Opportunistic Interference Cancellation in Cognitive Radio Systems

Petar Popovski*, Hiroyuki Yomo*, Kentaro Nishimori†,*, Rocco Di Taranto*, and Ramjee Prasad*
*Department of Electronic Systems, Aalborg University, Niels Jernes Vej 12, DK-9220 Aalborg, Denmark
Email: {petarp, yomo, in_nk, rdt, prasad}@kom.aau.dk
† NTT Network Innovation Laboratories, NTT Corporation, 1-1 Hikarinooka, Yokosuka-shi, 239-0847 Japan
Email: {nishimori.kentaro}@lab.ntt.co.jp

***Abstract*—In this paper we investigate the problem of spectrally efficient operation of a cognitive radio (CR), also called Secondary System (SS), under an interference from the primary system (PS). A cognitive receiver (CRX) observes a multiple (MA) access channel of two users, the secondary and the primary transmitter, respectively. We advocate that the SS should apply *Opportunistic Interference Cancellation (OIC)* and decode the PS signal when such an opportunity is created by the rate selected in the PS and the power received from the PS. We derive the achievable data rate in the SS when OIC is applied. When the PS is decodable, we devise a method applied by the SS to achieve the maximal possible secondary rate. This method has a practical significance, since it enables rate adaptation without requiring any action from the PS. We investigate the power allocation in the SS when OIC is applied over multiple channels. We show that the optimal power allocation can be achieved with *intercepted water–filling* instead of the conventional water–filling. The results show a significant gain for the rate achieved by OIC.**

I. Introduction

A cognitive radio (CR) [1] is allowed to *reuse* the frequency spectrum which is assigned to a primary system (PS). A CR network (or secondary system (SS)) is allowed to use certain radio resource if it is not causing an adverse interference to the PS. Furthermore, the CR should achieve a spectrally efficient operation under the interference from the PS. One strategy [2] is to treat the PS signals as a noise and use only the radio resources where CR link can meet the target Signal–To–Interference–and–Noise–Ratio (SINR).

In this paper, our departing point is that it is reasonable that the SS can decode the PS signals, as the PS is a legacy system. On Fig. 1, the secondary receiver (SRX) receives both the signal from the primary Base Station (BS) and the secondary transmitter (STX). Hence, SRX observes a *multiple access (MA) channel* of two users: the desired STX and the undesired primary TX. However, the PS adapts its data rate for the primary terminals and the chosen transmission rate in the PS is *independent* of the SNR at which the PS signal is received by the SRX. Therefore, the SS should adapt its data rate by first considering whether the PS signal can be decoded. This is done by observing the received powers and the region of the achievable rates in the multiple access channel. We call this *opportunistic interference cancellation (OIC)*, as the decodability of the PS signal at the SRX depends on the opportunity created by the selection of the data rate in the PS and the SNR on the link between the primary BS and the SRX.

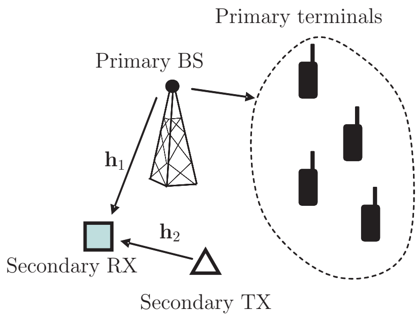

Fig. 1. The considered scenario where the primary transmitter is a Primary Base Station (BS), which adapts the transmission rates to the population of a Primary Terminals. The Secondary Transmitter (TX) knows the rates used in the primary system and accordingly adapts its transmission to the Secondary Receiver (RX).

We first derive the function by which the SS can adapt its rate by OIC over a single channel.When the PS signal is decodable, we introduce a method based on superposition coding by which any rate pair of the MA channel can be achieved without time sharing [3]. This has a practical significance, since the PS cannot be compelled to adapt the rate in a time–sharing manner. The derived rate adaptation function for the SS link is not a simple log–function of the power on SS the link. This has an impact on the power allocation in the SS when it is in case of multiple available channels, where the optimal power allocation can be achieved by *intercepted water–filling*.

II. Assumptions and System model

The primary BS is using M channels and adapts the rate in each channel according to the scheduling policy and the channel state information (CSI) of the PS terminals, see Fig. 1. We assume that the rate adaptation in the PS is independent of the SS. The communication in the SS does not cause an adverse interference to the PS, since the SS to be a short–range radio system which uses a regulated low power.

The symbol y_m received at SRX at the $m-$th channel is:

$$y_m = h_{s,m}\sqrt{\mathcal{E}_m}x_{s,m} + h_{p,m}x_{p,m} + z_m \tag{1}$$

where $h_{s,m}$ ($h_{p,m}$) is the complex channel gain on the $m-$th channel from the STX(BS) to the SRX; $\sqrt{\mathcal{E}_m}x_{s,m}$ is the signal transmitted by the STX, with a normalized expectation $E[|x_{s,m}|^2] = 1$, while $\mathcal{E}_m$ is proportional to the energy used

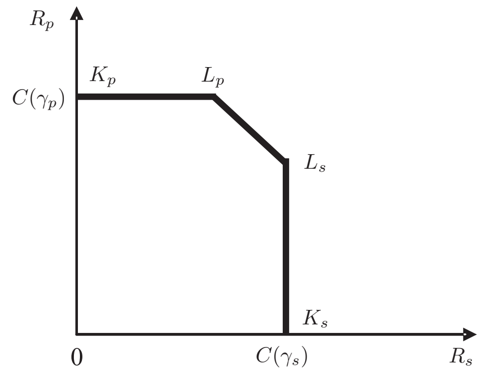

Fig. 2. The region of achievable rate pairs $\mathcal{R} = (R_s, R_p)$ in a two–user multiple access channel.

in channel m. $x_{p,m}$ is the normalized ($E[|x_{p,m}|^2] = 1$) signal from the primary BS. z_m is the complex Gaussian noise with variance σ^2. Each channel has a normalized bandwidth $W = 1$ [Hz], such that the time is measured in terms of number of symbols. The transmissions of the BS and the STX are assumed synchronized at the SRX, such that we consider the information–theoretic setting of the MA channel [3].

The PS serves the users in *scheduling epochs*. In each epoch, the primary BS decides accordingly the transmission rate $R_{p,m}$ for the $m-$th channel. This information is broadcasted by the BS, such that the CR terminals learn about $R_{p,m}$ for each m. Let $\beta_{p,m}$ be the minimal SNR that enables successful decoding of a message sent at rate $R_{p,m}$. Then:

$$R_{p,m} = \log_2(1 + \beta_{p,m}) = C(\beta_{p,m}) \quad \text{[bps]} \tag{2}$$

Due to the bandwidth normalization, the spectral efficiency [bps/Hz] and the rate [bps] are equivalent. A scheduling epoch lasts for N symbols, where N is sufficiently large such that the primary BS can apply capacity–achieving transmissions. During each epoch, the channels $h_{s,m}, h_{p,m}$ do not change. The *secondary SNR at the SRX for the channel* m is defined as $\gamma_{s,m} = \frac{\mathcal{E}_m |h_{s,m}|^2}{\sigma^2} = \frac{\mathcal{E}_m}{\nu_m}$, where ν_m is the normalized noise energy at the $m-th$ channel of the SRX. The *primary SNR at the SRX for the channel* m is $\gamma_{p,m} = \frac{|h_{p,m}|^2}{\sigma^2}$. The total average energy available for transmission on all channels is $\sum_{m=1}^{M} \mathcal{E}_m = \mathcal{E}$. In each scheduling epoch, the SS adapts the energy $\mathcal{E}_m$ and the data rate $R_{s,m}$ in each channel.

III. Opportunistic Interference Cancellation

The concept of OIC will be introduced for $M = 1$ channel (in this section we drop the subscript m). The SRX can reliably decode both the primary/secondary signal if the rates R_p/R_s are within the capacity region of the MA channel (Fig. 2):

$$R_s \leq C(\gamma_s) \quad R_p \leq C(\gamma_p) \quad R_p + R_s \leq C(\gamma_s + \gamma_p) \tag{3}$$

The rate pairs $\mathcal{R} = (R_s, R_p)$ at the points L_s and L_p are $\mathcal{R}(L_s) = \left(C(\gamma_s), C\left(\frac{\gamma_p}{1+\gamma_s}\right)\right)$ and $\mathcal{R}(L_p) = \left(C\left(\frac{\gamma_s}{1+\gamma_p}\right), C(\gamma_p)\right)$, respectively. The rate pairs at the border are achieved by successive interference cancellation. In addition, the rates on the segment L_pL_s can be achieved by *time–sharing* [3]: The two transmitters should use $\mathcal{R}(L_s)$ for a fraction of time θ, and $\mathcal{R}(L_p)$ for the fraction of time $1-\theta$. With $\theta \in [0, 1]$, any point on L_pL_s is achievable. However, in our scenario R_p is given *a priori* and time–sharing is not possible. Hence, the STX needs an alternative strategy to achieve the rate pairs $\mathcal{R} \in L_pL_s$. Let the PS have R_p, such that $C\left(\frac{\gamma_p}{1+\gamma_s}\right) \leq R_p \leq C(\gamma_p)$. We propose that STX uses *superposition coding* and transmits $x_s = (1-\alpha)x_s^{(1)} + \alpha x_s^{(2)}$, where $0 \leq \alpha \leq 1$ and $E[|x_s^{(1)}|^2] = E[|x_s^{(2)}|^2] = 1$, such that SRX receives $y = h_s\left((1-\alpha)x_s^{(1)} + \alpha x_s^{(2)}\right) + h_px_p + z$. The SRX decodes in three steps: **Step 1:** $x_s^{(1)}$ is decoded from y by treating $h_s\alpha x_s^{(2)} + h_px_p$ as noise, after which $y' = y - h_s(1-\alpha)x_s^{(1)}$ is obtained; **Step 2:** x_p is decoded from y' by treating $h_s\alpha x_s^{(2)}$ as a noise, after which $y'' = y' - h_px_p$ is obtained; **Step 3:** $x_s^{(2)}$ is decoded from y''. From Step 2, by setting $R_p = C\left(\frac{\gamma_p}{1+\alpha\gamma_s}\right)$, we can determine the coefficient $\alpha = \frac{\frac{\gamma_p}{\beta_p}-1}{\gamma_s}$, where $\beta_p = \frac{\gamma_p}{1+\alpha\gamma_s}$ from (2). The rate of $x_s^{(1)}$ is $R_s^{(1)} = C\left(\frac{(1-\alpha)\gamma_s}{1+\gamma_p+\alpha\gamma_s}\right)$ and the rate of $x_s^{(2)}$ is $R_s^{(2)} = C(\alpha\gamma_s)$. The total secondary rate is $R_s = R_s^{(1)} + R_s^{(2)}$ and it can be easily verified that the rate pair (R_s, R_p) lies on the segment L_pL_s: $R_s + R_p = C(\gamma_p + \gamma_s)$.

In our scenario, the STX observes γ_p and $R_p = C(\beta_p)$ as a priori given values and determines the maximal achievable rate R_s as a function of γ_s, with parameters γ_p and β_p:

$$R_s = F_{\gamma_p,\beta_p}(\gamma_s) \tag{4}$$

In absence of the PS signal, $F_{\gamma_p=0,\beta_p}(\gamma_s) = C(\gamma_s)$. The function $F_{\gamma_p,\beta_p}(\gamma_s)$ reflects the policy of *opportunistic interference cancellation (OIC)*, where the CR makes the best possible use of the knowledge about the PS. A less optimal strategy would be to treat the PS signal an undecodable interference, even when $\beta_p \leq \gamma_p$. In order to determine $F_{\gamma_p,\beta_p}(\gamma_s)$ we consider two regions for γ_p. When $\gamma_p < \beta_p$, SRX receiver cannot decode the PS signal, such that:

$$R_s = F_{\gamma_p,\beta_p}(\gamma_s)\big|_{\gamma_p<\beta_p} = C\left(\frac{\gamma_s}{1+\gamma_p}\right) \tag{5}$$

In the second region $\gamma_p \geq \beta_p$, the SRX can decode the PS signal and R_s is chosen such that (R_p, R_s) belongs to the achievable rate region, determined for the given γ_p and γ_s. Depending on γ_s, there are two different cases:

- $\gamma_s \leq \frac{\gamma_p}{\beta_p} - 1$: In this case the rate pair lies on the segment K_sL_s on Fig. 2:

$$R_s = F_{\gamma_p,\beta_p}(\gamma_s) = C(\gamma_s) \tag{6}$$

- $\gamma_s > \frac{\gamma_p}{\beta_p} - 1$: In this case the rate pair lies on the segment K_sL_s on Fig. 2 and we use the proposed strategy with superposition coding, such that

$$R_s = F_{\gamma_p,\beta_p}(\gamma_s) = \log_2\left(\frac{1+\gamma_p}{1+\beta_p}\right) + C\left(\frac{\gamma_s}{1+\gamma_p}\right) \tag{7}$$

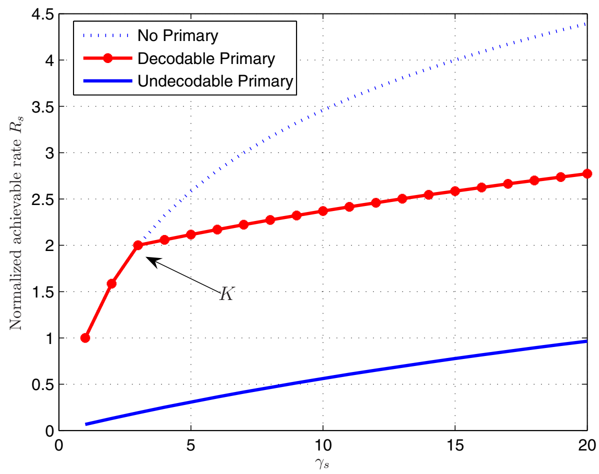

Fig. 3. Normalized achievable rate as a function of the secondary SNR γ_s. The abscissa is in the linear scale of γ_s. The case 'No primary' corresponds to $\gamma_p = 0$. In the other cases, $\gamma_p = 20$, the value $\beta_p = 5$ when the primary is decodable, while it is $\beta_p > 10$ when the primary is not decodable.

Fig. 3 exemplifies three different cases of the rate function $F_{\gamma_p,\beta_p}(\gamma_s)$. Note that, when $\beta_p < \gamma_p$, the function is non–differentiable at the point K with $\gamma_s = \frac{\gamma_p}{\beta_p} - 1$.

IV. OIC WITH MULTIPLE CHANNELS

In this section, we elaborate on how the energy should be allocated when the SS uses OIC over $M > 1$ channels. We first consider the case $M = 2$. In absence of the primary interference ($\gamma_p = 0$) the problem of rate/energy allocation in parallel Gaussian channels is [3]: Maximize $C\left(\frac{\mathcal{E}_1}{\nu_1}\right)+C\left(\frac{\mathcal{E}_2}{\nu_2}\right)$ for $\mathcal{E}_1 \geq 0, \mathcal{E}_2 \geq 0, \mathcal{E}_1 + \mathcal{E}_2 = \mathcal{E}$. This optimization problem has the *water–filling solution*: If $\mathcal{E} \leq \nu_2 - \nu_1$, then $\mathcal{E}_1 = \mathcal{E}$ and $\mathcal{E}_2 = 0$; while if $\mathcal{E} > \nu_2 - \nu_1$ then $\mathcal{E}_1 = \frac{\mathcal{E}+\nu_2-\nu_1}{2}$ and $\mathcal{E}_2 = \frac{\mathcal{E}-\nu_2+\nu_1}{2}$. This conventional water–filling (CWF) can be interpreted as follows: While $C\left(\frac{\mathcal{E}_1}{\nu_1}\right)$ is the faster–growing function, all the energy is poured in channel 1; when $\mathcal{E}_1 = \nu_2 - \nu_1$, then the rate in both channels starts to increase with identical pace, such that the energy $\Delta\mathcal{E} = \mathcal{E} - (\nu_2 - \nu_1)$ should be equally distributed to both channels.

Let now $\gamma_{p,1} = \gamma_{p,2} = \gamma_p > 0$, $\nu_1 = \nu_2 = \nu$ and $\beta_{p,1} = \beta_p > \gamma_p$, but $\beta_{p,2} < \gamma_p$. The optimization problem is: Maximize $\rho_s(\mathcal{E}_1, \mathcal{E}_2) = R_{s,1}(\mathcal{E}_1) + R_{s,2}(\mathcal{E}_2)$ for $\mathcal{E}_1 \geq 0, \mathcal{E}_2 \geq 0, \mathcal{E}_1 + \mathcal{E}_2 = \mathcal{E}$, where the achievable rates $R_{s,1}(\mathcal{E}_1)$ and $R_{s,1}(\mathcal{E}_2)$ are determined according to the function $F_{\gamma_p,\beta_p}(\gamma_s)$, discussed in the previous section. Here the Karush–Kuhn–Tucker conditions [4] cannot be directly applied, since the function $\rho_s(\mathcal{E}_1, \mathcal{E}_2)$ is not a continuously differentiable function of $(\mathcal{E}_1, \mathcal{E}_2)$, as $R_{s,1}(\mathcal{E}_1)$ is not a continuously differentiable function of $\mathcal{E}_1$. Still, due to the properties of the log–functions, the optimal solution can be described in the following way.

Region $\mathcal{E} < \mathcal{E}_{10}$. Here $R_{s,1} = C\left(\frac{\mathcal{E}_1}{\nu}\right)$ and, as it grows faster than $R_{s,2}$, CWF implies $\mathcal{E}_1 = \mathcal{E}$ and $\mathcal{E}_2 = 0$. For the CWF, such an allocation would have continued until $\mathcal{E} + \nu = \nu(1 + \gamma_p)$ i. e. $\mathcal{E} = \nu\gamma_p$. However, at $\mathcal{E} = \mathcal{E}_{10} = \nu\left(\frac{\gamma_p}{\beta_p} - 1\right) < \nu\gamma_p$ the rate $R_{s,1}$ starts to grow as a different function and we have to consider re–allocation.

Region $\mathcal{E} = \mathcal{E}_{10} + \Delta\mathcal{E}$, where $\Delta\mathcal{E} > 0$ is sufficiently small. Let $\mathcal{E}_1 = \mathcal{E}_{10} + \mathcal{E}_{11}$, such that we can write:

$$R_{s,1} = \log_2\left(\frac{1+\gamma_p}{1+\beta_p}\right) + \log_2\left(1 + \frac{\mathcal{E}_{10}+\mathcal{E}_{11}}{\nu(1+\gamma_p)}\right) = \\ = \log_2\left(\frac{\gamma_p}{\beta_p}\right) + \log_2\left(1 + \frac{\mathcal{E}_{11}}{\nu(1+\gamma_p)+\mathcal{E}_{10}}\right) \quad (8)$$

We can conclude that $R_{s,2}$ grows with $\mathcal{E}_2$ faster than $R_{s,1}$ with $\mathcal{E}_{11}$ for all points $(\mathcal{E}_{11}, \mathcal{E}_2) = (0, \mathcal{E}_2)$ with $0 \leq \mathcal{E}_2 < \mathcal{E}_{10}$. Now the CWF imposes that $\mathcal{E}_{11} = 0$ and $\mathcal{E}_2 = \Delta\mathcal{E}$ when $\Delta\mathcal{E} < \mathcal{E}_{10}$.

Region $\mathcal{E} = 2\mathcal{E}_{10} + \Delta\mathcal{E}$, where $\Delta\mathcal{E} > 0$. In this region, the energy of $\mathcal{E}_{10} + \frac{\Delta\mathcal{E}}{2}$ is allocated to each channel.

The described solution is similar, yet not identical with the CWF solution and it can be interpreted as an *intercepted water–filling (IWF*, see Figure 4. Note that in the absence of the upper "stone" block in channel 1, this figure would have represented a CWF. The region pinched between stone blocks of channel 1 and 2 can be thought of a leakage canal of zero volume, such that while $\mathcal{E} < \mathcal{E}_{10}$ the lower basin of channel 1 is being filled only.

IWF produces the optimal solution when $M > 2$ and the values of $\nu_m, \gamma_{p,m}$ and $\beta_{p,m}$ are arbitrary. We omit the rigorous proof here and provide only the main arguments. First, note that $F_{\gamma_p,\beta_p}(\gamma_s)$ is always a concave function of γ_s. When $\beta_p > \gamma_p$ the function is non–differentiable at one point, but is still concave, as it can be represented as a minimum of two concave functions [4]. In that case the IWF implements the steepest ascent algorithm, which leads to a globally optimal solution. The IWF implementation can be described by the following, rather visual, explanation. Based on $\nu_m, \gamma_{p,m}, \beta_{p,m}$ we have to determine the height of the "stone" blocks for each

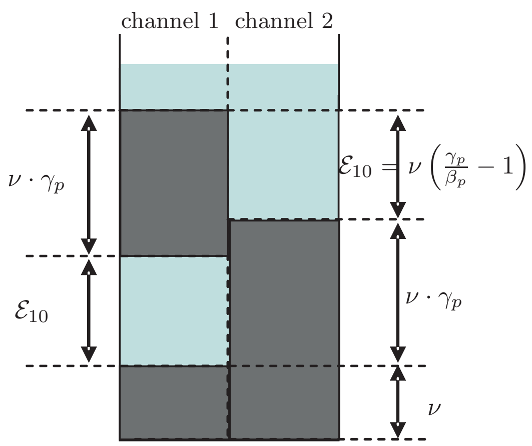

Fig. 4. Example of intercepted water–filling for two channels in which $\nu_1 = \nu_2 = \nu$, $\gamma_{p,1} = \gamma_{p,2} = \gamma_p$ and $\beta_{p,1} = \beta_p > \gamma_p, \beta_{p,2} < \gamma_p$.

TABLE I
THE BLOCK HEIGHTS FOR INTERCEPTED WATER–FILLING (IWF)

Per–channel blocks for Intercepted Water–Filling
• If $\gamma_{p,m} < \beta_{p,m}$, then the channel contains only one block of height $\nu_m(1+\gamma_{p,m})$ • If $\gamma_{p,m} \geq \beta_{p,m}$, then the channel contains two blocks. The lower block starts from the bottom and has a height ν_m. The upper block starts at a height of $\nu_m + \nu_m\left(\frac{\gamma_{p,m}}{\beta_{p,m}} - 1\right) = \nu_m \frac{\gamma_{p,m}}{\beta_{p,m}}$. The height of the upper block is $\nu_m \gamma_{p,m}$.

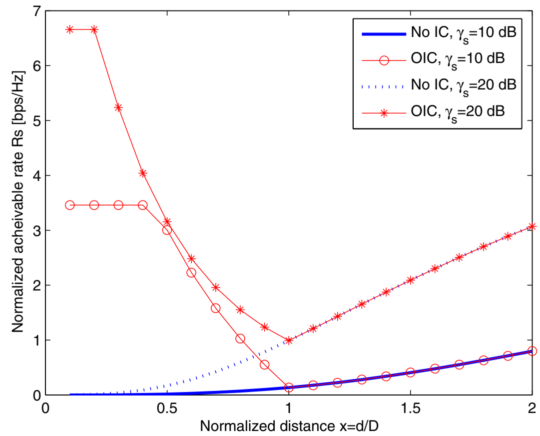

Fig. 5. Normalized achievable rate as a function of the normalized distance of the secondary RX from the primary BS. "No IC" is without Interference Cancellation. Here $\beta_p = 20$ [dB], propagation coefficient is $v = 3$.

channel, as well as the position of the upper stone block. This is summarized in Table I. The upper block appears only in the channels in which the primary signal is decodable. With determined block levels/positions, the energy allocation can be done by water–filling and considering that the water is leaking through the side walls of the upper blocks in the channels. The total block height in a channel is equal to $\nu_m(1+\gamma_{p,m})$. This implies that, when the energy is sufficiently high, such that the water–filling goes above the uppermost block, then the power allocation of IWF and CWF is identical.

V. NUMERICAL RESULTS

We first illustrate a scenario with $M = 1$. The PS has a range of D meters and it adjusts its power so as to have a predefined SNR of β_p for a primary receiver at a distance D with LOS link to the BS. The SRX is at the distance d from the BS and a primary SNR of $\gamma_p(x) = \frac{\beta_p}{x^v}$, where $x = \frac{d}{D}$ and v is the propagation coefficient. Fig. 5 depicts the normalized achievable rate as a function of the normalized distance x. Two values of γ_s are used, 10 and 20 dB, respectively and γ_s is a measure of the power applied in the SS. OIC leads to higher rate when $x < 1$, but is identical to the case without interference cancellation for $x > 1$, as the PS signal cannot be decoded when the SRX is at distances $d > D$. For OIC, the rate points in the region $\frac{1}{(1+\gamma_s)^{\frac{1}{v}}} < x < 1$ are achieved by the described strategy of superposition coding.

Fig. 6 compares IWF and CWF for $M = 10$ channels. For given $\mathcal{E}$, the normalized achievable rate is the sum of the rates

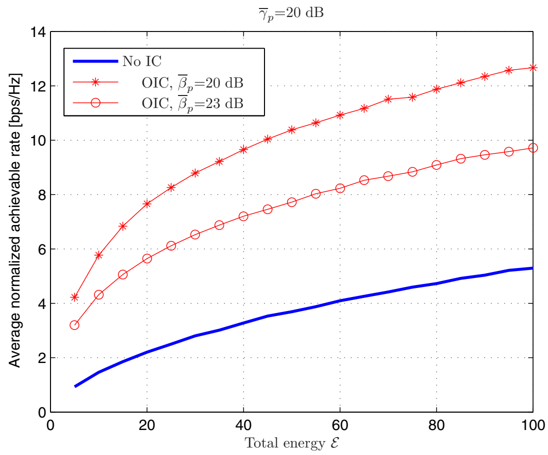

Fig. 6. Normalized average achievable rate in [bps/Hz] as a function of the average SNR $\overline{\gamma}_s = \frac{\mathcal{E}}{\nu}$ on the secondary link. The number of channels is $M = 10$.

for all 10 channels and the value is obtained by averaging over 10^4 iterations. In each iteration, $\nu_m = \frac{1}{\gamma_m}$ where γ_m is an exponentially distributed variable with average value 1, such that the average secondary SNR per channel is $\frac{\mathcal{E}}{M}$. In each iteration, the values $\gamma_{p,m}$ is generated from exponential random variable with mean $\overline{\gamma}_p = 20dB$. $\beta_{p,m}$ is generated from exponential random variable with mean value 20 dB and 23 dB, respectively, for each of the two OIC curves. We can see that IWF leads to significant rate improvements. When $\overline{\beta}_p > \overline{\gamma}_p$ the SRX has less opportunity to decode the primary, such that the improvement of IWF over CWF is decreased.

VI. CONCLUSION

We have investigated the problem of spectrally efficient operation in a cognitive radio (CR)(or secondary system (SS)) under interference from a primary system (PS). We have shown that the SS should apply *Opportunistic Interference Cancellation (OIC)* and cancel the interference from the PS whenever such opportunity is created by (a) selection of the data rate in the PS and (b) the link quality between the primary TX and the secondary RX. We devise a method to obtain a maximal achievable rate in the SS whenever the primary signal is decodable. The derived rate adaptation function is then applied in case the SS uses multiple channels interfered by the PS. We show that the solution to the power/rate allocation problem is intercepted water–filling rather than the conventional water–filling. The numerical results confirm that the OIC can bring rate gains in the CR systems.

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