# Recognizing Investment Opportunities at the Onset of Recoveries

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**Abstract**

Investment decision-making is modeled by means of a Kohonen neural net, where neurons represent firms. This is done in order to model investments in novel fields of economic activity, that according to this model are carried out when firms recognize the emergence of a new technological pattern. Combination of the equations of Kohonen model neuron with macroeconomic relationships yields disaggregated accelerator equations with flexible coefficients, that in the aggregate and fixed-coefficients case boil down to the traditional accelerator equations. A simulation tests the model in a situation that is remindful of the very beginning of economic recoveries.

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# 1 The Investment Acceleration Principle

The first clue of what later was to be known as *investment acceleration principle* can be found in a work published by Albert Aftalion at the beginning of the XX century [3]. Aftalion argued that, if a temporary increase of the demand for final goods triggers the production of capital goods, and if capital goods become available only when the demand for final goods is already back at its original level, then the economy finds itself with an excess of productive capacity and a crisis is likely to begin.

A few years later, Clark [14] added a distinction between demand per unit time, which he called *speed*, and the *acceleration* of this demand. Clark argued that, since firms adjust their productive capacity according to variations of demand, investments ultimately depend on the acceleration of demand. At that time, this was just a felicitous expression waiting tot be translated into formulas.

In the subsequent decades, mathematical formulations of the investment acceleration principle became a main component of business cycle models. Undoubtedly, it was Kalecki who provided the most refined mathematical models [27], but eventually the far simpler formulas proposed by Samuelson [39] and Hicks [26] gained a much wider acceptance. According to their proposals, aggregate investments either depend on aggregate variation of consumption (1) or, alternatively, on lagged aggregate variation of income (2):

$$I_t = \kappa(C_t - C_{t-1}) \tag{1}$$

$$I_t = \lambda(Y_{t-1} - Y_{t-2}) \tag{2}$$

where $I$, $C$ and $Y$ denote aggregate investments, consumption and income, respectively. Coefficients $\kappa$ and $\lambda$ are constants.[1]

Behind these aggregate magnitudes hide a large number of firms that carry out their investment plans independently of one another. Firms invest on innovations that open up new possibilities for competition, each firm hoping to increase its own market share. Since each firm seeks to exploit the whole increase of demand, for any single firm it is rational to commit to investment plans that are tailored for a larger market share. However, since the (eventually) higher demand must distribute itself among all firms, most of them will end up with an excess of productive capacity. Ultimately, the investment acceleration principle has its roots in

[1]In order to avoid confusion with other magnitudes used in this paper, constants have been denoted by different letters from those used by Samuelson and Hicks.

the very fact that what is rational for a single firm to do, may not be rational for the economic system as a whole [36].

Thus, the very rationale of the investments acceleration principle can only be seen at the microeconomic level. In fact, it is a rationale that involves expectations and convictions, including the idea that variation of demand is the relevant signal for investing.

The economic literature provides only one example of an accelerator with a microeconomic foundation. This is the one that we can find in Lucas' equilibrium business cycle model [33], where economic agents are distributed on "islands" that only occasionally communicate. In this model, the accelerator takes the form $k_{t+1} \propto (\hat{k}_t - k_t)$, where $k_t$ denotes the logarithm of aggregate capital at time $t$, and $\hat{k}_t$ is the (correctly) estimated mean value of the stochastic distribution of $k_t$ over the "islands".

Lucas carried out a thorough discussion of his accelerator equation, concluding that investment acceleration is pronounced if economic agents:

i. are responsive to perceived future returns of physical capital relative to money capital;

ii. are convinced that the current demand for physical capital relative to money capital is a good indicator of the future return of physical capital;

iii. are convinced that current price movements contain information about the current demand for physical capital relative to money capital.

The most remarkable feature of the above considerations is that Lucas spoke of "*perceived* future relative returns", and of being "*convinced* that ...". Interestingly, by moving from the macroeconomic to the microeconomic level Lucas came to focus on availability of information and the relative importance of different information sources to different decision-makers.

This article deepens further this line of reasoning. Specifically, it makes use of a neural net in order to model the formation of expectations in the minds of the managers who decide to invest. By proceeding along this path, it arrives at a generalization and microeconomic foundation of Goodwin's "flexible" accelerator [23]. Namely, Goodwin's model marked a major cornerstone in the history of investment acceleration equations.

Early empirical applications of (1) and (2) had shown that, in order to fit with empirical data, accelerator equations must take account of available capital stock [12]. Goodwin's accelerator (3) is a simple theoretical model where investments depend on the difference between available $K$ and *desired* capital $\xi$ stock.

It is a non-linear accelerator where investments $I$ switch between $K^*$ and $K^{**}$ according to the values taken by capital $K$:

$$I = \begin{cases} K^* & \text{if } K < \xi \\ 0 & \text{if } K = \xi \\ K^{**} & \text{if } K > \xi \end{cases} \tag{3}$$

where $\xi$ denotes *desired* capital stock.

Since Goodwin assumed that desired capital $\xi$ is proportional to income $Y$, and since capital is accumulated income, Goodwin's accelerator ultimately depends on past income variations, just like (2). Rather, its distinguishing feature is that aggregate investments react differently to income variations that take place at different levels of capital stock. Since this is equivalent to having an accelerator with variable coefficients (e.g. $\kappa$ or $\lambda$ in equation (1) or (2), respectively), Goodwin's has been called a *flexible accelerator*.

Goodwin introduced his flexible accelerator with an eye to the upper turning points of business cycles [23], where crises begin because of shortages of credit and labor force. However, a justification for investment acceleration to set in at the low turning points of business cycles has always been regarded as more problematic. Namely, why should firms invest if they still have an excess of productive capacity?

Goodwin's answer was that those investments that take an economy out of a recession involve machineries of a novel kind. According to Goodwin, it is investments on innovations that trigger economic recovery [24]. However, a modelization of innovations triggering investment acceleration has not been attempted hitherto.

This is namely the aim of the model presented herein, which employs a neural net in order to reproduce firms' cognitive processes. However, setting the investment acceleration principle on one's own research agenda may be regarded as an anomaly. In fact, with the notable exception of Robert Lucas, rational expectations theorists rejected the investment acceleration principle on the ground that it is not based on utility optimization. Thus, after Lucas' model [33], accelerators disappeared from theoretical economics.

Nonetheless, empirical literature continued to provide evidence that accelerator equations exhibited a much better predictive power than any competing model [15] [8] [1]. Rational expectations theorists provided two justifications for this.

Both of them are based on the observation that capital stock time series are less reliable than income time series. The first justification was provided by Sargent, who observed that since income time series are less affected by noise than capital stock time series, noise is likely to introduce a spurious causality link from income towards capital stock [40]. The second justification was provided by Acemoglu, who maintained that firms observe statistical reports when they make investments and, since they know that capital stock time series are not very reliable, they base their decisions on income time series [2].

However, a second tide of empirical studies stressed once again the ability of accelerator equations to track investments in the most diverse economies and times, including Malaysia from 1971 to 1988 [9], France from 1972 to 1991 [35], U.S. from 1948 to 1985 [7], France and U.S. from 1968 to 1993 [34], Cameroon, Ghana, Kenya and Zimbabwe from 1971 to 1995 [10] and the Czech Republic from 1992 to 1996 [32]. None of these studies rejected alternative models such as Tobin's *q*, but all of them ascribed the largest explanatory power to accelerator equations.

Notably, the power of accelerator equations seems not to be affected by differences in data reliability across countries and time. Although this consideration is far from being a definitive proof, persistence of the predictive power of accelerator equations *vis à vis* the enormous improvement in the quality of economic data in industrialized countries may cast doubts on the relevance of Sargent's argumentation. As far as it regards Acemoglu's argument, even if basing investment decisions on statistical data had ever been a meaningful strategy for firms operating in industrialized economies, it is surely irrelevant in developing and transition economies, where statistical data are generally available with lags of years and only in a very aggregate form.

On the theoretical side, Velupillai observed that multiplier-accelerator models actually do reflect rational decision-making because they arise from decision rules that, although eventually different from utility maximization, are not necessarily less *rational*. On the contrary, decision rules based on procedural rationality that include utility maximization as a special case are able to generate more realistic and general dynamics, including deterministic chaos [45] [46].

The paper is organized as follows. Firstly, Section 2 introduces a few basic concepts on cognition and neural nets. Subsequently, Section 3 derives disaggregated accelerator equations from an analysis of information flows in an economy with two production stages. The core of the paper is entailed in Section 4, which makes use of a neural net in order to link the variation of the accelerator coefficients to the evolution of the mental categories of the managers who make investment decisions. Finally, section 5 illustrates the meaning of the equations derived in the previous sections by means of a numerical example and Section 6 concludes.

## 2 A Few Concepts on Cognition and Neural Nets

In very general terms, one can claim that the mess of information generated by continuous technological innovation conflicts with the bounded rationality of economic agents, who are forced to operate some simplification in order to make sense of it [42]. However, since human reasoning is not quite the same as executing an algorithm, it is not altogether correct to liken bounded rationality to memory and time constraints on electronic computers. Rather, human beings simplify the enormous amount of information that they receive by classifying it into a maneageable number of mental categories.

Interestingly, mental categories are not defined by pre-specified similarity criteria that the objects to be classified should fulfill. In fact, since the qualitative features of objects like future goods and future technologies cannot be known in advance, classification criteria that are absolutely correct cannot exist. Rather, mental categories are continuously constructed and modified according to similarity of a just-received piece of information to the pieces of information that have already been stored in existing categories. Stored pieces of information that become guidelines to subsequent classification are called *prototypes* [5] [13] [25].

Notably, it is not even necessary to assume that all items classified in a category share common features. As an example, the reader is invited to find whatever feature all human occupations have in common, that are subsumed by the mental category labelled by the word *game*: a few minutes reflection are sufficient to realize that this is an impossible task! On the other hand, all we need in order to use the category ”game” is that we are able to evaluate the similarity of a new game to *some* of the items already stored in the category. Such items are acting as *prototypes* for future classification [31] [13].

Neural nets are able to reproduce these features of human cognition. Thus, neural nets model bounded rationality in terms of information categorization.

As such, neural nets could possibly become a viable alternative to utility maximization. A few attempts to use neural nets in order to model decision-making by economic agents have already been made [37] [11] [38] [19] [43] [44] [47] [48].

Neural nets fit into the framework of *case-based decision theory* [20] [21] [22], where individuals measure the similarity of a decision problem to the situations that they encountered in the past and take a course of actions that is similar to one that in the past, in a situation that is similar enough to the present one, had produced satisfactory results. In this context, decision-makers do not necessarily *maximize* utility, though they eventually approach the utility-maximization solution. Furthermore, in this framework experience with novel situations may possibly change utility values over time.

There exist many kinds of artificial neural nets, that are more or less close to the biological neural nets that inspired them. It is of paramount importance to distinguish neural nets where category formation is supervised by an external operator from Kohonen neural nets, where category formation is left to the net itself.

In the first case, a neural net is only used after it underwent a *training phase* where the external operator wires in the categories employed by the net. In practice, a human operator chooses which patterns the net will be able to recognize.

In the second case, no training phase takes place prior to the normal operation of the net. On the contrary, the net forms and modifies its categories according to the patterns contained in the information that it is classifying. Clearly, only Kohonen nets can give us a clue of the behavior of decision-makers who are facing novel situations and require continuous adaptation of their mental categories.

Kohonen neural nets [29] [30] base their flexibility on feed-back and feed-forward loops that allow adaptation to a changing environment. In this respect, Kohonen artificial neural nets are most similar to the biological ones [16].

Kohonen's model neuron produces an output $y \in \Re$ by summing inputs $x_1$, $x_2, \ldots x_N \in \Re$ by means of coefficients $a_1, a_2, \ldots a_N$:

$$y = \sum_{i=1}^{N} a_i x_i \tag{4}$$

Evidently, for any set of coefficients $a_i$ this simple device is able to distinguish at least some of the possible input vectors $\mathbf{x}$ from one another by yielding different outputs $y$. In fact, since there exist many vectors $\mathbf{x}$ whose weighted sum yields the same $y$, even a single neuron is able to classify input vectors into categories.

The ability of a neuron to adapt these categories to the patterns of input information stems from a feed-back from output $y$ and a feed-forward from input $\mathbf{x}$, towards coefficients $a_i$:

$$\frac{da_i}{dt} = \phi(\mathbf{a}, y) x_i - \gamma(\mathbf{a}, y) a_i \quad \forall i \tag{5}$$

where $\phi(\mathbf{a}, y)$ and $\gamma(\mathbf{a}, y)$ may be linear or non-linear functions.

Equation (5) differentiates operator-assisted neural nets from Kohonen neural nets. In operator-assisted neural nets, equation (5) does not exist. In fact, coefficients $a_i$ are fixed during a training phase that takes place *before* the normal operation of the net.

On the contrary, by means of equation (5) a Kohonen net is able to modify its own categories and learns to recognize novel patterns. Obviously, Kohonen nets pay a price for this flexibility: they are slower than operator-assisted neural nets. This is the reason why Kohonen nets are uncommon in commercial applications, although they constitute a basic research tool in artificial intelligence.

In equation (5), term $\phi(\mathbf{a}, y)x_i$ enables the neuron to learn input patterns. It entails both a feed-back (from $y$) and a feed-forward (from $x_i$). This *learning term* makes $a_i$ increase when *both* $y$ and $x_i$ take high values, thereby enhancing those coefficients that happened to yield a high $y$ when a particular $x_i$ was high. Thus, the structure of coefficients vector $\mathbf{a}$ ultimately depends on which vectors $\mathbf{x}$ appeared more often as input. These vectors are the *prototypes* around which the net constructs its categories (remark that categories, in a neural net, are embedded in coefficients $a_i$).

On the contrary, term $\gamma(\mathbf{a}, y)a_i$ in equation (5) enables the neuron to forget input patterns. It entails a feed-back from output $y$ and, most importantly, coefficient $a_i$ itself. By allowing the neuron to forget categories that refer to patterns that no longer appear, this *forgetting term* eases up the formation of novel categories that allow classification of novel events.

Simple, but non trivial examples of equation (5) are: $\dot{\mathbf{a}} = \mu y\mathbf{x} - \nu\mathbf{a}$, $\dot{\mathbf{a}} = \mu\mathbf{x} - \nu y\mathbf{a}$, $\dot{\mathbf{a}} = \mu y\mathbf{x} - \nu y\mathbf{a}$, $\dot{\mathbf{a}} = \mu y\mathbf{x} - \nu y^2\mathbf{a}$, where $\mu$ and $\nu$ are constants. Figure (1) illustrates the feed-backs and -forwards within a Kohonen model neuron.

In general, a net of neurons is able to discriminate input information according to much finer categories than a single neuron can do. As a rule, the greater the number of neurons, the finer the categories that the net constructs. However, a neural net is useful precisely because it is able to classify a huge amount of information into a few broad categories. If categories are so fine that they track exactly input information, a neural net becomes useless. Thus, the number of neurons that a net should possess depends on the variability of input information as well as on user needs.

However, the behavior of a neural net does not only depend on the number of its neurons but, to an even larger extent, on the structure of connections between them. In fact, just like the capabilities of Kohonen neurons depend on feed-backs and -forwards, the capabilities of a neural net depend on shortcuts that eventually enable information to circulate along loops that involve several neurons. If

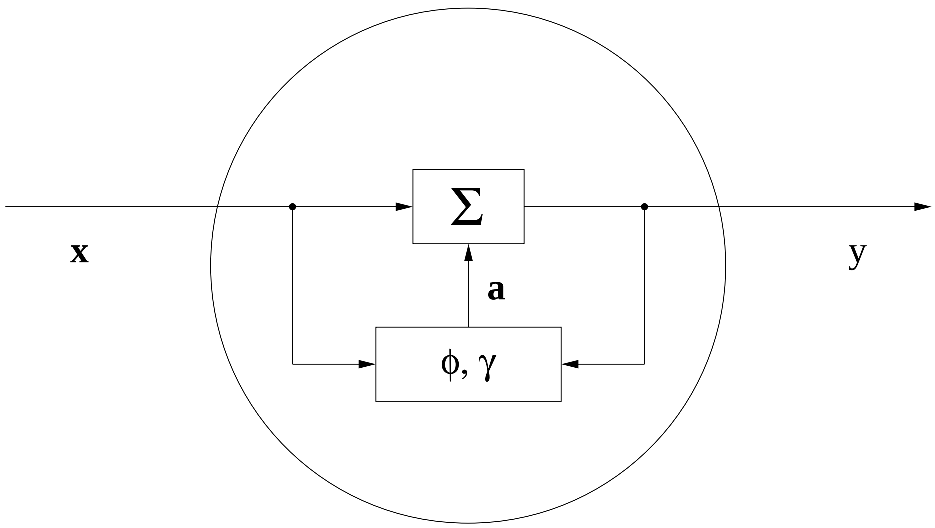


Figure 1: Kohonen model neuron. The feed-backs and -forwards are responsible for all notable properties of Kohonen nets, including the absence of a training phase. Actually, a training phase can be seen as a feed-back and -forward passing through a human operator.

information loops take place, then the net as a whole acquires a *memory*.

It is called a *distributed, associative* memory, and it is fundamentally different in nature from the more usual *localized* memories. Localized memories, such as books, disks, tapes etc., store information at a particular point in space. This information can only be retrieved if one knows where its support is (e.g. the position of a book in a library, or the address of a memory cell in a computer disk).

On the contrary, in a neural net each neuron may be part of a number of information circuits where information is ”memorized” as long as it does not stop to circulate. Although this *is* a memory, one cannot say that information is stored in any particular place. For this reason, one speaks of a *distributed* memory.

Obviously, information stored in a distributed memory cannot be retrieved by means of an address. However, a piece of information flowing in a particular loop can be retrieved by some other piece of information that is flowing close enough to it. Thus, in a distributed memory information can be retrieved by means of *associations* of concepts, with a procedure that reminds of human capabilities such as ”recognition” or ”intuition” [29] [13]. For this reason, one speaks of *associative* memory as well.

The importance of the capability of a neural net to implement an associative memory will become clear in the following sections, where it will be shown that the Keynesian multiplier and the accelerator arise out of information circuits that involve the outputs of at least two production stages. In the light of the above considerations, the ability of an economy to recognize the importance of innovations appears to be similar in nature to the ability of an individual to recognize patterns and trace similarities.

# 3 The Disaggregated Accelerator

The aim of this section is that of deriving disaggregated accelerator equations from an analysis of the structure of information flows within an economy. For this limited purpose, and only in this section, innovation will be assumed away.

The minimal economic structure that we need to consider involves households, firms that produce final goods (hereafter labelled *final goods sector*) and firms that produce capital goods (hereafter labelled *capital goods sector*). Eventually, existence of a banking system must be assumed in order to allow investments beyond internal financial resources, but this will not be modeled explicitely.

Within this framework, 'investments' are purchases of capital goods carried out by firms that produce final goods. For simplicity, let us suppose a constant number of firms in both sectors.

There are three markets in this scheme: the market for final goods, the market for capital goods and the labor market. The market for final goods is assumed to be in imperfect competition because of qualitative diversity of the goods exchaged, which can be complementary or substitutes of one another in any degree. On the contrary, it is assumed that at any point in time only one kind of capital good can be produced. Similarly, only one kind of job is available at any point in time. However, understanding which firms will need what amount of capital good in the next time step is not a trivial task so the situation is quite different from perfect competition.

Information is free to circulate, but only within certain institutional channels. These require, coherently with traditional assumptions surrounding the investment acceleration principle [3], that firms in the final goods sector only observe demand for final goods and firms in the capital goods sector only observe demand for capital goods.

Furthermore, let us assume that:

i. Firms react to changing demand by adjusting quantities, not prices;


Figure 2: The structure of information flows in an economy with two production stages. One feed-back through the labor market would be sufficient to generate the Keynesian multiplier, but at least two feed-backs are necessary for investments acceleration to take place.

ii. Population dynamics, increase of productivity and oscillations of production never combine to make labor a rationed good.

These two assumptions are not realistic in general, but they are realistic in the particular situation for which this model is thought, namely the onset of a recovery. In fact, in this situation labor force is likely to be abundant and inflationary pressures are likely to be low. Furthermore, demand is still increasing at a low pace so firms are able to satisfy all requests.

Note that, under the above assumptions, information flows are strictly unidirectional. In fact, increasing demand for a certain good never leads to price bargaining (which would imply information flowing back and forth), but rather to prompt delivering. Thus, information simply flows in the opposite direction of goods.

Ultimately, information conveyed by these markets regard: 1) Final goods requested by households; 2) Capital goods requested by the final goods sector; 3) Labor requested by the final goods sector; 4) Labor requested by the capital goods sector. Figure (2) illustrates the structure of information flows between these sectors.

It can be noted that, according to the scheme of figure (2), firms in the capital goods sector produce capital goods out of labor only. This may strike the reader as unrealistic, but it is a mere artifact of having condensed all production stages into two aggregates. Consequently, the capital goods sector actually encompasses all production stages from mining to production of capital goods for its own use.

In figure (2), two feed-back loops can be recognized. The inner one is due to labor requested by the final goods sector: through households consumption, this feed-back is sufficient to generate the demand multiplication effect. The outer one is due to labor requested by the capital goods sector: through households consumption and firms investments, this feed-back adds the investments acceleration effect.

Let us assume that both sectors are composed by $N$ firms. The final goods sector produces $N$ different goods for a demand that is not disaggregated across consumers. On the contrary, the capital good sector produces one single capital goods for $N$ firms that it distinguishes from one another.

Let $N$-dimensional vectors $\mathbf{c}$, $\mathbf{k}$, $\mathbf{l}'$, $\mathbf{l}''$ denote consumption of the $N$ goods, capital endowments in the $N$ firms of the final goods sector and employment in the $N$ firms of the final goods and the capital goods sectors, respectively. Note that, since in this scheme households savings do not exist, aggregate consumption coincides with aggregate income. Obviously, it must be $\mathbf{c} \geq \mathbf{0}$, $\mathbf{k} \geq \mathbf{0}$, $\mathbf{l}' \geq \mathbf{0}$, $\mathbf{l}'' \geq \mathbf{0}$.

According to Bateson [6], information is not carried by the values taken by physical magnitudes but rather by their change with respect to a reference level. For instance, Shannon's information theory [41] takes a message of equiprobable characters as a reference value of zero information.

When the investment acceleration principle states that firms react to *variations* of demand, it implicitely assumes that decision-makers consider past values as reference values for extracting information from the signals that they receive. Past level of demand for final goods is regarded as a stock variable related to consumption of generally short-lived goods that will have to be purchased again at the next time step. Thus, relevant information is carried by variations of this stock variable with time. By generalizing this approach we can state that firms in the capital goods sector react to requests of variations of the stock of capital goods in the final goods sector, as well as that households react to variations of employment levels.

Thus, let us define the following information vectors:

**$\Delta\mathbf{c}$:** The information carried by variations of consumption that, since we assumed savings away, reflect variations of income. Since according to the hypotheses of the investments acceleration principle the reference level of zero information is past demand, firms that produce final goods receive information from households by means of $\Delta\mathbf{c}_t = \mathbf{c}_t - \mathbf{c}_{t-1}$. The $i$-th component of this vector represents the variation of demand for the $i$-th final good.

**$\Delta\mathbf{k}$:** The information carried by variations of the capital stock, i.e. investments. Since capital goods by definition last longer than production time, we can take the capital stock (integrated by replacements due to wear and tear) as a reference level of zero information. Thus, information is carried by variations of capital stock $\Delta\mathbf{k}_t = \mathbf{k}_t - \mathbf{k}_{t-1}$. The $i$th component of this vector represents the variation of demand of the only capital good by the $i$-th firm of the final goods sector.

**$\Delta\mathbf{l}'$:** The information carried by variations of employment in the final goods sector. Since production time is generally shorter than the time needed to hire and fire workers, employment can be considered as a stock variable just like capital and its past level can be taken as the reference level of zero information. Thus, relevant information for households is carried by variations of employment $\Delta\mathbf{l}'_t = \mathbf{l}'_t - \mathbf{l}'_{t-1}$. The $i$-th component of this vector represents variations of employment of the only kind of labor in the $i$-firm of the final goods sector.

**$\Delta\mathbf{l}''$:** The information carried by variations of employment in the capital goods sector. Just like in the case of employment in the final goods sector, relevant information for households is carried by $\Delta\mathbf{l}''_t = \mathbf{l}''_t - \mathbf{l}''_{t-1}$. The $i$-th component of this vector represents variations of employment of the only kind of labor in the $i$-firm of the capital goods sector.

Since we assumed savings away, the outcome of utility maximization can be subsumed by a linear function $f$ that depends on current income only:

$$\Delta\mathbf{c} = f(\Delta\mathbf{l}) \tag{6}$$

where $\Delta\mathbf{l} = \Delta\mathbf{l}' + \Delta\mathbf{l}''$.

Likewise, let us assume that the labor requested by the final goods sector is linked to the amount of capital goods that it requests by means of a linear function $g$:

$$\Delta\mathbf{l}' = g(\Delta\mathbf{k}) \tag{7}$$

where $g$ ultimately depends on technical coefficients of capital and labor.

Functions $f$ and $g$ are black boxes that hide parts of information processing and decision-making. In order to understand investment acceleration, these black boxes can remain such. On the contrary, it is of paramount importance that we model (i) how firms in the final goods sector process information in order to make investment decisions, and (ii) how firms in the capital goods sector process information in order to make employment decisions.

By assuming constant returns to scale in the final goods sector we can introduce a coefficients matrix $\mathbf{A}$ and write:

$$\Delta\mathbf{k}_t = \mathbf{A}\Delta\mathbf{c}_{t-1} \tag{8}$$

where each line represents information processing by a different firm in the final goods sector.

In a similar way, by assuming constant returns to scale in the capital goods sector we can introduce a coefficients matrix **D** and write:

$$\Delta \mathbf{l}''_t = \mathbf{D} \Delta \mathbf{k}_{t-1} \tag{9}$$

where each line represents information processing by a different firm in the capital goods sector.

It is important to stress that the assumption of constant returns to scale is limited to this section only. In the ensuing sections, variable returns to scale will arise out of technological innovations that affect matrices **A** and **D**.[2]

Matrices **A** and **D** subsume firms decision-making. In the static framework of this section, we can think of **A** and **D** as arising from maximization of intertemporal profits $\pi_F = \sum_t \mathbf{p}_c^T \mathbf{c} - \mathbf{p}_k^T \mathbf{k} - \sum_t \mathbf{p}_l^T \mathbf{l}'$ and $\pi_K = \mathbf{p}_k^T \mathbf{k} - \sum_t \mathbf{p}_l^T \mathbf{l}''$, respectively. On the contrary, in the innovation-driven setting of the ensuing section **A** and **D** will evolve according to entrepreneurs' "animal spirits" concerning the future profitability of novel investment opportunities [28]. Financial considerations will eventually constrain the evolution of **A** and **D**, but they will not identify a unique path of development.

From observation of figure (2) and consideration of production time lags we can write:

$$\Delta \mathbf{k}_t = \mathbf{A} \Delta \mathbf{c}_{t-1} \tag{10}$$

$$\Delta \mathbf{k}_t = \mathbf{A} f(\Delta \mathbf{l}'_{t-1} + \Delta \mathbf{l}''_{t-1}) \tag{11}$$

$$\Delta \mathbf{k}_t = \mathbf{A} f(g(\Delta \mathbf{k}_{t-1}) + \mathbf{D} \Delta \mathbf{k}_{t-2}) \tag{12}$$

Equations (10), (11), (12) are disaggregated accelerator equations, equivalent to one another.

It is easy to show that (10) and (11) are disaggregated versions of (1) and (2), respectively. In fact, let us make the following positions in order to pass to the

[2]Neoclassical economics has a different notion of increasing (decreasing) returns to scale than the one employed herein. According to the hypotheses of neoclassical economics, a set of different technologies is given and each of them is appropriate to a particular scale of production. Consequently, neoclassical economics is concerned with equilibrium arising out of given technologies. On the contrary, here the focus is on recognition and adoption of novel technologies. Consequently, the model presented herein tells *stories* about striving to increase returns to scale with novel means.

macroeconomic level:

$$\mathbf{p}_c^T \Delta \mathbf{c}_t = C_t - C_{t-1} \tag{13}$$
$$\mathbf{p}_k^T \Delta \mathbf{k}_t = I_t \tag{14}$$
$$\mathbf{p}_l^T \Delta \mathbf{l}_t = Y_t - Y_{t-1} \tag{15}$$

where $C$, $I$ and $Y$ represent aggregate consumption, aggregate investment and aggregate income, respectively. Their aggregation was carried out by means of the corresponding price vectors $\mathbf{p}_c$, $\mathbf{p}_k$ and $\mathbf{p}_l$, respectively.

If the economy is close enough to perfect competition, we can write $\mathbf{p}_k \simeq \mathbf{A}\mathbf{p}_c$ and $\mathbf{p}_c \simeq f(\mathbf{p}_l)$. By combining these equations with (14), (10) and (11) one obtains:

$$I \simeq \mathbf{p}_c^T \mathbf{A}^T \mathbf{A} \Delta \mathbf{c} \tag{16}$$
$$I \simeq f^T(\mathbf{p}_l) \mathbf{A}^T \mathbf{A} f(\Delta \mathbf{l}) \tag{17}$$

which, keeping in mind equations (13) and (15), in the one-dimensional case boil down to (1) and (2), respectively.[3]

Goodwin's accelerator (3) is more complex than Samuelson's and Hicks', since its coefficients are allowed to change at the turning points of business cycles. In the ensuing section, we shall interpret the coefficients of a disaggregated accelerator as the coefficients of neurons that represent decision-making.

# 4 The Flexible Accelerator

Let us suppose that firms may face situations that they never met before, opportunities that involve producing and commercializing qualitatively novel goods, which in their turn require novel production technologies and imply novel consumption habits. If this is the case, undertaking an investment does not mean that a firm is making a plan about increasing its endowment of given machineries in order to increase its productive capacity of given goods. Rather, undertaking an investment means guessing the most recent development of consumers' desires, designing novel goods in order to meet these desires, and ordering construction of proper machineries in order to produce them.

[3] Actually, equation (2) depends on $Y_{t-1} - Y_{t-2}$, whereas the corresponding equation that can be derived from (17) depends on $Y_t - Y_{t-1}$. However, this difference would disappear if consumption would not be supposed to be istantaneous (i.e. if $f$ would introduce a time delay).

Thus, the crucial issue is classifying information provided by requests of existing goods as well as by novel techological possibilities. However novel this information might be, a firm must be able to form categories for kinds of investment that can be reasonably deemed to be more or less successful under several respects. Remarkably, the crucial step is not that of attaching probabilities of monetary returns to investments of different kinds, but rather that of defining ”kinds” that are able to distinguish successful investments from unsuccessful ones. Clearly, categories of investments are formed by highlighting patterns in incoming information, such as patterns of request of goods that entail new technologies.

In this section, the number of final goods will be kept fixed to $N$ and the number of capital goods will be kept fixed to one. However, as a consequence of technological innovation the qualitative features of goods may change with time. Information to be classified regards both the direction of technological change and the reception of goods that entail them by the public.

A Kohonen neural net will be used in order to reproduce classification of situations and investment decision-making. Each neuron will represent decision-making by a single firm, so the net as a whole will represent the productive system. Notably, in this model the behavior of the productive system as a whole depends on the structure of the connections between its components.

For each neuron, the learning term in equation (5) has a straightforward interpretation. In fact, firms classify information into different categories according to the market in which they specialized, where in its turn firm specialization depends on physical and human capital accumulated as a consequence of past investment decisions. This inertial factor is subsumed by $\phi(\mathbf{a}, y)x_i$ terms, which act as localized memories for firms decision-making.

However, renewal of capital goods is eased by natural ageing of existing machinery. Similarly, renewal of human capital is eased by personnel turnover. In equation (5), forgetting terms $\gamma(\mathbf{a}, y)a_i$ express this second effect. In other words, $\phi$-terms account for biases posed to decision-making by the existing capital stock, whereas $\gamma$-terms account for the new decision possibilities opened up by wear and tear.

Clearly, decision-making is strongly path-dependent in this model. What prevents firms that operate e.g. in the furniture market from entering e.g. the computer market is simply the fact that they never did this job: they are not acquainted with the computer market, they never developed the categories that would enable them to understand which items are most profitable in this market, they own completely different capital goods.

However, path-dependence does not mean that the role of each firm is fixed once and for all. In fact, firms continuously innovate their products as well as their production techniques, and occasionally it does happen that a technological breakthrough leads a firm into a completely different field of activity. Nevertheless, past experiences generally influence which innovations are carried out by which firms, and even firms that belong to the same industry may exhibit striking differences in their relative abilities to recognize the profitability of an innovation. Once again, this ability depends on the categories employed by a firm in order to classify information.

Let us suppose that the current state of technologies is subsumed by an $N$-dimensional, exogenous vector $\mathbf{e}$. The $i$-th component of $\mathbf{e}$ is the technological content of the $i$-th final good.

Let us assume that, coherently with the assumptions of the investment acceleration principle, managers are reactive to *variations* of technologies. Thus, information on new technologies is carried by a vector $\Delta\mathbf{e}$, where $\Delta\mathbf{e}_t = \mathbf{e}_t - \mathbf{e}_{t-1}$ with a vector of zeros as initial conditions. The $i$-th component of $\Delta\mathbf{e}$ represents the amount of technological innovation that can impact the $i$-th final good.

Let us assume that information carried by $\Delta\mathbf{e}$ is free and available to all firms. Note that vector $\Delta\mathbf{e}$ does not represent technological details that are developed by firms themselves and that are kept strictly private unless acquired under licensing agreement. Rather, $\Delta\mathbf{e}$ represents all publicly available information about new technologies which can induce managers to invest on a specific field, eventually developing private information as a consequence of this decision. It includes basic research made available by non-profit institutions, rumors about competitors' strategies, as well as information that was intended to be private but which is actually difficult to appropriate and to trade, e.g. because of reverse engineering [4].

Figure (3) illustrates the neural net that represents decision-making in the productive system. Firms in the final goods sector are represented by the first layer of neurons, the one on the left side. On the contrary, firms in the capital goods sector are represented by the second layer of neurons, the one on the right side. Just like in figure (2), inner and outer feed-backs give rise to the multiplier and the accelerator, respectively.

A difference with the previous Section is that now firms in the final goods sector receive exogenous information about innovations besides information on consumers demand. Thus, equations (10), (11), (12) become:

$$\Delta\mathbf{k}_t = \mathbf{A}\Delta\mathbf{c}_{t-1} + \mathbf{B}\Delta\mathbf{e}_{t-1} \qquad (18)$$

$$\Delta\mathbf{k}_t = \mathbf{A} f(\Delta\mathbf{l}'_{t-1} + \Delta\mathbf{l}''_{t-1}) + \mathbf{B}\Delta\mathbf{e}_{t-1} \qquad (19)$$

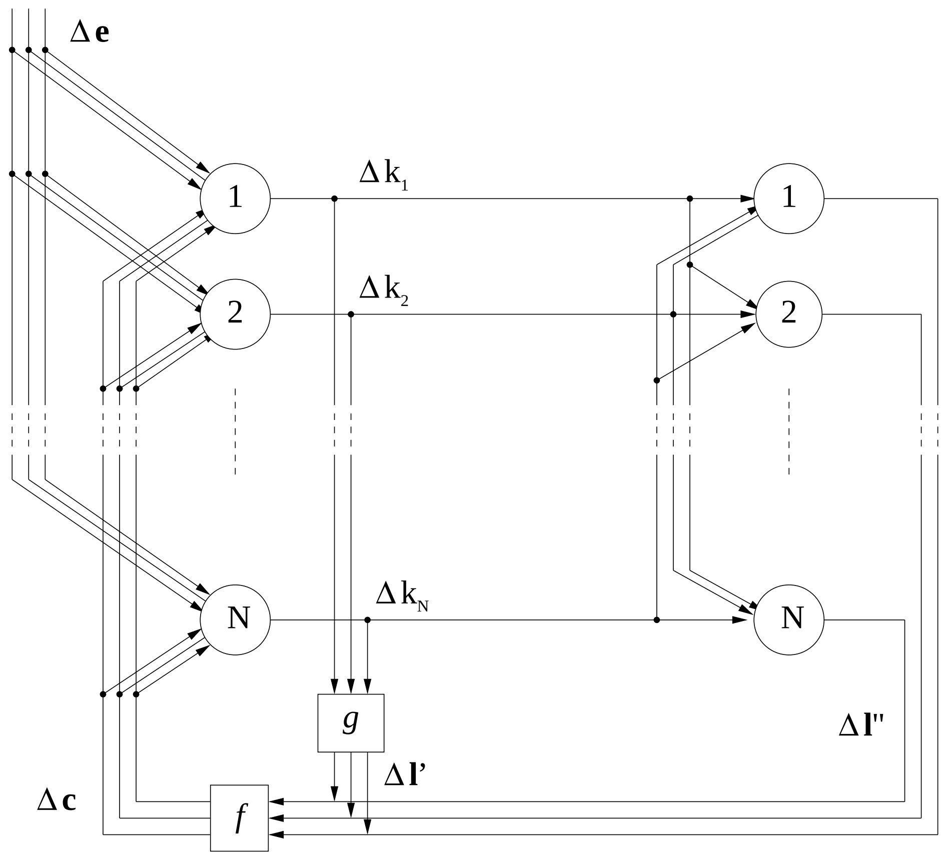


Figure 3: Decision-making in the productive system, described by means of a neural net. Each neuron represents decision-making by one firm. The left layer represents firms that produce final goods, the right layer represents firms that produce capital goods. Two information feed-backs through the labor market give rise to the multiplier ad the accelerator, respectively.

$$\Delta \mathbf{k}_t = \mathbf{A} f(g(\Delta \mathbf{k}_{t-1}) + \mathbf{D}\Delta \mathbf{k}_{t-2}) + \mathbf{B}\Delta \mathbf{e}_{t-1} \tag{20}$$

where **B** is the $N \times N$ matrix of the coefficients by which information on innovation is processed.

Ultimately, neurons are devices that operate linear combinations of the information vectors that they receive as input. Thus, matrices **A**, **B** and **D** can be interpreted as neurons coefficients.

In particular, the *i*-row of these matrices contains the coefficients of the *i*-th neuron of its industrial sector. In particular, the rows of matrices **A** and **B** contain coefficients of the neurons in the left layer (final goods sector), whereas the rows of matrix **D** contain coefficients of the neurons in the right layer (capital goods sector). Note that neurons in the left layer have two sets of coefficients, the first one for weighing information about consimers' demand and the second one for weighing information on new technologies.

Since matrices **A**, **B**, **D** change with time according to equation (5), equations (18), (19), (20) now describe a flexible accelerator. Note also that, since **A**, **B**, **D** have become variables, accelerator equations are no longer linear.

Equation (5) can be operationalized in many ways, according to the choice of $\phi$ and $\gamma$. A simple choice that is good enough in the early stages of pattern recognition is [29]:

$$\frac{d\mathbf{A}}{dt} = \mu \Delta \mathbf{k} \Delta \mathbf{c}^T - \nu \mathbf{A} \tag{21}$$

$$\frac{d\mathbf{B}}{dt} = \mu \Delta \mathbf{k} \Delta \mathbf{e}^T - \nu \mathbf{B} \tag{22}$$

$$\frac{d\mathbf{D}}{dt} = \mu \Delta \mathbf{l}'' \Delta \mathbf{k}^T - \nu \mathbf{D} \tag{23}$$

where matrix derivative applies element by element.

In equations (21), (22), (23) the learning term enhances coefficients that yield a high output for a high input. On the contrary, the forgetting term scales down coefficients exponentially with time.

Matrices **A**, **B**, **D** specify the structure of information circuits that, passing through the two feed-backs created by the labor market, can traverse the productive system along a number of different paths. Each particular structure of these paths corresponds to certain firms having specialized into certain technologies with varying degrees of success. Thus, matrices **A**, **B**, **D** specify the *distributed memory* of the productive system, its collective behavior when it is confronted with information on novel technologies injected by $\Delta \mathbf{e}$.

However, a productive system is likely to *learn* to deal with innovations and exploit the novel possibilities that they open up. Equations (21), (22), (23) tell us how the productive system can develop a new structure by transforming the paths of its information circuits and memorize a new configuration.

# 5 A Numerical Example

The accelerator equations derived so far aimed at describing investments at the very beginning of recovery phases. Thus, they should be evaluated when the productive system receives the first hints of the novel technologies that will trigger a new phase of expansion.

Such a situation is characterized by the slow emergence of patterns in a sea of indistinct chaos. Information on technological patterns is made of rumors and hints that, for instance, biotechnologies are going to have a future in the first decades of the XXI century. Thus, it makes sense to invest there.

In the simulation presented herein, the state of technology is represented by a sinusoid that is slowly emerging out of white noise. This sinusoid is defined over goods, and represents their development possibilities opened up by new technologies at each time step. Production of goods that are positively affected by novel technologies is likely to expand and their qualitative features is likely to change.

During 100 time steps this sinusoid spans 100 goods with 5 periods of 20 goods each. However, its amplitude $A_{min} = 0$ at time $t = 1$ increases linearly up to $A_{max} = 2$ at time $t = 100$. Thus, the pattern expressed by this sinusoid is invisible at $t = 1$ and becomes increasingly evident with time. Upon this pattern, a noise generated by a normal distribution with zero mean represents ambiguity regarding which goods will be blessed by novel technologies. However, the variance of this distribution decreases from $V_{max} = 1$ at time $t = 1$ to $V_{min} = 0$ at time $t = 100$. Thus, the overall effect is that of a sinusoidal pattern slowly emerging from chaos.

Figure (4) illustrates the sequence of vectors on the state of technology $\mathbf{e}$ that will be employed in the simulation. Since a three-dimensional graph would be difficult to read, this figure shows its horizontal section at $\mathbf{e} = \mathbf{0}$. Black areas denote the parts of the three-dimensional graph where $\mathbf{e} > \mathbf{0}$, white areas denote the parts of the three-dimensional graph where $\mathbf{e} \leq \mathbf{0}$. Thus, emergence of a sinusoid reflects in the formation of stripes out of irregular spots.

Information on technological novelties is carried by $\Delta\mathbf{e}$, which is obtained by differentiation of $\mathbf{e}$. However, since managers are likely to attach comparable importance to information on innovation and information stemming from demand,

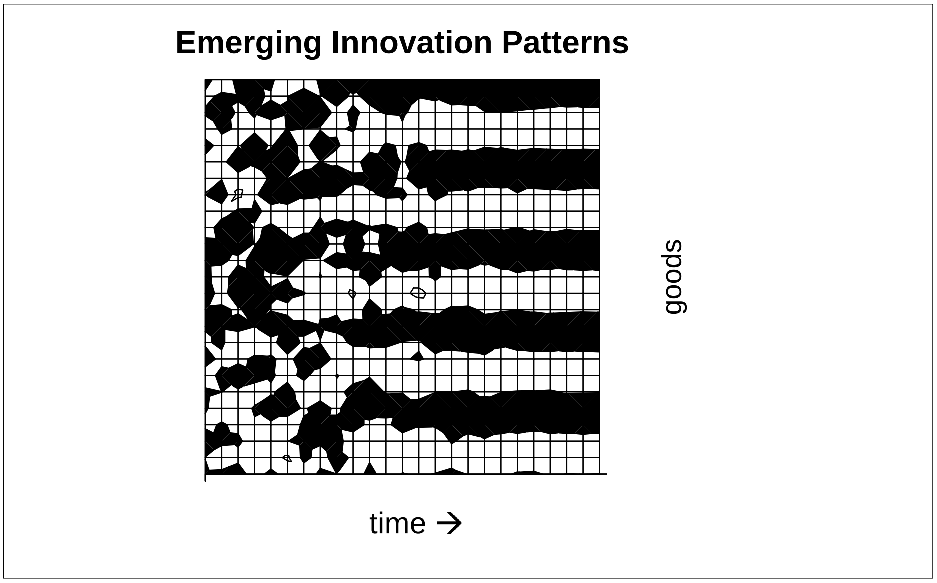


Figure 4: A sequence of vectors $\mathbf{e}$ fron $t = 1$ to $t = 100$, horizontal sections at $\mathbf{e} = 0$. Black areas correspond to $\mathbf{e} > \mathbf{0}$, white areas correspond to $\mathbf{e} \leq \mathbf{0}$. In order to simplify the image only one out of four firms and one out of four time steps have been shown, resulting in a $25 \times 25$ grid.

vectors $\Delta\mathbf{e}$ and $\Delta\mathbf{c}$ should be of similar size. Thus, at each simulation step the interval spanned by $\Delta\mathbf{e}$ has been adjusted to the one spanned by $\Delta\mathbf{c}$ and the median of $\Delta\mathbf{e}$ has been shifted to that of $\Delta\mathbf{c}$.

By inserting equations (6), (7) and (9) into (18), it is possible to obtain investments $\Delta\mathbf{k}$ from information on innovation $\Delta\mathbf{e}$ and previous values of $\Delta\mathbf{c}$, $\Delta\mathbf{k}$, $\Delta\mathbf{l}'$, $\Delta\mathbf{l}''$. Thus, simulations basically consist of feeding the above equations with a series of vectors $\Delta\mathbf{e}$ like the one illustrated in figure (4) and observing the corresponding $\Delta\mathbf{k}$.

Furthermore, one should consider that decision-making is *rational* only if it is channeled within a set of logical constraints [45], [46]. In this model, let the outcome of neurons be constrained by the following two rules:

1. Output is not allowed to be negative. Thus, in the short run capital equipment cannot be disinvested and workers cannot be fired.

2. Credit exists, but loans cannot be indefinetly large. Since it is likely that capital stock serves as collateral, it is assumed that the output of a neuron cannot be larger than cumulative output (this rule is not applied if cumulative output is zero).

Initial conditions, keeping in mind that we are describing the onset of a recovery, are obviously $\mathbf{c} = \mathbf{0}$, $\mathbf{k} = \mathbf{0}$, $\mathbf{l}' = \mathbf{0}$, $\mathbf{l}'' = \mathbf{0}$ and $\Delta\mathbf{c} = \mathbf{0}$, $\Delta\mathbf{k} = \mathbf{0}$, $\Delta\mathbf{l}' = \mathbf{0}$, $\Delta\mathbf{l}'' = \mathbf{0}$. Learning and forgetting parameters have been set at $\mu = 0.1$ and $\nu = 0.1$, respectively. Matrices $\mathbf{A}$, $\mathbf{D}$ and $\mathbf{B}$ have been initialized by means of a normal distribution with variance $W = 100$.

Figure (5) illustrates aggregate investments during a hundred time steps, in logarithmic scale. Dashed lines represent the outcome of ten different simulations whereas the thick line results from their average.

The most interesting feature of the investment curves illustrated in figure (5) is the discontinuity that they all exhibit between the $50^{th}$ and the $60^{th}$ time step. In fact, this point in time corresponds to the emergence of a pattern in information on innovation as it is illustrated in figure (4). It is evident that recognition of novel investment possibilities takes place at once, when firms suddenly understand what a pattern is emerging from chaos.

Figure (6) illustrates three indicators $\alpha$, $\beta$ and $\delta$ of the variation of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{D}$, respectively. Indicators $\alpha$, $\beta$ and $\delta$ have been defined as the sum of the absolute variations of all elements of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{D}$, respectively.

Figure (6) makes clear that $\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$ behave very similarly to one another. In fact, all three matrices vary according to an exponential path that has two sharp

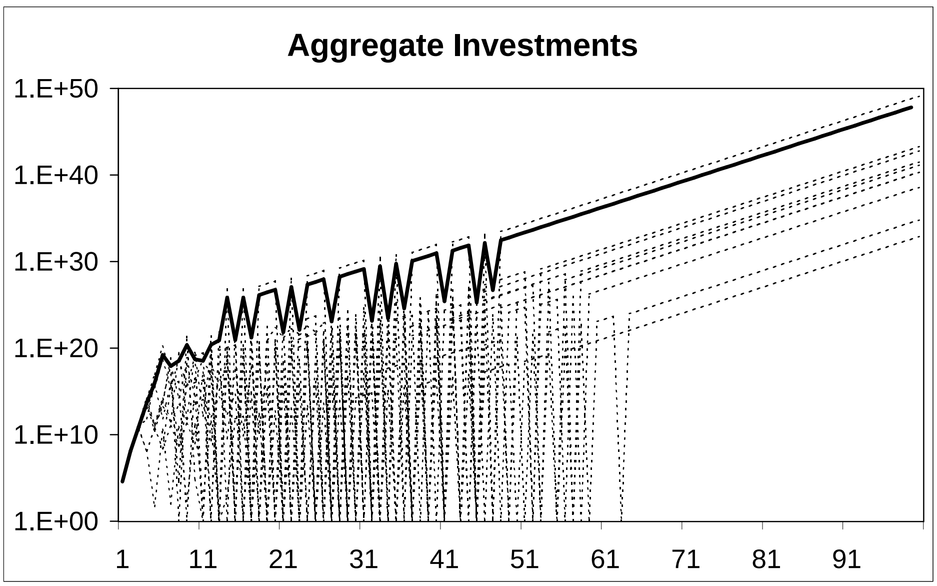


Figure 5: Aggregate investments in logarithmic scale, 100 time steps. Dashed lines illustrate aggregate investments during ten simulation runs with the same parameters set, the thick line results from their average. In order to represent zero values on a logarithmic scale, a one has been added to all values of aggregate investments.

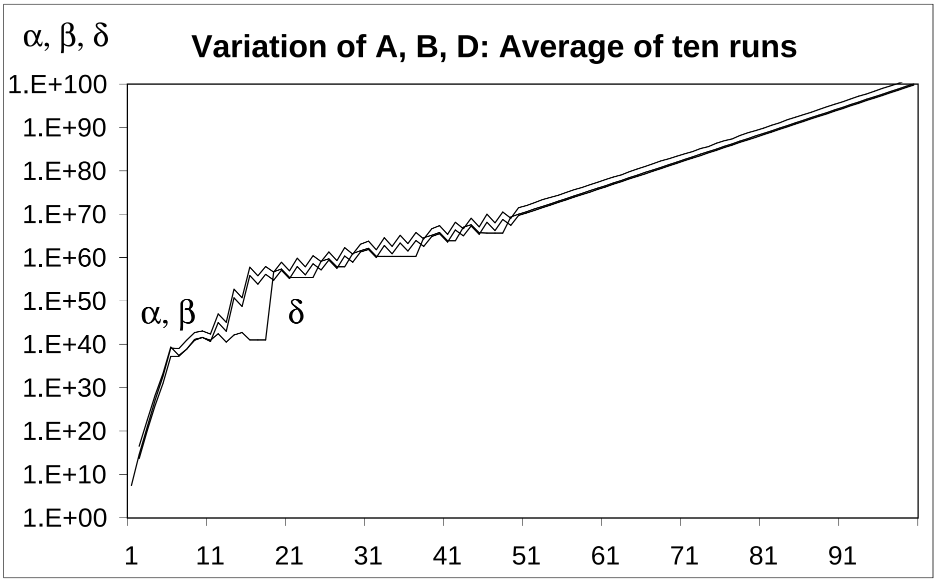


Figure 6: Variation of **A**, **B**, **D** in logarithmic scale. Indicators α, β and δ express the sum of the absolute variations of the elements of **A**, **B** and **D**, respectively. Values have been averaged over ten simulations.

discontinuities, the first one at the very beginning of the simulation, when coefficients move away from initial values that had been set at random and the second one around the $50^{th}$ time step, when firms recognize a novel technological pattern. Since we are observing the initial phase of an exponential dynamics and not a cycle, accelerator coefficients entailed by **A**, **B** and **D** tend to grow indefinetly. However, by detrending along the growth path we would obtain the values of a fixed-coefficients accelerator equation *before* and *after* recognition of technological innovations.

Aggregate dynamics arise out of microeconomic investments that are likely to be different across firms. Actually, the rationale for using a neural net is that firms specialize into different fields of activity, that are likely to be hit by technological innovation to varying extent and generate investments in varying degrees. Figure (7) illustrates investments by each firm during one simulation.

Figure (7) shows that, although all firms behave in phase because they all access the same information, different initial conditions with respect to physical and human capital expressed by **A**, **B**, **D** at time $t = 0$ make them grow according to exponential paths that may have very different slopes. In other words, depending on their initial endowment firms develop idiosyncratic knowledge that is specific

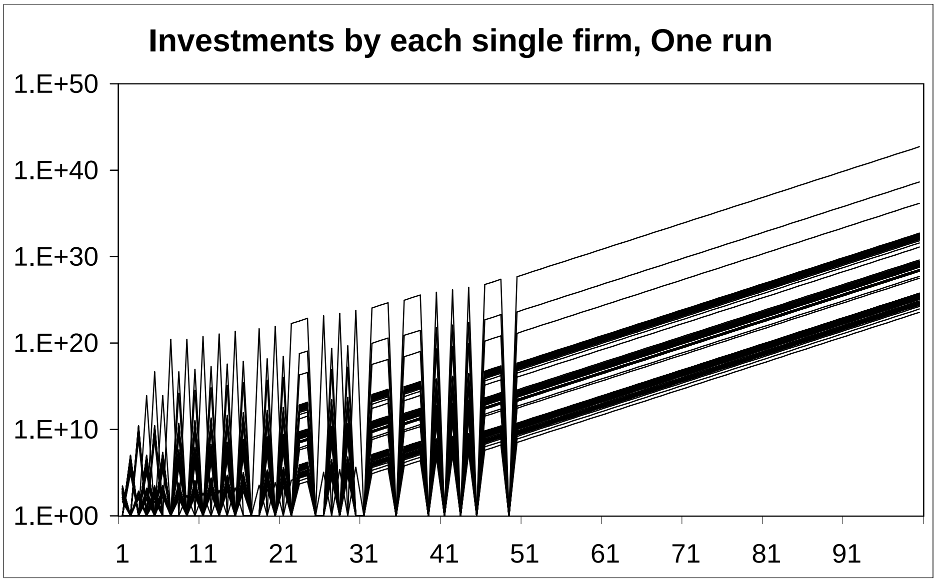


Figure 7: Investments by each single firm. This picture illustrates $\Delta k_i$ for $i = 1, 2, \ldots 100$ along 100 time steps of one single simulation. In order to represent zero values on a logarithmic scale, a one has been added to all components of $\Delta \mathbf{k}$.

to particular fields of activity characterized by different growth paths.

# 6 Conclusions

This article presented a cognitive model of the very beginning of the process of investments acceleration, a phase that is crucial to the onset of economic recoveries. Notably, it is a model that operates at the microeconomic and macroeconomic level at the same time. This result could be achieved because the *structure* of interactions between industrial sectors was described.

Structure embodies the distributed knowledge of an economy, representing which kinds of technologies it is able to exploit and implement. Ultimately, this depends on the history of an economy represented by the initial conditions of matrices **A**, **B**, **D** and later on by their evolution with time. Since this evolution depends on the sequence of exogenous vectors $\Delta \mathbf{e}$, this model is definitely path-dependent in spirit and practice.

The model prove to be quite stable with respect to parameters, but not with respect to decision rules (1) and (2) of Section 5. In fact, different decision rules implement alternative procedural rationalities that ultimately lead to opposite outcomes. In this paper, only very simple rules have been used.

Matrices **A**, **B**, **D** link the accelerator coefficient to firms' experiences embodied in their knowledge, both at the individual and the systemic level. Analytical treatment was kept at a basic level, but further investigations are available in a companion paper [17].

Possibilities for empirical applications are hindered by the evident difficulty of encoding rumors on technological novelties into strings of zeros and ones, as vectors $\Delta\mathbf{e}$ are. Note that this is not a difficulty in principle, but it is in practice because it is difficult to think of homogeneous empirical documentation of what managers, at any precise point in time, knew about technological perspectives.

However, it is easy to think of an application of a reduced version of the model presented herein, where the two information feed-backs are cut and empirical data on demand are used. By doing this, one can think of modeling managers' reasoning in order to derive disaggregated investments from disaggregated demand, where the performance of the model could be chacked against empirical data on investments. Actually, a first attempt in this direction yielded very encouraging results [18].

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