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Robust adaptive fuzzy tracking control for a class of perturbed strict-feedback nonlinear systems via small-gain approach

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Abstract

This paper presents a novel robust adaptive fuzzy tracking controller (RAFTC) for a wide class of perturbed strict-feedback nonlinear systems with both unknown system and virtual control gain nonlinearities. For unknown system nonlinearities, two types for them are included: one naturally satisfies the "triangularity condition" and may possess a class of unstructured uncertain functions which are not linearly parameterized, while the other is partially known and consists of parametric uncertainties and known "bounding functions". The Takagi–Sugeno type fuzzy logic systems are used to approximate unknown system nonlinearities and a systematic design procedure is developed for synthesis of RAFTC by combining the backstepping technique and generalized small-gain approach. The algorithm proposed is highlighted by three advantages: (i) the semi-global uniform ultimate bound of RAFTC in the presence of perturbed uncertainties and unknown virtual control gain nonlinearities can be guaranteed, (ii) the adaptive mechanism with minimal learning parameterizations is obtained and (iii) the possible controller singularity problem in some of the existing adaptive control schemes with feedback linearization techniques can be removed. Performance and limitations of proposed method are discussed and illustrated with simulation results.

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1. Introduction

In the past decade, there has been a rapid growth of research efforts aimed at the development of systematic design methods for the adaptive control of nonlinear systems with parametric uncertainty. Many remarkable results have been obtained owing to the advances in geometric nonlinear control theory, and in particular, feedback linearization techniques [1,2]. As a breakthrough in nonlinear control area, a recursive design procedure, adaptive backstepping approach, was presented to obtain global stability and asymptotic tracking for a large class of nonlinear systems which can be transformable to parametric strict-feedback canonical ones, e.g. [3–6]. The overparametrization problem was soon eliminated by Krstic et al. [7] by elegantly introducing the concept of tuning function. Recently, nonlinear damping was also introduced in the controller by Kanellakopoulos [8,9] to improve transient performance.

However, the common feature of the adaptive control algorithms discussed in [3–9] is to deal with the case of uncertainties of the systems in the linearly parameterized forms, that is, the system's nonlinearities are assumed to be known while parameters are unknown and linear with respect to those known nonlinear functions. Unfortunately, in industrial control environment, some systems are characterized by a wide class of uncertainties referred to as unstructured ones, which cannot be modelled or repeatable. In order to cope with such kind of uncertainties, as an alternative, approximator-based control approaches have been studied for those systems in a Brunovsky form using Lyapunov stability theory, e.g. [10–17]. Recently, the developed approximator-based adaptive control approaches were extended to strict-feedback nonlinear systems with unstructured uncertainties using the idea of adaptive backstepping by means of the neural network or fuzzy system approximators, e.g. [18–20].

In this paper, we present a fuzzy system approximator-based robust adaptive control design procedure for a class of strict-feedback nonlinear systems in the general form

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i(\bar{x}_i) + \Delta_i(t,x), \quad 1 \leqslant i \leqslant n-1 \\ \dot{x}_n = f_n(x) + g_n(x)u + \Delta_n(t,x) \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$ is the system state vector, $u \in R$ is the input, $y \in R$ is the output of system. Let $\bar{x}_i = [x_1, x_2, \ldots, x_i]^{\mathrm{T}}$. $f_i(\bar{x}_i)$'s are unknown smooth system functions with $f_i(0) = 0$ and $g_i(\bar{x}_i)$'s are unknown smooth functions which are referred to as the virtual control gain ones. All of functions $f_i(\bar{x}_i)$ and $g_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$ may not be linearly parameterized. $\Delta_i(t,x)$'s are the disturbance uncertain nonlinearities of the system.

Note that for the adaptive fuzzy control design of uncertain strict-feedback canonical system (1), there are two main difficulties. One comes from the uncertain virtual control gain functions $g_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$. When $g_i(\bar{x}_i)$'s are unknown nonlinearities, if feedback linearization type controllers $\alpha_i = (1/\hat{g}_i(\cdot))(-\hat{f}_i(\cdot) + v_i)$ are considered [45], where $\hat{f}_i(\cdot)$ and $\hat{g}_i(\cdot)$ are the estimates of $f_i(\cdot)$ and $g_i(\cdot)$, respectively, and v_i is a new control to be defined, the difficulty arises when $\hat{g}_i(\cdot) \to 0$, which is referred to the controller singularity problem. There are some robust adaptive control algorithms for system (1) have been developed in [21–25], when the virtual control gain functions $g_i(\bar{x}_i) = 1$, $i = 1, 2, \ldots, n-1$, for example, in [21], a stable adaptive fuzzy control was presented for strict-feedback nonlinear systems with virtual control gains equal to one and without the disturbance uncertain nonlinearities. The problem of adaptive control of systems with unknown virtual control gain functions has also received much attention in recent years. In [26], an adaptive control solution was presented for strict-feedback nonlinear systems with unknown virtual control gain constants. The other is that many parameters need to be tuned in the on-line learning laws when there are many state variables in the designed systems, particularly by use of fuzzy logic systems adopted by [21], when many rule bases are used in the fuzzy systems to approximate the uncertain nonlinear functions, so that the learning time tends to become unacceptably large for systems of higher order and time-consuming process is unavoidable when the controllers above are implemented.

In this paper, a new systematic design procedure will be developed for the synthesis of the stable robust adaptive fuzzy tracking controller for perturbed strict-feedback canonical nonlinear systems in the presence of unstructured uncertainties and unknown virtual control gain nonlinearities. Takagi–Sugeno type fuzzy logic systems [27] are used to approximate the unstructured uncertain functions. Then a stable robust adaptive fuzzy controller is proposed by use of input-to-state stability (ISS) theory [28] and by combining backstepping technique with generalized small-gain approach [29]. The controller proposed in this paper guarantees semi-global uniform ultimate boundedness in the presence of unstructured uncertainties. The outstanding features of derived controller are that it has the adaptive mechanism with minimal learning parameterizations, no matter how many states in the designed systems are investigated and how many rules in the fuzzy logic systems are used, the order of the dynamic compensator is only $2n$, where n is the dimensions of the state in the designed systems, such that the burdensome computation of the algorithm can be lightened and it is convenient to realize the algorithm in engineering, and meanwhile it can avoid the possible controller singularity problem in some of the existing adaptive control schemes with feedback linearization techniques.

This paper is organized as follows. In Section 2, we will review T–S fuzzy logic systems, some necessary definitions of input–to–state stability (ISS), and small-gain theorem. Section 3 proposes a motivating problem. In Section 4, a systematic procedure for the synthesis of robust adaptive fuzzy tracking controller is developed. In Section 5, an application is used to demonstrate the effectiveness of proposed schemes. The final section contains conclusions.

Notation: Throughout this paper, let $\|\cdot\|$ be any suitable norm. The vector norm of $x \in R^n$ is Euclidean, i.e., $\|x\|^2 = (x^{\mathrm{T}}x)$ and the matrix norm of $A \in R^{n\times m}$ is defined by $\|A\|^2 = \lambda_{\max}(A^{\mathrm{T}}A)$, where $\lambda_{\max(\min)}(\cdot)$ denotes the operation of taking the maximum (minimum) eigenvalue. The vector norm over the space defined by stacking the matrix columns into a vector, so that it is compatible with the vector norm, i.e., $\|Ax\| \leqslant \|A\| \cdot \|x\|$.

For any piecewise continuous function $u : R_+ \to R^m$, $\|u\|_\infty$ denotes $\sup\{|u(t)|, t \geqslant 0\}$, which stands for L_∞ supremum norm, and for any pair of times $0 \leqslant t_1 \leqslant t_2$, the truncation $u_{[t_1,t_2]}$ is a function defined on R_+ which is equal to $u(t)$ on $[t_1, t_2]$ and is zero outside the interval. In particular, $u_{[0,t]}$ is the usual truncated function u_t.

2. Preliminaries

2.1. T–S fuzzy systems

In the past few years, various types of fuzzy logic systems (e.g. Mamdani type and Takagi–Sugeno type) have been proved to be universal approximators in that they can uniformly approximate any continuous functions defined on compact domains to any degree of accuracy [30–33]. The fuzzy logic systems use the fuzzy IF–THEN rules to perform a mapping from an input linguistic vector $x^{\mathrm{T}} = (x_1, x_2, \ldots, x_n) \in R^n$ to an output linguistic variable $y \in R$. Generally, the fuzzy logic systems can be constructed by the following $K(K > 1)$ fuzzy rules

$$\begin{aligned} R_i: \quad & \text{If } x_1 \text{ is } \Psi^i_{h_1} \text{ AND } x_2 \text{ is } \Psi^i_{h_2} \text{ AND} \ldots \text{AND } x_n \text{ is } \Psi^i_{h_n} \\ & \text{THEN } y_i \text{ is } \Omega^i_{h_1 h_2 \cdots h_n}, \quad i = 1, 2, \ldots, K \end{aligned}$$

where $\Omega^i_{h_1 h_2 \cdots h_n}$ denotes an output fuzzy set. If $\Omega^i_{h_1 h_2 \cdots h_n}$ is a singleton fuzzy set, its membership function is 1 only at $y_i = \sigma_i$ (an arbitrary unknown constant) and 0 at other position, then that is called Mamdani type fuzzy system. If $\Omega^i_{h_1 h_2 \cdots h_n}$ is a function of $a_{i0} + a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n$ which a_{ij}, $i = 1, 2, \ldots, K$, $j = 0, 1, \ldots, n$ are the unknown constants, then that is called Takagi–Sugeno type fuzzy system, T–S fuzzy system for short. The product fuzzy inference is employed to evaluate the ANDs in the fuzzy rules. After being defuzzified by a typical center average defuzzifier, we can write the output of T–S fuzzy system in the vector form as follows

$$\hat{f}(x, A_x) = \xi(x)A_x\bar{x} \tag{2}$$

where $\xi(x) = [\xi_1(x), \xi_2(x), \ldots, \xi_K(x)]$, $\xi_i(x) = \prod_{j=1}^n \mu^i_{h_j}(x_j) / \sum_{i=1}^K \left[\prod_{j=1}^n \mu^i_{h_j}(x_j)\right]$, $i = 1, 2, \ldots, K$ is called as fuzzy basis function and $\bar{x} := [1, x]$. When the membership function $\mu^i_{h_j}(x_j)$ in $\xi_i(x)$ is denoted by some type of membership function. In Eq. (2), we have

$$A_z = \begin{bmatrix} a_{10} & a_{11} & \cdots & a_{1n} \\ a_{20} & a_{21} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{K0} & a_{K1} & \cdots & a_{Kn} \end{bmatrix}.$$

For any continuous function $f(x)$, there exists some type fuzzy logic system to approximate it to an arbitrary accuracy. Then there exists the lemma proved by Wang [32] and Cao et al. [33].

Lemma 1. *Suppose that the input universal of discourse U is a compact set in R^r. Then, for any given real continuous function $f(x)$ on U and* $\forall\varepsilon > 0$, *there exists a T–S type fuzzy system in the form of* (2) *such that*

$$\sup_{x\in U} \|f(x) - \hat{f}(x, A_x)\| \leqslant \varepsilon \tag{3}$$

Remark 1. For any n-dimensional continuous function $f(x)$, if $N_i + 1$ input fuzzy sets for each variable x_i are used, there will be $K = \prod_{i=1}^n (N_i + 1)$ IF-THEN fuzzy rules in the fuzzy system. If Mamdani type system is used to approximate the function $f(x)$, we observe that there are a total of $\prod_{i=1}^n (N_i + 1)$ parameters to describe it. And while T–S type fuzzy system is used there are a total of $(n + 1) \cdot \prod_{i=1}^n (N_i + 1)$ parameters to describe it.

2.2. ISS and small-gain theorem

The concepts of ISS and ISS-Lyapunov function proposed by [28,34,35] have recently been used in various control problems such as nonlinear stabilization, robust control and observer designs (see, e.g. [36–41]). In order to ease the discussion of the design of RAFTC scheme, two definitions with respect to input-to-state stability are reviewed in the following. First, we recall the class K, K_∞ and KL functions which are standard in the stability literature, see Khalil [42].

A class K-function γ is a continuous, strictly increasing function from R_+ into R_+ and $\gamma(0) = 0$. It is of class K_∞ if additionally $\gamma(s) \to \infty$ as $s \to \infty$. A function $\beta : R_+ \times R_+ \to R_+$ is of class KL if $\beta(\cdot, t)$ is of class K for every $t \geqslant 0$ and $\beta(s, t) \to 0$ as $t \to \infty$.

Definition 1. For the system $\dot{x} = f(x, u)$, it is said to be input-to-state practically stable (ISpS) if there exist a function γ of class K, called the nonlinear L_∞ gain, and a function β of class KL such that, for any initial condition $x(0)$, each measurable essentially bounded control $u(t)$ defined for all $t \geqslant 0$ and a non-negative constant d, the associated solutions $x(t)$ are defined on $[0, \infty)$ and satisfy:

$$\|x(t)\| \leqslant \beta(\|x(0)\|, t) + \gamma(\|u_t\|_\infty) + d \tag{4}$$

When $d = 0$ in (4), the ISpS property reduces to the input-to-state stability (ISS) property introduced in [34].

Definition 2. A C^1 function V is said to be an ISpS-Lyapunov function for the system $\dot{x} = f(x, u)$ if there exist functions α_1, α_2, α_3, α_4 of class K and a constant $d > 0$ such that

$$\alpha_1(\|x\|) \leqslant V(x) \leqslant \alpha_2(\|x\|), \quad \forall x \in R^n \tag{5}$$

$$\frac{\partial V(x)}{\partial x} f(x, u) \leqslant -\alpha_3(\|x\|) + \alpha_4(\|u\|) + d \tag{6}$$

When (6) holds with $d = 0$, V is referred to as an ISS-Lyapunov function.

Then it holds that one may pick a nonlinear L_∞ gain γ in (4) of the form [36]

$$\gamma(s) = \alpha_1^{-1} \circ \alpha_2 \circ \alpha_3^{-1} \circ \alpha_4(s), \quad \forall s > 0 \tag{7}$$

The following proposition establishes equivalence between ISpS and the existence of ISpS-Lyapunov function by Sontag and Wang [35], Praly and Wang [40].

Proposition 1. *The system $\dot{x} = f(x, u)$ is ISpS if and only if there exists an ISpS-Lyapunov function.*

A trivial refinement of the proof of the generalized small-gain theorem given by [29,41] yields the following variant.

Theorem 1. *Consider a system in composite feedback form*

$$\Sigma_{\tilde{z}\omega} : \begin{cases} \dot{x} = f(x, \omega) \\ \tilde{z} = H(x) \end{cases} \tag{8}$$

$$\Sigma_{\omega\tilde{z}} : \begin{cases} \dot{y} = g(y, \tilde{z}) \\ \omega = K(y, \tilde{z}) \end{cases} \tag{9}$$

of two ISpS systems. In particular, there exist two constants $d_1 > 0$, $d_2 > 0$, and let β_ω and β_ξ of class KL, and γ_z and γ_ω of class K be such that, for each ω in the L_∞ supremum norm, each $\tilde{z}$ in the L_∞ supremum norm, each $x \in R^n$ and each $y \in R^m$, all the solutions $X(x; \omega, t)$ and $Y(y; \tilde{z}, t)$ are defined on $[0, \infty)$ and satisfy, for almost all $t \geqslant 0$:

$$\|H(X(x; \omega, t))\| \leqslant \beta_\omega(\|x\|, t) + \gamma_z(\|\omega_t\|_\infty) + d_1 \tag{10}$$

$$\|K(Y(y;\tilde{z},t))\| \leqslant \beta_{\xi}(\|y\|,t) + \gamma_{\omega}(\|\tilde{z}_t\|_{\infty}) + d_2 \tag{11}$$

Under these conditions, if

$$\gamma_z(\gamma_\omega(s)) < s \quad (\textit{resp.}\ \gamma_\omega(\gamma_z(s)) < s) \quad \forall s > 0, \tag{12}$$

then the solution of the composite systems (8) *and* (9) *is ISpS.*

3. A motivating problem

The primary goal of this paper is to track a given reference signal $y_d(t)$ while keeping the states and control bounded. That is, the output tracking error $z_1 = y(t) - y_d(t)$ should be small. The given bounded reference signal $y_d(t)$ is generated from the following smooth model

$$\begin{cases} \dot{x}_{di} = f_{di}(x_d), & 1 \leqslant i \leqslant m \\ y_d = x_{d1}, & n \geqslant m \end{cases} \tag{13}$$

where $x_d = [x_{d1}, x_{d2}, \ldots, x_{dm}]^{\mathrm{T}} \in R^m$ are the states, $y_d \in R$ is the system output, $f_{di}(\cdot)$'s are known nonlinear functions. Assume that the states of the reference model remain bounded, i.e., $x_d \in \Omega_d$, $\forall t \geqslant 0$. We can define the tracking error vector $e(t) = x(t) - \bar{x}_{d(n)}$.

Remark 2. Under the assumptions that the virtual control gain functions $g_i(\bar{x}_i) = 1$, $i = 1, 2, \ldots, n-1$ and the unknown functions $f_i(\bar{x}_i)$'s are linearly parameterized as $\theta_i^{\mathrm{T}}\psi_i(\bar{x}_i)$ with θ_i being the unknown constant parameters vector, several adaptive robust control algorithms for strict-feedback nonlinear systems (1) have been developed in [22–24]. If the unknown functions $f_i(\bar{x}_i)$'s cannot be linearly parameterized, the fuzzy logic system approximators were used to approximate functions $f_i(\bar{x}_i)$'s and $g_n(x)$ and an adaptive robust control algorithm was presented in [21]. In [26], an adaptive robust control algorithm was presented for strict-feedback nonlinear systems with unknown virtual control gain constants. However, for more general class of nonlinear uncertain systems like (1), few results are available in the literature. In this paper, we will address this problem.

For the system (1), the following assumptions are introduced.

Assumption 1. The uncertain virtual control gain functions $g_i(\bar{x}_i)$'s are confined within a certain range such that

$$0 < b_{\min} \leqslant |\, g_i(\bar{x}_i) \,| \leqslant b_{\max} \tag{14}$$

where $b_{\min}$ and $b_{\max}$ are the lower and upper bound parameters respectively.

The above assumption implies that smooth functions $g_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$ are strictly either positive or negative. From now onwards, without loss of generality, we shall assume $g_i(\bar{x}_i) \geqslant b_{\min} > 0$, $i = 1, 2, \ldots, n$. Assumption 1 is reasonable because $g_i(\bar{x}_i)$'s being away from zero is the controllable conditions of system (1). It should be emphasized that the upper and lower bound parameters, $b_{\max}$ and $b_{\min}$, are required for analytical purposes, theirs true value is not necessarily known.

Assumption 2. For $1 \leqslant i \leqslant n$, there exists an unknown positive constant p_i^* such that $\forall (t, x) \in R_+ \times R^n$

$$| \Delta_i(t, x) | \leqslant p_i^* \phi_i(\bar{x}_i)$$

where $\phi_i(\cdot)$ is a known nonnegative smooth function.

Generally speaking, the control objective is to find a robust adaptive fuzzy tracking controller for the system (1) in the following form

$$\dot{\chi} = \varpi(\chi, \xi(e), e), \quad \chi \in R^p \tag{15}$$

$$u = u(\chi, \xi(e), e) \tag{16}$$

where $\xi(e)$ is a known fuzzy base function vector. In such a way that all the solutions of the closed-loop system (1), (15) and (16) are globally uniformly ultimately bounded. Furthermore, the tracking error vector e can be rendered small.

Remark 3. From Eqs. (15) and (16), we can observe that it is a dynamic feedback controller and p is the order of the dynamic part χ of the controller. An important quality of the controller is of course the property that the order p of χ should be as small as possible, and in particular doses not depend on the dimensions of the state. Therefore, the dynamic part of the controller is the adaptive law for estimating the unknown parameters on-line and the order p of χ is equal to the number of parameters to be estimated. In the conventional adaptive fuzzy controller, the order p of χ is equal to the number of parameters to be used for describing the fuzzy logic system, which is employed to approximate the unknown uncertain functions in the designed systems.

Remark 4. Wang et al. [21] presented an adaptive fuzzy controller for the system (1) with $g_i(\bar{x}_i) = 1$, $i = 1, 2, \ldots, n-1$. The authors used the Mamdani type fuzzy systems to approximate the functions $f_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$ and $g_n(x)$. According to Remark 1, we know that there are $\sum_{j=1}^{n} \prod_{i=1}^{j} (N_i + 1)$ parameters for $f_i(\bar{x}_i)$, $i = 1, 2, \ldots, n$ and $\prod_{i=1}^{n} (N_i + 1)$ parameters for $g_n(x)$ needed to be estimated in the adaptive fuzzy control scheme. In order to make our point clearer, we give a simple example here, i.e., let us discuss a system which has the order $n = 3$ and use $N_i + 1 = 5$ to be continuous input fuzzy sets, as a result there will be $5+5\times5+5\times5\times5+5\times5\times5=280$ parameters needed to be estimated in the adaptive fuzzy controller proposed by Wang et al. [21]. From this small example, we can see that there will be many parameters needed to be estimated in the on-line adaptive law when there are many state variables in the designed systems and many rule bases in the fuzzy system are used to approximate the uncertain nonlinear functions. It will result in that the learning time tends to become unacceptably large and time-consuming process is unavoidable when the controller is implemented.

Remark 5. In [21], it was assumed that the approximation errors and external disturbances were square integrable. However, the square-integrable property of the approximation error and external disturbance is difficult to show for given plant and this calculation may require knowledge of plant dynamics.

Remark 6. In this paper, we will present a robust adaptive fuzzy tracking control algorithm with the structure as Eqs. (15) and (16) for a class of perturbed strict-feedback uncertain nonlinear system (1) by use of input-to-state stability theory and by combining backstepping technique with generalized small-gain approach. The outstanding feature of the algorithms proposed in this paper is that the order p of χ is $2n$, no matter how many states in the designed systems are investigated and how many rules in the fuzzy system are used, i.e., for the small example given above, there will be only 6 parameters needed to be estimated in the derived algorithms.

4. Design of robust adaptive fuzzy tracking control

The backstepping design procedure contains n steps. At each step, an intermediate control function α_k shall be developed using an appropriate Lyapunov function V_k. We give the proceeding of the backstepping design as follows:

Step 1. Define the error variables $z_1 = x_1 - y_d$ and $z_2 = x_2 - \alpha_1 - \dot{y}_d$, then

$$\begin{aligned}\dot{z}_1 &= g_1(x_1)x_2 + f_1(x_1) + \Delta_1(t,x) - \dot{y}_d \\ &= g_1(x_1)(z_2+\alpha_1) + f_1(x_1) + \Delta_1(t,x) + (g_1(x_1)-1)\dot{y}_d \end{aligned} \tag{17}$$

Since $f_1(x_1)$ is an unknown continuous function, according to Lemma 1, T–S fuzzy system $\widehat{f_1}(x_1, A_1)$ with input vector $x_1 \in U_{x_1}$ for some compact set $U_{x_1} \subset R$ is proposed here to approximate the uncertain term $f_1(x_1)$ where A_1 is a matrix containing unknown constants. Then $f_1(x_1)$ can be expressed as

$$\begin{aligned}f_1(x_1) &= \xi_1(x_1)A_1x_1 + \varepsilon_1 = \xi_1(x_1)A_1z_1 + \xi_1(x_1)A_1y_d + \varepsilon_1 \\ &= c_{\theta 1}\xi_1(x_1)\omega_1 + \xi_1(x_1)A_1y_d + \varepsilon_1\end{aligned} \tag{18}$$

where $\omega_1 = A_1^m z_1$ and ε_1 is a parameter denoting approximating accuracy. Let $c_{\theta 1} = \|A_1\|$, such that $A_1^m = c_{\theta 1}^{-1} A_1$ and $\|A_1^m\| \leqslant 1$.

Substituting (18) into (17), we get

$$\dot{z}_1 = g_1(x_1)(z_2 + \alpha_1) + c_{\theta 1}\xi_1(x_1)\omega_1 + v_1 \tag{19}$$

where $v_1 = \xi_1(x_1)A_1 y_d + \varepsilon_1(x_1) + \Delta_1(t,x) + (g_1(x_1) - 1)\dot{y}_d$ and $c_{\theta 1}$ is an unknown constant. In light of Assumption 2, we can obtain a bound for v_1 as follows

$$\begin{aligned} \|v_1\| &\leqslant \|\xi_1(x_1)A_1 y_d + \varepsilon_1 + \Delta_1(t,x) + (g_1(x_1) - 1)\dot{y}_d\| \\ &\leqslant \|A_1 y_d\|\|\xi_1(x_1)\| + \|\varepsilon_1\| + p_1^*\phi_1(x_1) + (b_{\max} + 1)\|\dot{y}_d\| \\ &\leqslant b_{\min}\theta_1\psi_1(x_1) \end{aligned} \tag{20}$$

where $\theta_1 = b_{\min}^{-1}\max(\|A_1 y_d\|, \|\varepsilon_1\|, p_1^*, (b_{\max}+1)\|\dot{y}_d\|)$ and $\psi_1(x_1) = 1 + \|\xi_1\| + \phi_1(x_1)$.

Consider the stabilization of the subsystem (19) and the Lyapunov function candidate is

$$V_1(z_1, \lambda_1, \hat{\theta}_1) = \frac{1}{2}z_1^2 + \frac{1}{2}b_{\min}\Gamma_{11}^{-1}\tilde{\lambda}_1^2 + \frac{1}{2}b_{\min}\Gamma_{12}^{-1}\tilde{\theta}_1^2 \tag{21}$$

where Γ_{11} and Γ_{12} are the positive definite constants. $\tilde{\lambda}_1 = (b_{\min}^{-1}c_{\theta 1}^2 - \lambda_1)$ and $\tilde{\theta}_1 = (\theta_1 - \hat{\theta}_1)$. λ_1 and $\widehat{\theta_1}$ are the estimates of $b_{\min}^{-1}c_{\theta 1}^2$ and θ_1, respectively. The time derivative of V_1 is

$$\dot{V}_1(z_1, \lambda_1, \hat{\theta}_1) = z_1[g_1(x_1)(z_2 + \alpha_1) + c_{\theta 1}\xi_1(x_1)\omega_1 + v_1] - \Gamma_{11}^{-1}\tilde{\lambda}_1\dot{\lambda}_1 - \Gamma_{12}^{-1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \tag{22}$$

Let $\gamma_1 > 0$, we can get

$$\begin{aligned} c_{\theta 1}\xi_1(x_1)\omega_1 z_1 &= c_{\theta 1}\xi_1(x_1)\omega_1 z_1 - \gamma_1^2\omega_1^{\mathrm{T}}\omega_1 + \gamma_1^2\omega_1^{\mathrm{T}}\omega_1 \\ &= -\gamma_1^2\left(\omega_1 - \frac{c_{\theta 1}}{2\gamma_1^2}\xi_1 z_1\right)^2 + \frac{c_{\theta 1}^2}{4\gamma_1^2}\xi_1\xi_1^{\mathrm{T}}z_1^2 + \gamma_1^2\omega_1^{\mathrm{T}}\omega_1 \\ &\leqslant \frac{c_{\theta 1}^2}{4\gamma_1^2}\xi_1\xi_1^{\mathrm{T}}z_1^2 + \gamma_1^2\omega_1^{\mathrm{T}}\omega_1 \\ &\leqslant b_{\min}\frac{\lambda_1}{4\gamma_1^2}\xi_1\xi_1^{\mathrm{T}}z_1^2 + b_{\min}\frac{\tilde{\lambda}_1}{4\gamma_1^2}\xi_1\xi_1^{\mathrm{T}}z_1^2 + \gamma_1^2\omega_1^{\mathrm{T}}\omega_1 \end{aligned} \tag{23}$$

Using (20), we can get

$$v_1 z_1 \leqslant b_{\min}\theta_1\psi_1(x_1)\|z_1\| \leqslant b_{\min}\hat{\theta}_1\psi_1(x_1)\|z_1\| + b_{\min}\tilde{\theta}_1\psi_1(x_1)\|z_1\| \tag{24}$$

Therefore, we can get the intermediate stabilizing function α_1 as follows

$$\alpha_1 = -k_1 z_1 - \frac{\lambda_1}{4\gamma_1^2}\xi_1(x_1)\xi_1^{\mathrm{T}}(x_1)z_1 - \hat{\theta}_1\psi_1(x_1)\tanh\left(\frac{\hat{\theta}_1\psi_1(x_1)z_1}{\delta_1}\right)$$

where $k_1 > 0$ and $\delta_1 > 0$ are the design constants. Then we get

$$\begin{aligned} g_1(x_1)\alpha_1 z_1 &= g_1(x_1)\left(-k_1 z_1^2 - \frac{\lambda_1}{4\gamma_1^2}\xi_1(x_1)\xi_1^{\mathrm{T}}(x_1)z_1^2 - \hat{\theta}_1\psi_1(x_1)z_1\tanh\left(\frac{\hat{\theta}_1\psi_1(x_1)z_1}{\delta_1}\right)\right) \\ &\leqslant b_{\min}\left(-k_1 z_1^2 - \frac{\lambda_1}{4\gamma_1^2}\xi_1(x_1)\xi_1^{\mathrm{T}}(x_1)z_1^2 - \hat{\theta}_1\psi_1(x_1)z_1\tanh\left(\frac{\hat{\theta}_1\psi_1(x_1)z_1}{\delta_1}\right)\right) \end{aligned} \tag{25}$$

Using (23)–(25), $\dot{V}_1$ can be written as

$$\begin{aligned} \dot{V}_1(z_1,\lambda_1,\hat{\theta}_1) \leqslant & -b_{\min}k_1 z_1^2 + g_1(x_1)z_1z_2 + b_{\min}\hat{\theta}_1\psi_1(x_1)\|z_1\| \\ & - b_{\min}\hat{\theta}_1\psi_1(x_1)z_1\tanh\left(\frac{\hat{\theta}_1\psi_1(x_1)z_1}{\delta_1}\right) + \gamma_1^2\omega_1^{\mathrm{T}}\omega_1 \\ & + b_{\min}\Gamma_{11}^{-1}\tilde{\lambda}_1\left(\frac{\Gamma_{11}}{4\gamma_1^2}\xi_1\xi_1^{\mathrm{T}}z_1^2 - \dot{\lambda}_1\right) + b_{\min}\Gamma_{12}^{-1}\tilde{\theta}_1\left(\Gamma_{12}\psi_1(x_1)\|z_1\| - \dot{\hat{\theta}}_1\right) \end{aligned} \tag{26}$$

As in [18], in order to prevent parameters drift, we present the following adaptive laws incorporating a leakage term based on a variation of σ-modification. Let the parameter adaptive laws for λ_1 and $\hat{\theta}_1$ now chosen as

$$\begin{cases} \dot{\lambda}_1 = \Gamma_{11}\left[\frac{1}{4\gamma_1^2}\xi_1(x_1)\xi_1^{\mathrm{T}}(x_1)z_1^2 - \sigma_{11}(\lambda_1 - \lambda_1^0)\right] \\ \dot{\hat{\theta}}_1 = \Gamma_{12}\left[\psi_1(x_1)\|z_1\| - \sigma_{12}(\hat{\theta}_1 - \theta_1^0)\right] \end{cases} \tag{27}$$

where λ_1^0, θ_1^0, σ_{11} and σ_{12} are design parameters.

And we deal with the relative term in Eq. (26). By using the lemma proved by [18], we can get

$$\hat{\theta}_1\psi_1(x_1)\|z_1\| - \hat{\theta}_1\psi_1(x_1)z_1\tanh\left(\frac{\hat{\theta}_1\psi_1(x_1)z_1}{\delta_1}\right) \leqslant \delta_1 \tag{28}$$

Let $c_1 = \min\{2b_{\min}k_1, \sigma_{11}b_{\min}\Gamma_{11}, \sigma_{12}b_{\min}\Gamma_{12}\}$. Then $\dot{V}_1$ is converted into

$$\dot{V}_1(z_1,\lambda_1,\hat{\theta}_1) \leqslant -c_1V_1(z_1,\lambda_1,\hat{\theta}_1) + g_1(x_1)z_1z_2 + \gamma_1^2\omega_1^{\mathrm{T}}\omega_1 + \mu_1 \tag{29}$$

where $\mu_1 = b_{\min}\left(\delta_1 + \frac{1}{2}\mid c_{\theta 1}^2 - \lambda_1^0\mid^2 + \frac{1}{2}\mid \theta_1 - \theta_1^0\mid^2\right)$.

Step 2.

$$\dot{z}_2 = g_2(\bar{x}_2)x_3 + f_2(\bar{x}_2) + \Delta_2(t,x) - \dot{\alpha}_1 - \ddot{y}_d \tag{30}$$

Then the time derivative of α_1 is

$$\begin{aligned}\dot{\alpha}_1 &= \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial\alpha_1}{\partial\lambda_1}\dot{\lambda}_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d \\ &= \frac{\partial\alpha_1}{\partial x_1}(g1(x1)x_2 + f_1(x_1) + \Delta_1(t,x)) + \frac{\partial\alpha_1}{\partial\lambda_1}\dot{\lambda}_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d \\ &= f_{12}(z_1,\bar{x}_2) + \frac{\partial\alpha_1}{\partial x_1}\Delta_1(t,x) + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d \end{aligned} \tag{31}$$

Substituting (31) into (30), we get

$$\begin{aligned}\dot{z}_2 &= g_2(\bar{x}_2)x_3 - g_1z_1 + f_2(\bar{x}_2) + g_1z_1 + \Delta_2(t,x) - f_{12}(z_1,\bar{x}_2) \\ &\quad - \frac{\partial\alpha_1}{\partial x_1}\Delta_1(t,x) - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d - \ddot{y}_d \\ &= g_2(\bar{x}_2)x_3 - g_1z_1 + f_2'(z_1,z_2,y_d) + \Delta_2(t,x) - \frac{\partial\alpha_1}{\partial x_1}\Delta_1(t,x) - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d - \ddot{y}_d \end{aligned} \tag{32}$$

We also use a T–S fuzzy system to approximate the unknown function $f_2'(z_1,z_2,y_d)$ and obtain

$$\begin{aligned} f_2'(z_1,x_2,y_d) &= \xi_2(z_1,x_2,y_d)A_2[z_1,x_2,y_d]^{\mathrm{T}} + \varepsilon_2 = \xi_2A_2^1[z_1,x_2]^{\mathrm{T}} + \xi_2A_2^2y_d + \varepsilon_2 \\ &= c_{\theta2}\xi_2\omega_2 + d_2 \end{aligned}$$

where $\omega_2 = A_2^m\bar{z}_2$ and $c_{\theta2} = \|A_2^1\| = \lambda_{\max}^{1/2}(A_2^{1T}A_2^1)$, such that $A_2^1 = c_{\theta2}A_2^m$ and $\|A_2^m\| \leqslant 1$. $d_2 = \xi_2A_2^{12}(\alpha_1 + y_d) + \xi_2A_2^2y_d + \varepsilon_2$.

Defining a error variable z_3 as $z_3 = x_3 - \alpha_2 - \ddot{y}_d$, Eq. (32) can be written as

$$\dot{z}_2 = g_2(\bar{x}_2)(z_3 + \alpha_2) - g_1z_1 + c_{\theta2}\xi_2\omega_2 + v_2 \tag{33}$$

where $v_2 = d_2 + \Delta_2 - \frac{\partial\alpha_1}{\partial x_1}\Delta_1 - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + (g_2(\bar{x}_2) - 1)\ddot{y}_d$.

Choosing Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}b_{\min}\Gamma_{12}^{-1}\tilde{\lambda}_2^2 + \frac{1}{2}b_{\min}\Gamma_{22}^{-1}\tilde{\theta}_2^2$$

A similar procedure with (23) and (24) is used and the time derivative of V_2 becomes

$$\begin{aligned}\dot{V}_2 \leqslant & -c_1V_1 + g_1z_1z_2 + \gamma_1^2\omega_1^{\mathrm{T}}\omega_1 + \mu_1 + z_2[g_2(z_3 + \alpha_2) - g_1z_1] \\ & + b_{\min}\frac{\lambda_2}{4\gamma_2^2}\xi_2\xi_2^{\mathrm{T}}z_2^2 + b_{\min}\hat{\theta}_2\psi_2\|z_2\| + b_{\min}\Gamma_{21}^{-1}\tilde{\lambda}_2\left(\frac{\Gamma_{21}}{4\gamma_2^2}\xi_2\xi_2^{\mathrm{T}}z_2^2 - \dot{\lambda}_2\right) \\ & + b_{\min}\Gamma_{22}^{-1}\tilde{\theta}_2\left(\Gamma_{22}\psi_2\|z_2\| - \dot{\hat{\theta}}_2\right) + \gamma_2^2\omega_2^{\mathrm{T}}\omega_2 \end{aligned}$$

where $\|v_2\| \leqslant \theta_2\psi_2$ and $\psi_2 = 1 + (1 + \|\alpha_1\|)\|\xi_2\| + \phi_2(\bar{x}_2) + \left\|\frac{\partial\alpha_1}{\partial x_1}\right\|\phi_1(x_1) + \left\|\frac{\partial\alpha_1}{\partial y_d}\right\|$.

Now, choose the intermediate stabilizing function α_2 and adaptive laws as

$$\alpha_2 = -k_2 z_2 - \frac{\lambda_2}{4\gamma_2^2}\xi_2\xi_2^{\mathrm{T}} z_2 - \hat{\theta}_2\psi_2 \tanh\left(\frac{\hat{\theta}_2\psi_2 z_2}{\delta_2}\right) \tag{34}$$

$$\begin{cases} \dot{\lambda}_2 = \Gamma_{21}\left[\frac{1}{4\gamma_2^2}\xi_2\xi_2^{\mathrm{T}} z_2^2 - \sigma_{21}\left(\lambda_2 - \lambda_2^0\right)\right] \\ \dot{\hat{\theta}}_2 = \Gamma_{22}\left[\psi_2\|z_2\| - \sigma_{22}\left(\hat{\theta}_2 - \theta_2^0\right)\right] \end{cases} \tag{35}$$

where k_2, δ_2, λ_2^0, θ_2^0, σ_{21} and σ_{22} are design constants.

Let $c_2 = \min\{c_1, 2b_{\min}k_2, \sigma_{21}b_{\min}\Gamma_{21}, \sigma_{22}b_{\min}\Gamma_{22}\}$, then $\dot{V}_2$ is converted to

$$\dot{V}_2 \leqslant -c_2 V_2 + g_2 z_2 z_3 + \sum_{i=1}^{2}\gamma_i^2\omega_i^{\mathrm{T}}\omega_i + \mu_2 \tag{36}$$

where $\mu_2 = \mu_1 + b_{\min}\left(\delta_2 + \frac{1}{2}\left|c_{\theta 2}^2 - \lambda_2^0\right|^2 + \frac{1}{2}\left|\theta_2 - \theta_2^0\right|^2\right)$.

A similar procedure is employed recursively for each step $k(3 \leqslant k \leqslant n-1)$. By considering the equation of system (1) for $i = k, \dot{x}_k = g_k x_{k+1} + f_k(\bar{x}_k) + \Delta_k(t,x)$, and the Lyapunov function candidate

$$V_k = V_{k-1} + \frac{1}{2}z_k^2 + \frac{1}{2}b_{\min}\Gamma_{k1}^{-1}\tilde{\lambda}_k^2 + \frac{1}{2}b_{\min}\Gamma_{k2}^{-1}\tilde{\theta}_k^2$$

where $\tilde{\lambda}_k = (b_{\min}^{-1}c_{\theta k}^2 - \lambda_k)$ and $\tilde{\theta}_k = (\theta_k - \hat{\theta}_k)$.

We may design the control function α_k, and learning laws for λ_k and $\hat{\theta}_k$, which take similar forms of (27) and (35), respectively. The controller u for the system (1) shall be constructed in step n.

Step n: Define the error variable as $z_n = x_n - \alpha_{n-1} - y_d^{(n-1)}$. We have

$$\dot{z}_n = g_n(x)u + f_n(x) + \Delta_n - \dot{\alpha}_{n-1} - y_d^{(n)}$$

Using the similar way to (31) in Step 2, we have

$$\begin{aligned} \dot{\alpha}_{n-1} &= \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left\{g_{j+1}(\bar{x}_{j+1})x_{j+1} + f_j(\bar{x}_j) + \Delta_j\right\} \\ &\quad + \frac{\partial\alpha_{n-1}}{\partial\lambda_{n-1}}\dot{\lambda}_{n-1} + \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_{n-1}}\dot{\hat{\theta}}_{n-1} + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}}y_d^{(j)} \\ &= f_{(n-1)n}(\bar{z}_{n-1}, \bar{x}_n, \bar{x}_{d(n-1)}) + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\Delta_j + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}}y_d^{(j)} \end{aligned}$$

Then

$$\begin{aligned} \dot{z}_n &= g_n(x)u - g_{n-1}z_{n-1} + f_n'(\bar{z}_{n-1}, x_n, \bar{x}_{d(n-1)}) + \Delta_n - y_d^{(n)} \\ &\quad - \sum_{j=1}^{n-1}\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\Delta_j + \frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}}y_d^{(j)}\right) \end{aligned}$$

where $f_n' = f_n(x) - f_{(n-1)n}(\bar{z}_{n-1}, \bar{x}_n, \bar{x}_{d(n-1)}) + g_{n-1}z_{n-1}$.

We also use a T–S fuzzy system to approximate the unknown function $f'_n(\bar{z}_{n-1}, x_n, \bar{x}_{d(n-1)})$ and obtain

$$\begin{aligned} f'_n &= \xi_n A_n [\bar{z}_{n-1}, x_n, \bar{x}_{d(n-1)}]^{\mathrm{T}} + \varepsilon_n \\ &= \xi_n A_n^1 [\bar{z}_{n-1}, x_n]^{\mathrm{T}} + \xi_n A_n^2 \bar{x}_{d(n-1)}^{\mathrm{T}} + \varepsilon_n \\ &= \xi_n A_n^1 \bar{z}_n^{\mathrm{T}} + d_n \end{aligned}$$

where $d_n = \xi_n A_n^{12}\left(\alpha_{n-1} + y_d^{(n-1)}\right) + \xi_n A_n^2 \bar{x}_{d(n-1)}^{\mathrm{T}} + \varepsilon_n$. Let $\omega_n = A_n^{m1}\bar{z}_n$, $c_{\theta n} = \|A_n^{m1}\|$ and $A_n^1 = c_{\theta n} A_n^{m1}$. Let $u = \alpha_n + y_d^{(n)}$, we get

$$\dot{z}_n = g_n \alpha_n - g_{n-1} z_{n-1} + c_{\theta n} \xi_n w_n + v_n$$

where $v_n = d_n + \Delta_n + (g_n(x) - 1) y_d^{(n)} - \sum_{j=1}^{n-1}\left(\frac{\partial \alpha_{n-1}}{\partial x_j}\Delta_j + \frac{\partial \alpha_{n-1}}{\partial y_d^{(j-1)}} y_d^{(j)}\right)$.

Taking the following Lyapunov function candidate

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} b_{\min} \Gamma_{n1}^{-1} \tilde{\lambda}_n^2 + \frac{1}{2} b_{\min} \Gamma_{n2}^{-1} \tilde{\theta}_n^2$$

its time derivative is

$$\begin{aligned} \dot{V}_n = {} & \dot{V}_{n-1} + z_n(g_n\alpha_n - g_{n-1}z_{n-1} + c_{\theta n}\xi_n w_n + v_n) - b_{\min}\Gamma_{n1}^{-1}\tilde{\lambda}_n\dot{\lambda}_n \\ & - b_{\min}\Gamma_{n2}^{-1}\tilde{\theta}_n\dot{\hat{\theta}}_n \leqslant -c_{n-1}V_{n-1} + \sum_{i=1}^{n}\gamma_i^2\omega_i^{\mathrm{T}}\omega_i + \sum_{i=1}^{n-1}\mu_i + b_{\min}\delta_n \\ & + z_n\left(g_n\alpha_n + b_{\min}\frac{\lambda_n}{4\gamma_n^2}\xi_n\xi_n^{\mathrm{T}}z_n + b_{\min}\hat{\theta}_n\psi_n\tanh\left(\frac{\hat{\theta}_n\psi_n z_n}{\delta_n}\right)\right) \\ & + b_{\min}\Gamma_{n1}^{-1}\tilde{\lambda}_n\left(\frac{\Gamma_{n1}}{4\gamma_n^2}\xi_n\xi_n^{\mathrm{T}}z_n^2 - \dot{\lambda}_n\right) + b_{\min}\Gamma_{n2}^{-1}\tilde{\theta}_n\left(\Gamma_{n2}\psi_n\|z_n\| - \dot{\hat{\theta}}_n\right) \end{aligned}$$

where

$$\|v_n\| \leqslant \theta_n\psi_n$$

and

$$\psi_n = 1 + (1 + \|\alpha_{n-1}\|)\|\xi_n\| + \phi_n + \left\|\sum_{j=1}^{n-1}\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\phi_j + \frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}}\right)\right\|$$

Now, we get $k_n > 0$ as a design constant and are ready to choose the controller as

$$u = \alpha_n + y_d^{(n)} = -k_n z_n - \frac{\lambda_n}{4\gamma_n^2}\xi_n\xi_n^{\mathrm{T}}z_n - \hat{\theta}_n\psi_n\tanh\left(\frac{\hat{\theta}_n\psi_n z_n}{\delta_n}\right) + y_d^{(n)} \quad (37)$$

and adaptive laws in step n as

$$\begin{cases} \dot{\lambda}_n = \Gamma_{n1}\left[\frac{1}{4\gamma_n^2}\xi_n\xi_n^{\mathrm{T}}z_n^2 - \sigma_{n1}\left(\lambda_n - \lambda_n^0\right)\right] \\ \dot{\hat{\theta}}_n = \Gamma_{n2}\left[\psi_n\|z_n\| - \sigma_{n2}\left(\hat{\theta}_n - \theta_n^0\right)\right] \end{cases} \tag{38}$$

According to the recursive control design procedure above, at the last step (i.e., $i = n$), picking the robust adaptive fuzzy control u in (37) and the adaptive laws in (38), we arrive at

$$\begin{aligned} \dot{V}_n &\leqslant -c_nV_n + \sum_{i=1}^{n}\gamma_i^2\omega_i^{\mathrm{T}}\omega_i + \mu_n \\ &\leqslant -c_nV_n + \gamma^2\|\omega\|^2 + \mu_n \end{aligned} \tag{39}$$

where $\mu_n = \sum_{i=1}^{n-1}\mu_i + b_{\min}\left(\delta_n + \frac{1}{2}\left|c_{\theta n}^2 - \lambda_n^0\right|^2 + \frac{1}{2}\left|\theta_n - \theta_n^0\right|^2\right)$, $\omega = [\omega_1, \omega_2, \ldots, \omega_n]^{\mathrm{T}}$ and $\gamma = \left(\gamma_1^2 + \gamma_2^2 + \cdots + \gamma_n^2\right)^{1/2}$.

We are now in a position to state our main result on semi-global robust adaptive fuzzy controller.

Theorem 2. *Consider the system* (1) *with unknown system and virtual control gain nonlinearities, and suppose that the packaged uncertain functions* $f_i'(\bar{z}_{i-1}, x_i, \bar{x}_{d(i)})$, $i = 1, 2, \ldots, n$ *can be approximated by T–S fuzzy systems in the sense that* ε_i *is bounded. If we pick* $\gamma < 1$ *and* $k_i > \frac{1}{b_{\min}}$, $i = 1, 2, \ldots, n$ *in* (39), *then the robust adaptive fuzzy tracking control law* $u = \alpha_n + y_d^{(n)}$ *with the intermediate stabilizing functions* α_i, $i = 1, 2, \ldots, n$ *and adaptive laws for* λ_i *and* $\hat{\theta}_i$ *can make all the solutions* $(z(t), \lambda, \hat{\theta})$ *of the derived closed-loop system uniformly ultimately bounded. Furthermore, given any* $\varpi > 0$, *we can tune our controller parameters such that the output error* $z_1 = y(t) - y_d(t)$ *satisfies* $\lim_{t\to\infty} | z_1(t) | \leqslant \varpi$.

Proof. In order to use Theorem 1 (small-gain theorem), it is necessary to construct a system in composite feedback form with $\Sigma_{\tilde{z}\omega}$-subsystem and $\Sigma_{\omega\tilde{z}}$-subsystem. We begin with the $\Sigma_{\tilde{z}\omega}$-subsystem. According to the error variables $z_i = x_i - \alpha_{i-1} - y_d^{(i-1)}$, $i = 1, 2, \ldots, n$, $\alpha_0 = 0$ defined in Section 4, we substitute z_i into (1) and use T–S fuzzy systems to approximate the packaged uncertain functions $f_i'(\bar{z}_{i-1}, x_i, \bar{x}_{d(i)})$, $i = 1, 2, \ldots, n$, then the closed-loop system can be given as follows

$$\Sigma_{\tilde{z}\omega} : \begin{cases} \dot{z}_1 = g_1(\bar{x}_1)(\alpha_1 + z_2) + c_{\theta 1}\xi_1\omega_1 + v_1 \\ \dot{z}_i = g_i(\bar{x}_i)(\alpha_i + z_{i+1}) - g_{i-1}z_{i-1} + c_{\theta i}\xi_i\omega_i + v_i, \quad 2 \leqslant i \leqslant n-1 \\ \dot{z}_n = g_n(x)u - g_{n-1}z_{n-1} + c_{\theta n}\xi_n\omega_n + v_n \\ \tilde{z} = H(z) = z \end{cases} \tag{40}$$

where $\omega = [\omega_1, \omega_2, \ldots, \omega_n]^{\mathrm{T}}$ is considered as the virtual input and $\tilde{z}$ as the output.

For subsystem $\Sigma_{\tilde{z}\omega}$, if picking $k_i > 1/b_{\min}$, $i = 1, 2, \ldots, n$ from (39), we obtain

$$\dot{V}_n \leqslant -z^2 + \gamma^2 \|\omega\|^2 + \delta'_n$$

By Definition 2, we propose the robust adaptive fuzzy tracking controller such that the requirement of ISpS for system $\Sigma_{\tilde{z}\omega}$ can be satisfied with the functions $\alpha_3(s) = s^2$ and $\alpha_4 = \gamma^2 s^2$ of class K_∞. According to (7), we can get a gain function $\gamma_z(s)$ of $\Sigma_{\tilde{z}\omega}$-subsystem

$$\gamma_z(s) = \alpha_1^{-1} \circ \alpha_2 \circ \alpha_3^{-1} \circ \alpha_4, \quad \forall s > 0$$

where $\alpha_1(z) \leqslant V_n(z) \leqslant \alpha_2(z)$.

For $\Sigma_{\omega\tilde{z}}$-subsystem, it is

$$\Sigma_{\omega\tilde{z}} : \begin{cases} \omega_1 = A_1^m z_1 \\ \omega_2 = A_2^m [z_1, z_2]^{\mathrm{T}} = A_2^m \bar{z}_2 \\ \vdots \\ \omega_n = A_n^m [z_1, z_2, \ldots, z_n]^{\mathrm{T}} = A_n^m \bar{z}_n \end{cases} \tag{41}$$

We can rewrite the above equations as

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix} = K(z) = \begin{bmatrix} A_1^m & 0 & \cdots & 0 \\ A_2^{m1} & A_2^{m2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ A_n^{m1} & A_n^{m2} & \cdots & A_n^{mn} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} = Az$$

and obtain

$$\|\omega\| \leqslant \|A\| \|z\| = \gamma' \|z\| \tag{42}$$

Then the gain function γ_w for system $\Sigma_{\omega\tilde{z}}$ is $\gamma_w(s) = \gamma' s$. In order to check the requirement $\gamma_z(\gamma_\omega(s)) < s$ in small-gain theorem 1, we select (40) as (8), and (42) as (9), and obtain $\gamma\gamma' < 1$. Due to $\gamma' = \|A\| \leqslant 1$, the condition of small-gain theorem 1 can be satisfied by choosing $\gamma < 1$, such that it can be proven that the composite closed-loop system is ISpS. Therefore, direct use of Definition 1 yields that the composite closed-loop system has bounded solutions over $[0, \infty)$. More precisely, there exists a class KL-function β and a positive constant d_1 such that

$$\|z(t), \lambda(t), \hat{\theta}(t)\| \leqslant \beta(\|z(0), \lambda(0), \hat{\theta}(0)\|, t) + \delta'_n$$

where $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]^{\mathrm{T}}$ and $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_n]^{\mathrm{T}}$.

This, in turn, implies that the tracking error vector $z(t)$ is bounded over $[0, \infty)$. According to Proposition 1, there exists an ISpS-Lyapunov function for the composite closed-loop system. By substituting (42) into (39), the ISpS-Lyapunov function is satisfied as follows

$$\begin{aligned}\dot{V}_n &\leqslant -z^{\mathrm{T}}Qz - \frac{1}{2}\tilde{\lambda}^{\mathrm{T}}Q_1\tilde{\lambda} - \frac{1}{2}\tilde{\theta}^{\mathrm{T}}Q_2\tilde{\theta} + \gamma'^2\gamma^2\|z\|^2 + \mu_n \\ &\leqslant -z^{\mathrm{T}}Qz - \frac{1}{2}\tilde{\lambda}^{\mathrm{T}}Q_1\tilde{\lambda} - \frac{1}{2}\tilde{\theta}^{\mathrm{T}}Q_2\tilde{\theta} + \|z\|^2 + \mu_n \\ &\leqslant -c_1V_n + \mu_n \end{aligned} \tag{43}$$

where

$$Q = \mathrm{diag}[k_1, k_2, \ldots, k_n]$$

$$Q_1 = \mathrm{diag}[\sigma_{11}, \sigma_{12}, \ldots, \sigma_{1n}]^{\mathrm{T}}$$

$$Q_2 = \mathrm{diag}[\sigma_{21}, \sigma_{22}, \ldots, \sigma_{2n}]^{\mathrm{T}}$$

$$c_1 = \min\{2(\lambda_{\min}(Q) - 1/b_{\min}), \lambda_{\min}(Q_1)/\lambda_{\max}(\Gamma_1^{-1}), \lambda_{\min}(Q_2)/\lambda_{\max}(\Gamma_2^{-1})\}$$

$$\Gamma_1 = [\Gamma_{11}, \Gamma_{12}, \ldots, \Gamma_{1n}]^{\mathrm{T}}$$

and

$$\Gamma_2 = [\Gamma_{21}, \Gamma_{22}, \ldots, \Gamma_{2n}]^{\mathrm{T}}$$

From (44), we obtain

$$V_n(t) \leqslant \frac{\mu_n}{c_1} + \left(V_n(t_0) - \frac{\mu_n}{c_1}\right)\mathrm{e}^{-(t-t_0)}$$

It results that the solutions of composite closed-loop system are uniformly ultimately bounded, and implies that, for any $\varpi_1 > (\mu_n/c_1)^{1/2}$, there exists a constant $T > 0$ such that $\|z_1(t)\| \leqslant \varpi_1$ for all $t \geqslant t_0 + T$. The last statement holds readily since $(\mu_n/c_1)^{1/2}$ can be made arbitrarily small if the design parameter vectors λ^0, θ^0, δ, σ_1, σ_2 are chosen appropriately. Finally, we have proved Theorem 2. □

Remark 7. Since the function approximation property of fuzzy systems is only guaranteed within a compact set, the stability result proposed in this paper is semi-global in the sense that, for any compact set, there exists a controller with sufficiently large number of fuzzy rules such that all the closed-loop signals are bounded when the initial states are within this compact set. In practical applications, the number of fuzzy rules usually cannot be chosen too large due to the possible computation problem. This implies that the fuzzy system approximation capability is limited, that is, the approximating accuracy ε_i's for the estimated the packaged uncertain functions $f_i'(\bar{z}_{i-1}, x_i, \bar{x}_{d(i)})$'s will be greater when chosen small number of fuzzy rules. But we can choose appropriately the design parameter vectors λ^0, θ^0, δ, σ_1, σ_2 to improve both stability and performance of the closed-loop systems.

5. Illustrative example

In order to reveal the control performance of the proposed RAFTC, the following simulation example, an uncertain chaotic nonlinear system, i.e., Brusselator model in dimensionless form, is considered in this section.

$$\begin{cases} \dot{x}_1 = A - (B+1)x_1 + x_1^2 x_2 \\ \dot{x}_2 = Bx_1 - x_1^2 x_2 \end{cases} \tag{44}$$

where x_1 and x_2 denote the concentrations of the reaction intermediates: $A, B > 0$ are parameters describing the (constant) supply of "reservoir" chemicals. The Brusselator model is a simplified model describing a certain set of chemical reactions. This model was introduced by Turing [43] and studied in detail by Prigogine and coworkers [44]. This model was named Brusseltor because its originators worked in Brussels. It has become one of the most popular nonlinear oscillatory models of chemical kinetics, as well as one of the paradigms in the research of chaos.

As a simplified model depicting chemical reactions, the Brusselator model is derived from partial differential equations after a series of approximations. Thus, there must exist modelling errors and other types of unknown nonlinearities in the practical chemical reactions. The controller Brusselator with disturbance [45] is assumed as

$$\begin{cases} \dot{x}_1 = A - (B+1)x_1 + x_1^2 x_2 + \Delta_1(x_1, x_2, t) \\ \dot{x}_2 = Bx_1 - x_1^2 x_2 + (2 + \cos(x_1))u + \Delta_2(x_1, x_2, t) \\ y = x_1 \end{cases} \tag{45}$$

where Δ_1 and Δ_2 are the disturbance terms, the nonlinearities $f(x_1) = A - (B+1)x_1$, $g_1(x_1) = x_1^2$, $f_2(\bar{x}_2) = Bx_1 - x_1^2 x_2$, $g_2(\bar{x}_2) = 2 + \cos(x_1)$ are assumed unknown to the controller u. In the simulation, we get $\Delta_1(x_1, x_2, t) = 0.7x_1^2 \cos(1.5t)$ and $\Delta_2(x_1, x_2, t) = 0.5(x_1^2 + x_2^2)\sin^3 t$.

The control objective is to guarantee (i) all the signals in the closed-loop system remain bounded, and (ii) the output y follows the reference signal $y_d = 3 + \sin(0.5t) + 0.5\sin(1.5t)$. The robust adaptive fuzzy tracking controller is chosen according to Theorem 2 as follows:

Define five fuzzy sets for each variable x_1, z_1, z_2, y_d and so on with labels Ψ_{hi}^1 (NL), Ψ_{hi}^2 (NM), Ψ_{hi}^3 (ZE), Ψ_{hi}^4 (PM), Ψ_{hi}^5 (PL) which are characterized by the following membership functions

$$\begin{aligned} \mu_{\Psi_{hi}^1} &= \exp[-(x+1)^2] \\ \mu_{\Psi_{hi}^2} &= \exp[-(x+0.5)^2] \\ \mu_{\Psi_{hi}^3} &= \exp[-x^2] \\ \mu_{\Psi_{hi}^4} &= \exp[-(x-0.5)^2] \\ \mu_{\Psi_{hi}^5} &= \exp[-(x-1)^2] \end{aligned} \tag{46}$$

The first stabilizing function α_1 is

$$\alpha_1 = -50z_1 - \lambda_2\xi_1\xi_1^{\mathrm{T}}z_1 - \hat{\theta}_1\psi_1 \tanh\left(\hat{\theta}_1\psi_1 z_1/1.0\right) \tag{47}$$

where $\psi_1(x_1) = 1 + \|\xi_1(x_1)\| + x_1^2$ and $z_1 = y - y_d$. Then the adaptive laws are given as follows

$$\begin{cases} \dot{\lambda}_1 = 1000\left[\xi_1\xi_1^{\mathrm{T}}z_1^2 - 0.3(\lambda_1 - 0.1)\right] \\ \dot{\hat{\theta}}_1 = 0.5\left[\psi_1\|z_1\| - 0.3\left(\hat{\theta}_1 - 0.1\right)\right] \end{cases} \tag{48}$$

and we obtain the controller law as

$$u = -15z_2 - \lambda_2\xi_2\xi_2^{\mathrm{T}}z_2 - \hat{\theta}_2\psi_2 \tanh\left(\hat{\theta}_2\psi_2 z_2/1000\right) + \ddot{y}_d \tag{49}$$

where $z_2 = x_2 - \alpha_1 - \dot{y}_d$.

Then adaptive laws are

$$\begin{cases} \dot{\lambda}_2 = 30\left[\xi_2\xi_2^{\mathrm{T}}z_2^2 - 0.3(\lambda_2 - 0.3)\right] \\ \dot{\hat{\theta}}_2 = 5\left[\psi_2\|z_2\| - 0.3\left(\hat{\theta}_1 - 1\right)\right] \end{cases} \tag{50}$$

where $\psi_2(x_1, x_2) = 1 + (1 + |\alpha_1|)\|\xi_2\| + x_1^2 + x_2^2 + \left|\frac{\partial\alpha_1}{\partial x_1}\right|x_1^2 + \left|\frac{\partial\alpha_1}{\partial y_d}\right|$.

Simulation results in Figs. 1–4 show the effectiveness of the proposed robust adaptive fuzzy tracking control design for Brusselator model with the reference signal $y_d = \sin(t) + 0.5\sin(1.5t)$. Fig. 1 shows that the systems output and the boundedness of control u. Fig. 2 shows that the tracking error converges to a small neighborhood around zero. Figs. 3 and 4 show the variations of adaptive parameters λ_1, $\hat{\theta}_1$, λ_2 and $\hat{\theta}_2$, which are also bounded.

6. Conclusion

In this paper, the tracking control problem has been considered for a class of perturbed strict-feedback uncertain nonlinear systems with both unknown system and virtual control gain nonlinearities. We have discussed that the systems possess two types of uncertainties: one naturally satisfies the "triangularity condition" and is not linearly parameterized, while the other is partially known and consists of parametric uncertainties and known "bounding functions", and used Takagi–Sugeno type fuzzy logic systems to approximate uncertain functions. Combining backstepping technique with small-gain approach, we have proposed a robust adaptive fuzzy tracking control (RAFTC) algorithm which can guarantee that the closed-loop system is semi-globally uniformly ultimately bounded. The main feature of the algorithm proposed is the adaptive mechanism with minimal learning parameterizations, that is, no matter how many states in the system are investigated and how many rules in

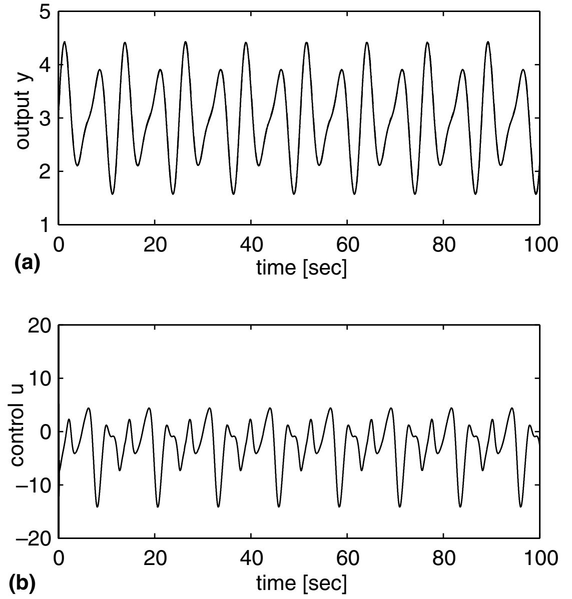

Fig. 1. Simulation results for Brusselator model with $y_d = \sin(t) + 0.5\sin(1.5t)$. (a) System output y and reference signal y_d (solid line: y and dashed line: y_d), (b) control u.

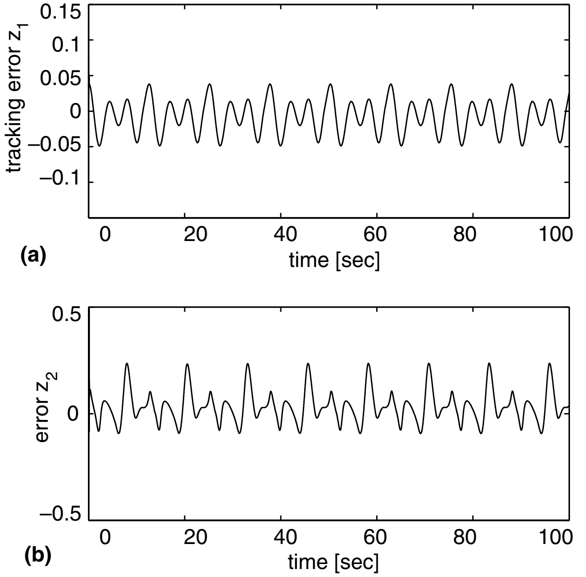

Fig. 2. Simulation results for Brusselator model with $y_d = \sin(t) + 0.5\sin(1.5t)$. (a) Tracking error z_1, (b) intermediate error variable z_2.

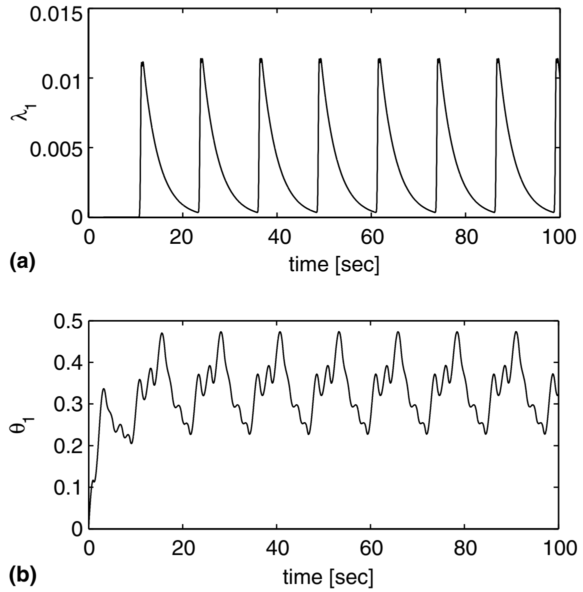

Fig. 3. Simulation results for Brusselator model with $y_d = \sin(t) + 0.5\sin(1.5t)$. (a) Adaptive parameter λ_1, (b) adaptive parameter $\hat{\theta}_1$.

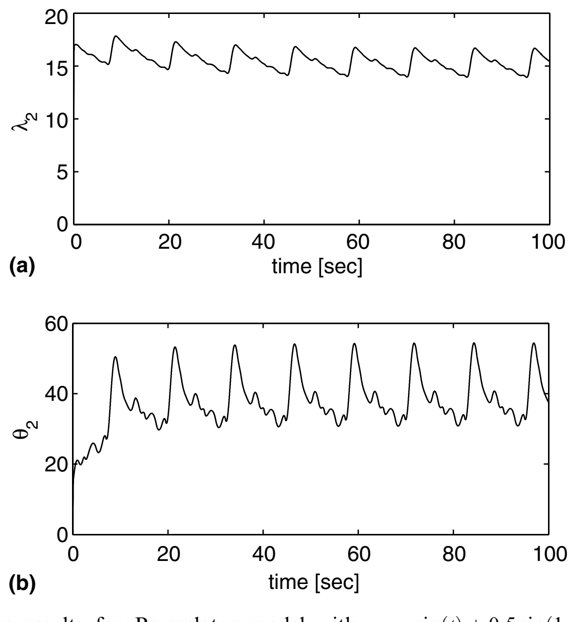

Fig. 4. Simulation results for Brusselator model with $y_d = \sin(t) + 0.5\sin(1.5t)$. (a) Adaptive parameter λ_2, (b) adaptive parameter $\hat{\theta}_2$.

the fuzzy system are used, only $2n$ parameters are needed to be adapted on-line. Then the computation load of the algorithm can be reduced, and it is a convenience to realize this algorithm for engineering. Finally, a simulation example has been presented to illustrate the tracking and stabilization performance of the closed-loop systems by use of the proposed RAFTC algorithm.

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