other neighboring OBUs on its current status. The inter vehicle and vehicle-roadside communications are carried out through Dedicated Short Range Communications (DSRC) which has recently been assigned a 75 MHz bandwidth in the 5.9 GHz frequency band. The RSUs may be connected to each other through wired connections or through high capacity wireless backhaul. The data analysis in the RSUs is performed in one of the following ways. For delay sensitive safety applications, data gathered from nearby vehicles is locally analyzed at the RSU and appropriate alarm messages are disseminated accordingly. For applications such as long term traffic monitoring, data of all RSUs are backhauled and gathered in a central management unit where they are summarized and the results are rendered to vehicles through the Internet.

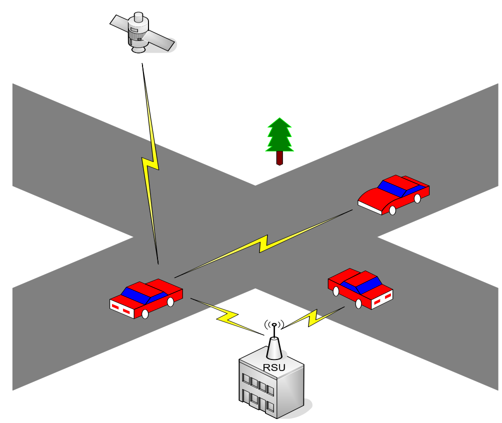

Fig. 1. A generic VANET.

Despite the increasing amount of research on VANETs, a rigorous mathematical framework to study their fundamental limits and scaling laws is still in its infancy. There exists a considerable literature on the fundamental limits of wireless networks and some papers address MANETs. However, our results show that VANETs have some unique characteristics and thus their scaling laws differ significantly from other wireless networks.

As Shown in [1], We observed that the road geometry plays a significant role in the fundamental properties of VANETs. As it will be seen, even a single isolated road (e.g. rural area) can potentially have every possible capacity scaling just based on its path geometry. For example the capacity of an inter-state single road resembles that of a line network, whereas the capacity of a more turned and twisted urban area street can be as large as a wireless network with random node placement. (Figure 2)

Another issue is that a unique mobility paradigm exists in VANETs. There is some interesting literature on the effects of mobility on the capacity of wireless networks [2-8]. In these analyses it is usually assumed that there is high delay tolerance, nodes have huge buffer sizes, and the network topology changes over time-scale of packet delivery. Indeed none of the assumptions hold in VANETs. For example, emergency and safety-related messages are extremely delay sensitive in VANETs. More importantly, a very common assumption in the literature is that the nodes move independently of each other. This assumption by no means holds in VANETs. In fact our results suggests unlike the existing literature, mobility does *not* improve capacity scaling in VANETs. This is in contrast with the previous conception on mobility and capacity.

Finally, new capacity and throughput metrics should be defined for VANETs. In the study of transport capacity it is usually assumed that each node has a random destination chosen uniformly from the available nodes in the network. In VANETs, different applications give rise to the need for diversified capacity declarations. For example, as stated before, in safety applications vehicles need to communicate with vehicles that are in their vicinities other than just picking a destination at random. This significantly affects the scaling laws for throughput.

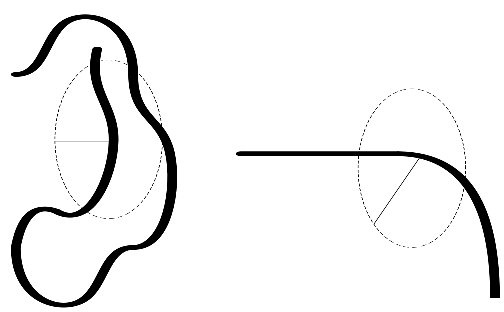

Fig. 2. A single isolated highly sparse road (right) and a less sparse topology of an isolated road leading to more capacity (left).

II. Formulation and Preliminaries

To provide a rigorous analysis of VANETs, we provide some definitions. We show a lane on a road by a parameterized smooth continuous curve $X_n(s) = (x_n(s), y_n(s)), s \in [0, 1]$ on the plane, see Figure 2. The length of each section of the curve is obtained using the Hausdorff one-dimensional measure [9]. The subscript n shows the number of vehicles on the road and can be dropped for simplicity, if it is clear from the context. The curve shows the trajectory of the road. $X_n(0)$ is the beginning of the lane and $X_n(1)$ shows the end. It is assumed a road can intersect itself only a finite number of times. Multi-lane roads are indicated by several parallel curves. A transportation network is usually consisted of several roads. The geometry of the roads plays an important role in the performance of the corresponding VANET. It is assumed that the movements of vehicles on the roads follow a stationary stochastic process. In particular, the *density*, $k_n(s)$, of a road is defined by the average number of vehicles per unit length at point $X_n(s)$. At any part of the road, the density of cars is assumed to be a bounded positive number as in reality the density is limited by the physical size of cars. In this paper, for simplicity we assume $k_n(s) = k$, for all $s \in [0, 1]$ in all proofs. However, the results are easily extendable to the general case. For transportation networks consisting of several roads, the values of densities are chosen in a way that the flow conservation principle is satisfied at the intersections.

The mobility model for vehicles is an important factor in vehicular ad hoc networks. Note that in VANETs, the vehicles do *not* move independently from each other. However, it has been observed that at any time t, the positions of vehicles can be modeled based on a Poisson process on the road, thus the spacing between them has exponential distribution [10-11]. In this paper we follow this assumption, however, it can be shown that the results hold for more general mobility models that satisfy some specific conditions. To define our model rigorously, we extend the lane $X_n(s)$ from both ends to infinity. Then, we place a Poisson point process with density k on the extended curve. Any point of the process will correspond to a vehicle. At time $t = 0$, all vehicles on the same lane will choose a common speed $v \in [0, v_{max}]$ uniformly at random. It is assumed that v_{max} is a fixed and bounded real number. It is assumed that the vehicles do not change their speed. Thus, at any time t, the positions of vehicles is still a Poisson process. Since we assume Poisson distribution and are interested in scaling laws, we can often combine parallel lanes to obtain one curve whose density is given by the summation of densities, i.e, $k = k_1 + k_2 + ... + k_l$ to simplify the analysis. However, it is important to note that this is possible only when we are providing macroscopic analysis, otherwise we need to consider each lane separately and account for the interactions between lanes. We assume $B(X, r)$ is the closed ball with radius r centered at X in $\mathbb{R}^2$. Also, $C(X, r)$ is the circle with radius r centered at X. We consider transportation networks that consist of n cars equipped with OBUs. We are interested in the fundamental limits of these networks as n grows large. Since the density of cars is a bounded positive number, to have a large number of nodes, the total lengths of the roads are assumed to be large, $L = \frac{n}{k} = \Theta(n)$. We also make the assumption that the roads are not highly dense on the plane. We call this the *sparseness* condition. To make this rigorous, for any point $Y \in \mathbb{R}^2$, let $l(Y, r)$ be the Hausdorff one-dimensional measure (combined length) of the sections of the roads inside $B(Y, r)$. It is assumed that $l(Y, \alpha r_t) = O(r_t)$ for all $Y \in \mathbb{R}^2$ and any constant $\alpha > 0$. For any point X on a road let $n(X)$ be the number of times that $C(X, r_t)$ intersects with the road curves. It is assumed that $n(X)$ is bounded. Let A_{r_t} be the sections of the roads consisting of points with $n(X) > 2$. We say that the road system is *sparse* if the combined length of A_{r_t} is $o(L)$. If $n(X) \leq 2$ for all points on the roads, then the system is said to be *highly sparse*. At any intersection, it is assumed that only a bounded number of roads can intersect with each other. In Figure 2, the road depicted on the right is a highly sparse road, whereas the other one has $n(X) > 2$ in some points.

III. Results and Discussions

A. Distance-Limited Capacity

In [1] we have focused on the transport capacity of VANETs, in which it is assumed that every node communicates with a randomly chosen node in the network. In many applications of VANETs, nodes usually need to communicate to nodes that are within a certain distance d from them. For example, in accident warning systems, a car would need to exchange messages with cars that are in its vicinity. To study the performance of VANETs for these scenarios, we define *Distance-Limited Capacity*, $\Lambda_d(n)$, as the highest achievable throughput, when the cars are limited to communicate with cars that are within a distance d from them. The following Theorem provides the distance-limited capacity of VANETs for isolated roads and grids.

Theorem 1. *Assume the sparseness condition is satisfied.*

- *If $d(n) \geq r_t(n)$, the distance-limited capacity of a single road $X(s), s \in [0,1]$, is given by $\Lambda_d(n) = \Theta(\frac{1}{d})$. If $d(n) \leq r_t(n)$, $\Lambda_d(n) = \Theta(\frac{1}{r_t(n)})$.*
- *If $d(n) \geq r_t(n)$, the distance-limited capacity of the $Grid(m)$ is given by $\Theta(\frac{1}{d(n)})$, and $d(n) = O(\frac{n}{m})$. If $d(n) \leq r_t(n)$, $\Lambda_d(n) = \Theta(\frac{1}{r_t(n)})$.*

Proof: First we consider the case where $d(n) \geq r_t(n)$ in a single road. The following lemma is utilized in our proof:

Lemma 1. *Consider a transportation network that consists of u single roads with finite lengths $l_1, l_2, ..., l_u$. Suppose that we divide the roads to sections of lengths βr_t, where β is a constant. We can place these sections into a bounded finite number of non-interfering groups.*

This lemma states that we can schedule parallel transmissions in the network as long as the transmissions belong to different groups. This is a standard method used to obtain a lower bound on the capacity of wireless networks. The lemma can be proved using graph coloring which is omitted here due to simplicity.

It's important to note that in the case of a single road, mobility does not guarantee capacity increase. For example, imagine a multi-lane road where a fraction of the vehicles reside in the highest speed lane. If we assume all the vehicles keep their constant speed and follow each other continuously, the distance between the source and target vehicles does not change in time, this means that the capacity of the system cannot grow faster than the capacity of a static system with the same parameters.

Note that because we assume that the road is highly sparse, all transmissions have to occur along the road. That is, all transmissions consume at least $\Theta(r_t)$ length on the road, where r_t is the range of transmission and is dependent on n, the number of nodes. Also, due to the distance limited communications, two randomly chosen cars are $O(d)$ away from each other. Thus we can use the method in [12] to obtain an upper bound on the achievable throughput. Specifically, if the throughput $\Lambda_d(n)$ is achieved, we need at least $n\Lambda_d(n)\frac{d}{r}$ concurrent transmissions. However, the number of concurrent transmissions is limited by $O(\frac{L}{r})$. Replacing $L = O(n)$ in the latter, we would have: $\Lambda_d(n) = O(\frac{1}{d})$.

To show that $\Theta(\frac{1}{d})$ is achievable we provide a routing strategy. Since the topology of the network changes much more slowly than the packet delivery rate, we can have dynamic routing protocols, in which the routes need to be slowly adjusted as the vehicles move. Choose $r_t = \frac{2\ln n}{k}$. Divide the road into sections of length $\frac{r_t}{2}$. Due to the properties of the Poisson process, in each section, there exists at least one node.

$$
\begin{aligned}
P(\textit{no car exists in a specific section}) &= e^{-\frac{r_t}{2}k} \\
&= \frac{1}{n}
\end{aligned}
$$

Also, as the number of sections of the road is $O(\frac{n}{log(n)})$, utilizing the union bound, it's quite obvious that the probability that there is a section without a car in it tends to zero as n goes to infinity. Hence the connectivity of the network is preserved. Now, divide the sections into a finite number of non-interfering groups. This is possible based on Lemma 1. Route the messages along the road through the sections. Each section has to support at most $\Theta(d)$ routes, thus $\Lambda_d(n) = \Theta(\frac{1}{d})$ is achievable.

For the case of a single road case, when we have $d(n) \leq r_t(n)$, there is no need for multi-hop routing. We divide the road into sections of length $2r_t$ as in Figure 3 to ensure that each transmission opportunity falls within at least one section, so it's guaranteed a time unit for transmission. First we show that $\Lambda_d(n) = O(\frac{1}{r_t(n)})$. Exploiting the method introduced in [13] for the single-hop case, leads to the constraint:

$$n \times \Lambda_d(n) \times 1 \leq \frac{n}{r} \rightarrow \Lambda_d(n) = O(\frac{1}{r_t(n)})$$

Now we examine the achievability of $\Lambda_d(n) = \Theta(\frac{1}{r_t(n)})$. Since the length of the sections is $\Theta(r_t(n))$, we can divide the sections into a finite number of non-interfering groups by

Lemma 1. Also it is easy to see that all sections have $\Theta(r_t)$ nodes with high probability. Thus, we can schedule each link within $\Theta(r_t(n))$ unit times. Also, $\omega(\frac{1}{r_t(n)})$ throughput is not achievable because that would need $\omega(\frac{n}{r_t(n)})$ concurrent transmissions.

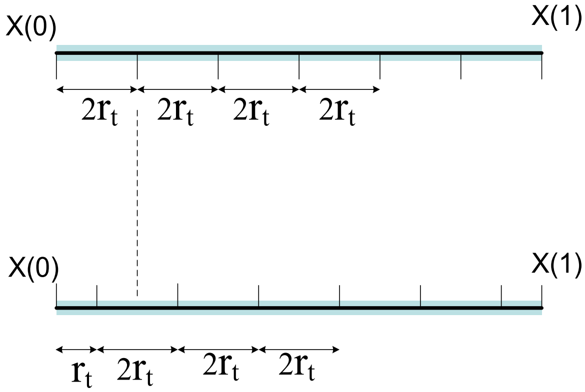

Fig. 3. Divisions of a single road to be used for routing when $d(n) \leq r_t(n)$.

We now consider a more realistic transportation topology which is the grid model. To make the argument rigorous, we define a transportation grid of order m, as a set of m parallel streets intersected with another set of m parallel streets of length l each. We assume that the two sets of roads are orthogonal to each other, see Figure 4. We refer to this structure as $Grid(m)$. Note that due to the intersections, it is virtually impossible to have a highly sparse grid structure. Yet to obtain a sparse grid topology, the transmission range of vehicles has to be much less than the distance between two parallel roads. i.e: $r_t(n) = o(\Delta) = o(\frac{l}{m})$; Again we investigate the distance limited capacity of VANETs for both the cases $d(t) \geq r_t(n)$ and $d(t) \leq r_t(n)$, this time for the grid structure.

We first deal with the achievability of the given bound when $d(n) \geq r_t(n)$. Choose $r_t(n) = \frac{2ml}{n} \ln n$. Each street is divided into sections of length $\frac{r_t(n)}{2}$. In the intersections, the sections consist of four parts of length $\frac{r_t(n)}{8}$. The sections are divided to a finite number of non-interfering groups using Lemma 1. The algorithm is as follows. Assume a car located at point X_s wants to transmit to a car located at X_d. The information is transferred through the closest vertical street to X_s, and the closest horizontal road to X_d. The packets are transferred from each section to the neighboring sections until they reach the destination. To find the achievable capacity, we need to obtain the amount of traffic passed through each section. Indeed, when studying non distance limited communications, it can be easily seen that the number of information paths traveling through each section is (like the one colored in red in Figure (4))$O(\frac{n}{m})$. This is because- due to the previously described routing strategy- the data of all the vehicles that reside on the black portion of the grid, will pass through the specified red section, if its destination is chosen on the opposite side. In the same way, if we are studying distance limited communications, then at most the data of $d(n)$ vehicles will pass through the desired section, where $d(n) = O(\frac{n}{m})$. Hence a distance limited capacity of $\Lambda_d(n) = \frac{1}{d(n)}$ is achievable. Note that due to the spareness condition, we have assumed that we can neglect the interference caused by the transmission of vehicles in other roads, hence the bound obtained for M in a single-road scenario is also applicable here.

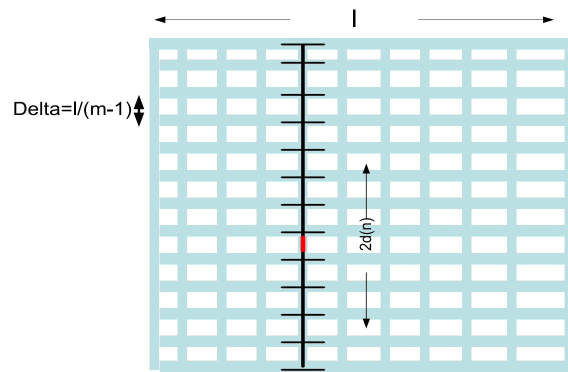

Fig. 4. A grid with m parallel lines of length l each.

We now show that throughput higher than $\Theta(\frac{m}{n})$ is not achievable. Again, the maximum number of concurrent transmissions is limited by $\Theta(\frac{L}{r})$. Due to the distance limited nature of communications, two randomly chosen points have distance $O(d)$. Thus, from a sender to its destination, the packets should go through $O(\frac{d}{r})$ hops. Thus

$$n\Lambda_d(n)O(\frac{d(n)}{r}) \leq \Theta(\frac{L}{r}) \leq \Theta(\frac{n}{r})$$

Thus we cannot achieve throughput higher than $\Lambda_d(n) = \Theta(\frac{1}{d(n)})$.

Now we turn to the case where $d(t) \leq r_t(n)$ for grid topologies. In this case, grid lines are divided in the same way as in Figure 3 which was initially developed for the single road case. The same arguments as for the single road case are true for grid topology, leading to a distance limited capacity of $\Lambda_d(n) = \Theta(\frac{1}{r_t(n)})$.

■

B. Cooperative Distance-Limited Capacity

We now consider the case in which each node has to communicate with all the nodes that are within distance $d(n)$ from it. This corresponds to the situations in which all vehicles that are located close to each other try to cooperatively accomplish a task, such as accident avoidance. The highest achievable rate is called the cooperative distance-limited capacity and is shown by $\Delta_d(n)$. The following Theorem provides the distance-limited capacity of VANETs for isolated roads and grids.

Theorem 2. *Assume the sparseness condition is satisfied.*

- *If* $d(n) \geq r_t(n)$*, the cooperative distance-limited capacity of a single road* $X(s), s \in [0,1]$*, is given by* $\Delta_d(n) = \Theta(\frac{1}{d(n)^2})$*. If* $d(n) \leq r_t(n)$*, then* $\Delta_d(n) = \Theta(\frac{1}{r_t(n)d(n)})$
- *If* $d(n) \geq r_t(n)$*, the cooperative distance-limited capacity of the* $Grid(m)$ *is given by* $\Delta_d(n) = \Theta(\frac{1}{d^3(n)})$*, and* $d(n) = O(\frac{n}{m})$*. If* $d(n) \leq r_t(n)$*, then* $\Delta_d(n) = \Theta(\frac{1}{r_t(n)d^2(n)})$*.*

As expected, the achievable per node communication rate here is much smaller than that of the distance limited capacity.

Proof: The ideas of proof here are similar to the distance-limited capacity case. Thus we just highlight the differences. The main difference is that for example in the case where $d(n) \leq r_t(n)$ for a single road, after applying a division like the one given in Figure 3, it is easy to see that all sections have $\Theta(r_t)$ nodes with high probability. Thus, we can schedule each link within $\Theta(r_t(n)d(n))$ unit times. This is because in each section each vehicle needs to communicate with $d(n)$ other vehicles. For the case $d(n) \geq r_t(n)$, following the same lines as the one in Theorem 1 and also taking in mind the definition of cooperative distance-limited capacity, we would easily come up with $\Delta_d(n) = \Theta(\frac{1}{d(n)^2})$. For the grid, we must note that any node needs to communicate with $O(d^2)$ nodes. The proof is simple and omitted here due to the lack of space. ■

IV. Conclusion

In this paper we introduced a novel capacity definition, specially developed for safety applications in VANETs. That is, VANET nodes which are vehicles riding along roads, mainly need to communicate with other parties which are within a distance $d(n)$ of their vicinity. We studied the capacity of the VANET when there is this limitation on node transmissions and we called it *Distance-Limited Capacity*. Also we rendered capacity scaling laws as the number of users increases. Moreover specific analysis was carried out for two famous transportation topologies, which are the single road and grid geometries. Also for specific emergency applications, a vehicle needs to send high priority data to *all* other vehicles in its vicinity. We've also derived formulas to study the VANET capacity under those circumstances. We have shown that the VANET capacity results differ significantly from the known capacity results obtained for MANETs. In particular, it is observed that the road geometry plays an important role in the capacity of VANETs. This paper opens up an important direction for future research on understanding VANETs. Indeed, several simplifying assumptions regarding the mobility models, geometric properties, communication models, and capacity definitions are adopted in this paper. Future work will consider developing and analyzing more realistic models.

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