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Intuitionistic Fuzzy Ideals with Thresholds (α, β) of Rings

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Abstract

In this paper, we apply the concept of intuitionistic fuzzy sets to rings. The notion of intuitionistic fuzzy ideals with thresholds (α, β) of a ring is introduced and some related properties are investigated. Also, the characterizations of intuitionistic fuzzy ideals with thresholds (α, β) are obtained. Finally, we show that the set of all the intuitionistic fuzzy ideals with thresholds (α, β) of a ring forms a modular lattice.

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1 Introduction

The concept of the fuzzy set was introduced by Zadeh[10]. Since then, many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, groupoids, real analysis, measure theory, topology, ect. Many notions of mathematics are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. The study of the fuzzy algebraic structures has started with the introduction of the concept of fuzzy subgroups in the pioneering paper of Rosenfeld[8]. Afterwards, many authors further studied fuzzy subsemigroups, fuzzy subrings, fuzzy ideals and so on. After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept. The notion of of intuitionistic fuzzy sets introduced by Atanassov[1] is one among them. Biswas[3] applied the concept of intuitionistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group. Kim et al.[6] introduced the concept of intuitionistic fuzzy subquasigroups of a quasigroup. Also, Kim and Jun[7] introduced the concept of intuitionistic fuzzy ideals of a semigroup. Recently, Dudek et al.[5] and Davvaz et al.[4] considered the intuitionistic fuzzy sub-hyperquasigroups of hyperquasigroups and intuitionistic fuzzy H_v-submodules, respectively. Also, Yuan et al.[9] introduced the definition of a fuzzy subgroup with thresholds which is a generalization of Rosenfeld's fuzzy subgroup[8] and Bhakat and Das's fuzzy subgroup[2]. In this paper, we apply the concept of intuitionistic fuzzy sets to rings. The notion of intuitionistic fuzzy ideals with thresholds (α, β) of rings is introduced and some related properties are investigated. Also, the characterizations of intuitionistic fuzzy ideals with thresholds (α, β) are obtained. And it is shown that the set of all the intuitionistic fuzzy ideals with thresholds (α, β) of a ring forms a modular lattice.

2 Intuitionistic fuzzy sets and rings

Let X be a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X. The complement of μ, denoted by μ^c, is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$. For any $P \subseteq X$, κ_P denote the characteristic function of P.

For any fuzzy set μ in X and any $r \in [0,1]$, define two sets

$U(\mu; r) = \{x \in X | \mu(x) > r\}$ and $L(\mu; r) = \{x \in X | \mu(x) < r\}$,

which are called an upper and lower r-strong level cut of μ, respectively.

As an important generalization of the notion of fuzzy sets, Atanassov introduced in [1] the concept of an intuitionistic fuzzy set as follows.

Definition 2.1 [1] *An intuitionistic fuzzy set A in a non-empty set X is an object having the form*

$$A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of *membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \lambda_A(x) \le 1$ for all $x \in X$.*

Definition 2.2 [1] *Let $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$ and $B = \{(x, \mu_B(x), \lambda_B(x)) | x \in X\}$ be intuitionistic fuzzy sets in X and let $r, t \in [0, 1]$. Then*

(1) $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\lambda_A(x) \ge \lambda_B(x)$ for all $x \in X$,
(2) $A \cap B = \{(x, \mu_A(x) \wedge \mu_B(x), \lambda_A(x) \vee \lambda_B(x)) | x \in X\}$,
(3) $A \cup B = \{(x, \mu_A(x) \vee \mu_B(x), \lambda_A(x) \wedge \lambda_B(x)) | x \in X\}$,
(4) $\Box A = \{(x, \mu_A(x), \mu_A^c(x)) | x \in X\}$,
(5) $\Diamond A = \{(x, \lambda_A^c(x), \lambda_A(x)) | x \in X\}$.

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$.

A ring is an algebraic system $(R, +, \cdot)$ consisting of a non-empty set R together with two binary operations on R called addition and multiplication (denoted in the usual manner) such that $(R, +)$ is a commutative group and $(R, \cdot)$ is a semigroup and the following distributive laws

$$a(b + c) = ab + bc \text{ and } (a + b)c = ac + bc$$

are satisfied for all $a, b, c \in R$. A non-empty subset P in R is called a left(resp. right) ideal of R if it satisfies $RP \subseteq P$(resp.$PR \subseteq P$). Both a left and right ideal of R is called an ideal of R.

A fuzzy set μ in a ring R is called a fuzzy left ideal of R if (1) $\mu(x - y) \ge \min\{\mu(x), \mu(y)\}$, and (2) $\mu(xy) \ge \mu(y)$ for all x, y in R. A fuzzy subset μ of a ring R is a fuzzy right ideal of R if (1) $\mu(x - y) \ge \min\{\mu(x), \mu(y)\}$, and (2) $\mu(xy) \ge \mu(x)$ for all x, y in R. If μ is both a fuzzy left and right ideal of R, then μ is a fuzzy ideal of R.

Definition 2.3 *Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be two intuitionistic fuzzy sets in R. Define $A + B = (\mu_A \check{+} \mu_B, \lambda_A \hat{+} \lambda_B)$, the sum of A and B by*

$$\mu_A \check{+} \mu_B(x) = \bigvee_{x=y+z} \mu_A(y) \wedge \mu_B(z)$$

and

$$\lambda_A \hat{+} \lambda_B(x) = \bigwedge_{x=y+z} \lambda_A(y) \vee \lambda_B(z)$$

for all $x \in R$.

In the sequel, unless otherwise stated, R always denotes a ring.

Proposition 2.4 *Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic fuzzy sets in R. Then so is $A + B$.*

Proof. It is sufficient to show that $0 \le (\mu_A \check{+} \mu_B)(x) + (\lambda_A \hat{+} \lambda_B)(x) \le 1$ for all $x \in S$. Let $x \in S$. We have

$$(\mu_A \check{+} \mu_B)(x) = \bigvee_{x=y+z} \mu_A(y) \wedge \mu_B(z) \le \bigvee_{x=y+z} (1-\lambda_A(y)) \wedge (1-\lambda_B(z)))$$

$$\text{(since } \mu_A(y) \le 1-\lambda_A(y) \text{ and } \mu_B(z) \le 1-\lambda_B(z) \text{ for all } y,z \in R\text{)}$$

$$=1 - \bigwedge_{x=y+z} \lambda_A(y) \vee \lambda_B(z) = 1 - (\lambda_A \hat{+} \lambda_B)(x).$$

Therefore, $A+B$ is also an intuitionistic fuzzy set in R. □

3 Intuitionistic fuzzy ideals with thresholds (α, β) of rings

Definition 3.1 *An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in R is called an intuitionistic fuzzy ideal with thresholds (α, β) of R if it satisfies:*
(i) $\mu_A(x-y) \vee \alpha \ge (\mu_A(x) \wedge \mu_A(y)) \wedge \beta$,
(ii) $\mu_A(xy) \vee \alpha \ge (\mu_A(x) \vee \mu_A(y)) \wedge \beta$,
(iii) $\lambda_A(x-y) \wedge (1-\alpha) \le (\lambda_A(x) \vee \lambda_A(y)) \vee (1-\beta)$,
(iv) $\lambda_A(xy) \wedge (1-\alpha) \le (\lambda_A(x) \wedge \lambda_A(y)) \vee (1-\beta)$,
for all $x, y \in R$.

Note that any intuitionistic fuzzy ideal with thresholds $(0,1)$ of R is an intuitionistic fuzzy ideal with thresholds (α, β) of R, where $\alpha, \beta \in (0,1)$ and $\alpha < \beta$, but the converse does not hold in general as shown in the following example.

Example 3.2 *Let $R = \{0, a, b, c\}$ be a set with a addition operation $(+)$ and a multiplication operation $(.)$ as follows:*

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

and

·	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	b	b
c	0	0	b	b

Then R is a ring. Let μ_A and λ_A be fuzzy sets in R such that

$$\mu_A(0) = r, \mu_A(a) = r, \quad \mu_A(b) = \alpha, \quad \mu_A(c) = \alpha,$$

and

$$\lambda_A(0) = 1-\beta, \lambda_A(a) = 1-\beta, \quad \lambda_A(b) = t, \quad \lambda_A(c) = t,$$

respectively, where $0 < \alpha < \beta < 1, r \in [0, \alpha)$ *and* $t \in [0, 1-\beta]$. *Clearly,* $A = (\mu_A, \lambda_A)$ *is an intuitionistic fuzzy ideal with thresholds* (α, β) *of* R*, but is not an intuitionistic fuzzy ideal with thresholds* $(0, 1)$ *of* R*, since* $r = \mu_A(0) = \mu_A(bc) < \mu_A(b) \vee \mu_A(c) = \alpha$.

Theorem 3.3 *An intuitionistic fuzzy set* $A = (\mu_A, \lambda_A)$ *in* R *is an intuitionistic fuzzy ideal with thresholds* (α, β) *of* R *if and only if all the non-empty sets* $U(\mu_A; r)$ *and* $L(\lambda_A; t)$ *are ideals of* R *for all* $r \in [\alpha, \beta)$ *and* $t \in (1-\beta, 1-\alpha]$*, respectively.*

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal with thresholds (α, β) of R. Let $r \in [\alpha, \beta), x \in R$ and $y \in U(\mu; r)$. Then $\mu(xy) \vee \alpha \geq (\mu(x) \vee \mu(y)) \wedge \beta > r \geq \alpha$, hence $\mu(xy) > r$ and so $xy \in U(\mu; r)$. Similarly, $yx \in U(\mu; r)$. Further, if $x, y \in U(\mu; r)$, then $\mu(x-y) \vee \alpha \geq (\mu(x) \wedge \mu(y)) \wedge \beta > r \geq \alpha$, hence $\mu(x-y) \geq r$ and so $x - y \in U(\mu; r)$. Therefore, $U(\mu_A; r)$ is an ideal of R. In a similarly way we may prove that non-empty set $L(\lambda_A; t)$ is an ideal of R for all $t \in (1-\beta, 1-\alpha]$.

Conversely, assume that the given conditions hold. If possible, let $x, y \in R$ and $\mu_A(x-y) \vee \alpha < (\mu_A(x) \wedge \mu_A(y)) \wedge \beta$. Choose r such that $\mu_A(x-y) \vee \alpha < r < (\mu_A(x) \wedge \mu(y)) \wedge \beta$. Then $r \in [\alpha, \beta)$ and $\mu_A(xy) < r < \mu_A(x) \wedge \mu_A(y)$, that is, $x \in U(\mu_A; r)$ and $y \in U(\mu_A; r)$ but $x - y \notin U(\mu_A; r)$, a contradiction. Hence $\mu_A(x-y) \vee \alpha \geq (\mu_A(x) \wedge \mu_A(y)) \wedge \beta$ for all $x, y \in S$. Similarly, we may prove that the conditions (ii), (iii) and (iv) of Definition 3.1 are valid. Therefore, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal with thresholds (α, β) of R. □

Corollary 3.4 *Let* P *be an ideal of* R*. Define fuzzy sets* μ_A *and* λ_A *in* R *by*

$$\mu_A(x) = \begin{cases} r_1 & \text{if } x \in P \\ r_2 & \text{otherwise,} \end{cases} \qquad \lambda_A(x) = \begin{cases} t_1 & \text{if } x \in P \\ t_2 & \text{otherwise,} \end{cases}$$

where $0 \leq r_2 < r_1, t_1 < t_2 \leq 1$ *and* $r_i + t_i \leq 1$ *for* $i = 1, 2$. *Then* $A = (\mu_A, \lambda_A)$ *is an intuitionistic fuzzy ideal with thresholds* (α, β) *of* R.

Proposition 3.5 *If* $A = (\mu_A, \lambda_A)$ *is an intuitionistic fuzzy ideal with thresholds* (α, β) *of* R*, then so are* $\Box A$ *and* $\Diamond A$.

Proof. It is sufficient to show that μ_A^c satisfies the conditions (iii) and (iv) of Definition 3.1. For any $x, y \in R$, we have

$$\mu_A(x-y) \vee \alpha \geq (\mu_A(x) \wedge \mu_A(y)) \wedge \beta,$$

and so

$$(1 - \mu_A^c(x-y)) \vee \alpha \geq ((1 - \mu_A^c(x)) \wedge (1 - \mu_A^c(y)) \wedge \beta.$$

Hence

$$1-\mu^c_A(x-y)\wedge(1-\alpha)\geq 1-(\mu^c_A(x)\vee\mu^c_A(y))\vee(1-\beta),$$

that is

$$\mu^c_A(x-y)\wedge(1-\alpha)\leq(\mu^c_A(x)\vee\mu^c_A(y))\vee(1-\beta).$$

Thus the condition (ii) of Definition 3.1 is valid. Similarly, the condition (iii) of Definition 3.1 is satisfied. Therefore, $\Box A$ is an intuitionistic fuzzy ideal with thresholds (α,β) of R. In a similar way, we get $\Diamond A$ is also an intuitionistic fuzzy ideal with thresholds (α,β) of R. $\Box$

In view of Proposition 3.5, it is easy to verify that the following Theorem is valid.

Theorem 3.6 *An intuitionistic fuzzy set $A=(\mu_A,\lambda_A)$ in R is an intuitionistic fuzzy ideal with thresholds (α,β) of R if and only if $\Box A$ and $\Diamond A$ are intuitionistic fuzzy ideals with thresholds (α,β) of R.*

Corollary 3.7 *An intuitionistic fuzzy set $A=(\mu_A,\lambda_A)$ in R is an intuitionistic fuzzy ideal with thresholds (α,β) of R if and only if μ_A and λ^c_A are fuzzy ideals with thresholds (α,β) of R.*

Corollary 3.8 *$A=(\kappa_R,\kappa^c_R)$ is an intuitionistic fuzzy ideal with thresholds (α,β) of R.*

In view of Corollary 3.8, we may have the following Theorem.

Theorem 3.9 *A non-empty set P in R is an ideal of R if and only if $A=(\kappa_P,\kappa^c_P)$ is an intuitionistic fuzzy ideal with thresholds (α,β) of R.*

Theorem 3.10 *Let $A=(\mu_A,\lambda_A)$ and $B=(\mu_B,\lambda_B)$ be intuitionistic fuzzy ideals with thresholds (α,β) of R. Then so is $A+B$.*

Proof. By proposition 2.4, we know that $A+B$ is an intuitionistic fuzzy set in R. Now, let $x, y \in S$. We have

$$\begin{aligned}(\mu_A \check{+} \mu_B)(x-y) \vee \alpha =&(\bigvee_{x-y=(a-c)+(b-d)} \mu_A(a-c) \wedge \mu_B(c-d)) \vee \alpha \\ =& \bigvee_{x-y=(a-c)+(b-d)} (\mu_A(a-c) \vee \alpha) \wedge (\mu_B(b-d) \vee \alpha) \\ \geq& \bigvee_{x-y=(a-c)+(b-d)} (\mu_A(a) \wedge \mu_A(c) \wedge \beta) \wedge (\mu_B(b) \wedge \mu_B(d) \wedge \beta) \\ \geq& \bigvee_{x=a+b, y=c+d} (\mu_A(a) \wedge \mu_B(b) \wedge \beta) \wedge (\mu_A(c) \wedge \mu_B(d) \wedge \beta) \\ =&(\bigvee_{x=a+b} (\mu_A(a) \wedge \mu_B(b) \wedge \beta) \wedge (\bigvee_{y=c+d} \mu_A(c) \wedge \mu_B(d) \wedge \beta) \\ =&((\mu_A \check{+} \mu_B)(x) \wedge \beta) \wedge ((\mu_A \check{+} \mu_B)(y) \wedge \beta) \\ =&((\mu_A \check{+} \mu_B)(x) \wedge (\mu_A \check{+} \mu_B)(y)) \wedge \beta.\end{aligned}$$

$$\begin{aligned}(\mu_A \check{+} \mu_B)(x) \wedge \beta =&(\bigvee_{x=a+b} \mu_A(a) \wedge \mu_B(b)) \wedge \beta = \bigvee_{x=a+b} (\mu_A(a) \wedge \beta) \wedge (\mu_B(b) \wedge \beta) \\ \leq& \bigvee_{x=a+b} (\mu_A(ay) \vee \alpha) \wedge (\mu_B(by) \vee \alpha) = (\bigvee_{x=a+b} (\mu_A(ay) \wedge \mu_B(by)) \vee \alpha \\ \leq&(\bigvee_{xy=c+d} \mu_A(c) \wedge \mu_B(d)) \vee \alpha = (\mu_A \check{+} \mu_B)(xy) \vee \alpha.\end{aligned}$$

Similarly, $(\mu_A \check{+} \mu_B)(y) \wedge \beta \leq (\mu_A \check{+} \mu_B)(xy) \vee \alpha$. Hence $(\mu_A \check{+} \mu_B)(xy) \vee \alpha \geq (\mu_A \check{+} \mu_B)(x) \vee \mu_A \check{+} \mu_B)(y)) \wedge \beta$. In a similar way, we get

$$(\lambda_A \check{+} \lambda_B)(x-y) \wedge (1-\alpha) \leq (\lambda_A \check{+} \lambda_B)(x) \vee (\lambda_A \check{+} \lambda_B)(y)) \vee (1-\beta)$$

and

$$(\lambda_A \check{+} \lambda_B)(xy) \wedge (1-\alpha) \leq (\lambda_A \check{+} \lambda_B)(x) \wedge (\lambda_A \check{+} \lambda_B)(y)) \vee (1-\beta)$$

for all $x, y \in R$. Therefore, $A+B$ is an intuitionistic fuzzy ideal with thresholds (α, β) of R. □

Theorem 3.11 *Let* $\{A_i = (\mu_{A_i}, \lambda_{A_i}) | i \in I\}$ *be a family intuitionistic fuzzy ideals with thresholds* (α, β) *of* R*, then* $A = \bigcap_{i \in I} A_i = (\bigcap_{i \in I} \mu_{A_i}, \bigcup_{i \in I} \lambda_{A_i})$ *is also intuitionistic fuzzy ideal with thresholds* (α, β) *of* R*.*

Proof. It is straightforward. □

Theorem 3.12 *Let $\mathcal{IFI}(R)$ be the set of all intuitionistic fuzzy ideals with thresholds $(0,1)$ and the same tip "t" (i.e. $\mu_A(0) = \mu_B(0), \lambda_A(0) = \lambda_B(0)$ for all $A, B \in \mathcal{IFI}(R)$) of R. Then $(\mathcal{IFI}(R), \cap, +)$ is a modular lattice.*

Proof. Let $A, B \in \mathcal{IFI}(S)$. Clearly, $A \cap B$ is the greatest lower bound of A and B. By Theorem 3.10, we have $A + B \in \mathcal{IFI}(S)$. Now, we prove that $A + B = A \vee B$. Let $z \in S$. Since $x = 0 + x$, $\mu_A(0) = \mu_B(0) \geq \mu_A(y)$ and $\lambda_A(0) = \lambda_B(0) \leq \lambda_A(y)$ for all $y \in R$, then we have

$$(\mu_A \check{+} \mu_B)(x) = \bigvee_{x=y+z} \mu_A(y) \wedge \mu_B(z) \geq \mu_A(x) \wedge \mu_B(0) = \mu_A(x)$$

and

$$(\lambda_A \hat{+} \lambda_B)(x) = \bigwedge_{x=y+z} \mu_A(y) \vee \mu_B(z) \leq \mu_A(x) \vee \mu_B(0) = \mu_A(x),$$

which imply that $A \subseteq A + B$. Similarly, $B \subseteq A + B$ and so $A + B$ is an upper bound of A and B. Now, let $C \in \mathcal{IFI}(S)$ containing A and B. It is easy to see $A + B \subseteq C$. So, $A + B = A \vee B$. Therefore, $(\mathcal{IFI}(S), \cap, +)$ is a lattice. For modularity, let $A, B, C \in \mathcal{IFI}(S)$ be such that $A \supseteq C$. We have to show $A \cap (B + C) = (A \cap B) + C$. But, the relation $A \cap (B + C) \supseteq (A \cap B) + C$ is clear. It remains to show $A \cap (B + C) \subseteq (A \cap B) + C$. Now let $x \in S$. Then we have

$$\begin{aligned} ((\mu_A \cap \mu_B) \check{+} \mu_C)(x) &= \bigvee_{x=y+z} \mu_A(y) \wedge \mu_B(y) \wedge \mu_C(z) \geq \bigvee_{x=y+z} \mu_A(x) \wedge \mu_A(z) \wedge \mu_B(y) \wedge \mu_C(z) \\ &= \bigvee_{x=y+z} \mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z) = \mu_A(x) \wedge (\mu_B \check{+} \mu_C)(x) = (\mu_A \cap (\mu_B \check{+} \mu_C))(x). \end{aligned}$$

and

$$\begin{aligned} ((\lambda_A \cup \lambda_B) \hat{+} \lambda_C)(x) &= \bigwedge_{x=y+z} \lambda_A(y) \vee \lambda_B(y) \vee \lambda_C(z) \leq \bigwedge_{x=y+z} \lambda_A(x) \vee \lambda_A(z) \vee \lambda_B(y) \vee \lambda_C(z) \\ &= \bigwedge_{x=y+z} \lambda_A(x) \vee \lambda_B(y) \vee \lambda_C(z) = \lambda_A(x) \vee (\lambda_B \hat{+} \lambda_C)(x) = (\lambda_A \cup (\lambda_B \hat{+} \lambda_C))(x). \end{aligned}$$

That is, $(\mu_A \cap \mu_B) \check{+} \mu_C \supseteq \mu_A \cap (\mu_B \check{+} \mu_C)$ and $(\lambda_A \bigcup \lambda_B) \hat{+} \lambda_C \subseteq \lambda_A \bigcup (\lambda_B \hat{+} \lambda_C)$. Hence $A \cap (B + C) \subseteq (A \cap B) + C$. Thus $A \cap (B + C) = (A \cap B) + C$ and so $(\mathcal{IFI}(S), \cap, +)$ is a modular lattice. □

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References

[1] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20(1986), 87-96.

[2] S.K. Bhakat, P. Das, On the definition of a fuzzy subgroup, Fuzzy sets Syst. 51(1992), 235-241.

[3] R. Biswas, Intuitionistic fuzzy subgroups, Math. Forum. 10(1989), 37-46.

[4] B. Davvaz, W.A. Dudek, Y.B. Jun, Intuitionistic fuzzy H_v-submodules, Inform. Sci. 176(2006), 285-300.

[5] W.A. Dudek, B. Davvaz, Y.B. Jun, On intuitionistic fuzzy sub-uasihypergroups of quasihypergroups, Inform. Sci. 170(2005), 251-262.

[6] K.H. Kim, W.A. Dudek, Y.B. Jun, On intuitionistic fuzzy subquasigroups of quasigroups, Quasigroups Relat Syst 7(2000), 15-28.

[7] K.H. Kim, Y.B. Jun, Intuitionistic fuzzy ideals of semigroups, Indian T. Pure Appl. Math. 33(4)(2002), 443-449.

[8] A. Rosenfeld, fuzzy groups, J.Math.Anal.Appl.35(1971), 512-517.

[9] X. Yuan, C. Zhang, Y. Ren, Generalized fuzzy groups and many-valued implications, Fuzzy sets Syst. 138(2003), 205-211.

[10] L.A.Zadeh, Fuzzy sets, Inform and control, 8(1965), 338-353.

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