

Curvelet based Image Compression using Support Vector Machine and Core Vector Machine – A Review

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Abstract

Images are very important documents nowadays. To work with them in some applications they need to be compressed, more or less depending on the purpose of the application. To reduce transmission cost and storage requirements, competent image compression schemes without humiliation of image quality are required. Several image coding techniques were developed so far for both lossless and lossy image compression. Extensions of 1-D transforms such as wavelet transform have limitations of capturing geometry of image edges. Functions that have discontinuities along straight lines cannot be effectively represented by normal wavelet transforms but natural images have geographic lines such as edges, textures which cannot be well reconstructed if compression is done by 1-D Transforms. Nowadays image coding is done, using Curvelet Transform since it supports different orientations of image textures. An investigation is done on various types of image coding techniques based on Curvelet Transform that exist. This paper deals with study of image compression techniques using Curvelet Transform based on Support vector machine and Core vector machine with their performance results.

Keywords

Curvelet Transform, Support vector machine, Core vector machine, Image Compression, Image Coding.

1. Introduction

The technique used to decrease data storage requirements and communication costs, is data compression, which reduces redundancies in data representations. Due to the pervasive distribution

digital image contents, compression of images or data outcomes inconsiderable reduction in the storage capacity of the memory devices. Transferring images without compression over digital networks needs very high bandwidth. Due to that the need for efficient storage and transferring medical images is noticeably increasing, image compression is essential. The heart of any image processing tasks is an efficient representation of visual information lies in the image. The medical data is articulated as images or other types of digital signals, such as Magnetic Resonance Imaging, Computer Tomography, Ultrasound, Positron Emission Tomography etc.

Transferring image information into transform domain will be more competent rather the image itself. Image transformations can be done using various transforms such as Discrete Cosine Transform, Discrete Fourier Transform, Wavelet Transform, Contourlet Transform etc. Compression can be achieved by transferring data into transform domain, quantizing the transformed coefficients and encoding the quantized coefficients.

To avoid redundancy, the transform must be at least biorthogonal, and to save CPU time, the transform's algorithm must be fast. Various image compression techniques are exists so far. Even so observations have noted that wavelets may not be best choice presenting natural images. This is because the tendency of wavelets to ignore to smoothness along the edges (cannot provide 'sparse' representation). This property has been taken advantage by some novel transforms like ridge let and curvelet transform. These can be formed into elements which are anisotropic and exhibit high directional sensitivity.

The curvelet discrete transform are theoretically simpler, faster and less redundant than earlier implementations. In this paper, we have reviewed some Curvelet based image compression techniques with Support vector machine and Core vector machine which approximates the curvelet coefficients using a fewer support vectors and weights.

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2. The Curvelet Transform

To overcome the drawbacks of wavelet transform, a new multi-resolution transform was developed by Candés and Donoho in 1999. The transform that is a two-dimensional anisotropic extension of wavelet, originally designed to represent edges and other singularities beside curves better than wavelet transforms [5]. Although curvelets is an extension of wavelets, there exists an association between curvelet and wavelet sub bands.

In wavelet transform, the elements have only location and scale parameters. In curvelet transform, the elements have location, scale and orientation parameters. The fundamental defect reside in wavelet transform is that, unable to represent edge discontinuities along curves. In compression process, limited number of coefficients are required but in case of reconstructing the edges properly along curves several wavelet coefficients are employed. This is mainly because of the reason that, in case of mapping large wavelet coefficients, edges are repeated at scale after scale [12]. It required a transform that can handle 2D singularities along the curves sparsely distributed. As a result new multi resolution wavelet transform is produced [13]. Curvelet basis elements possess wavelet basis function qualities but these also oriented at various directions as a result of it edge discontinuities and other singularities are well defined by it when compared to wavelet transform [14]. Curvelet transform comes under multiscale geometric transform. One of the special member in it wavelet transform. This transform has multiscale pyramid with many directions at each length and scale.

The superiority of curvelets over wavelets in cases such as,

- i. Optimally sparse representation of objects with edges.
- ii. Optimal image reconstruction in several ill-posed problems.
- iii. Optimal sparse representation of wave propagators.

3. The Second Generation of Curvelet Transform

The Curvelet transform has been taken into two major revisions. At beginning the curvelet transform (“curvelet 99”transform now) used a computer series of steps involving the ridgelet analysis of the radon transform of the image [candes & Donoho, 2000]. It was found that it performance was slowly exceeding.

Soon after they introduced, researchers developed numerical algorithms for their implementation [Donoho Duncan, 2000] and reported on a series of practical successes [Starck, Murtagh, Candes & Donoho, 2003].

In order to make use of curvelets and easy understanding curvelets are redesigned. In this new method, the use of the ridgelet transform was eliminated, as a result redundancy is reduced and speed is increased. It is faster, simpler and less redundant than “curvelet 99” transform [2] and [10].

3.1 Continuous-Time Curvelet Transforms

Authors have worked throughout in 2D(i.e., R^2) with spatial variables ‘x’, with ‘w’ a frequency domain variable, and with r and θ polar coordinates in the frequency domain. They start with a pair of windows w(r) called as “radial window” and v(t) called as “angular window”[7]. These windows are smooth, non-negative and real-valued, with W taking positive real arguments and supported on $r \in (1/2, 2)$ and ‘v’ taking real arguments and supported on $t \in [-1, 1]$. For each $j \geq j_0$, the frequency window U_j is defined in the fourier domain as,

$$U_j(r, \theta) = 2^{-3j/4} W(2^{-j}r) V(2^{[j/2]} \theta/2\pi) \quad \text{-----(1)}$$

According to equation 1, U_j is a polar “wedge” window, as show in Fig. 1.

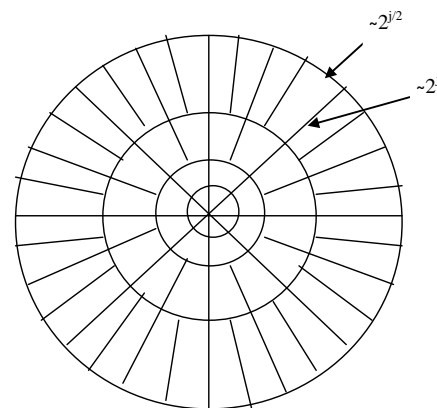


Figure 1: Continuous curvelet support in the frequency domain

To define the waveform $\phi_j(x)$ by means of its Fourier transform, $\phi_j(x)$ is a “mother” curvelet in the sense that all curvelets at scale 2^{-j} can be obtained by rotations and translations of $\phi_j(x)$. Introduce the equispaced sequence of rotation angles $\theta_l = 2\pi \cdot 2^{-[j/2]} \cdot l$ with $l = 0, 1, \dots$ such that $0 \leq \theta_l < 2\pi$, and the sequence of translation parameters $k = (k_1, k_2) \in Z^2$.

With these notations, curvelets are defined by

$$\omega_{j,l,k}(x) = \omega(R_{\theta}(x-x_c)^{j,l}) \quad \text{-----}(2)$$

where R_{θ} is the rotation by θ radians.

A curvelet coefficient is then simply the inner product between an element $f \in L^2(\mathbb{R}^2)$ and a curvelet $\phi_{j,l,k}$,

$$c(j,l,k) := \langle f, \phi_{j,l,k} \rangle = \int_{\mathbb{R}^2} f(x) \overline{\phi_{j,l,k}(x)} dx \quad \text{---}(3)$$

Reconstruction formula is

$$f = \sum \langle f, \phi_{j,l,k} \rangle \phi_{j,l,k} \quad \text{-----}(4)$$

3.2 Digital Curvelet Transform

The window U_j smoothly extracts frequencies near the dyadic corona and near the angle in continuous time. For cartesian array, the corona and rotations are not especially adapted. It is convenient to replace them by Cartesian equivalent. It is done based on concentric squares and shears [1] and [8]. The ‘‘Cartesian window’’ can be defined as

$$\tilde{U}_j(\omega) := W_j(\omega) V_j(\omega) \quad \text{-----}$$

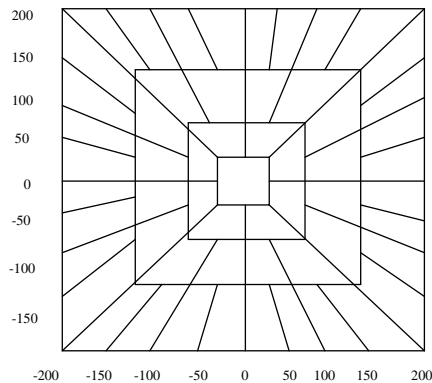


Figure 2: Digital curvelet tiling of space and frequency

$W_j(\omega)$ is a window of the form

$$W_j(\omega) = (\varphi_{j+1}^2(\omega) - \varphi_j^2(\omega))^{1/2}, \quad j \geq 0 \quad \text{-----}(6)$$

where φ is defined as the product of low-pass one-dimensional windows

$$\varphi_j(\omega_1, \omega_2) = \varphi(2^{-j} \omega_1) \varphi(2^{-j} \omega_2) \quad \text{-----}(7)$$

The function φ obeys $0 \leq \varphi \leq 1$, might be equal to 1 on $[-1/2, 1/2]$, and vanishes outside of $[-2, 2]$ [11].

4. Curvelet Transform and Support Vector Machine for Image Compression

4.1. SVM Regression for Image Compression

Because of good generalization ability the SVM has been widely used. At first, it is designed to solve pattern recognition problem. Regression is an extension use of classification. It is a non-separable classification that each data point can be taught of being as its own class.

In regression process, a set of training points are given, the real function is approximated with in a predefined error ϵ by choosing the minimum number of training points. There is a corresponding weight for each training point chosen by the SVM (support vector). Number of Vectors and Weights is less than training points, which is that SVM regression can carry out data compression.

The regression problem can be formulated as follows:

$$f(x,w) = \sum_{i=1}^N w_i \phi_i(x) \quad \text{-----}(8)$$

SVM attempts to learn the input - output relationship from the given training points $(x_1, y_1), (x_2, y_2), \dots, (x_l, y_l)$ where $x_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}$. In the case of regression, vapnik’s linear loss function is used with insensibility zones as a measure of the error between $f(x)$ and y .

$$\begin{aligned} \text{Error} &= |f(x,w) - y| = 0 \quad \text{if } |y - f(x,w)| \leq \epsilon \\ &= |y - f(x,w)| - \epsilon \quad \text{if } |y - f(x,w)| > \epsilon \quad \text{---}(9) \end{aligned}$$

4.2. Compression of Curvelet Coefficients using SVM

Initially, the original image is decomposed into number of sub-bands. These sub-bands are the representation of image in different frequency range and has different importance. Most of the image energy is resides in lowest sub-band, in the reconstruction of image it plays a vital role because low frequency information is highly sensible to human eyes. So the authors have used different compression technique for different sub-bands to import information with given bit rate. The lowest sub-band is encoded by DPCM, which is nearly lossless. The SVM regression compressed the finer scale sub-bands which approximates the curvelet coefficients using a fewer support vectors and weights. In addition, some of the finer scale sub-bands are discarded directly due to the reason that

they only contain a little amount of energy and have little noticeable effect on image quality.

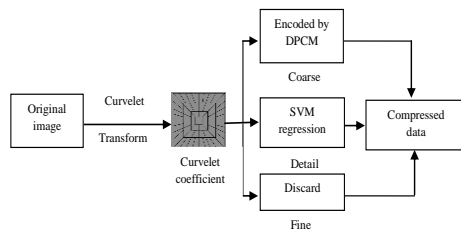


Figure 3: Compression Scheme

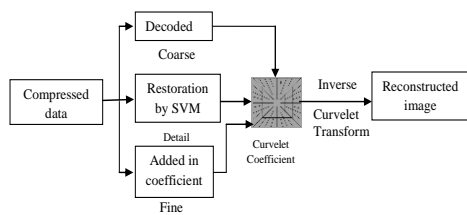


Figure 4: Decompression Scheme

4.3. Curvelet Coefficients Organization

By means of fast discrete curvelet transform the original image was decomposed into frequency domain. Based on decomposition rules, we can get the scale number ($n_{scales} = \log_2 n - 3$, where $[m, n] = \text{size}(\text{image})$). In this paper, n_{scales} equals 6, because the size of image they used was 512 pixels \times 512 pixels.

After the process of decomposition the original image was divided into three levels: coarse Detail and fine. The low frequency coefficients were assigned to coarse (inner level). The high frequency coefficients were assigned to fine (outer most level). The middle frequency coefficients were assigned to Detail. The detailed structure of the curvelet coefficients are shown in Table 1.

The characteristics of curvelet coefficients are given below:

- (1) Most of the image energy is compressed into the lowest sub-band. The rest energy is spread over other sub-bands, reducing from low frequency to high frequency.
- (2) The highest value of coefficients focused on the first level.
- (3) The lowest value of coefficients focused on the final level.
- (4) With the enhanced number of scale, coefficients include more zero.

4.4. Curvelet Coefficients Normalization

The important step of image compression is normalization. Normalizing curvelet coefficients is used in SVM regression method. It will produce the weight that are lower in magnitude but having similar value and also makes the weight more compressible. It is mainly used to overcome the coefficients of different subbands variation.

Coefficients normalization can be done using the relation

$$c' = \frac{c - c_{\min}}{c_{\max} - c_{\min}} \quad \text{-----(10)}$$

Where c_{\min} and c_{\max} are the minimal and the maximal curvelet coefficients in the sub-band, respectively, c is the coefficient to be normalized and c' is the value after normalization.

4.5. Curvelet Coefficient Encoding

We can get the support vectors and weights after SVM regression and they should be encoded in the coding bit stream. In decoding process, to produce the original curvelet coefficients by the regression modes, the sum of regression modes is used with the support vectors and weight [9].

The weight in the SVM regression provides the support vector one by one. If any input training point is chosen as the support vector, it should have some corresponding weight. The support vector has two meanings in the SVM regression method: One is input and second one is the position of input. Finally the support vectors and weight are combined and encoded together.

5. Curvelet Transform and Core Vector Machine for Image Compression

5.1. Core Vector Machine for Image Compression

SVM has been widely used in many areas because of its good generalization ability. In SVM implementations, the training time complexity is scales between $O(m)$ and $O(m^{2.3})$ and it can be further driven down to $O(m)$ alongwith the use of parallel mixture[15]. By generalizing the underlying minimum enclosing ball problem by CVM algorithm, it can be used with any linear or non-linear kernels and also it obtains optimal solutions appropriately provable. In the number of training patterns 'm', the asymptotic time complexity of CVR is linear, while the space complexity of CVR is independent of 'm'.

Authors show that CVR is independent of ‘m’. Authors have shown that CVR has comparable performance with SVR, the CVR method is much faster and it produces fewer core vectors on very

large data sets. So the CVR method can be inserted into the image compression algorithm to gain much improvement of compression ratio [1].

Table 1: Structure of the Curvelet Coefficients

Levels	Scales	Orientations	Matrix form				
Coarse	Cell[1]	1	32 x 32				
Detail	Cell[2]	32 (4 x 8)		16X12	12x16	16x12	12x16
	Cell[3]	32 (4 x 8)		32x22	22x32	32x22	22x32
	Cell[4]	64(4 x 16)		64x22	22x64	64x22	22x64
	Cell[5]	64(4 x 16)		128x44	44x128	128 x 44	44x128
Fine	Cell[6]	1	512 x 512				

5.2. The CVM Algorithm

For the Core-set ‘St’, the ball’s center ‘ct’ and radius ‘Rt’ at the tth iteration, the CVM algorithm follows as given below:

- i. Initialize S0, c0 and R0.
- ii. Stop if there is no training point z such that $\phi(z)$ falls outside the $(1+\epsilon)$ -ball $B(ct, (1+\epsilon)Rt)$.
- iii. Find z (core vector) such that $\phi(z)$ is furthest away from ct. Set $St+1 = St \cup \{z\}$. This can be made more competent by using the probabilistic speedup method that finds a ‘z’ which is only approximately the outermost.
- iv. Find new MEB(St+1) and set $ct+1 = cMEB(St+1)$ and $Rt+1 = rMEB(St+1)$.
- v. Increment t by step1 and go back to Step2.

5.3. Compression achieved by CVM

Corresponding to the orientation characteristics of each subband, the proper scan order is used to map from two dimension block into one dimension vector. It is called as ‘y’ for convenient. To form the vector ‘x’ by the positions of the elements in ‘y’. The input is ‘x’ and the output ‘y’ in the CVM regression model. In training process, moreover the \mathcal{E} , kernel type and kernel parameters affects the compression efficiency in the model. Different types of data is suited by different kernels. Due to that the coefficient from one block is almost considered as Gaussian function, the Gaussian function is choosed as the regression kernel[1].

6. CVR versus SVR

CVR algorithm can be used with any linear non-linear kernels and also it obtains optimal solutions appropriately provable. In the number of training patterns ‘m’, the asymptotic time complexity of CVR is linear, while the space complexity of CVR is independent of ‘m’. The CVR method is much faster and it produces fewer vectors on very large data sets. The performance of CVR is better than the SVR implementation when the data set is large, and it produces fewer support vectors. More over all the core vectors are useful support vectors. On the very large data set , the time required to find theoretical is constant with respect to the training set.

7. Performance Parameters

Performance of any image compression can be obtained by PSNR (Peak Signal-to-Noise Ratio) and CR (Compression Ratio) parameters.

$$PSNR(dB) = \frac{20 \times \log_{10}(\text{Maximum pixel value})}{(MSE)^{1/2}} \quad \text{---(11)}$$

where, MSE represents the mean squared error

$$MSE = 1/N \times \sum_i \sum_j (f(i,j) - F(i,j))^2 \quad \text{----(12)}$$

f(i,j) denotes the pixel value in the original image.

F(i,j) denotes the pixel value in the reconstructed image.

$$CR(bpp) = \frac{\text{Size of the compressed image}}{\text{Total number of pixels}} \quad \text{-----(13)}$$

reconstructed image. Some of the readings were taken from the above literature to represent the performance parameter.

Table 2: Comparison of image compression techniques based on Curvelet Transform

Reference	Technique used	PSNR /dB	CR %	CPU Time/s
[1]	Curvelet Transform and Core Vector Machine	27	22	30.2
[2]	Curvelet Transform and Support Vector Machine	26.93	22	34.4

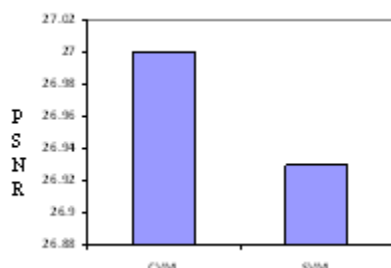


Figure 5: Graphical Representation of PSNR from Table 2

8. Conclusion

Image quality of the image after compression is the main criteria that all the compression techniques should hold. Here we discussed some existing image compression techniques based on Curvelet Transform with their performance results. In section 4, Curvelet Transform and support vector machine for Image compression is discussed. In section 5, Curvelet Transform and core vector machine for Image compression is discussed. Experimental results

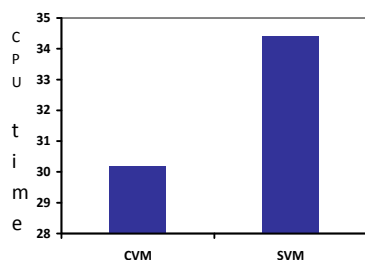


Figure 6: Graphical Representation of CPU time from Table 2

from both the papers shows that the curvelet transform along with core vector machine gains better compression performance than that of curvelet transform along with support vector machine both in PSNR and CPU time. At the same time, the algorithm works fairly well for declining block effect at higher compression ratios. The results are only a preliminary investigation of compressing curvelet coefficient using CVM regression, and there is much can be done to improve the performance. For example, the method of encoding curvelet sub-bands should be more flexible, which makes CVM learn data dependency more efficiently.

For real-time image transmission or storage process, all the compression techniques are useful. Each one of the coding techniques is different from the other. The selection of high PSNR value will lead to maintain the quality of the image and success in compression process.

References

- [1] Yuancheng Li, Yiliang Wang, Rui Xiao, Qiu Yang, “ Curvelet based image compression via core vector machine” , ELSEVIER – Optik – International Journal for Light Electron Optics, Volume 124, Issue 21, November 2013, pp.4859-4866.
- [2] Yuancheng Li, Qiu Yang, Runhai Jiao, “ Image compression scheme based on curvelet transform and support vector machine” , ELSEVIER – Expert Systems with Applications, Volume 37, Issue 4, April 2010, pp.3063-3069.
- [3] Arvind Kourav, Prashant Singh, “Advanced Technique for Feature Extraction and Image Compression”, International Journal of Computer Applications, Volume 68, No.21, April 2013, pp.22-27.
- [4] A.Sivanantha Raja, D.Venugopal, S.Navaneethan, “An Efficient Coloured Medical Image Compression Scheme using Curvelet Transform”, European Journal of Scientific Research, ISSN 1450-216X Vol.80, No.3, 2012, pp.416-422.
- [5] Nilima D.Maske, Wani V.Patil, “Comparison of Image Compression using Wavelet for Curvelet Transform & Transmission over Wireless Channal”, International Journal of Scientific and Research Publications, Volume 2, Issue 5, May 2012, pp.1-5.
- [6] Yan Zhang, Tao Li, Qingling Li, “Defect detection for tire laser shearography image using curvelet transform based edge detector”, ELSEVIER – Optics & Laser Technology 47 (2013), pp.64-71.

- [7] Walaa M. Abd-Elhafiez, "Image Compression Algorithm using a Fast Curvelet Transform", International Journal of Computer Science and Telecommunications, Volume 3, Issue 4, April 2012, pp.43-47.
- [8] K.Siva Nagi Reddy, L.Koteswara Rao, B.R.Vikram, P.Ravikanth, "Image Compression by Discrete Curvelet Wrapping Technique with Simplified SPIHT", International Journal of Computer Applications, Volume 39, No.18, February 2012, pp.1-9.
- [9] T.Rammohan, K.Sankaranarayanan, Shalakh Rajan, "Image Compression using Fusion Technique and Quantization", International Journal of Computer Applications, Volume 63, No.22, February 2013, pp.8-11.
- [10] Anil Balaji Gonde, R.P.Maheswari, R.Balasubramanian, "Modified curvelet transform with vocabulary tree for content based image retrieval", ELSEVIER-Digital Signal Processing 23 (2013), pp.142-150.
- [11] Rashid Minhas, Abdul Adeel Mohammed, Q.M.Jonathan Wu, "Shape from focus using fast discrete curvelet transform", ELSEVIER-Pattern Recognition 44 (2011), pp.839-853.
- [12] Starck, J.L.Murtagh, F.Candes, E.J.Donoho, "Gray and Color image contrast enhancement by the curvelet transform", IEEE Transaction on Image Processing, 12(6), 2003, pp.706-717.
- [13] Iqbal, M.A.Javed, M.Y.Qayyum, "Curvelet-based image compression with SPIHT", International Conference on convergence information technology, IEEE Computer Society, 2007.
- [14] J.L.Starck, E.J.Candes, D.L.Donoho, "The Curvelet transform for image denoising", IEEE Transaction on Image processing, 11(6), 2002, pp.670-684.
- [15] K.K.Seo, "An Application of one-class support vector machines in content-based image retrieval", ELSEVIER - Expert Systems with Applications, Volume 33, 2007, pp.491- 498.



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