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Matrix Fitting Approach to Direction of Arrival Estimation with Imperfect Spatial Coherence of Wavefronts

Alex B. Gershman, Christoph F. Mecklenbräuker, and Johann F. Böhme

***Abstract*— The performance of high-resolution direction of arrival (DOA) estimation methods significantly degrades in several practical situations where the wavefronts have imperfect spatial coherence. The original solution to this problem was proposed by Paulraj and Kailath, but their technique requires *a priori* knowledge of the matrix characterizing the loss of wavefront coherence along the array aperture. In this correspondence, a novel solution to this problem is proposed, which does not require *a priori* knowledge of the spatial coherence matrix.**

I. Introduction

The majority of high-resolution DOA estimation methods [1]–[3] is model-based and, therefore, very sensitive to various types of model errors [4]. Usually, when implementing DOA estimation algorithms, each wavefront is assumed to be perfectly coherent within the array aperture, i.e., the amplitude and phase of any wavefront are supposed to be fully correlated between any two sensors of the receiving array. Such perfect coherence of the wavefront implies that it contributes a rank-one component to the array covariance matrix. However, in many practical situations, as, for example, in sonar and radar, wavefront coherence suffers with increasing spatial separation between array sensors [5]–[13]. Such wavefront decorrelation can result from signal propagation through randomly inhomogeneous media [9], [10], from scattering at randomly varying surfaces [8], [11], [12], and from other types of stochastic model deviations [13]. As a result, the high-resolution DOA estimation and detection methods are no longer applicable. In their work [8], Paulraj and Kailath have developed a statistical model for sources with partial wavefront coherence and have studied how the performance of the MUSIC DOA estimator degrades if spatial coherence is ignored in the signal model. They also proposed the elegant technique that exploits the model developed for improving the estimation performance of the MUSIC algorithm.

The main drawback of their algorithm is the requirement of full *a priori* knowledge of spatial coherence matrix characterizing the loss of wavefront coherence along the array aperture. In practical situations, this matrix may be unknown.

In this correspondence, a new matrix fitting technique is proposed as a solution to DOA estimation problem in the presence of imperfectly coherent wavefronts. Unlike the Paulraj–Kailath technique, our algorithm does not require *a priori* knowledge of the spatial coherence matrix because the elements of this matrix are estimated simultaneously with signal DOA's.

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The authors are with the Signal Theory Group, Department of Electrical Engineering, Ruhr University, Bochum, Germany.

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II. Problem Formulation

Consider a uniformly spaced linear array of n sensors. Assume that there are $q < n$ narrowband stationary zero-mean mutually uncorrelated far-field sources with central frequency ω_0. In this correspondence, we only address the source localization problem, i.e., the number of sources is assumed to be known *a priori*. First of all, consider the familiar case of perfect wavefront coherence. The ith array vector snapshot can be modeled as [2]–[4]

$$\boldsymbol{r}(i) = \boldsymbol{A}\boldsymbol{s}(i) + \boldsymbol{n}(i) \tag{1}$$

where

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_q)]$$

is the $n \times q$ matrix of the wavefront vectors of each source

$$\boldsymbol{a}(\theta) = (1, e^{-j\omega_0 d \sin\theta/c}, \cdots, e^{-j\omega_0 (n-1) d \sin\theta/c})^T$$

is the $n \times 1$ wavefront vector corresponding to the direction $\theta, \{\theta_l\}_{l=1,2,\cdots,q}$ are the signal DOA's, $\boldsymbol{s}(i)$ is the $q \times 1$ vector of random source waveforms, $\boldsymbol{n}(i)$ is the $n \times 1$ vector of random sensor noise, d is the interelement spacing, c is the propagation speed, and $(\cdot)^T$ denotes the transpose. The array covariance matrix is given by [1]–[3]

$$\boldsymbol{R} = \mathrm{E}\{\boldsymbol{r}(i)\boldsymbol{r}^H(i)\} = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H + \sigma^2\boldsymbol{I} \tag{2}$$

where

- $\boldsymbol{S}$ $q \times q$ covariance matrix of signal waveforms;
- $\boldsymbol{I}$ $n \times n$ identity matrix,
- σ^2 noise variance;
- $\mathrm{E}\{\cdot\}$ expectation operator;
- $(\cdot)^H$ Hermitian transpose.

Let us now assume that the wavefronts have imperfect coherence within the array aperture and briefly revisit the underlying model [8]. Wavefront perturbation can be represented as multiplicative noise, leading to the snapshot model

$$\boldsymbol{f}(i) = (\boldsymbol{G}(i) \odot \boldsymbol{A})\boldsymbol{s}(i) + \boldsymbol{n}(i) \tag{3}$$

where $\boldsymbol{G}(i)$ is the $n \times q$ matrix of random wavefront perturbations, and $\odot$ denotes the Schur–Hadamard (element-by-element) matrix product. The elements of matrix $\boldsymbol{G}(i)$ describe the amplitude and phase fluctuations of wavefronts, i.e.,

$$[\boldsymbol{G}(i)]_{lk} = \zeta_{lk}(i)e^{j\phi_{lk}(i)}.$$

It should be noted that unlike (1), the vector process (3) is always non-Gaussian. This is the main reason why one cannot exploit the maximum likelihood technique [3] in the situation considered.

Following [8], we assume isotropic coherency loss, i.e., we consider the case when the loss across the array is the same for all wavefronts, irrespective of their DOA's. The phase field of emitted signals can often be modeled as a Wiener–Levy process in various practical situations. In such cases, the stated isotropic coherency loss is an exact result [12]. Random phase-velocity fluctuations in a time-varying propagation medium may result in situations where this assumption is a reasonable approximation. Typical situations arise in long-range ocean acoustic propagation and electromagnetic propagation in the lower troposphere. In addition, this assumption may be reasonable when modeling stochastic array deviations [13].

The assumption of isotropic coherence loss means that the spatial coherence function is independent of the wavefront index k

$$\begin{aligned} b_{lm} &= \mathrm{E}\{[\boldsymbol{G}(i)]_{lk}[\boldsymbol{G}(i)]^*_{mk}\} \\ &= \mathrm{E}\{\zeta_{lk}(i)\zeta_{mk}(i)e^{j(\phi_{lk}(i)-\phi_{mk}(i))}\} \end{aligned} \tag{4}$$

where $(\cdot)^*$ denotes the complex conjugate. From isotropic model, it follows that function (4) depends on the separation between the lth and mth sensors only, i.e., for a uniform linear array $b_{lm} = b_{l-m}$, whereas the assumption of zero-mean phase fluctuations gives that all b_{l-m} have real values. Additionally, assume that the random wavefront perturbations, the additive sensor noises, and the source waveforms are all mutually statistically independent. Thus, the array covariance matrix for the data model (3) can be expressed as

$$\boldsymbol{F} = \mathrm{E}\{\boldsymbol{f}(i)\boldsymbol{f}^H(i)\} = (\boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H) \odot \boldsymbol{B} + \sigma^2\boldsymbol{I} \tag{5}$$

where $[\boldsymbol{B}]_{lm} = b_{l-m}$ and, without loss of generality, we assume that $b_0 = 1$. This normalization of the matrix $\boldsymbol{B}$ is equivalent to multiplying all snapshot vectors by a constant, and obviously, it does not cause any change of the model. Therefore, $\boldsymbol{I} \odot \boldsymbol{B} = \boldsymbol{I}$, and (5) can be rewritten as

$$\boldsymbol{F} = \boldsymbol{R} \odot \boldsymbol{B}. \tag{6}$$

Summarizing, we conclude that $\boldsymbol{B}$ can be modeled as a real-valued symmetric Toeplitz positive definite matrix[1] [8].

III. Paulraj–Kailath Method

To improve the MUSIC algorithm in a situation of imperfect wavefront coherence and *a priori* known spatial coherence matrix $\boldsymbol{B}$, Paulraj and Kailath [8] introduced and exploited the so-called restored array covariance matrix

$$\tilde{\boldsymbol{R}} = \hat{\boldsymbol{F}} \oslash \boldsymbol{B} \tag{7}$$

where

$$\hat{\boldsymbol{F}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{f}(i)\boldsymbol{f}^H(i) \tag{8}$$

is the sample estimate of the matrix $\boldsymbol{F}$, N is the number of snapshots, and $\oslash$ denotes the inverse of Schur–Hadamard product, i.e.,

$$[\boldsymbol{C} \oslash \boldsymbol{D}]_{lm} = [\boldsymbol{C}]_{lm}/[\boldsymbol{D}]_{lm}.$$

This preprocessing operation allows one to find a consistent estimate of the matrix $\boldsymbol{R}$. After that, the MUSIC algorithm can be applied straightforwardly [8] to the restored covariance matrix $\tilde{\boldsymbol{R}}$.

The main drawback of this approach is the requirement of exact *a priori* knowledge of the spatial coherence matrix $\boldsymbol{B}$. In practice, this condition may be unrealistic. With imprecise knowledge of matrix $\boldsymbol{B}$, serious problems can occur, especially when some elements of this matrix are close to zero.

IV. Proposed Matrix-Fitting Technique

The non-Gaussian array data vector model (3) does not allow for applying the maximum likelihood algorithms for DOA estimation in the situation of imperfect wavefront coherence. However, the natural cost function whose global minimum corresponds to the required estimates of parameters may be chosen as

$$Z(\boldsymbol{\Theta}) = \|\tilde{\boldsymbol{R}} - \boldsymbol{R}\|_F^2 = \|\hat{\boldsymbol{F}} \oslash \boldsymbol{B} - \boldsymbol{R}\|_F^2 \tag{9}$$

where the minimization is performed over the matrices $\boldsymbol{R}$ and $\boldsymbol{B}$. The minimizer of $Z(\boldsymbol{\Theta})$ can be rewritten as

$$\min_{\boldsymbol{\Theta}} \ \mathrm{tr}\{(\hat{\boldsymbol{F}} \oslash \boldsymbol{B} - \boldsymbol{R})^2\} \tag{10}$$

which corresponds to a least-squares fit and provides a statistically consistent estimator of the $M \times 1$ vector $\boldsymbol{\Theta}$ of unknown parameters [3].

We need to estimate q DOA's, q^2 real independent parameters of Hermitian matrix $\boldsymbol{S}$, the noise variance σ^2, and $n-1$ real independent parameters of the matrix $\boldsymbol{B}$. Therefore, the total number of estimated parameters is $M = q(q+1) + n$. Taking into account that the Hermitian array covariance matrix is defined by n^2 real independent parameters, we have that our estimation problem is well posed if $q(q+1) \le n(n-1)$. This is, however, always fulfilled because $q < n$.

Let us now reduce the dimension of the multidimensional search implied by (10). For fixed DOA's and matrix $\boldsymbol{B}$, the optimum of (10) is achieved for

$$\hat{\boldsymbol{S}} = \boldsymbol{A}^\dagger(\hat{\boldsymbol{F}} \oslash \boldsymbol{B} - \hat{\sigma}^2\boldsymbol{I})\boldsymbol{A}^{\dagger H} \tag{11}$$

$$\hat{\sigma}^2 = \frac{1}{n-q}\mathrm{tr}\{\boldsymbol{P}_{\boldsymbol{A}}^\perp(\hat{\boldsymbol{F}} \oslash \boldsymbol{B})\} \tag{12}$$

$$\boldsymbol{A}^\dagger = (\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H, \quad \boldsymbol{P}_{\boldsymbol{A}} = \boldsymbol{A}\boldsymbol{A}^\dagger, \quad \boldsymbol{P}_{\boldsymbol{A}}^\perp = \boldsymbol{I} - \boldsymbol{P}_{\boldsymbol{A}}. \tag{13}$$

Using (11)–(13), we can rewrite the minimization problem (10) as

$$\begin{aligned} &\min_{\boldsymbol{\Theta}} \ \mathrm{tr}\{(\hat{\boldsymbol{F}} \oslash \boldsymbol{B} - \boldsymbol{P}_{\boldsymbol{A}}(\hat{\boldsymbol{F}} \oslash \boldsymbol{B} - \frac{1}{n-q}\ \mathrm{tr}\{\boldsymbol{P}_{\boldsymbol{A}}^\perp(\hat{\boldsymbol{F}} \oslash \boldsymbol{B})\}\boldsymbol{I})\boldsymbol{P}_{\boldsymbol{A}} \\ &\qquad - \frac{1}{n-q}\ \mathrm{tr}\{\boldsymbol{P}_{\boldsymbol{A}}^\perp(\hat{\boldsymbol{F}} \oslash \boldsymbol{B})\}\boldsymbol{I})^2\} \\ &\quad = \min_{\boldsymbol{\Theta}} \ \mathrm{tr}\{(\hat{\boldsymbol{F}} \oslash \boldsymbol{B} - \boldsymbol{P}_{\boldsymbol{A}}(\hat{\boldsymbol{F}} \oslash \boldsymbol{B})\boldsymbol{P}_{\boldsymbol{A}} \\ &\qquad - \frac{1}{n-q}\ \mathrm{tr}\{\boldsymbol{P}_{\boldsymbol{A}}^\perp(\hat{\boldsymbol{F}} \oslash \boldsymbol{B})\}\boldsymbol{P}_{\boldsymbol{A}}^\perp)^2\} \end{aligned} \tag{14}$$

where the $(q+n-1) \times 1$ vector

$$\boldsymbol{\Theta} = (\boldsymbol{\theta}^T, \boldsymbol{b}^T)^T \tag{15}$$

[1] It should be pointed out that this is not a necessary requirement for the proposed algorithm. Other models of the matrix $\boldsymbol{B}$ can be involved as well.

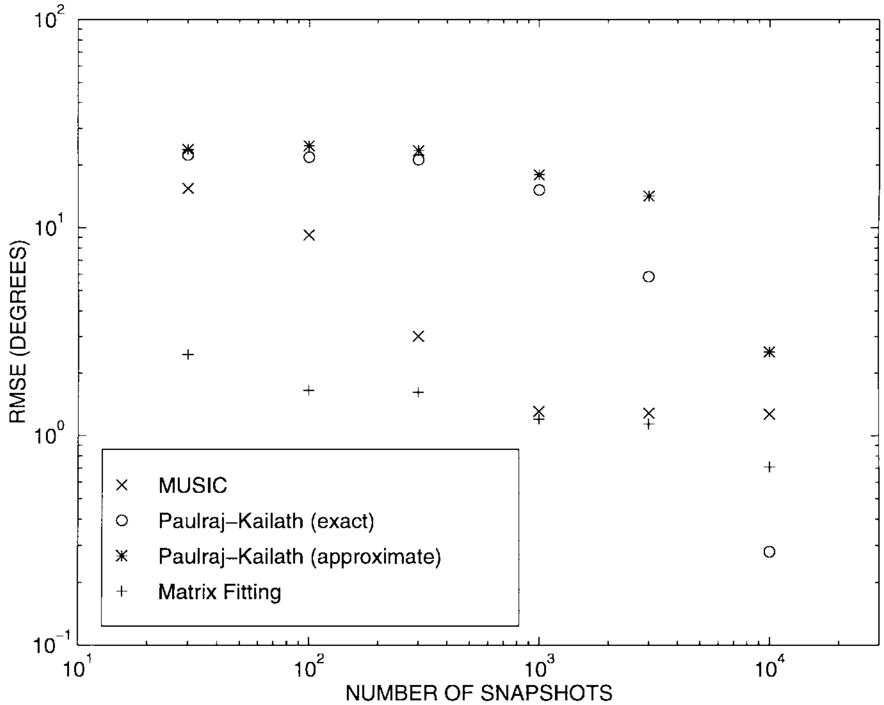

Fig. 1. Experimental RMSE of DOA estimation versus the number of snapshots. SNR = 20 dB. Three techniques are compared: the conventional MUSIC algorithm, the Paulraj–Kailath modification of MUSIC, and the proposed matrix fitting technique. The exact Paulraj–Kailath technique corresponds to precise *a priori* knowledge of the spatial coherence matrix $\boldsymbol{B}$, whereas the approximate Paulraj–Kailath technique corresponds to the case where this matrix is known with a small error.

contains the reduced set of estimated parameters

$$\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_q)^T, \quad \boldsymbol{b} = (b_1, b_2, \cdots, b_{n-1})^T.$$

If the global minimization of (14) over $\boldsymbol{\Theta}$ is already performed, then, according to (11), the final estimates of the source powers $\sigma_l^2, l = 1, 2, \cdots, q$ can be found as

$$\hat{\sigma}_l^2 = [\hat{\boldsymbol{A}}^\dagger (\hat{\boldsymbol{F}} \oslash \hat{\boldsymbol{B}} - \frac{1}{n-q} \operatorname{tr}\{\boldsymbol{P}_{\hat{\boldsymbol{A}}}^\perp (\hat{\boldsymbol{F}} \oslash \hat{\boldsymbol{B}})\} \boldsymbol{I}) \hat{\boldsymbol{A}}^{\dagger H}]_{ll} \qquad (16)$$

where $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ are the final estimates of the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$.

Unlike the Paulraj–Kailath algorithm, the presented technique does not require *a priori* knowledge of spatial coherence matrix $\boldsymbol{B}$ because this matrix is estimated simultaneously with the source DOA's.

V. Simulation Results

Computer simulations have been carried out to compare the DOA estimation performances of the matrix fitting technique, the conventional MUSIC estimator, and the Paulraj–Kailath modification of MUSIC. We assume a uniformly spaced linear array with $n = 8$ sensors and half-wavelength spacing and two mutually uncorrelated equipower signal sources impinging on the array from the directions $\theta_1 = 11^\circ$ and $\theta_2 = 15^\circ$. The additive Gaussian noise is uncorrelated with the sources and between array sensors and has the same variance σ^2 in each sensor. We assume that wavefront amplitudes do not fluctuate, whereas the wavefront phases have Gaussian independent fluctuations with sensor-to-sensor phase increment variance σ_ϕ^2. In other words, the spatial coherence function (4) is modeled as [6], [8], [12]

$$b_{l-m} = \mathrm{E}\{e^{j(\phi_{lk}(i) - \phi_{mk}(i))}\} = e^{-\sigma_\phi^2 |l-m|/2}. \qquad (17)$$

In all simulation examples, $\sigma_\phi^2 = 0.25$ has been taken corresponding approximately to a -1.086 dB coherency loss at one-wavelength separation.

Minimization of the cost function (14) has been performed over the parameters (15) using the genetic algorithm (GA), which is known to converge to a global minimum [14]. This algorithm seems to be suitable for solving the multidimensional parameter estimation problems in array processing [15]–[17]. At the same time, GA is known to be computationally expensive. For reduction of the computational burden, the domain of variation of the estimated parameters $\boldsymbol{b} = (b_1, b_2 \cdots, b_{n-1})^T$ has been bounded between

$$\boldsymbol{b}_1 = (e^{-\sigma_{\phi\ \min}^2/2}, e^{-2\sigma_{\phi\ \min}^2/2}, \cdots, e^{-(n-1)\sigma_{\phi\ \min}^2/2})^T \quad \text{and}$$
$$\boldsymbol{b}_2 = e^{-\sigma_{\phi\ \max}^2/2}, e^{-2\sigma_{\phi\ \max}^2/2}, \cdots, e^{-(n-1)\sigma_{\phi\ \max}^2/2})^T \qquad (18)$$

where $\sigma_{\phi\ \min}^2 = 0.09$ and $\sigma_{\phi\ \max}^2 = 0.49$, respectively. Similarly, the estimated DOA's have been bounded as well, i.e., they have been assumed to belong to the interval $6^\circ \div 20^\circ$. This corresponds to the very rough pre-estimation of the DOA localization sector by conventional beamformer, which is relatively insensitive to the coherency loss compared with the high-resolution methods [5].

A total of 100 independent simulation runs have been performed to compute the experimental root-mean-square error (RMSE) and the bias of DOA estimation for each algorithm and simulated point. In all examples, the Paulraj–Kailath method has been tested in two different modes. The first one, which is referred to as the *exact* Paulraj–Kailath method, corresponds to precise *a priori* knowledge of the coherence matrix $\boldsymbol{B}$. The second mode, which is referred to as *approximate* Paulraj–Kailath method, corresponds to the case where this matrix is known with a small error that can easily occur in practice. Namely, in the second mode, we assume that the restored array covariance matrix (7) is calculated using the imprecisely known matrix $\boldsymbol{B}$. In turn, this matrix is calculated using the model (17) and the measured value of σ_ϕ^2, i.e., $\tilde{\sigma}_\phi^2 = 0.27$ (recall that the true value of σ_ϕ^2 is 0.25). This corresponds to the 8% measurement error of σ_ϕ^2.

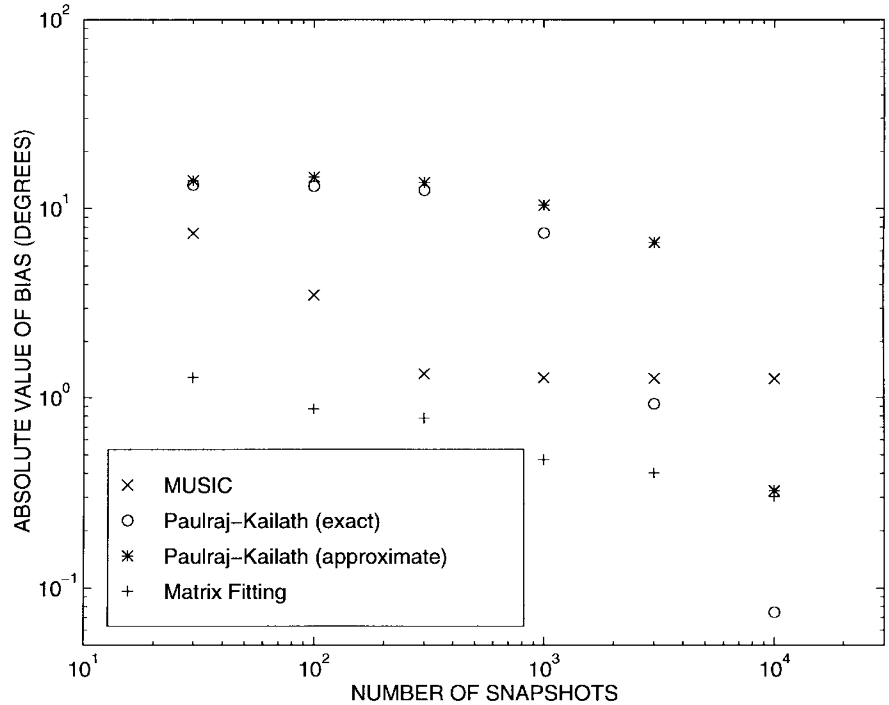

Fig. 2. Experimental absolute value of bias of DOA estimation versus the number of snapshots. SNR = 20 dB. Three techniques are compared: the conventional MUSIC algorithm, the Paulraj–Kailath modification of MUSIC, and the proposed matrix fitting technique. The exact Paulraj–Kailath technique corresponds to precise *a priori* knowledge of the spatial coherence matrix $\boldsymbol{B}$, whereas the approximate Paulraj–Kailath technique corresponds to the case where this matrix is known with a small error.

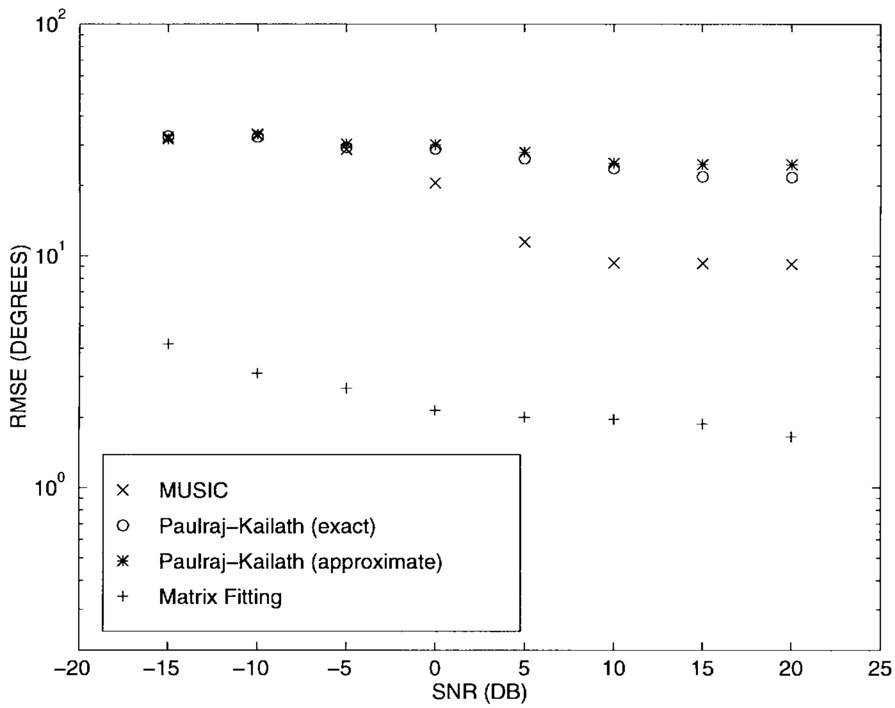

Fig. 3. Experimental RMSE of DOA estimation versus SNR. The number of snapshots $N = 100$. Three techniques are compared. The conventional MUSIC algorithm, the Paulraj–Kailath modification of MUSIC, and the proposed matrix fitting technique. The exact Paulraj–Kailath technique corresponds to precise *a priori* knowledge of the spatial coherence matrix $\boldsymbol{B}$, whereas the approximate Paulraj–Kailath technique corresponds to the case where this matrix is known with a small error.

Fig. 1 shows the comparison of experimental RMSE's of DOA estimation for the conventional MUSIC, Paulraj–Kailath, and matrix fitting techniques versus the number of snapshots for the fixed signal to noise ratio (SNR) equal to 20 dB for each source. SNR is defined as $10\log(\sigma_S^2/\sigma^2)$, where σ_S^2 is the power of each signal in single sensor. Fig. 2 shows the same curves as in Fig. 1 but for the absolute value of DOA estimation bias. Fig. 3 compares the experimental RMSE's of DOA estimation for the conventional MUSIC, Paulraj–Kailath,

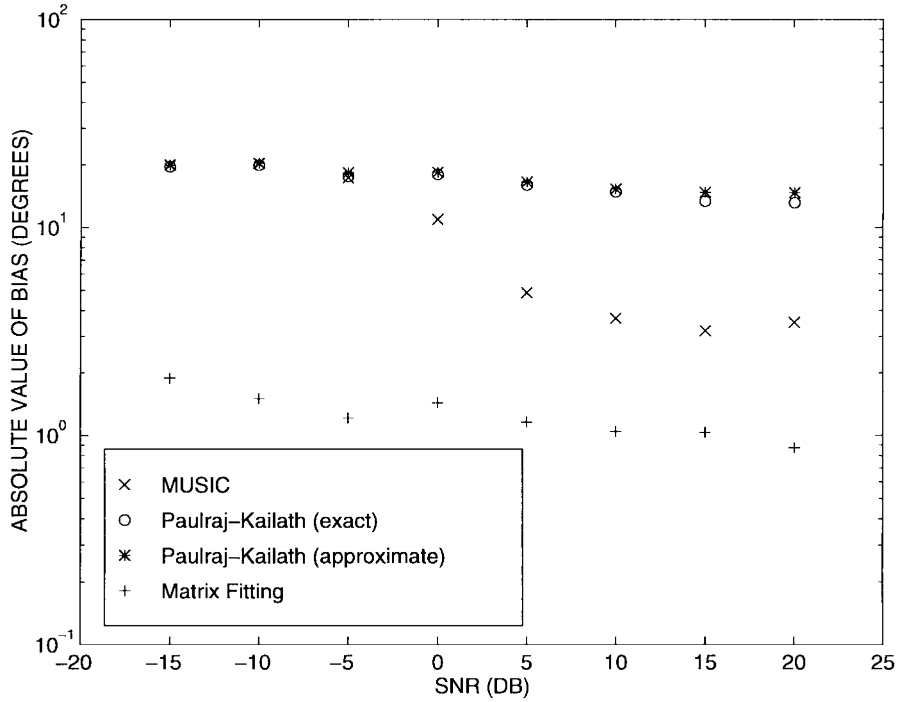

Fig. 4. Experimental absolute value of bias of DOA estimation versus SNR. The number of snapshots $N = 100$. Three techniques are compared. The conventional MUSIC algorithm, the Paulraj–Kailath modification of MUSIC, and the proposed matrix fitting technique. The exact Paulraj–Kailath technique corresponds to precise *a priori* knowledge of the spatial coherence matrix $\boldsymbol{B}$, whereas the approximate Paulraj–Kailath technique corresponds to the case where this matrix is known with a small error.

and matrix fitting techniques versus SNR for the fixed number of snapshots $N = 100$. Fig. 4 shows the same curves as in Fig. 3 but for the absolute value of DOA estimation bias.

It follows from Figs. 1 and 2 that for high SNR, the proposed matrix fitting technique significantly outperforms both the conventional MUSIC and Paulraj-Kailath algorithms in the case of a moderate and a large number of snapshots. The exact Paulraj–Kailath method has superior performance only in the case of a very large number of snapshots (i.e., $N \geq 10^4$). However, in the presence of the small measurement error of the matrix $\boldsymbol{B}$, the performance of the Paulraj–Kailath technique degrades significantly. A surprising fact following from Figs. 1 and 2 is that in the case of a moderate and even a large number of snapshots (i.e., for $N \leq 3000$), the conventional MUSIC algorithm can perform better than both the exact and approximate Paulraj–Kailath techniques. This fact can be explained by the weak statistical consistency of the estimate (7) based on the inverse of Schur–Hadamard product. It should be noted that this fact is pointed out in [8] as well, namely, it is mentioned there that the Paulraj–Kailath method provides improved results relative to conventional MUSIC only when the number of snapshots exceeds a certain threshold.

Figs. 3 and 4 demonstrate that for a moderate number of snapshots $(N = 100)$, only the proposed matrix fitting technique can provide satisfactory performance up to SNR $\simeq 0$ dB, namely, in this situation, the performances of conventional MUSIC and both the exact and approximate Paulraj–Kailath techniques severely degrade in the whole range of SNR.

Unfortunately, it is not possible to compare the computational cost of our technique and the Paulraj–Kailath method in terms of number of operations because the computational complexity of GA severely depends on the optimization function profile as well as on the internal parameters of algorithm and the choice of convergence criterion [14]. In order to provide insights regarding the relative complexity, we compared the computational time of the matrix fitting and Paulraj–Kailath techniques in our simulations for typical parameters of GA (the number of generations = 100, the number of individuals in one generation = 30, the binlength = 20, the probability of crossover = 0.75, and the probability of mutation = 0.001) and of MUSIC (the spectral function has been calculated with the angular grid 0.1° in the whole array field of view $[-90^\circ, 90^\circ]$). Our comparison shows that the matrix fitting technique is more expensive in the situation considered (approximately with the factor $10 \div 20$). In fact, this is the payment for the improved performance. However, it seems that the proposed technique can be used at least in nonreal-time processing applications. This conclusion is based on the positive experience when using GA in practical sonar and seismic problems [15]–[17].

VI. Conclusions

A novel matrix fitting approach to the DOA estimation problem with imperfect wavefront coherence is proposed. Unlike the well-known Paulraj–Kailath method, our algorithm does not require *a priori* knowledge of the spatial coherence matrix because the elements of this matrix are estimated simultaneously with signal DOA's. Moreover, computer simulations have shown significant improvement of the DOA estimation performance of the proposed technique compared with the conventional MUSIC and Paulraj–Kailath methods. The payment for the improved performance is higher computational complexity.

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