

Interest, Price, and Profit: An Overview of Mathematical Economics in YBC 4698¹

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§1. General presentation

§1.1. Tablet YBC 4698 provides an interesting overview of the mathematical approach to economic problems in the Old Babylonian period. Although well preserved (see the photo posted at <<http://cdli.ucla.edu/P255010>>), the text has been poorly published. In *MKT* 1, Neugebauer confessed that he was not able to publish the text.² In *MKT* 3, he made little progress. He transliterated the text leaving many signs unidentified, and translated only a small portion of it (*MKT* 3, 42-45). Neugebauer recognized that his difficulties came mainly from the fact that he was not familiar with economic terminology.³ Th-

ureau-Dangin improved the reading of some words, and provided a transcription and translation of section 16, the rest of the text remaining quite obscure (Thureau-Dangin 1937b, 89-90). In *TMB*, he only quoted the text (p. 214) and referred to *MKT* 3 and to his previous paper.⁴ Friberg, in different chapters of his 2005 monograph (2005: 23 n. 10, 60-61, 67-68, and 215-218), made substantive improvements to YBC 4698's reading and translations, as well as interpretation of problems 3, 6, 7, 8, 9 and 10. However, he considered the problems as isolated entities. In this article, we try to look at the text as a whole, and to understand how the different problems are interconnected.

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² "The text bears the single line colophon x x *tuppu* 3-kamma. The number of subjects (or examples) is 17. These are grain (*še*), silver (*kù-babbar*), interest (*máš*) and the like, but in detail there are still so many difficulties that I do not wish to give a full transcription" (*MKT* 1, 513 [translation of the German by the authors]).

³ "Several points in this text remain unclear to me. Aside

§1.2. Few mathematical texts dealing with economic matters are known to date. As noted by Neugebauer and Sachs (1945: 106), "the published mathematical texts concerning prices are badly preserved or obscure for other reasons." Among these economic mathematical texts are VAT 7530, MLC 1842,⁵ as well as some texts discovered after the publication of *MKT* and *MCT*, such

from the difficult readability, this is due above all to the fact that obvious conventions and terms of the economy to be taken as a given are not known (at least to me)" (*MKT* 3, 43 [translation of the German by the authors]).

⁴ The tablet is mentioned in Legrain 1937: 1947 in connection with *UET* 3, 1377. See also Thureau-Dangin 1937a: 80-86; Bruins & Rutten 1961 in connection with TMS 13; Nemet-Nejat 1993: 57, 60, 61-63, 86, with some comments.

⁵ VAT 7530 is published in *MKT* 1, 287-289, and in *TMB* 100; MLC 1842 is published in *MCT* 106-107. See also the list of problems dealing with compound interests in Nemet-Nejat 1993: 58-61, AO 6770 #2, VAT 8521, VAT 8528, YBC 4669 #11.

as texts from Susa (TMS 13 and 22⁶), from the Diyala Valley (IM 54464),⁷ and from southern Mesopotamia (MS 3895).⁸ Nowadays, the reading and interpretation of YBC 4698 can be improved as we profit from a larger comparative material, from a better understanding of economic texts and mathematical series texts, and from Friberg's work on "combined market rate exercises" (Friberg 2007: 157-168). Thus, in this article we can provide an almost complete transliteration, translation and possible interpretations of the preserved portions of the text.

§1.3. The type of tablet is M(2,2) (multi-column tablet, with 2 columns on the obverse and 2 columns on the reverse). The text contains 17 problem statements on interest rates, prices and profit, and ends with a colophon which indicates that the tablet is the third of a series (dub 3-kam-ma). Thus, YBC 4698 belongs to the small collection of 20 mathematical series tablets known to date. The origin of this tablet is unknown. However, the series tablets probably date from the end of the Old Babylonian period (late 18th century or early 17th), and may come from central Mesopotamia (perhaps Sippar or Kish). (See discussion on date and provenience of mathematical series tablets and related bibliography in Proust 2009b: 169-170.)

§1.4. Although damaged, one guesses that the colophon contained the number of sections (17 im-šu) as well, though this reference seems to have been erased in antiquity. Each of the 17 sections contains the statement of a problem, without any indication about the procedure of resolution. All of the statements deal with the same general topic, namely economic issues. The writing uses mainly sumerograms, but it is not clear if the text is written in Akkadian, in Sumerian, or in a completely artificial language (see discussion in Proust 2009b: 170-171).

§1.5 The list of 17 problem statements found in YBC 4698 can be divided into several homogeneous groups (for more details, see §6):

#1-2: The interest is given, find the principal.

#3-5: The rate in-kind or values of two kinds of goods are different; find the price of each of them in various conditions.

#6-11: The purchase price and the selling price of a good are given; find the profit (and reciprocal problems).

#12-14: similar to #3-5, but the resolution of the problem involves division by non-regular numbers.

#15-16: Different ratios are used to evaluate goods. Find the values or rates.

#17: Rates are used to evaluate value of two kinds of stones. Find the weight of the stones.

§2. Terminology

A glossary is provided in §8. Here, we emphasize the technical meaning of expressions used in economic problems.

§2.1. The suffix -ta. As an economic mathematical text, YBC 4698 exhibits some features and terminology similar to those found in economic archives. For instance, the ablative suffix -ta is used to indicate rate per unit. The same layout is described by Snell (1982, 39) for Ur III documents. According to Snell (ibid., 116), the silver implied with this formula is 1 *gin*₂. This rendered the market rate, x unit quantity per 1 *gin*₂ of silver (example: "9 *sila*₃-ta" means "9 *sila*₃ per (*gin*₂ of silver).") This is seen here where the silver given (written 'sum') in problems 3, 11 – 13 and probably 9 is stated as 1 *gin*₂ of silver per unit, as well as problems 5-8, 10, and probably 15-17, where 1 *gin*₂ given per unit is implied. Only problem 14, a combined market rate exercise where the market rate is stipulated, differs from this formula (see §6.14). When identified as a rate per unit, we translate the suffix -ta as "per unit"; this unit is generally 1 *gin*₂, thus, we can often translate as "per *gin*₂."

§2.2. *sa*₁₀. 'sa₁₀' is an interesting verb in these texts, and its exact understanding is essential for the interpretation of the problems. Neugebauer and Sachs first suggested the basic meaning of this verb as "to be equivalent," with the nominalized form "equivalent" (written 'šam₂' in *MCT* 97). This is taken up by Jacobsen who further illustrates its use in Sumerian texts and suggests a factitive meaning "to make equivalent." (Jacobsen 1946: 18). The use and development of *sa*₁₀ is elaborated further in Steinkeller 1989: 153-162. When *sa*₁₀ was used, a procedure was implied, namely, making the equivalence of one item, expressed as a quantity in a standard metrological system, to another item, expressed as a quantity in another standard metrological system. One literally made the equivalent. In the example of YBC 4607 (problems dealing with the volume of bricks) discussed by Neugebauer and Sachs in *MCT* p. 97, "i₃ šam₂ sahar-bi," is translated "oil, the (capacity-) equivalent of its (i.e. the brick's) volume." The reading *sa*₁₀ rather than the conventional šam₂ is consis-

⁶ TMS 13 and 22 are published in Bruins and Rutten 1961: 82-83, 111-114.

⁷ Baqir 1951; see also the bibliography provided by Gonçalves forthcoming: 68.

⁸ Friberg & George 2010: 149-154.

tent with Steinkeller's interpretation (1989: 155). The translation would be then "the equivalent volume," which makes sense in the context of the calculation of brick volumes, a context where oil is somehow incongruous (see Proust 2007: 214, and Friberg 2011: 262). In YBC 4698 problem 6 we see the operation of making equivalent played out with 30 *gur* of grain. The rate of equivalence is specified as (1 *gin*₂ silver) per 1 *gur* grain.

The nuance, "to make equivalent" fits well with many of *sa*₁₀'s appearances in YBC 4698. However, a sale of one good for a price in silver is, indeed, implied in the texts as well. Most striking in this regard is problem 6, just mentioned, which employs this verb together with the verb *bur*₂ (see §2.3.) to mark a purchase and a sale respectively using silver as the medium of exchange. Moreover, problem 6 requests that one compute the gain in silver of the second transaction (represented by *bur*₂). We see then, after the first use of *sa*₁₀, the actor in the problem gains the right to sell the property acquired at a higher price and a payment is made in both transactions with a single medium of exchange: silver.

Merriam-Webster's translates "buy" as: "1: to acquire possession, ownership, or rights to the use or services of by payment especially of money" and "4: to be the purchasing equivalent of." Both uses of the term "to buy" are relevant here for *sa*₁₀. As just seen, the acquisition of ownership, possession, and rights is certainly implied, and silver is used as the purchasing equivalent. This nuance of *sa*₁₀ as "to buy" is supported by the Akkadian understanding of a purchase (Steinkeller 1989: 156-157) which the author of YBC 4698, as an Akkadian speaker, would most certainly have been aware of. We thus retain the use of "to buy" for *sa*₁₀ in our translation, while making full note of the understanding "to make equivalent" for this verb.

§2.3. *bur*₂. In problems #6-11 the verb *bur*₂ is used opposite *sa*₁₀ to denote the (re)sale of a specified product. The verb *bur*₂, equated in the lexical tradition with the Akkadian verb *pašāru* has several different uses assigned to it often based on context (see, for instance, Sb Voc. II 170 presented in *MSL* 3, 142: *bu-ur* : *bur*₂ = *pa-ša₂-ru*). In mathematical texts, the Akkadian verb *pašārum* can take on two different meanings: "to solve" (see Friberg 2007, 296: MS 3049 #1, line 9 where *pa-šar* is tentatively translated "solve") or "to sell."⁹ The meaning to sell is

played out in TMS 13: 4 where we read *kī maši ašām u₃ kī maši ap-šu-ur₂*, "at what price did I buy and at what price did I sell?" (see §6.9. below) *pašāru* is used to mark the (re)sale of the purchase marked by *šāmum*, the Akkadian equivalent of *sa*₁₀ (see §2.2. above). We see a similar use of *pašārum* in TMS 13: 12 and perhaps 15; MLC 1842: 7; VAT 6469 i: 5; VAT 6546 ii: 7; and MS 3895 (Friberg 2010: 149). What we do not see, however, is the use of the Sumerian verb *bur*₂ in any of these examples. Moreover, we are not aware of any instance where *bur*₂ is employed in an economic or administrative text from the Old Babylonian period. Indeed, it seems to be an academic and literary term.

§2.4. *ganba*. This term corresponds to the Akkadian *maḥīrum* for which the *CAD* translates as "3. tariff, price equivalent, rate" (M1, 92, 94-97, especially 96-97 3f). In mathematical texts, the 'ganba' of a given good is either the quantity of a good equivalent to 1 *gin*₂ of silver (rate in-kind), or the equivalent in silver of 1 unit of a good (rate in-silver). The *ganba* is a key concept in economic mathematical texts.¹⁰ Its translation as "market price" in mathematical texts is quite confusing for our purposes as it is actually an equivalency rate. We prefer to translate 'ganba' as "rate (in-kind)" or "rate (in-silver)" according to the context. Note that in #14-15, the value is estimated in-grain instead of in-silver.

We see this use born out in economic texts from the Old Babylonian period. For instance, a rate in-kind appears in HE 111 (*RA* 15, 191: 16-18) (among others) dated to the fifth year of Samsu-iluna of Babylon:

2(aš) sum ^{sar} šum-sikil-	2 <i>gur</i> garlic, š.-plant,
lum ^{sar} za-ḥa-din	z.-plant,
ganba 3(barig)-ta	rate (in-kind) 3 barig per
	(<i>gin</i> ₂ silver),
ku ₃ -bi 3 1/3 ḡ <i>gin</i> ₂ ḡ	its silver value: 3 1/3 <i>gin</i> ₂

Moreover, a rate in-silver is clear in *YOS* 14, 290: 1-3 (NBC 8014) dated to Samsu-iluna year 6, where we

lished *MKT* 1, p. 269) dealing with purchases. Waschow translates *pšr* by "einlösen, für Geld weggeben" [to redeem, to give away for money]. In translating this same text, Friberg equates *bur*₂ with "to sell" (2005: 216).

¹⁰ See *TMB* 221 under "*maḥīru*," *MCT* 106 and note 276e. Particularly interesting is the example cited by Neugebauer and Sachs in note 276e: "In the case of HE 113 (published by Scheil, *RA* 15, 184-185; republished by Boyer *CH* pl. 6 and pp. 33ff.), line 6, the phrase *ganba a-na* 1 *gin*₂ heads a column containing entries which give the quantity of fish of various sorts corresponding to 1 *gin*₂ of silver." We come back in §3 on the use of tabular presentation for rates in-kind and in-silver in mathematical texts.

⁹ See *MKT* 3, 74: *pašārum* = "verkaufen" [to sell]. In *MCT* 107, note 276h states "As pointed out by Waschow [2] p. 246a, the verb *pšr* occurs in two badly preserved mathematical texts (VAT 6469 and VAT 6546, both pub-

read:

$\lceil 1/2 \rceil$ ma-na 6 $1/2$ gin ₂ ku ₃ -babbar sa ₁₀ $3\frac{2}{3}$ gin ₂ ku ₃ -sig ₁₇ ganba-a 10 gin ₂ -ta-am ₃	$1/2$ mana 6 $1/2$ gin ₂ silver, equivalent of $3\frac{2}{3}$ gin ₂ gold, rate in silver 10 gin ₂ per (gin ₂ gold)
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In both examples we note ganba's appearance with the -ta (-am₃) suffix to denote an equivalency rate.

§2.5. ib₂-sa₂. This verb corresponds to the Akkadian *maḥārum*, to make equal (see VAT 7530, obv. 10 and rev. 6). The procedure of making the quantities of various goods of different value (for example fine oil and common oil, iron and gold) equal is the basis of problems 3-5 and 12-14 of YBC 4698, as well as most other mathematical economic problems.

§2.6. e₃ and e₁₁. The expression “še ḥe₂-e₃ u₃ ḥe₂-e₁₁-ma ganba ib₂-sa₂,” “let the grain rise and fall so that the rates are equal,” found in #14, is the exact parallel of the Akkadian expression “ku₃-babbar li-li li-ri-id-ma ganba li-im-ta-ab-ra” found in VAT 7530, 19-20. A variant of the same expression appears as well in #15 (§6 below). This reading, which improves the understanding of the three last problems of the tablet, was suggested to us by Antoine Cavigneaux (personal communication).

§2.7. a-na. The term ‘a-na’ may be understood either as the Akkadian *a-na* (“to” or “for”), or Sumerian ‘a-na’ (corresponding to the Akkadian word *mala* and translated “as much as” or “as”). In the context of series texts, the second meaning is probable. Moreover, in our text, ‘a-na’ seems to connect two equivalent elements. For these reasons, we translate ‘a-na’ from Sumerian, following Thureau-Dangin,¹¹ although Akkadian *ana* cannot be excluded. However, note the use of *ma-la* in #16.

§2.8. Verbal forms. Here, as is often the case in mathematical series texts, verbal roots do not bear any grammatical element. This could lead one to leave the verbs unconjugated in the translation (as did Neugebauer in 1935; see *MKT* ch. 7 and Friberg 2005: 60). However, the alternation of conjugated and unconjugated Sumerian forms in series texts suggests that unconjugated roots may

be abbreviations (see for example the case of AO 9072, analyzed in Proust 2009b: 171-172). Moreover, in the Old-Babylonian mathematical texts, generally written in Akkadian, the statements are almost always expressed in the first person singular simple past. Respecting the uses in Old-Babylonian mathematical tradition, we decided to translate the verbal forms in first person preterit, as did Thureau-Dangin (see for example *TMB* 148ff.), as well as Neugebauer from 1945 (see for example *MCT* 112ff.).

§2.9. Translation of units of measurement. In mathematical texts, the notations of units of measurement are highly standardized, and almost always appear as Sumerian ideograms (a few exceptions are found in texts from Diyala Valley). In modern publications, some of these units are translated (e. g. še is translated in English by grain, gin₂ by shekel, ma-na by *mina*), while others are not (e. g. sila₃, gur). This raises three problems. The first is consistency: if some of the units’ name cannot be translated, it is better to translate none of them. The second is the representation of ancient standardization: when units are translated in different modern languages (e. g. shekel in English, sicle in French), we artificially replace a unified ancient system by our Babel of language. The third and more important is the meaning of the modern translations: shekel is a unit of weight; however, in ancient texts, gin₂ may represent not only a unit of weight (the sixties of one *mana*), but also a unit of capacities (the sixties of one *sila*₃), or surface (the sixties of one *sar*). Thus, following Neugebauer and Sachs in *MCT*, we decided to leave the Sumerian names of the units untranslated.

§3. Methodological remarks

§3.1. Our text contains only statements of problems, without procedure for resolution, and, often, with neither question nor answer. Thus, the goal of the problems is not always explicit. However, the full understanding of the statement requires the reader to solve the supposed problem. As much as possible, we tried to implement calculations leading to the solution. This raises a methodological issue: what is the historical relevance of calculations for which we have little trace in our text? This question refers to a symmetrical one: in the corpus of mathematical cuneiform texts, we find a lot of school exercises with numerical calculations, but without indication of their goal. These exercises probably correspond to statements of problems which are absent from the text. Thus, we have on the one hand, statements without resolution in series texts, and on the other hand, resolutions without statements in some school texts. Of course it would be highly speculative to decide that some of the school exercises correspond to the resolution of some of the state-

¹¹ “Those texts of Yale that make up series employ a term frequently written *a-na* where N. [Neugebauer] sees the Akkadian preposition *ana* “to,” “for,” etc.. This preposition is never supposed “to govern” anything, for which N. does not seem surprised. In reality, *a-na* is here the ideogram *mala*.” (Thureau-Dangin 1936: 57 [translation of the French by the authors]).

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I Rate in-silver	$\text{II} \times 28.48$ Value in-silver	$\text{I} \times \text{III}$ Value in-kind
Good 1	1	1	28.48	28.48
Good 2	2	30	14.24	28.48
Good 3	3	20	9.36	28.48
Good 4	4	15	7.12	28.48
Total		2.5	1	

Table 1: tabular presentation of exchange problems based on data of MS 2830

ments found in series texts. Even so in some cases, given the similarities of numerical data, we may be tempted to do so. The important point for us is that some school exercises present a specific pattern offering a powerful tool for solving mathematical economic problems. This pattern uses a tabular format to represent the algorithm for making equivalences of values, with fixed sequences of operations represented by columns.

§3.2. Interesting examples of such “combined market rate exercises” were analyzed by Friberg (2007: 157-168). Eight of such exercises noted on “square tablets,” kept at Yale University, were published in *MCT* 17, and gathered again by K. Nemet-Nejet (2002: 253-258) with photos.¹² These “combined market rate exercises” were not necessarily produced in the same mathematical context as series texts. However, they offer a relevant framework of calculation for solving most of the problems of YBC 4698. Thus, instead of implementing calculations and reasonings based on modern algebraic or arithmetic methods, we try to solve problems of our tablet by using the tabular presentation, attested in related school texts. The striking parallel between YBC 4698 and “combined market rate exercises” was first underlined by Friberg (2005: 60-61). However, Friberg does not utilize this parallel to implement his own calculation. This entails a different approach for interpretation. For example, the interpretation of problems 3 and 6-10 by Friberg is mathematically equivalent to ours, but the manner in which the calculations are explained is not the same.

§3.3. The examples provided by Friberg are purely nu-

merical texts, without any word indicating the context, except MS 2830, which states the total value in silver by mention of “1 gin_2 ku_3 -babbar” (1 gin_2 of silver) on the bottom edge.¹³ The first section of the reverse of MS 2830 reads as follows:

1 gin_2 ku_3 -babbar			
1	1	28.48	28.48
2	30	14.24	28.48
3	20	9.36	28.48
4	15	7.12	28.48

The relationships between data of the four columns are the following:

Column II contains the reciprocals of column I.

Column III contains the product of the values of column II by a constant coefficient (here 28.48).

Column IV contains the product of values of column I by the values of column III (that is, the coefficients of column II; thus, these products are equal).

The total of values of column III is 1, that is the number in sexagesimal place value notation corresponding, in metrological tables (see outline in §8.3), to the value “1 gin_2 of silver” noted on the bottom edge.

Diverse mathematical situations may be represented by these relationships. The observation of interest for us is that these situations are described in many sections of the tablet YBC 4698.

§3.4. According to various parallels presented by Friberg, including YBC 4698, the meaning of the rows and the columns of the first section of the reverse of MS 2830 may be the following:

Each row corresponds to a given good (named as good 1, good 2, etc. in Table 1).

Column I contains the rate in-kind of the goods, that is, the quantity equivalent to 1 gin_2 of silver.

Column II contains the reciprocal of rates in-kind, that is, the rate in-silver of the goods, or, in other terms, the value in silver of 1 unit (generally, 1 sila_3) of the goods.

Column III contains the value in silver of the actual quantity of the goods (of which the total is usually 1 gin_2).

¹² See also Nemet-Nejet 1995: 256 and Proust 2007: 202-205 for other examples of similar texts.

¹³ Note that the inscription on the bottom edge appears to be above the table, which is on the reverse. Indeed, the tablets are usually turned around the bottom edge. The obverse of the tablet is a list of statements of five problems dealing with sharing an amount of silver between four brothers. Friberg doesn’t establish a relationship between the obverse and the reverse.

Column IV contains the actual quantities of the several goods which have been made equal.

Moreover, an important feature of the table is that the values provided in each cell are expressed in sexagesimal place value notations (henceforth SPVN).

§3.5. It is important for the following calculations to note that the relationships between columns are not always transmissible to sums and differences: they are for multiplication by a coefficient (for example, items of column III are the products of items of column II by a coefficient, and it is the same for the sum), and are not for reciprocals and products of columns (for example, items of column II are the reciprocals of items of column I, but the sum of column II is not the reciprocal of the sum of column I).

§3.6. Many of the problems of our tablet are structured according to a unique mathematical framework, represented by Table 1 or variants. A similar framework can be used as well for completely different mathematical situations, for example sets of rectangles of equal area. In this case, column I contains the widths of the rectangles, column II the reciprocals of the widths, column III the lengths, and column IV the equal areas (see examples in Proust 2007: 202-205).

§3.7. Organization of our calculations. In our commentary of YBC 4698, in particular in the reconstruction of the calculations expected for solving the problems (§6), we attempt, as much as possible, to follow the mathematical framework attested in OB mathematical texts, especially in school texts. This implies the following chart to represent the entire mathematical framework:

1. The metrological data provided by the statements of the problems are transformed into SPVN according to the metrological tables. Thus, the reader is invited to refer to these metrological tables throughout the reading of this article (see §8.3).
2. The calculations in SPVN are represented in tabular form, where each column represents an operation (for example, column II represents reciprocals of data of column I, or column III represents the multiplication by a coefficient of data of column II, and so on).
3. The results of the calculations are transformed into measures according to the metrological tables, with a mental control of the orders of magnitude.

It is important to keep in mind that this framework includes both the use of metrological tables and tabular presentation of numbers in SPVN. When relevant from a mathematical point of view, we use the three-step chart above in our reconstructions of the possible calculations

performed by ancient scribes (see conventions below in §3.8). We prefer to use charts attested in Old Babylonian sources instead of modern formalism because we are convinced that the organization of calculations made by modern historians has a substantial impact on the understanding of ancient texts. This impact is as important (and even more so) as the choice of notations in the transliterations, or the terminology used to translate the ancient terms.

§3.8. Conventions. To be clear in the tabular presentation, to distinguish the data provided explicitly or implicitly by the original text from the results of our calculations, we note in **bold** the former and in plain the latter. Correspondences between metrological data and abstract numbers as provided by metrological tables are represented by an arrow (\rightarrow). For example: 1 gin₂ \rightarrow 1, or 1 še \rightarrow 20. An outline of the metrological tables can be found in §8.3, and the complete set of metrological tables attested in Nippur is provided in Proust 2009.

§4. Transliteration and translation

obverse i

1	1	še a-na 1(aš) še gur	(When the principal in) grain (is) as much as 1 gur of grain,
	2	1(barig) še a-na maš ₂ šum ₂	1 barig of grain as the interest I give.
	3	še u ₃ maš ₂ en-nam	The grain and the interest are how much?
2	4	še a-na 1(aš) še gur	(When the principal in) grain (is) as much as 1 gur of grain,
	5	1(barig) 4(ban ₂) a-na maš ₂ šum ₂	1 barig 4 ban ₂ as the interest I give.
	6	še u ₃ maš ₂ en-nam	The grain and the interest are how much?
3	7	3 sila ₃ -ta i ₃ sag	(Rates in-kind are) 3 sila ₃ of first quality oil per (gin ₂)
	8	1(ban ₂) 2 sila ₃ -ta i ₃ -geš	(and) 1 ban ₂ 2 sila ₃ of common oil per (gin ₂).
	9	1 gin ₂ ku ₃ -babbar šum ₂	1 gin ₂ of silver I gave.
	10	i ₃ -geš u ₃ i ₃ sag	Common oil and first quality oil
	11	ib ₂ -sa ₂ -ma sa ₁₀	I made equal and I bought.
4	12	1(geš ₂) 3(u) DUG an-bar	60+30 DUG of iron
	13	9 ² DUG ku ₃ -sig ₁₇	9 (×60 ²) DUG of gold,
	14	1 ma-na ku ₃ -babbar šum ₂	1 mana silver I gave.
	15	an-bar u ₃ ku ₃ -sig ₁₇	The iron and gold

	16	1 gin ₂ -ma sa ₁₀	(is) 1 gin ₂ and I bought.
5	17	1/2 ma-na ku ₃ -babbar šum ₂	1/2 mana of silver I gave.
	18	1(aš) gur-ta sa ₁₀	1 gur per (gin ₂) I bought.
<i>obverse ii</i>			
6	1	3(u) še gur	30 gur grain
	2	1(aš) še gur-ta sa ₁₀ -ma	1 gur of grain per (gin ₂) I bought,
	3	4(barig) še-ta bur ₂ -ra	4 barig of grain per (gin ₂) I sold.
	4	ku ₃ diri en-nam	The silver profit is how much?
	5	7 1/2 gin ₂ ku ₃ -babbar diri	7 1/2 gin ₂ of silver is the profit.
7	6	sag ku ₃ -bi en-nam	The initial price is how much?
	7	1(aš) gur i ₃ -geš	1 gur of common oil.
	8	1(ban ₂)-ta sa ₁₀ -ma	1 ban ₂ per (gin ₂) I bought and,
	9	8 sila ₃ -ta bur ₂ -ra	8 sila ₃ per (gin ₂) I sold.
	10	ku ₃ diri en-nam	The silver profit is how much?
	11	7 1/2 gin ₂ ku ₃ diri	7 1/2 gin ₂ is the silver profit.
9	12	1(aš) gur i ₃ -geš	1 gur of common oil.
	13	i-na sa ₁₀ 1 ¹ gin ₂	When the purchase is 1 ¹ (text: 2) gin ₂ ,
	14	2 sila ₃ šuš ₄	2 sila ₃ I cut.
	15	7 1/2 gin ₂ ku ₃ diri	7 1/2 gin ₂ is the profit.
	16	en-nam sa ₁₀ -ma	How much did I buy?
	17	en-nam bur ₂ -ra	How much did I sell?
	18	1(ban ₂) sa ₁₀ -ma	1 ban ₂ I bought and
	19	8 sila ₃ bur ₂ -ra	8 sila ₃ I sold.
<i>reverse i</i>			
10	1	1(aš) gur i ₃ -geš	1 gur of common oil.
	2	9 sila ₃ -ta sa ₁₀ -ma	9 sila ₃ per (gin ₂) I bought,
	3	7 1/2 sila ₃ bur ₂ -ra	7 1/2 sila ₃ per (gin ₂) I sold.
	4	ku ₃ diri en-nam	The profit is how much?
	5	6 2/3 gin ₂ ku ₃ diri	6 2/3 gin ₂ is the silver profit.
11	6	1(aš) gur i ₃ -geš	1 gur of common oil.
	7	i-na sa ₁₀ 1 gin ₂ 1 1/2 sila ₃ šuš ₄	When the purchase is 1 gin ₂ , 1 1/2 sila ₃ I cut.
	8	6 2/3 gin ₂ ku ₃ diri	6 2/3 gin ₂ is the silver profit.
	9	en-nam sa ₁₀ -ma	How much did I buy?
	10	en-nam bur ₂ -ra	How much did I sell?
	11	9 sila ₃ sa ₁₀ -ma 7 1/2 sila ₃ bur ₂ -ra	9 sila ₃ I bought, and 7 1/2 sila ₃ is sold.
12	11	7 ma-na u ₃ 11 ma-na siki geš ²	(Per) 7 mana and 11 mana of... wool
	12	1 gin ₂ ku ₃ šum ₂ ib ₂ -	1 gin ₂ silver I paid. I made

		sa ₂ -ma sa ₁₀ -ma	equal and I bought.
	13	4.16.40 sa ₁₀	4.16.40 I bought.
13	14	7 sila ₃ -ta i ₃ -geš	Per 7 sila ₃ of common oil
	15	1(ban ₂) 2(diš) sila ₃ -ta i ₃ -šah ₂	per 1 ban ₂ 2 sila ₃ of lard,
	16	1(diš) gin ₂ šum ₂ ib ₂ - sa ₂ -ma sa ₁₀ 2(ban ₂) 3 1/3 sila ₃	1 gin ₂ I gave. I made equal and I bought: 2 ban ₂ 3 1/3 sila ₃ .
14	17	ganba a-na 1 2 3 4 5	(When) rates (in-kind are) as much as 1 2 3 4 5
	18	6 7 8 9 ganba	6 7 8 9, the rate (in grain)
	19	1(barig) še šum ₂ še he ₂ -e ₃	1 barig I give. Let the grain rise
	20	u ₃ he ₂ -e ₁₁ -ma	or fall so that the
	21	ganba ib ₂ -sa ₂	rates (in-kind) are equal.
<i>reverse ii</i>			
15	1	ganba 3 ku ₆ -a 5 sila ₃	Rate (in-kind) for 3 fish, 5 sila ₃
	2	5 ku ₆ -a ku ₆ [...]	for 5 fish, fish ...
	3	še gar-gar-ma 4(aš) 1(barig) 4(ban ₂) 5(2 ²) <sila ₃ gur>	The grain I added: 4 (gur) 1 barig 4 ban ₂ 5(2 ²) (sila ₃)
	4	ku ₆ -a gar-gar-ma ku ₆	The fish I added. Let the fish
	5	še he ₂ -e ₃ u ₃ he ₂ -e ₁₁ -ma	and the grain rise or fall
	6	ku ₆ -a ib ₂ -sa ₂	so that the fish are equal.
16	7	ma-la ganba hi-a nagga	As much as the rates in-kind of the lead
	8	ku ₃ i ₃ ² -la ₂ -ma	silver I weighed and
	9	nagga sa ₁₀	lead I bought.
	10	ku ₃ u ₃ nagga gar- gar-ma	The silver and the lead I added:
	11	7 ma-na ku ₃ u ₃ nagga	7 mana. The silver and the lead
	12	en-nam	are how much?
17	13	na ₄ šu-ti-a-ma	Stones I received and
	14	ki-la ₂ -bi nu-zu	their weight I don't know.
	15	6-ta sa ₁₀	6 (stones) per (gin ₂) I bought.
	16	6-ta ku ₃ -sig ₁₇ gar-ra	6 gold inlaid (stones) per (gin ₂) I bought).
	17	ku ₃ -sig ₁₇ gar-ra	Gold inlaid (stones),
	18	a-na 6-ta sa ₁₀	as much as 6 (stones) per (gin ₂) I bought.
	19	na ₄ ku ₃ -babbar en-nam	The stone, the silver how much
	20	gar-gar-ma 1/2 ma- na 1 1/2 gin ₂	did I add? 1/2 mana 1 1/2 gin ₂
	21	x ma na ² gar-gar-ma	... I added and
	22	1 ma-na 3 gin ₂	1 mana 3 gin ₂ .
<i>bottom edge</i>			
	1	(erasure) dub 3-kam-ma	Tablet number 3.

§5. Philological notes

- #3 In obv. i 3 and *passim*, we read i_3 , following Thureau-Dangin (1937b: 89) and Friberg (2005: 61), but Neugebauer reads NA_4 under Goetze's suggestion (see Archives Aaboe-Britton in §9 below). In obv. i 11 and *passim*, we read sa_{10} following Thureau-Dangin (1937b: 89) and Friberg (2005: 61) who chooses the nominalized form šam_2 , but Neugebauer reads $AN\text{ŠE}$.
- #4 In obv. i 12-13, we read "DUG," following Neugebauer (*MKT* 3, 42), which is confirmed by our collations as well as a reading performed by Cavigneaux. However, Friberg (2005: 61) reads "-bi," thus, his interpretation is completely different (*ibid.*, 67-68 and §6.4 below). We may understand this situation as one dealing with objects, the nature of which we cannot currently identify, made of iron and gold. Since iron was a very rare and precious metal in the Old Babylonian period, it is surprising that the proportion of iron and gold is 90 to 9. However, it is possible that the proportion is 90 to 9×60 (or 1 to 6). Thus 9 may represent 9 sixties instead of 9 units (according to another suggestion of Cavigneaux). In this case, we should restore "9 <šū-ši>." Another possibility would be that "DUG" refers to a container, perhaps alluding to leftovers from the metallurgical process. For a textual example of this latter use of a metal, see *YOS* 2, 112 11, an-na *hi-im-mi* u_3 *ša-ak-ti-šu*, which we translate "tin sweepings and its powder."
- #5 In obv. i 18, we read "1(aš) gur-ta," following Neugebauer (*MKT* 3, 42), but Friberg (2005: 61) reads "1/2 gin₂-ma."
- #6 In obv. ii 3 and *passim*, we read "bur₂" following Friberg (2005: 61) and Cavigneaux (*pers. com.*), but Neugebauer (*MKT* 3, 42) reads "bala."
- #9 In obv. ii 14 and *passim*, we read the sign "ŠUM" following Neugebauer (*MKT* 3, 42), which we transliterate as "šuš₄," to cut, because of the parallel with *kašāṭum*, to cut, found in an analogous text, TMS 13. However Friberg (2005: 61) reads "si₃."
- #12 In rev. i 11, we read "siki geš?" (wool of a certain kind) where Neugebauer reads (*MKT* 3, 42) "RU ...," and Friberg (2005: 61) "*ru-qa*."
- #13 In rev. i 14, Friberg and Neugebauer read 6 sila₃. However, we distinguished clearly the number "7"

on the tablet. In line 15, the last sign is probably sag, as in other similar contexts, but this sign is not clear; Friberg reads "šah."

In rev. i 16, the signs 2 ban₂ 3 1/3 sila₃ are visible on the right edge of the tablet, but they do not appear in Neugebauer's copy, nor in Friberg's transliteration (Friberg 2005: 61). According to the Archive Aaboe-Britton now kept at ISAW (see §9), Neugebauer used the photos of the obverse and reverse of the tablet, but did not have the photos of the edges, while Friberg's analysis of YBC 4698 was based solely on Neugebauer's hand copy.

- #14 The reading of this statement by Friberg is quite far from ours (and from Neugebauer's): in rev. i 18, Friberg reads "šu-ši" instead of "ganba"; line 19 he reads "1 še si₃ še šam₂-ma" instead of Neugebauer's "1 gun₂ sum še ib₂-sa₂ du₆." Following Neugebauer, however, he reads "GIŠ" instead of "DU" in line 20.

In the beginning of rev. i 19, the reading 1(barig) še is probable, but not certain. Neugebauer (*MKT* 3, 42) transliterates "3(gur)(?)," which seems quite improbable, and Friberg (2005: 61) "1 (gur) še," which is possible, but not consistent with the other data of the statement.

- #15 In "he₂-e₃ u_3 he₂-e₁₁" (rev. ii 5), the reading of e₁₁ (DU₆.DU) is uncertain. There is the possibility of reading this e₃ (UD.DU), though taken with problem 14 ll. 19-20 it is clear that e₁₁ was intended by the scribe. Note that the -ma placed in line 6 belongs in fact to line 5 that lacked space for it.

In rev. ii 4, we believe that the last sign is clearly 'ku₆,' while Friberg (*pers. comm.*) suggests another possibility: "this should be a number, possibly 2 ner = 20.00." We exclude this possibility because the graphy of the sign here is the same as in ll. 1 and 2 of this section, and the number "2(geš'u)" should have a slightly different graphic form (wedges and Winkelhaken more neatly separated—see for example HS 1703 rev. v). Moreover, we do not expect a number noted in system S here.

- #17 Rev. ii 16-17, we translate 'ku₃-sig₁₇ gar-ra' as 'gold inlaid,' following Cavigneaux's suggestion.

§6. Comments

- #1 The statement gives the interest to be paid for the loan of a given amount of grain that is, 1 *barig* per *gur*. The question asks principal and interest, or, perhaps, the total principal + interest. It seems that

some information concerning the principal is missing, so we cannot solve the problem. Perhaps the missing data were given in previous tablets of the series. Or, as another hypothesis, suggested by Neugebauer (*MKT* 3 43), the question concerns the interest **rate**, that is, the interest by unit of grain (1 *sil*₃). In this case, the calculation would be quite simple.

1 gur → 5
1 (barig) → 1

The interest rate is thus $1/5$, that is, 12, which corresponds to 12 *gin*₂ per *sil*₃.

Note that the interest rate of $1/5$ (or 20%) is the standard value for annual silver loans. The customary value for grain is $1/3$ per year though texts do vary. See, for instance, YOS 14, 178: 2, a loan where the interest rate is stipulated at 1 *barig* per *gur*, the rate presented here.

Interestingly, Friberg (personal communication) suggests an even simpler solution, that is, 1 gur is the actual principal, 1 barig is the actual interest, and the question is “what is the principal plus (*u*₃) the interest.” The answer should be 1 *gur* 1 *barig*.

- #2 This statement is identical to the previous one, with an interest of 1 *barig* 4 *ban*₂ per *gur*. The calculation of the interest rate would be:

1 gur → 5
1 (barig) 4 (*ban*₂) → 1.40

The interest rate is thus $1.40/5$, that is, $1.40 \times 12 = 20$, which corresponds to 20 *gin*₂ per *sil*₃. Note that this corresponds to a rate of $1/3$, which is the customary rate for a grain loan as stated above.

- #3 The rate in-kind of first quality oil is given as 3 *sil*₃ (this means that the value of 3 *sil*₃ of first quality oil is 1 *gin*₂ of silver), and the rate in-kind of common oil is given as 1 *ban*₂ 2 *sil*₃ (the value of 1 *ban*₂ 2 *sil*₃ of common oil is 1 *gin*₂ of silver). The same quantity of both oils is bought. However, this common quantity is not specified, and there is no question, nor answer, so that we may consider the following hypothesis:

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I Rate in-silver	II × 2.24 Value in-silver	I × III Value in-kind
First quality oil	3	20	48	2.24
Common oil	12	5	12	2.24
Total		25	1	

Table 2: calculations for solving #3

The total value in silver of the oils bought is 1 *gin*₂ (line 9). The problem can be solved by using the chart as explained in §3: the metrological data are converted into SPVN using metrological tables, the numbers in SPVN are displayed in tabular format and finally, the result is converted into metrological notation by means of a metrological table. Note that in Table 2 and thereafter, bold numbers correspond to data provided by the text, either as SPVN, or as metrological notation, while the use of metrological tables is represented by an arrow (→). Here, metrological tables provide:

3 *sil*₃ → 3
1 (*ban*₂) 2 *sil*₃ → 12
1 *gin*₂ → 1

The reader has to calculate the value of each kind of oil (column III of Table 2), and/or the common quantity (column IV). To solve the problem, we first note that the rate in-silver (column II) is the reciprocal of the rate in-kind (column I) and fill in accordingly. The total of items of column II is 25, and the total of the items of column III is 1. Thus, the coefficient of column III is 2.24 (reciprocal of 25) and column III is easy to fill. We can then fill in column IV which is the product of data of columns I and III, that is, the coefficient 2.24. According to metrological tables and a mental control of the orders of magnitude, the results correspond to the following weights and capacity:

48 → $2/3$ *gin*₂ 24 *še*
12 → $1/6$ *gin*₂ 6 *še*
2.24 → 2 $1/3$ *sil*₃ 4 *gin*₂

For 2 $1/3$ *sil*₃ 4 *gin*₂ of premium oil, $2/3$ *gin*₂ 24 *še* silver is given and for the same amount of common oil, $1/6$ *gin*₂ 6 *še* silver is given. This seems to be the solution espoused by Friberg (2005: 60-61).

- #4 Statement #4 seems to be similar to #3, except for two key points: the suffix -ta is not used nor does it state that the two goods “I made equal and I bought” but instructs that the two goods (here iron and gold) “is 1 *gin*₂ and I bought,” in the same position. Like in #3, the transaction is expressed by the verbs ‘sum’ (to

give) and 'sa₁₀' (to buy). However, this transaction is not completely clear to us. Only an attempt of interpretation is offered here.

Lines 12-13 seem to indicate that we deal with two lots of objects (DUG): 90 (60+30) iron objects and 9 gold objects. Line 14 states "1 *mana* silver I gave," thus we understand that 1 *mana* is the total value in-silver of the two lots, like in #3. Lines 15-16 refer to '1 *gin*₂' in relation to iron and gold. The nature of this relation is not clear. 1 *gin*₂ can hardly be the total weight of the two lots, or its total value in-silver (a weight of about 8 g for 99 objects is hard to imagine; and a value in-silver of 8 g for these 99 precious objects is even more unrealistic). We can guess that 1 *gin*₂ represents the weight of each object. Even so, this is not made explicit in the text. If this suggestion is correct, the total weight of iron bought is 1 ½ *mana* (90 times 1 *gin*₂) and the total weight of gold bought is 9 *gin* (9 times 1 *gin*₂).

The goal of the problem may be the value in silver of the 90 iron objects and of the 9 gold objects. Thus we need to know the rate in-silver of both metals. It then seems that data needed to evaluate rates is missing. This data could have been well known by the reader of the text, or it may have been provided in previous tablets of the series. Note that, if we suppose that the rate in-silver of iron is 8, a widely attested value (see below), then the rate in-silver of gold is 5.20 (because we know that the total value of iron plus gold is 1 *mana*). 5.20 is a realistic value for the rate in-silver of gold. Moreover, the rates 8 and 5.20 are regular numbers, as well as their sum (13.20), and the sum of their reciprocals 7.30 and 11.15 (18.45).

The understanding of the problem by Friberg (2005, 67), is different. For him, "iron and gold are 90 and 9 times more valuable than silver, and if 1 shekel of iron and gold together is bought for 1 mina of silver, what are then the amount of iron and gold, respectively?" In modern language, this should lead to a system of two linear equations. The solution should

be 37.46.40 and 22.13.20 (;37 46 40 and ;22 13 20 with Friberg's notations). We do not believe that the first two lines of #4 state that "iron and gold are 90 and 9 times more valuable than silver," because the syntax of the cuneiform text should be different. We should have: "1(geš₂) 3(u) gin₂-ta an-bar / 9(diš) gin₂-ta ku₃-sig₁₇." Moreover, the values would be completely unrealistic. According to Cécile Michel (pers. comm.), "In the Old Babylonian period, the ratio iron : silver is 1:8 in Southern Mesopotamia; 1:12 in Mari, and 1:40 in Aššur. The ration gold : silver is between 1:3 and 1:6 in Southern Mesopotamia, between 1:4 and 1:6 in Mari, and between 1:4 and 1:8 in Aššur."

- #5 Problem #5 seems to be a variant of #4. According to a system of notation widely used in series texts, only the modified information is noted in the variant, the unchanged information being omitted. The complete statement of #5 would be the following (the information given in #4 but omitted in #5 is noted between triangular brackets):

<1(geš₂) 30 DUG an-bar>
 <9(geš₂?) DUG ku₃-sig₁₇>
 1/2 ma-na ku₃-babbar sum
 <an-bar u₃ ku₂-sig₁₇>
 1(aš) gur-ta sa₁₀

The variant should be:

The total value in-silver of the two lots is 1/2 *mana* instead of 1 *mana*

The weight in *gin*₂ is replaced by an equivalent value in-grain (which means that the weight of iron objects is not the same as the weight of gold objects). The interpretation raises the same difficulties already witnessed in #4.

- #6 Neugebauer interpreted the statement as asking for a profit made in purchasing 30 *gur* of grain, knowing expected losses (5 *barig*), and actual losses (4 *barig*) (MKT 3, 44).

The situation is much simpler if we consider that the

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I Rate in-silver	II × 2.30 Price in-silver	I × III Value in-kind
Purchase	5	12	30	2.30
Sale	4	15	37.30	2.30
difference		3	7.30	

Table 3: calculations for solving #6

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I Rate in-silver	II × 5 Price in-silver	I × III Value in-kind
Purchase	10	6	30	5
Sale	8	7.30	37.30	5
difference		1.30	7.30	

Table 4: calculations for solving #8

transaction consists in buying and selling 30 *gur* of grain, and that the question asks for the profit in silver. This is also the understanding of Friberg (2005: 217), against Neugebauer. The text states that if 1 *gur* of grain is bought for a given price (sa_{10}), 4 *barig* of this grain is sold (bur_2) for the same price. Data consistency requires that the purchase price of 1 *gur* of grain is 1 *gin*₂ of silver, but this value is not provided in the statement. Indeed, since the market prices are usually given as capacity by *gin*₂ (rate in-kind) it is not necessary to precise “per 1 *gin*₂,” which is implicit. This interpretation assumes that bur_2 here has the sense ‘to sell,’ that is, the contrary of to buy (reverse operation, see §2.3).

The calculations may run as follows. First, the metrological data are transformed into SPVN using the metrological table of capacities:

30 gur	→ 2.30
1 gur	→ 5
4(<i>barig</i>)	→ 4

Then, the calculations are performed with the aid of tabular format indicated in table 3.

Finally, the profit is 7.30, the difference between the sale and purchase prices (column III). The metrological table of weights provides the metrological expression of the profit:

$$7.30 \rightarrow 7 \frac{1}{2} \text{gin}_2$$

The profit is 7 $\frac{1}{2}$ *gin*₂, as stated in the answer (obv. col. ii, line 5).

Note that the profit (7.30) may be calculated in two ways:

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I Rate in-silver	II × 5 Price in-silver	I × III Value in-kind
Purchase	10 (a)	6 (a')		5
Sale	8 (b)	7.30 (b')		5
difference	2	1.30	7.30	

Table 5: calculations for solving #9

a) Multiplying the difference of col. II (3) by the coefficient 2.30

b) Calculating the difference of purchase and selling prices in col. III.

#7 This section seems to be an additional question related to the statement of #6. If we consider that the situation is the same as in #6, the additional question probably asks the purchase price of the 30 *gur* of grain. This is an indication that, in #6, the profit is calculated by method a). The purchase price of the 30 *gur* is obtained by multiplying 12 by 2.30 (see Table 3). The result is 30, which corresponds to a purchase price of 30 *gin*₂.

#8 This problem is similar to #6, but the good purchased is 1 *gur* of oil instead of 30 *gur* of grain. As in the previous section, the metrological data are transformed into SPVN using the metrological table of capacities:

1 gur	→ 5
1(<i>ban</i> ₂)	→ 10
8 <i>sil</i> ₃	→ 8

Then, the calculations are performed with the aid of tabular format indicated in table 4.

Finally, the profit is 7.30, the difference of the sale and purchase prices (column III). The metrological table of weights provides the metrological expression of the profit:

$$7.30 \rightarrow 7 \frac{1}{2} \text{gin}_2$$

The profit is 7 $\frac{1}{2}$ *gin*₂, as stated in the answer (obv. ii 11).

#9 A new operation appears in line 14: 2 *sil*₃ are “cut” (šuš₄) from the good sold. This translation of ‘šuš₄’ is suggested by TMS 13, which contains a similar problem with a complete resolution. The statement TMS 13 (lines 1-4) runs as follows, according to our collation of the tablet, which corresponds to Friberg’s transliteration (2010: 154), except in line 2, where Friberg left out the sequence ‘i₃-geš’.

2(gur) 2(barig)	2 gur 2 barig 5 ban ₂ of
5(ban ₂) i ₃ -geš sa ₁₀	common oil I bought. When the
i-na sa ₁₀ 1 gin ₂ ku ₃ -	purchase (costs) 1 gin ₂ silver
babbar	(of oil),
4 sila ₃ -ta-am ₃ i ₃ -geš	4 sila ₃ of common oil I cut and
ak-ši ₂ -iṭ-ma	
2/3 ma-na 20 še ku ₃ -	2/3 mana 20 še silver of profit
babbar ne-me-la	I see.
a-mu-ur ₂	
ki ma-ši a-ša-am u ₃	How much did I buy? How
ki ma-ši ap-šu-ur ₂	much did I sell?

Parallel terms in TMS 13 and YBC 4698, resp.:

kašāṭum	šuš ₄	to cut
šāmum, sa ₁₀	sa ₁₀	to buy
pašārum	bur ₂	to sell

‘šuš₄’ is parallel to the Akkadian ‘kašāṭum’ used in TMS 13. The 2 *sil*₃ “cut” represents the difference between the purchase and the selling rates in-kind.

Thus, we can understand the statement of YBC 4698 #9 as a reverse problem of the previous one: the difference between the purchase and the selling rates in-kind being 2 *sil*₃, and the profit for 1 *gur* being 7 1/2 *gin*₂, find the purchase and the selling rates in-kind. However, this interpretation assumes that the purchase rate is given per 1 *gin*₂, not per 2 *gin*₂ as noted in the tablet (obv. col. i, line 13)

1 gur	→ 5
2 sila ₃	→ 2
7 1/2 gin ₂	→ 7.30

Since the difference of price in-silver is 7.30 (col.

III of table 5), the difference of rates in silver is 7.30/5, that is, 1.30 (col. II). The problem consists, then, of finding two numbers, *a* and *b*, from their difference (2) and the difference of their reciprocals *a*' and *b*' (1.30).

In modern language, this problem may be represented by the system of equations:

$$\begin{array}{rcl} a - b & & = 2 \\ b' - a' & & = 1.30 \end{array}$$

This leads to the problem of finding two numbers knowing their difference (2) and their product (2/1.30, that is, 1.20) (for the method of resolution of this quadratic problem, see the explanation of TMS 13 in Høyrup 2002: 206-209). The solution is *a* = 10 and *b* = 8.

$$\begin{array}{rcl} 10 & & \rightarrow 1(\text{ban}_2) \\ 8 & & \rightarrow 8 \text{ sila}_3 \end{array}$$

The purchase and the selling rates in-kind are thus 1 *ban*₂ and 8 *sil*₃. This solution corresponds to the answer given in the text (obv. col. ii, lines 18-19).

#10 This problem is similar to #8, with other purchase and sale rates. The purchase and the selling rates in-kind are 9 *sil*₃ and 7 1/2 *sil*₃. The metrological data are (in SPVN):

$$\begin{array}{rcl} 1 \text{ gur} & & \rightarrow 5 \\ 9 \text{ sila}_3 & & \rightarrow 9 \\ 7 \frac{1}{2} \text{ sila}_3 & & \rightarrow 7.30 \end{array}$$

See table 6 for calculations. The result 6.40 provided by col. III, corresponds to 6 2/3 *gin*₂:

$$6.40 \rightarrow 6 \frac{2}{3} \text{ gin}_2$$

The profit is 6 2/3 *gin*₂, as stated in the answer (rev. i 5).

#11 As in the case of #9, the problem #11 presents the reverse of the previous one.

$$\begin{array}{rcl} 1 \text{ gur} & & \rightarrow 5 \\ 1 \frac{1}{2} \text{ sila}_3 & & \rightarrow 1.30 \end{array}$$

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I Rate in-silver	II × 5 Price in-silver	I × III Value in-kind
Purchase	9	6.40	33.20	5
Sale	7.30	8	40	5
difference		1.20	6.40	

Table 6: calculations for solving #10

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I Rate in-silver	II × 5 Price in-silver	I × III Value in-kind
Purchase	9 (a)	6.40 (a')		5
Sale	7.30 (b)	8 (b')		5
difference	1.30	1.20	6.40	

Table 7: calculations for solving #11

$$6\frac{2}{3} \text{ sila}_3 \rightarrow 6.40$$

Since the difference of prices in-silver is 6.40 (col. III of table 7), the difference of rates in-silver is $6.40/5$, that is, 1.20 (col. II). The problem consists then, of finding two numbers, a and b , from their difference (1.30) and the differences of their reciprocals a' and b' (1.20).

In modern language, this problem may be represented by the system of equations:

$$\begin{aligned} a - b &= 1.30 \\ b' - a' &= 1.20 \end{aligned}$$

The solution of this quadratic problem is $a = 9$ and $b = 7.30$.

$$\begin{aligned} 9 &\rightarrow 9 \text{ sila}_3 \\ 7.30 &\rightarrow 7\frac{1}{2} \text{ sila}_3 \end{aligned}$$

The purchase and the selling rates in-kind are thus 9 sila_3 and $7\frac{1}{2} \text{ sila}_3$. This solution corresponds to the answer given in the text (rev. i 10).

#12 Problem 12 provides the rate in-kind of two types of wool, 7 *mana* and 11 *mana* respectively, as well as their total value in silver, 1 *gin*₂. There is no question, but it seems that the problem is to find the equal quantities of wool 1 and wool 2, and that the answer is given in the last line: this quantity corresponds to 4.16.40.

In this problem, the rates in-kind correspond to non-regular numbers (7 and 11), which means that the reciprocal cannot be found. One finds the same situation in VAT 7530, as well as in several

“combined market rate exercises” (see Friberg 2007: 162, 165). These texts suggest that, as the reciprocal of the rates in-silver cannot be found, column II does not provide the rates in-silver, but instead the product of the rates in-silver by 7×11 .

$$\begin{aligned} 7 \text{ ma-na} &\rightarrow 7 \\ 11 \text{ ma-na} &\rightarrow 11 \\ 1 \text{ gin}_2 &\rightarrow 1 \end{aligned}$$

Column II of table 8 contains the product of the reciprocal of 7 by 7×11 , which is 11, and the product of the reciprocal of 11 by 7×11 , which is 7. The total of column II is 18. The number 18 is a false value which must be compared to the actual total value in silver of 1 *gin*₂ (total column III) in order to produce a coefficient of 3.20 (the reciprocal of 18). This coefficient is necessary to multiply the rates in column II in order to produce column III. Thus we see:

$$\begin{aligned} 11 \times 3.20 &= 36.40 \\ 7 \times 3.20 &= 23.20 \end{aligned}$$

The next step is to multiply the rates in column I by the values in silver in column III to produce the values in kind of column IV, 4.16.40 in both cases. Thus we have filled in the table and obtained the required equal quantities of wool 1 and wool 2:

$$4.16.40 \rightarrow 4 \text{ ma-na } 16\frac{2}{3} \text{ gin}_2$$

The number 4.16.40 corresponds to the answer given in line 13. Curiously, the answer is expressed in place value notation and not, as usual, in metrological notation. This suggests that the author of the text actually solved the problem.

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I × 7 × 11 Rate in-silver × 7 × 11	II × 3.20 Value in-silver	I × III Value in-kind (equal)
Wool 1	7	11	36.40	4.16.40
Wool 2	11	7	23.20	4.16.40
Total		18	1	

Table 8: sketch of calculations for solving #12

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I \times 7 Rate in-silver \times 7	II \times 1.40 Value in-silver	I \times III Value in-kind (equal)
Common oil	7	1	1.40	11.40
Lard	12	35	58.20	11.40
Total		36	1	23.20

Table 9: calculations for solving #13

#13 The rates in-kind of two types of oil are given (7 *mana* per *gin*₂ and 1 *ban*₂ 2 *sil*₃ per *gin*₂) that correspond to the following SVPN:

7 ma-na	→ 7
1(ban ₂) 2 sila ₃	→ 12
1 gin ₂	→ 1

The same quantity of each type of oil is bought for the price of 1 *gin*₂ of silver (total of column III of table 9). There is no question, but we may suppose that the reader was expected to calculate the price of each kind of oil (column III), and/or the common quantity (column IV).

The rate in-kind of common oil is 7, a non-regular number, thus column II is replaced with the products of the rates in-silver by 7.

The total of items of column II is 36, and the total of the items of col. III is 1. Thus, the coefficient of col. III is 1.40 (reciprocal of 36). The rest of the table is then easy to fill. The value in-kind of common oil is 11.40, of lard is 11.40, thus the total is 23.20, which corresponds to 2 *ban*₂ 3 1/3 *sil*₃, the answer provided on the edge of the tablet (which does not appear

in Neugebauer's hand copy – see note on #13 in §5).

#14 Statement #14 displays similarities to N 3914 discussed in (Friberg 2007, 163-165) and to VAT 7530 #5 (obv. 17-21) discussed in (Friberg 2007, 166). The purpose of this problem is to find each quantity that corresponds to each given rate in-kind, from 1 to 9 in this problem, that will add up to a total amount paid, probably stipulated as “1(barig) še” in this problem. One of the rates in problem 14 is the number 7, a non-regular number in the sexagesimal system as in #12 and 13. Unlike the problems discussed in Friberg, problem 14 added an extra step: the scribe had to round in order to find the answer. Also, unlike both N 3914 and VAT 7530 § 5, problem 14 uses grain to evaluate the unnamed goods instead of silver, a significant deviation from the prior problems in this series.

There are nine sorts of goods, for which the rates in-kind are respectively 1 (unit), 2 (units), ..., 9 (units) (quantities which equivalent value in-grain is 1 *sil*₃).

I	II	III	IV
Rate in-kind	(reciprocal of I \times 7) False value in-grain	(II \times 3) Value in-grain	(I \times III) Value in-kind (equal)
1	7	21	21
2	3.30	10.30	21
3	2.20	7	21
4	1.45	5.15	21
5	1.24	4.12	21
6	1.10	3.30	21
7	1	3	21
8	52.30	2.37 30	21
9	46.40	2.20	21
Total	19.48.10	59.24.30	3.9
Rounded	20	1	
Difference	11.50	35.30	
Error	1%		

Table 10: calculations for solving #14

Col.	I	II	III	IV
	Rate in-kind	Rate in-grain	Value in-grain	Value in-kind
Fish 1	36 (3/5)	1.40 (5/3)		
Fish 2	? (5/x)	? (x/5)		
Total			21.45	

Table 11: calculations for solving #15

The value in-kind of the goods (column IV of table 10) must be a multiple of 7, the unique non-regular rate in-kind of the problem. Thus column II is replaced with the products of the rates in-grain by 7. Following the idea of Friberg (cf. N 3914 in Friberg 2007: 163-165), we designate these products as “false value in-grain.” The coefficient providing the actual value is calculated using the information given in line 19 of the text: the total value in grain of the nine goods bought is 1 *barig* (corresponding to 1). The total false value in-grain is 19.48.10 (approximately 20). The total value of the purchase to be found must be 1 *barig* (1). The coefficient is approximately the reciprocal of 20, that is, 3. Multiplying the values of column II by 3 gives the actual values in-grain of the nine goods (column III). The value in-kind is the false quantity, 7, multiplied by the coefficient, 3, that is, 21. It is also the product of column I, the rates in-kind, by column III, the values in-grain.

Since #14 exhibits some similarities with VAT 7530 obv. 17-21, this text deserves to be quoted (following Neugebauer, *MKT* 1, 288):

1 ma-na ta-am ₃ 2 ma-na [3 ma]- ^Γ na ^Γ 4 ma- ^Γ na ^Γ [5 ma-na]	Per 1 mana, 2 ma-na, 3 mana, 4 mana, 5 mana
6 ma-na 7 ma-na 8 ma-na 9 ma-na 10 ^Γ ma-na ^Γ	6 mana, 7 mana, 8 mana, 9 mana, 10 mana,
10 gin ₂ igi 4-gal ₂ u ₃ ši-za-a-at ¹⁴ še ku ₃ - babbar	10 1/4 gin ₂ and 1/3 še silver
ku ₃ -babbar li-li li-ri- da-ma	Let the silver rise and fall,
ganba li-im-ta-hi-ra	so that the rate in-kind is equal.

¹⁴ Note that Friberg (2007: 166) reads in line 19 “igi 4-a-at,” translated “1/4th part,” against Neugebauer’s “ši-za-at,” translated “ein Sechstel (??).” Friberg argues convincingly that “the solution to the problem VAT 7530 §3 is given in the form of a tabular array, where the number 1 03 45 corresponds to 1 gin₂ 11 še igi 4-a-at še” (Friberg, personal communication); see also Friberg 2007: 167.

#15 Problem 15 provides both a connection with problem 14 and a significant departure from the previous sections. It is connected to problem 14 in that it continues the use of value in-grain rather than value in-silver. However, a significant departure is seen in that the exchange rate is no longer stated as X in-kind quantity per 1 *gin*₂ of silver, but X in-kind quantity per Y *sila*₃ grain. The value in-kind is a number of fish. Thus we see two rates in problem 15: 3 ku₆-a 5 *sila*₃ “for 3 fish, 5 *sila*₃,” and 5 ku₆-a ku₆ [...] “for 5 fish” The exchange rate is a ratio of 3/5 (36) and 5/x respectively. These ratios can be easily converted into the same formula as in the previous problems. The data are converted in SPVN as follows:

$$5 \text{ sila}_3 \rightarrow 5 \\ 4(\text{aš}) 1(\text{barig}) 4(\text{ban}_2) 5 (\text{sila}_3) \rightarrow 21.45$$

Table 11 provides the following interpretation. Column I is the rate in-kind, 3/5 or 36. Column II is then the reciprocal, that is, 5/3 or 1.40. It is interesting to see that the goal of the problem is the same as in #14, to find the values in-grain and in-kind. However, to underline that there is a significant difference in the ratio, the -ta suffix is not employed, and the problem states in lines 5-6, ku₆ še he₂-e₃ u₃ he₂-e₁₁ ku₆-a ib₂-sa₂-ma, “Let the fish and the grain rise or fall (so that) the fish are equal.” Unfortunately the second rate is broken and so we are unable to solve this problem completely. Interesting is the use of ganba to describe this problem in line 1. One would expect a different term if the use of a different form or ratio defined by both the in-kind and in-grain rate is employed. However, this use makes sense when it is understood that the problem is a rate in-kind exercise, that is, that the use of the rate in-kind must be found and employed to find the answer. If this is true, then the use of ganba as described in §2.4 is justified. This is, in the end, a rate in-kind problem and ganba’s use here informs us on how the author of this text expected to deal with an odd exchange rate.

Further, we see a similar use of ganba in economic texts. As an example, we turn again to *RA* 15, 191 (mentioned above §2.4), this time to lines 1-3:

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I × 7 Rate in-silver × 7	II × 30 Value in-silver	I × III Value in-kind
Lead	7	1	30	3.30
Silver	1	7	3.30	3.30
Total		8	4	7

Table 12: calculations for solving #16

8 gu₂ siki sag-ga₂ 8 gu₂ select white wool,
ganba 1 gu₂-e 7 1/2 rate in-kind to 1 gu₂ 7 1/2 gin₂
ku₃-bi 1 ma-na its value 1 mana.

Note that in this example the explicit lack of the -ta suffix which is present in lines 16-18 of the same document to describe another rate. The difference between RA 15, 191, and our text would be the directive/locative -e in RA 15, 191, which is not explicitly mentioned in YBC 4698.

- #16 This statement was correctly read and translated by Thureau-Dangin (1937b: 89-90; our rendering of the French):

*As much as the price of lead
I paid (literally weighed) some silver,
I bought some lead,
I added silver and lead:
7 mana. The silver and lead
are what?*

As noted by Thureau-Dangin, the market rate of lead is missing, and the problem cannot be solved. However, it seems to us that arithmetical arguments actually demand that the rate in-kind of lead be 7.

Indeed, lines 10-11 seem to indicate that the weight of lead bought and the weight of silver used for buying the lead are added and that the total weight of metal is 7 mana (total value in-kind, corresponding to 7—see table 12, col. IV). As it is stated that “As much as the rates in-kind of the lead silver I weighed and lead I bought,” the weight of lead must be equal to the weight of silver, that is, 3.30 (col. IV). Thus,

Col.	I	II	III	IV
	Rate in-kind (weight of stones per 1 gin)	Reciprocal of I × 7 Rate In-silver × 7	II × 4.30 Value in-silver (price of the stones)	I × III Value in-kind (equal weights of stones)
Common stones	6 a	7 × 10 × a'	5.15 a'	31.30
Gold inlaid stones	6 b	7 × 10 × b'	5.15 b'	31.30
Total			1	1.3

Table 13: sketch of calculations for solving #17

the weights of both metals are 3.30. The goal of the problem is probably to find the value in silver of lead (col. III). Of course, the rate in-kind of silver is 1 and the value in-silver of silver is the same as the value in-kind of silver, that is, 3.30. For the lead, it seems that the lead's rate is missing, as noted by Thureau-Dangin. However, we can rely on the fact that 3.30 = 7×30, thus the factor 7 appears in columns I, II and III. Therefore, the rate in-kind of lead must be 7. Since 7 is not regular, column II cannot provide reciprocals of rates in-kind, but instead provides 7 times the reciprocals of rates in-kind. The coefficient providing values in-silver (col. III) from rates in-silver is 3.30, thus the coefficient which produces col. III from col. II is 30 (because col. II is 7 time the rates in-silver). The table can be filled, and the problem can be solved.

30 → 1/2 ma-na

The value in-silver of lead is 1/2 mana.

Note the use of the verb la₂ to mark the means of payment. The actor weighed out silver against the lead, and literally found the exchange ratio in the purchase.

- #17 Even if the situation described by this statement is not completely clear to us, it seems quite sure that:

- 1) We are dealing with two kinds of stones: common stones and gold inlaid (ku₃-sig₁₇ gar-ra) stones. 6 stones of each kind are bought. As the value of 6 common stones is 1 unit (probably 1 gin₂ of silver, as usual), and the value of 6 gold inlaid stones is the

Col.	I	II	III	IV
	Rate in-kind	Reciprocal of I \times 7 Rate in-silver \times 7	II \times 4.30 Value in-silver	I \times III Value in-kind
Common stones	6×21	3.20	15	31.30
Gold inlaid stones	6×7	10	45	31.30
Total			1	1.3

Table 14: possible sketch of calculations for solving #17

- same, the weight of one common stone is not the same as the weight of one gold inlaid stone.
- The question is probably to find the weight of each kind of stones, that is, the rate in-kind of 6 common stones and of 6 inlaid stones, as well as the corresponding rates in-silver (line 19).
 - Lines 17 and 18 seem to describe the process of making equal the quantities of the two kinds of stones. Thus, the weight of common stone is $\frac{1}{2}$ mana $1 \frac{1}{2}$ gin₂ (31.30), as well as the weight of inlaid stones; the total weight is 1 mana 3 gin₂ (1.3), as stated in line 22.

Now, if we observe that 31.30 is 7×4.30 , we guess that the factor 7 appears in rates in-kind, and that the situation is similar to #12. Thus, column II of Table 13 contains the products of reciprocals of column I by 7 (in the table, a is the weight of one common stone, and a' the reciprocal of a ; b is the weight of one inlaid stone, and b' the reciprocal of b).

We can now calculate the factor k providing column III from column II. Indeed, since IV is $I \times III$, we have:

$$31.30 = 6a \times 7 \times 10 \times a' \times k$$

Thus

$$k = 4.30$$

The values of a and b can be calculated, assuming that the total of column III is 1, as usual:

$$\begin{aligned} 5.15(a' + b') &= 1 \\ 7 \times 45(1/a + 1/b) &= 1 \\ (a + b)/ab &= 1/7 \times 1.20 \end{aligned}$$

This means that a or b is a multiple of 7. Of course, an infinity of solutions is possible. Say, for simplicity, that $b = 7$; thus $a = 21$.

The starting point of this analysis of the problem is the answer (the equal weights of the two kinds of stones is $\frac{1}{2}$ mana $1 \frac{1}{2}$ gin₂), that is, we filled in Table 13 from col. IV to col. I. Some data seem missing in order to fill the table from col. I to col. IV.

However, suppose that it was already stated in a pre-

vious tablet of this series that inlaid stone has three times more value than common stone: if the weight of one inlaid stone is n , then, the weight of one common stone is $3n$. Moreover, suppose that, in order to solve the problem, the column II provides the false value equal to "Reciprocal of $I \times n$." In this way the problem can be solved. A crucial test, to evaluate this hypothesis, should be to decipher the first signs of line 21, which remain unclear for us.

§7. Conclusion

§7.1. Some of the problems of the text remain unclear or uncertain. However, the improvement of the understanding of the text we tried to offer, compared to Neugebauer, Thureau-Dangin and Friberg's previous publications, lies in the meaning of the whole text. This set of 17 problems presents a strong consistency, and most of the statements appear as variations of a basic mathematical sketch. We tried to underline this single mathematical framework by referring systematically to the same chart, represented by conversions and a table of data, which emerges from school exercises (§3.2 and §3.3). In diverse situations, direct and reverse problems are built by suggesting different paths in filling the corresponding table of data.

§7.2. Some of the uncertainties of interpretation originate in our lack of pieces of information. This probably results from the fact that this tablet is just one tablet in a series (the third one), and that relevant information may have been provided in the first two tablets of the series. This detail underlines the importance of considering texts as wholes, when possible. It must be noted that some of the statements have parallels in the known corpus of mathematical procedure texts and school exercises. This means that the author of the tablet reused old mathematical material. But, unlike catalogue texts, the list of statements noted on YBC 4698 doesn't seem to be a compilation gathering statements from different sources, but rather a systematic elaboration of new material from old material. This is a typical feature of series texts (Proust 2009b).

§7.3. Another striking feature of this text is the way in which Akkadian expressions used elsewhere in math-

ematical texts are translated word for word into a quite artificial Sumerian language. The trace of Akkadian expressions is particularly clear in problems 14 and 15 (še₂-e₃ u₃ še₂-e₁₁-ma ganba ib₂-sa₂). The use of Sumero-grams such as bur₂, e₃ and e₁₁ seems to be specific to this text, since only the Akkadian counterparts are attested in other mathematical texts. These linguistic features probably reflect the habits of a highly erudite milieu.

§8. Indices

§8.1. Glossary

a-na	<i>as much as</i>
an-bar	<i>iron</i>
bur ₂	<i>to sell</i>
DUG	<i>object (unclear meaning)</i>
e ₃	<i>to rise</i>
e ₁₁	<i>to fall</i>
en-nam	<i>how much?</i>
ganba	<i>rate (in-kind or in-silver)</i>
gar-gar	<i>to add</i>
i ₃ -geš	<i>common oil</i>
i ₃ sag	<i>first quality oil</i>
i ₃ -šah ₂	<i>lard</i>
ib ₂ -sa ₂	<i>to make equal</i>
ki-la ₂	<i>weight, to weigh</i>
ku ₃ -babbar	<i>silver</i>
ku ₃ -babbar diri	<i>silver profit</i>
ku ₃ diri	<i>silver profit (abbreviation)</i>
ku ₃ -sig ₁₇	<i>gold</i>
ku ₃ -sig ₁₇ gar-ra	<i>inlaid gold</i>
ku ₃ -sum	<i>to pay</i>
ku ₃ la ₂	<i>to weigh silver, to pay</i>
ku ₆	<i>fish</i>
la ₂	<i>to weigh</i>
maš ₂	<i>interest</i>
na ₄	<i>stone</i>
nagga	<i>lead</i>
sa ₁₀	<i>v. to buy, to make equivalent; n. purchase, equivalent</i>
siki	<i>wool</i>
šum ₂	<i>to give</i>
šu-ti-a	<i>received</i>
šuš ₄	<i>to cut</i>
zu	<i>to know</i>

§8.2. Metrological systems

Units of capacities (1 sila₃ ≈ 1 liter)

gur	←×5←	barig	←×6←	ban ₂	←×10←	sila ₃
sila ₃	←×60←	gin ₂	←×180←	še		

Units of weight (1 ma-na ≈ 500 g)

gu ₂	←×60←	ma-na	←×60←	gin ₂	←×180←	še
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§8.3. Metrological tables (outline)

Capacities

1 še	20
1 gin ₂	1
1 sila ₃	1
1(ban ₂)	10
1(barig)	1
1 gur	5

Weights

1 še	20
1 gin ₂	1
1 ma-na	1
1 gu ₂	1

The complete OB metrological tables according to Nipur sources can be found in Proust 2009a: §8.

§9. Neugebauer archives

A copy of the folder “YBC 4698” from the Aaboe-Britton Archives is provided in figure 1. These archives, which contain the documents used by Neugebauer for the publication of *MKT* and *MCT*, are kept at the Institute for the Study of the Ancient World, New York University. We thank Alexander Jones for scanning these documents for us and allowing us to present them here.

The folder contains

- 1) the transliteration of the reverse of YBC 4698 by Neugebauer, with annotations by Albrecht Goetze
- 2) a postcard sent by Goetze to Neugebauer on April 17, 1935
- 3) the photographs used by Neugebauer to publish the text in *MKT* 3, 42-45

These photographs are not included in *MKT*. Thureau-Dangin used copies of the same photographic negative, sent to him by the curator, Ferris J. Stephens. More detail on Aaboe-Britton Archives and on the way in which Neugebauer and Thureau-Dangin worked with photographs can be found in (Proust & Rougemont forthcoming).

YBC 4698

Rs. I.

10 1 gur NI-PA
2 qa ANŠE-ma
3 1/2 qa BAL-PA
4 ku dirig en-nam
5 2/3 gin ku dirig
11 1 gur NI-PA
7 1-na ANŠE 1 gin 1 1/2 qa
8 2/3 gin ku dirig
9 en-nam ANŠE-ma en-nam BAL-PA
10 qa ANŠE-ma 7(?) 1/2 qa BAL-PA
12 17 ma-na BAL(?) RA 11 ma-na ANŠE
12 1/2 gin ku sum ib-sig-ma ANŠE
13 4, 16 še ANŠE
14 5 qa NI-PA
15 1(ban) 2 qa NI...3
16 1 gin sum ib-sig-ma ANŠE
17 malba a-na 1 2 3 4 5
18 7 8 9 malba
19 NI-PA(?) sum ib-sig
20 1/2 ib-sig-0iš-ma
21 TE(?) IM ib-sig

Rs. II.

15 1 malba 3 BA-A 3 qa
2 5 BA-A ANŠE(?)
3 gar-gar-ma ... 1, 10
4 BA-A gar-gar-ma 2, 20
5 1/2
6 BA-A ib-sig-ma
16 7 malba-bi-a AN-NA
8 NI 1 gin -ma
9 AN-NA ANŠE
10 ku 1/2 AN-NA gar-gar-ma
11 ma-na ku 1/2 AN-NA
12 en-nam
13 NI-PA(?)
14 -bi BA-SU

3) Hier steht ein mir unverständliches Zeichen (s.d. Autographie).

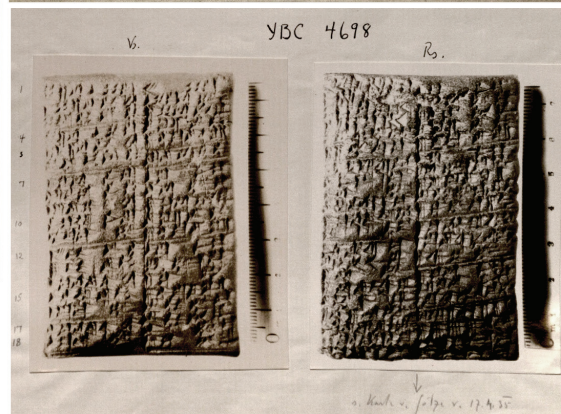
YBC 4698

15 6 ša ANŠE
16 5 ša kh máš gar-ra
17 ku máš gar-ra
18 a-na 6 ša ANŠE
19 NI-PA(?)
20 gar-gar-ma 1/2 ma-na 1 1/2 gin
21 gar-gar-ma
22 šu-ši 3 gin(?)

Unterschrift:

4) dub(tappu) 3-kam-ma

4) Kein Platz für [17 IM-SU].



NEW HAVEN
APR 16
5 PM
1935
CGNY

THIS SIDE OF CARD ADDRESS

Prof. Otto Neugebauer
Kjellerstr. - København
Bldg. Lund 4
Denmark.

April 7th 1935

Dear Mr. Neugebauer,

Your inscription has proved to be quite correct.
On 4667 and 4676 no subings. However on
4698 on the edge (1 under last column!) :
as best as possible.

Best love
A. Göpfert

Below some multiplication tablets follow, after
some exp. Yours very sincerely
A. Göpfert

Figure 1: Aaboe-Britton Archives, folder "YBC 4698" (courtesy of ISAW/NYU)

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MKT 1 = Neugebauer 1935

MKT 3 = Neugebauer 1937

TMB = Thureau-Dangin 1938

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