A DIGITAL COMPUTER MODEL OF A MULTIPLE EFFECT EVAPORATOR

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Abstract

A dynamic mathematical model of a multiple effect evaporator has been developed for a digital computer using the IBM Continuous Systems Modelling Program. The model will be incorporated in an overall simulation of a cane sugar factory.

The mathematical model was derived from material and energy balance equations assuming that (i) the liquid phase is perfectly mixed; (ii) the vapour phase is always in equilibrium with its liquid phase.

An empirical equation for the heat transfer coefficient was derived from first principles and fitted by non-linear regression to a set of factory data. Other physical parameters were defined by empirical equations derived from data in the literature.

Graphs are presented showing the effect of various changes in the feed conditions on exit brix and flowrate.

1. Introduction

A dynamic mathematical model of a multiple effect evaporator (MEE) is presented. The model was based on the MEE at the Jaagbaan factory of the Noodsberg Sugar Company, Limited, but it is not specific to this installation. It will form part of a complete sugar factory simulation at present under development by the CSIR.

Computer programs for multiple effect evaporation found in the literature (Coates,[1] Jernqvist[4] and Withers[8]) were unsuitable for the present need. As far as the author is aware, no dynamic mathematical model of an MEE has been published.

The model represents a four-effect evaporator. The first effect consists of two calandria evaporators connected in parallel and the other effects are all single calandria evaporators. Figure 1 shows a typical calandria evaporator and Figure 2 is a block diagram representation of the MEE. Each effect is connected to its neighbour so that the vapour from one effect supplies heat to the next. The first effect is operated at a pressure higher than one atmosphere and it is heated with exhaust steam from the turbines. The vapour pressure in the first effect is maintained constant by manually controlling the amount of exhaust steam fed to the calandrias. The last effect is operated under vacuum, and its vapour pressure is kept approximately constant by manual control of the cooling water fed to the condenser following this effect. The syrup in each vessel boils at a pressure and temperature lower than that in the preceding effect, so that heat transfer across the calandria tubes is possible. For reasons of economy, vapour is bled from the first and second effects and used for pre-heating the juice, and for heating the vacuum pans, where crystallisation occurs.

2. Nomenclature

The nomenclature used for the derivation of the mathematical model is listed below. It applies to a single calandria evaporator only. Reference to Figure 3 will help to clarify the significance of each stream variable. (SI units are used).

	Label	*Name*	*Units*
State variables	B	brix	°Brix
	F	flowrate	t/s
	L	syrup volume in vessel	m^3
	t	time	s
	T	temperature	°C
	V	vapour rate	t/s
Dependent variables	C_p	heat capacity	J/t°C
	D	density	t/m^3
	ΔT_b	boiling point elevation	°C
	H	enthalpy	J/t
	Q_{atm}	heat lost to surroundings	W
	U	heat transfer coefficient	$W/m^{2}{}^\circ C$
Constants	A	calandria heat transfer area	m^2
Subscripts	i	refers to syrup feed stream	
	o	refers to syrup exit stream	
	s	refers to steam supply stream	
	v	refers to vapour stream	

3. Derivation of the mathematical model

Multiple-effect evaporation is a multi-stage process. The dynamic mathematical model was derived from the mass and enthalpy balance equations valid for each single evaporator, expressed in the standard form:

$$\left\{\begin{matrix}\text{mass}\\ \text{or}\\ \text{energy}\end{matrix}\right\} \text{in} = \left\{\begin{matrix}\text{mass}\\ \text{or}\\ \text{energy}\end{matrix}\right\} \text{out} + \left\{\begin{matrix}\text{mass}\\ \text{or}\\ \text{energy}\end{matrix}\right\} \text{accumulated.}$$

If the liquid phase is assumed perfectly mixed and entrainment of sucrose in the vapour is neglected, the following expressions describe the physical behaviour of the evaporator (refer to Figure 3).

$$\text{total mass balance: } F_i = V + F_o + \frac{d(DL)}{dt} \qquad (1)$$

$$\text{brix balance: } F_iB_i = F_oB_o + \frac{d(DLB_o)}{dt} \qquad (2)$$

enthalpy balance:

$$F_iH_i + UA(T_s - T_o) = F_oH_o + VH_v + Q_{atm} + \frac{d(DLH_o)}{dt} \qquad (3)$$

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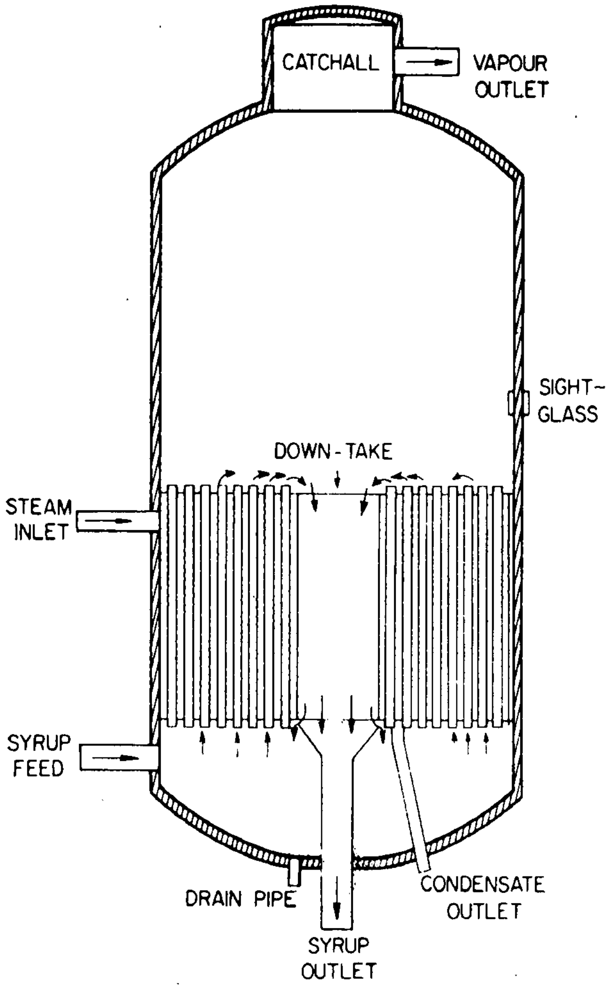
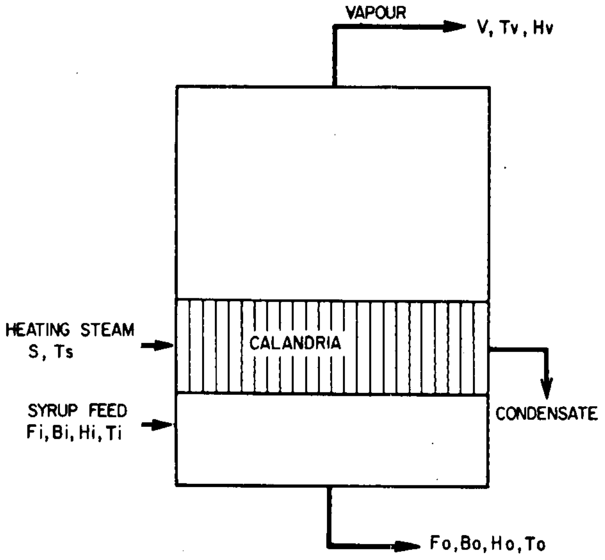
* Seconded from the Chemical Engineering Research Group, C.S.I.R.





Above:

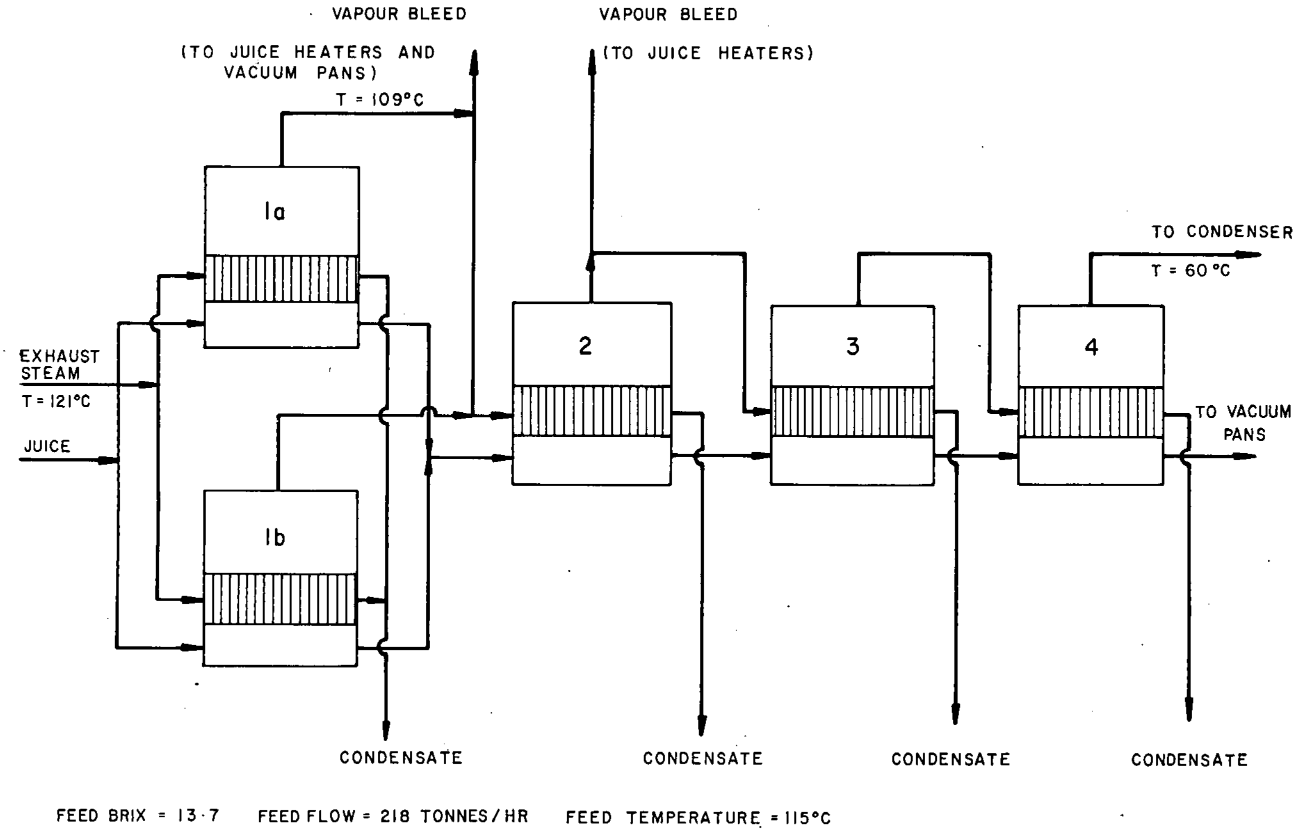
FIGURE 3. Block diagram representation of a Calandria evaporator showing the parameters associated with each stream.

Left:

FIGURE 1. Calandria evaporator.

Below:

FIGURE 2. Multiple effect evaporator.



Multiplying equation (1) by H_o and subtracting from equation (3) gives:

$$F_i(H_i-H_o)+UA(T_s-T_o) = V(H_v-H_o)+Q_{atm}+DL\frac{dH_o}{dt} \quad (4)$$

From thermodynamic theory, $H = \int_0^{T+273} CpdT$. Therefore, assuming that the heat capacity is a weak function of temperature, the following approximation was used:

$$H_i-H_o = \int_0^{T_i+273} Cp_i dT - \int_0^{T_o+273} Cp_o dT$$

$$\simeq Cp_i(T_i+273)-Cp_o(T_o+273)$$

Further: $\frac{dH_o}{dt} = Cp_o\frac{dT_o}{dt}$

and: $H_v-H_o = \Delta H$=latent heat of evaporation.

Substituting these values into equation (4), gives:

$$F_i(Cp_i(T_i+273)-Cp_o(T_o+273))+UA(T_s-T_o) = V\,\Delta H+Q_{atm}+DLCp_o\frac{dT_o}{dt} \quad (5)$$

Equations (1), (2) and (5) form the basis of the model, and have to be solved simultaneously.

4. Numerical solution of the model differential equations

To solve equations (1), (2) and (5), auxiliary equations were needed expressing physical parameters in terms of known quantities. Density, heat capacity and boiling point elevation were expressed as functions of the brix (see Appendix 2). The relevant equations were obtained either directly from the literature (Hugot,[3] Kern[5] and Spencer-Meade[7]) or derived from data in the literature. An equation for the heat transfer coefficient was derived from first principles and fitted by non-linear regression to a set of factory operating data for the MEE. (See Appendix 1). A proportionality factor for the heat loss to the surroundings was also found in the same correlation.

Since density is a weak function of the brix, the term $\frac{dD}{dt}$ was assumed to be negligibly small. Initially, the latent heat of evaporation was assumed constant at $2{,}326\times10^3$ J/t.

Superheating of the vapour and subcooling of the condensate were ignored because they will have been accounted for in the factory data used to correlate the formula for the heat transfer coefficient.

If the vapour pressure is fixed (as was assumed for the first and fourth effects) the saturated vapour temperature, T_v, can be found from steam tables (Perry[6]). The liquid temperature is then given by $T_o=T_v+\Delta T_b$. Since ΔT_b is a strong function of the brix, $\frac{dT_o}{dt}$ will be a function of $\frac{dB_o}{dt}$. Then equation (5) can be used to solve for V since all other quantities in it are known.

If the pressure is not fixed, as is the case in the second and third effects, equation (5) can be used to solve for $\frac{dT_o}{dt}$ provided V is already known. T_o is then found by integration.

Provided large changes in the pressure do not occur rapidly, accumulation of vapour in the vapour space can be ignored. Then:

(i) the third effect vaporisation rate will be equal to the steam condensation rate in the fourth effect calandria;

(ii) the second effect vaporisation rate will be equal to the vapour bleed rate plus the vapour condensation rate in the third effect calandria.

This enables a value for V to be found. Until further data could be gathered, the vapour bleed rate from the second effect was equated to a function of its vapour temperature.

The level of syrup in each vessel is usually controlled automatically. It was therefore assumed that the syrup volume was constant in each vessel. Thus, the term $\frac{dL}{dt}$ is zero. Equation (1) was then used to solve for F_o by substituting a value for V found either from equation (5) (fixed pressure case) or from the calculations for the condensation rate in the succeeding effect (variable pressure case).

Equation (2) was solved in the form

$$\frac{dB_o}{dt} = \frac{F_iB_i - F_oB_o}{DL}$$

and B_o was found by integration.

5. Description of the computer program

The dynamic model of the MEE was programmed for the digital computer using the high level language CSMP (Continuous Systems Modelling Program) developed by IBM.

CSMP was designed to facilitate the digital simulation of continuous processes and it can be used to solve sets of simultaneous linear or non-linear differential equations. CSMP has the additional advantage that it can accept "functional blocks". A functional block is a set of calculations forming a unit which is common to different areas of a system. Thus, systems can be programmed as a number of functional blocks connected together by simple statements.

The pressures of the first and fourth effects were assumed fixed, so their vapour temperatures were constant, as shown in Figure 2. The vapour bleed from the first effect was assumed unknown while that from the second effect, which is used to heat the mixed juice, was equated to a function of its vapour temperature. The second and third effects were evaluated as variable pressure vessels. The computer program thus contained two functional blocks: one representing fixed pressure operation of a single calandria evaporator, and the other representing variable pressure operation. Vessels 1a and 1b were assumed to be identical.

A plotting subroutine was included to provide a graphical display of the output variables as a function of time for given feed conditions.

The program uses 126K of core and simulates 2,5 hours of real time in about 46 seconds of computer time, on an I.B.M. 360/65 machine.

The computer program can also be used for the steady-state analysis of various process running conditions. If the feed conditions are all constant the output values tend to a steady state. This "natural" convergence plays the same role as an artificial iterative algorithm that would be necessary in a purely static mathematical model.

6. Explanation of the graphical output from the computer program

To test the computer program, a number of runs were made with various input disturbances. The graphs in Figures 4, 5 and 6 were plotted by the computer. Each curve is labelled with an alphabetic haracter and can be identified through the cross-eference provided. (Note flow units are t/h).

Figure 4 shows the response to a step change in the feed flowrate (curve H). A 5% increase in the flowrate results in an 8% decrease in the brix from the fourth effect (curve G).

Figure 5 represents the effect of a complex sinusoidal disturbance in the feed brix (curve A). The input signal was a combination of two sine waves, one of period 12 minutes and the other 30 minutes. These periods were selected to show that such a disturbance is damped: the high frequency signal is completely filtered out while the low frequency signal is attenuated (curve G).

Figure 6 shows the influence of the first effect vapour pressure on the system. A sinusoidal disturbance of amplitude 1°C in the vapour temperature (curve P) has little effect on the final brix (curve G). It does, however, have a major influence on the first effect vapour flowrate (curve V), which, in turn, influences the fourth effect syrup exit flowrate (curve N).

7. Discussion and conclusion

A mathematical model of a multiple effect evaporator has been developed in which each evaporator was modelled as a perfectly mixed tank. This assumption requires justification because, as is apparent from Figure 1, the syrup actually travels in a circular path, ascending from the lower portion through the calandria tubes and descending in the downtake. Here part of it is recycled back into the lower portion and the rest leaves the vessel. A more realistic model would have treated the flow in the tubes as plug flow, and accounted for the change in pressure due to hydrostatic head. However, it would have been necessary to develop a complicated iterative procedure for calculating the circulation flow and velocity, and the vapour produced in the tubes. These calculations would have been similar to those for a thermosyphon reboiler (Fair[2]). This method was not used because it was felt that the added complexity, which

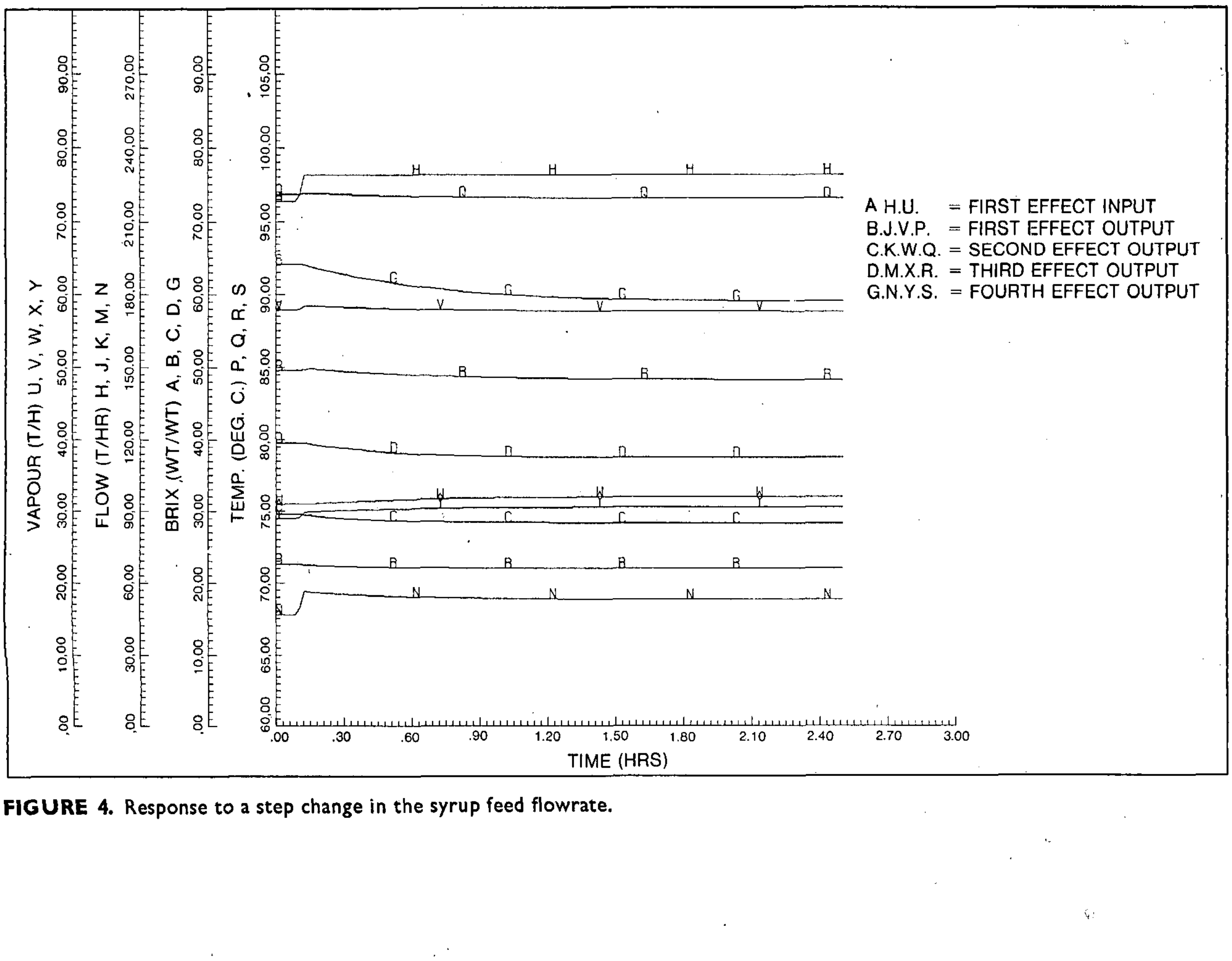


FIGURE 4. Response to a step change in the syrup feed flowrate.

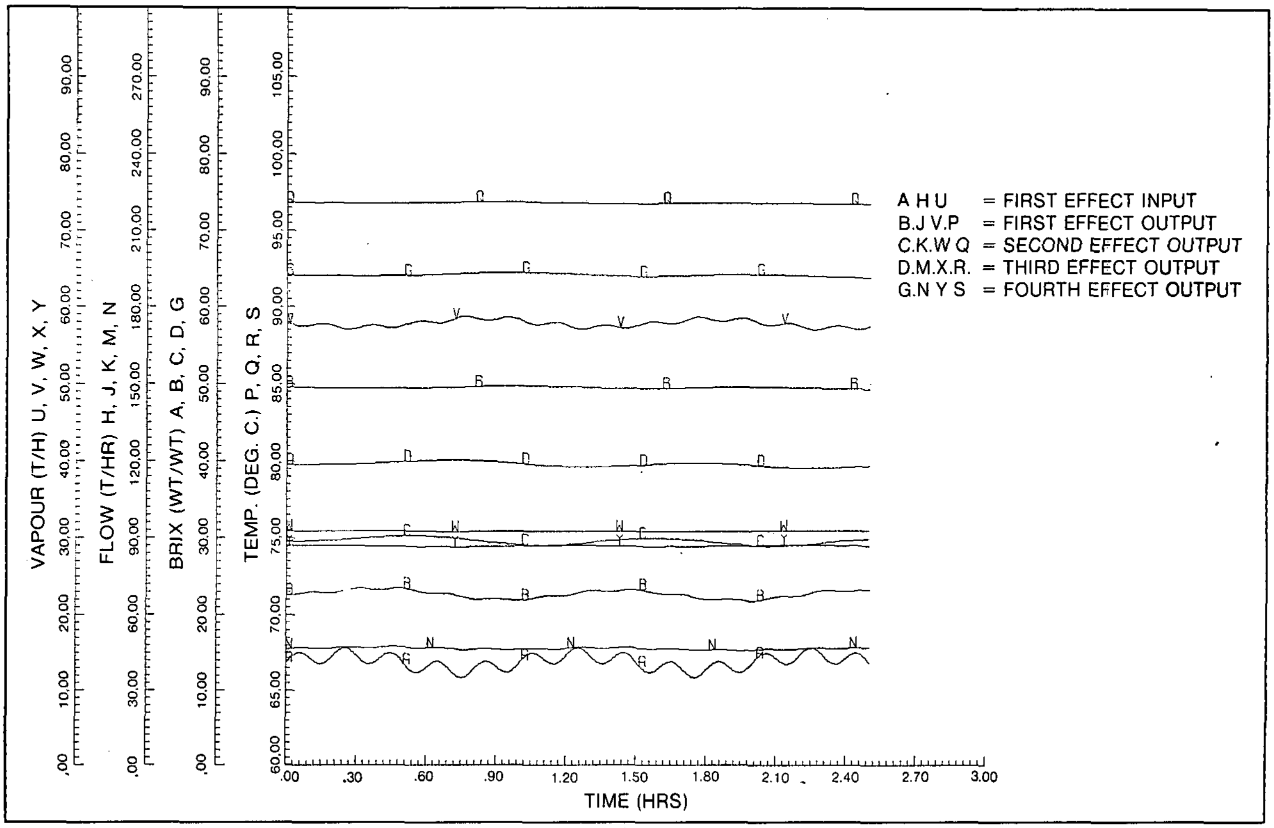


FIGURE 5. Response to a disturbance in the brix of the feed syrup.

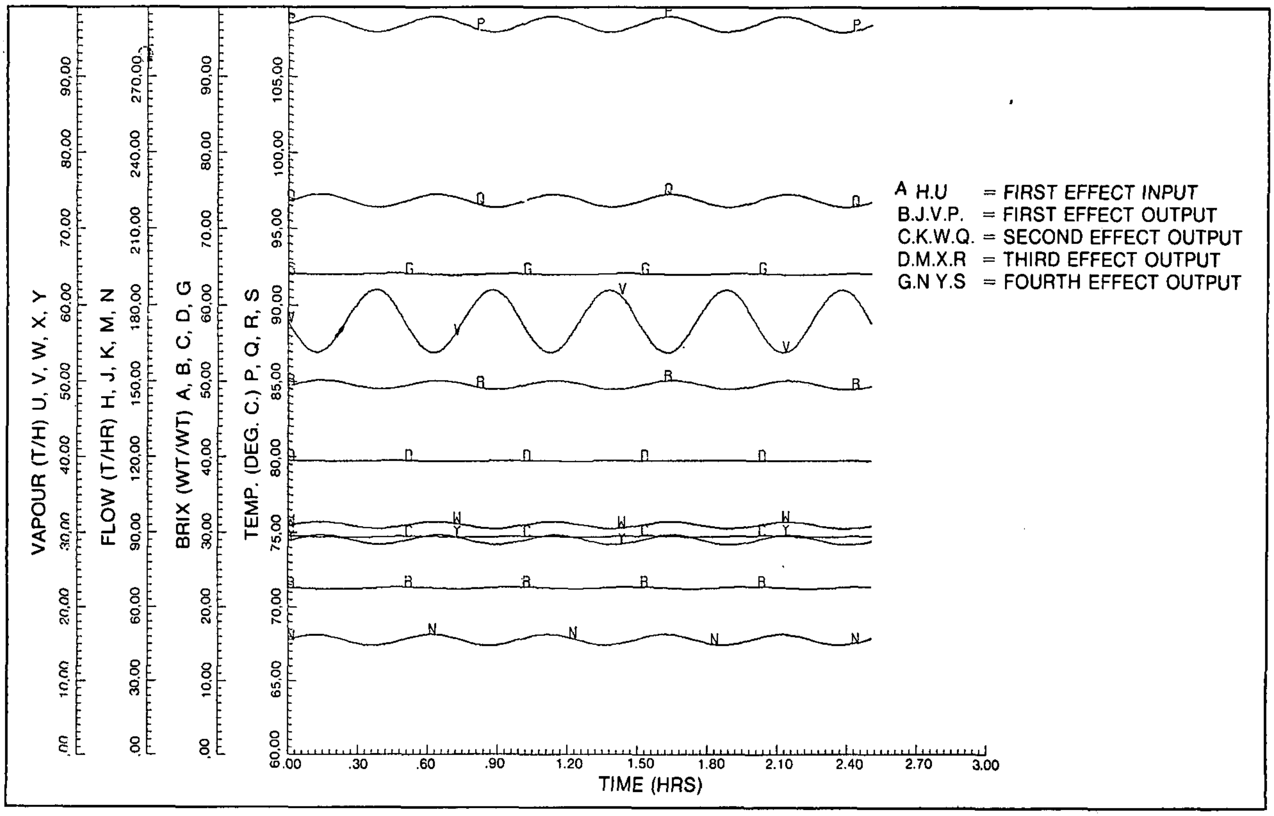


FIGURE 6. Response to a disturbance in the first effect vapour pressure.

would cause it to have taken up much more computer time, was not justified at this stage of development. By assuming perfect mixing, only the average conditions in the vessel (e.g. average pressure, temperature and heat transfer coefficient) need be considered.

The computer program of this model was tested by imposing disturbances on the input variables of flowrate, brix and pressure. The responses to these disturbances were as expected, and no instability was evident.

By adjusting the parameters in the heat transfer coefficient formula, the model could be fitted to any steady-state operating conditions. Owing to the lack of data, the range of operating conditions over which, for a fixed set of parameters, the model represents the process to a reasonable degree of accuracy has not yet been established. If this range is found to be sufficiently large, then the model could be incorporated in an overall factory simulation, and, with the addition of an optimising criterion and a parameter updating subroutine, could be used to find the optimum set of operating conditions.

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REFERENCES

1. Coates, J. and Pressburg, B. S. 21 March, 1960. Multiple Effect Evaporators. *Chemical Engineering*, pp. 157-160.
2. Fair, J. R., February, 1960. What you need to design thermosyphon reboilers. *Petroleum Refiner:* 39 (1960), pp. 105-123.
3. Hugot, F., 1972. Handbook of Sugar Cane Technology, 2nd Edition, Elsevier.
4. Jernqvist, A., Olgard, G. and Hedstrom, B., 15 August, 1966. A digital computer program for multiple effect evaporation. *Svensk Papperstidning*, 69 (1966), pp. 477-483.
5. Kern, 1950. Process Heat Transfer. McGraw-Hill, New York.
6. Perry, 1963. Chemical Engineering Handbook, 4th Edition, McGraw-Hill.
7. Spencer-Meade, 1963. Cane Sugar Handbook, 9th Edition, Wiley.
8. Withers, R. M. J., Bass, R. J. and Branch, M. F., June, 1968. Process Optimisation. *British Sugar Corporation, Ltd.*, 19*th Tech. Conf., Folkestone.*

Appendix 1

DERIVATION OF HEAT TRANSFER COEFFICIENT (U)

Owing to the lack of information concerning impure sugar solutions, and because conventional formulae, such as the Dittus-Boelter equation for heat transfer (Perry[6]) gave unsatisfactory results, a new equation was derived from first principles and fitted by non-linear regression to a set of factory data for a multiple-effect evaporator.

It was assumed that all the usual parameters influencing heat transfer, such as viscosity, thermal conductivity, heat capacity, etc., which are included in published formulae, could be expressed as functions of concentration and temperature. The proportion of vapour in the stream and the stream velocity also needed to be included. Therefore three basic parameters were used: brix, temperature and vapour mass flow rate.

Intuitively, an increase in the concentration would cause an increase in the viscosity and a decrease in U. Since the relation was not likely to be linear, the form $(1-\alpha B)^a$ was chosen to represent the dependence of U on the brix.

Likewise, an increase in temperature would cause an increase in U, so $(1+\beta T)^b$ was chosen as the functional form.

Lastly, an increase in the vapour rate would yield a higher flow velocity within the calandria tubes and hence a larger value for U, so the term $(1+\gamma V)^c$ was included. These three functions were combined in the form

$$U=K(1-\alpha B)^a\,(1+\beta T)^b\,(1+\gamma V)^c.$$

Initial values for the constants ^{a}, ^{b}, ^{c}, α, β, γ and K were then found by non-linear regression using values of U calculated from data in the table below. Boiling point elevation was taken into account, but flash vaporisation and heat loss to the atmosphere were initially neglected. The formula so obtained was then used in a steady state computer model of the multiple effect evaporator, where the same seven constants, together with a proportionality constant for the heat loss to the surroundings, were adjusted automatically to give the best fit to the same set of physical data.

The final formula used in the computer program was:

$$U=49{,}093\,(1+15{,}6V)^{0{,}776}\,(1-8{,}998\times 10^{-3}B)^{0{,}400}\,(1+0{,}2518T)^{1{,}036},$$

where V is expressed in t/s.

TABLE 1

Data for the multiple effect evaporator*

Effect No.	Vapour rate t/h	Heat transfer area (m^2)	Vap. temp. (°C)	Exit flowrate t/h	Brix in exit stream	U W/m^2°C
1	41,8	1 390	110	134	22,2	1 884
2	33,7	1 210	98	101	29,6	1 634
3	26,6	1 210	86	74	40,2	1 367
4	27,1	1 210	60	47	63,4	663

Feed brix = 13,7

Feed flowrate = 218 t/h

Exhaust steam temperature = 121°C

*Average operating conditions of the Jaagbaan installation during the 1968 season.

Appendix 2

SOURCE OF AUXILIARY EQUATIONS USED IN THE COMPUTER PROGRAM

(i) *Density*

By assuming that the volume of sucrose solution is equal to the sum of the volumes of the pure water and pure sucrose of which it consists, the following expression can be derived: $D=1/(1-0{,}00358\ B)$.

The density of pure water was taken as 1 t/m^3 and that of sucrose as 1,555 t/m^3.

(ii) *Boiling point elevation* (^{3,5,7})

$\Delta T_B=2{,}42\ B/(100-B)$ °C

(iii) *Heat capacity* (^{3,5,7})

$C_p=(1-0{,}0055\ B)\times 4{,}19\times 10^6$ J/t.

(iv) *Heat loss to the atmosphere*

$Q_{atm}=k(T_o-T_{amb})$

where k=(heat transfer coefficient) ×(surface area of vessel).

T_{amb}=ambient temperature (taken as 25°C).

The proportionality constant k was assumed to be the same for all vessels and was found to be 897,6 by the non-linear regression procedure mentioned earlier (see "Derivation of heat transfer coefficient").