Fuzzy Model-Based Robust Networked Control for a Class of Nonlinear Systems

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Fuzzy Model-Based Robust Networked Control for a Class of Nonlinear Systems

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***Abstract*—In this paper, the robust stability of a networked control system via a fuzzy estimator (FE) is studied, where the controlled plant is a class of nonlinear systems with external disturbances, which can be represented by a Takagi–Sugeno fuzzy model. Both network-induced delay and packet dropout are concerned. In the developed control scheme, the FE is used to estimate the states of the controlled plant for the purpose of effectively reducing the network burden. Based on the limited knowledge of a controlled plant in the presence of a network, a disturbance attenuation term is also employed to attenuate the influence of modeling errors and external disturbances on the system. The sufficient condition for the robust stability with H_∞ performance of the closed-loop system is obtained. The simulation results show the validity of the proposed control scheme.**

***Index Terms*—Fuzzy H_∞ control, fuzzy estimator (FE), networked control system (NCS), network-induced delay, packet dropout.**

I. Introduction

With the rapid development of digital control and communication network technology, feedback control systems in which control loops are closed via a real-time network are becoming increasingly important. Such systems are called networked control systems (NCSs). In NCSs, sensors, actuators, and controllers are interconnected via communication networks, which makes systems easier to install and maintain. Recently, much attention has been paid to the stability analysis and controller design of NCSs [8]–[13], [17], [18], [20], [22]–[24], where network-induced delay and packet dropout are two crucial issues.

While some interesting techniques and results have been presented in the aforementioned publications, the control of NCSs still remains an open problem. For example, most of the control schemes previously mentioned were developed only focusing on linear NCSs; nonlinear NCSs have received little attention, although some issues related to nonlinear NCSs have been investigated such as asymptotic behavior [18], input-to-state stability [12], input-to-output $\mathcal{L}_p$ stability with disturbances [13], and model-based method [11]. It should be noted that these results on nonlinear NCSs are only for the stability analysis without addressing controller design.

It is well-known that Takagi–Sugeno (T–S) fuzzy models are qualified to represent a certain class of nonlinear dynamic systems [15], [16] and many corresponding control techniques have been developed in the literature. A typical approach for controller designs is via the so-called parallel-distributed-compensation method [16]. Using the T–S fuzzy model, some results on NCSs' controller designs have recently been published [22]–[24]. In [24], a fault detection method for NCSs with Markov delays was addressed, where a linear plant was modeled in the discrete-time domain, and a set of T–S fuzzy rules were used to deal with network-induced delays. In contrast to controller design methods in the discrete-time domain, results in [22], [23] were formulated in the continuous-time domain, where the T–S fuzzy systems with norm-bounded uncertainties were utilized to characterize the nonlinear NCSs. The robust H_∞ control scheme [22] and the guaranteed cost control scheme [23] were developed. However, the control signal in [22] and [23] is not a continuous function but a piecewise constant function, which may reduce the robustness of the NCSs to some degree due to the sampling behavior. Therefore, a novel fuzzy model-based control method is needed to guarantee the control signals being a continuous function in the nonlinear NCSs.

In this paper, a system framework is first introduced (see Fig. 1) and a corresponding robust control scheme is developed based on a fuzzy estimator (FE), where the network is modeled as a sampler placed between the controlled plant and the controller/actuator. In the developed approach, the unknown nonlinear plant is first expressed by a T–S fuzzy model. Then, an FE is proposed to estimate the plant states in a network-based environment where the transmission of sensor data is not instantaneous, but as a data packet to the FE. In addition, the instant of packets arriving at the FE is uncertain because of network-induced delay and packet dropout. Due to these limited sampling data, an FE is needed to estimate the plant

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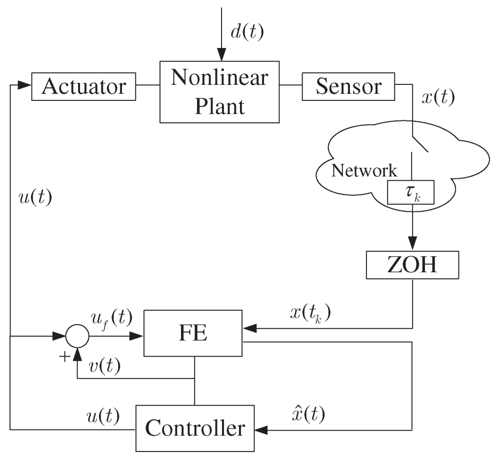

Fig. 1. Framework of FE-NCS.

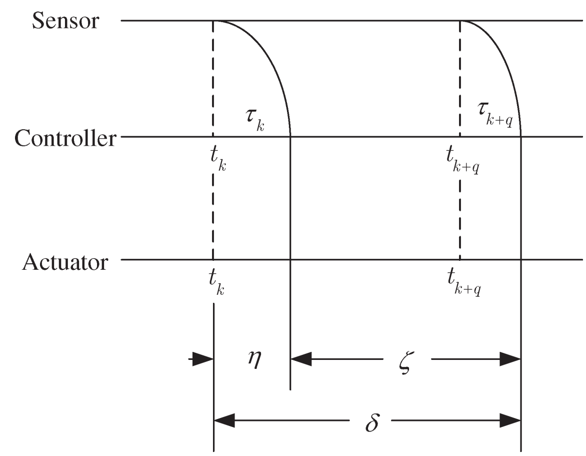

Fig. 2. Time-sequence diagram of the signals in the NCSs.

states, including sampling instant and during two sequential *effective* packets since the packets may drop out or disorder. In particular, the estimation can be updated when each effective packet arrives at the FE. The feedback control is then performed by using the states of the FE. Similar to other model-based approaches [11], the FE-based approach will also reduce the number of data packets transmitted. The FE is designed with two additional terms. The first is a disturbance attenuation term, which is to attenuate the influence of modeling errors and external disturbances on the system. The other is called the estimator gain term, which is introduced to improve the estimation precision of the FE. Due to these two additional terms, the states of the FE are continuous, being convenient to provide the control signal as a continuous function in the continuous-time domain, although actual plant states transmitted via the network are piecewise constant functions as a result of the existence of zeroth-order hold (ZOH). There are several important advantages of the proposed results that are worthy of mentioning. First, the disturbance attenuation problem for nonlinear NCSs is dealt with via the FE-based method. Second, the robust control scheme is studied in the continuous-time domain, i.e., the intersampling behavior is taken into account. Third, a sufficient condition of the fuzzy H_∞ control scheme is proposed by solving a set of linear matrix inequalities (LMIs), which is convenient for the controller design.

Before presenting the results, some notations are required. Throughout this paper, the superscript T stands for matrix transposition, and $*$ always denotes the symmetric block in one symmetric matrix. The notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$, means that X is symmetric and positive definite (respectively, positive semidefinite). Identity and zero matrices, of appropriate dimensions, will be denoted by I and 0, respectively.

II. FE-NCS Description

The framework of the FE-based NCS (FE-NCS) is shown in Fig. 1. The sensor is connected with the controller/actuator via a network which is shared by other NCSs and subjected to data packet dropout and network-induced delay. The FE is used for the controller/actuator to estimate the plant states and offer the continuous control input signals even if the sensors data are not available during the intersampling period.

Let $t_k, t_{k+1}, \ldots, t_{k+q} (k = 0, 1, 2, \ldots)$ be the sampling instants, $h = t_{k+1} - t_k$ be the sampling period, and $\tau_k, \tau_{k+1}, \ldots, \tau_{k+q}$ be the corresponding network-induced delays, respectively, where q is a positive integer. It is assumed that the computation delay is negligible and $\tau_0 = 0$. Thus, the control input signals may be obtained at the instant $t_k + \tau_k, t_{k+1} + \tau_{k+1}, \ldots, t_{k+q} + \tau_{k+q}$.

Two definitions are often used when developing the controller for an NCS. One is the maximum allowable delay bound η, which is defined as the maximum allowable interval from the instant when sensor nodes sample sensor data from a plant, to the instant when actuators output the transferred data to the plant [9]. The other is the maximum allowable transfer interval ζ, which is defined as a deadline if a transmission of control data takes place at time $t_k + \tau_k$, then another one must occur within the time interval $[t_k + \tau_k, t_k + \tau_k + \zeta)$ [18]. In this paper, the maximum allowable control interval is defined as $\delta = \eta + \zeta$ by combining the above two definitions, which is used to analyze the network-induced delay and packet dropout problems (see Fig. 2).

Remark 1: If a transmission of packet takes place at time t_k, the packet will reach the FE after τ_k, namely, the instant $t_k + \tau_k$. Simultaneously, the control data will be sent to the nonlinear plant. Then, the next control data should take place within the time interval $(t_k, t_k + \delta]$. Therefore, it is shown that δ can be defined as a bound in order to guarantee the system stability. Notice that the maximum allowable control interval δ is relative to both network-induced delay and packet dropout cases, while [9] and [18] only concern network-induced delay case.

Consider the situation that two sampling data packets arrive at the controller at instant $t_k + \tau_k$ and $t_{k+q} + \tau_{k+q}$ in sequence.

If $q = 1$, no packet dropout occurs during the time interval $[t_k, t_{k+1} + \tau_{k+1})$.

If $q > 1$, $q - 1$ packets are lost during the time interval $[t_k, t_{k+q} + \tau_{k+q})$.

For a given δ, if the following inequality:

$$t_{k+q} + \tau_{k+q} - t_k < \delta \quad (1)$$

holds, then the stability of the closed-loop system can be guaranteed, considering both network-induced delays and packet dropouts.

Before further discussion, we make the following assumptions.

Assumption 1: The sensor is time driven; the controller and the actuator are event driven; the clocks among them are synchronized, and the signal transmission is with a single packet.

Assumption 2: There exists a maximum bound of the network-induced delay in the FE-NCS, i.e., $\tau_{\max} \geq \tau_k$, $(k = 0, 1, 2, \ldots)$.

The maximal value of q that satisfies (1) can be derived as follows:

$$q_{\max} = int\left[\frac{\delta - \tau_{\max}}{h}\right] \tag{2}$$

where $int[\cdot]$ denotes the nearest integer part of $[\cdot]$.

Furthermore, we define the maximal allowable packet dropout rate $r_{\max}$ as

$$r_{\max} = \frac{q_{\max} - 1}{q_{\max}}. \tag{3}$$

Remark 2: In this paper, the purpose of (2) and (3) is to choose the proper sampling period h for a given δ based on the real network condition. If the actual network packet dropout rate r can be measured by experiments, we should choose $r_{\max}$ satisfying $r_{\max} \geq r$ by (3). Then, using this $q_{\max}$, the sampling period h can thus be obtained from (2), which in turn implies the condition (1).

For a given δ, a smaller h will lead to a larger $q_{\max}$, so the allowable packet dropout rate may be higher, but the amount of communication will be increased greatly. However, a larger h will lead to a lower allowable packet dropout rate, which may degrade the performance of the system. The relationship between δ and performance of the NCSs will be analyzed in the next section.

Now, consider a nonlinear plant of the following form:

$$\dot{x}(t) = f(x) + g(x)u(t) + d(t) \tag{4}$$

where $x(t) \in \mathbb{R}^n$ is a state vector, $u(t) \in \mathbb{R}^m$ is a control input vector, $d(t) \in \mathbb{R}^n$ is a bounded external disturbance vector, and $f(x)$, $g(x)$ are unknown nonlinear function vectors depending on $x(t)$. The system of (4) can be represented by a T–S fuzzy plant model, which expresses the nonlinear system as a weighted sum of linear systems. The ith rule is of the following format:

Plant rule i :

IF $x_1(t_k)$ is $F_{i1}, \ldots,$ and $x_s(t_k)$ is F_{is}

THEN $\dot{x}(t) = A_i x(t) + B_i u(t) + d(t)$, for $i = 1, 2, \ldots, r$ (5)

where F_{ig} is a fuzzy set $(g = 1, 2, \ldots, s)$; r is the number of rules; $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are the known system matrix and input matrix, respectively, of the ith rule subsystem. t_k is the sampling instant, and $x(t_k)$ is the state vector of plant at the instant t_k. The inferred system is described by

$$\begin{aligned}\dot{x}(t) &= \frac{\sum_{i=1}^{r} \mu_i\left(x(t_k)\right)\left[A_i x(t) + B_i u(t) + d(t)\right]}{\sum_{i=1}^{r} \mu_i\left(x(t_k)\right)} \\ &= \sum_{i=1}^{r} h_i\left(x(t_k)\right)\left[A_i x(t) + B_i u(t)\right] + d(t)\end{aligned} \tag{6}$$

where $h_i(x(t_k)) = \mu_i(x(t_k))/\sum_{i=1}^{r}\mu_i(x(t_k))$, $\mu_i(x(t_k)) = \prod_{g=1}^{s} F_{ig}(x_g(t_k))$, and $F_{ig}(x_g(t_k))$ is the grade of membership function F_{ig} [15], [16]. Usually, we assume that $1 \geq \mu_i(x(t_k)) \geq 0$, and $\sum_{i=1}^{r}\mu_i(x(t_k)) > 0$ for all t_k. Then, we can see that $h_i(x(t_k)) \geq 0$, and $\sum_{i=1}^{r} h_i(x(t_k)) = 1$.

From (4) and (6), the plant model can be rewritten as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\left(x(t_k)\right)\left(A_i x(t) + B_i u(t)\right) + \Delta f + \Delta g + d(t) \tag{7}$$

where $\Delta f = f(x) - \sum_{i=1}^{r} h_i(x(t_k))A_i x(t)$, $\Delta g = (g(x) - \sum_{i=1}^{r} h_i(x(t_k))B_i)u(t)$, denote the bounded modeling errors between the nonlinear plant (4) and the fuzzy model (6). Now, we use $\omega(t) = \Delta f + \Delta g + d(t)$ to denote the bounded modeling errors and the external disturbances. Thus, (4) can be rewritten as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\left(x(t_k)\right)\left(A_i x(t) + B_i u(t)\right) + \omega(t). \tag{8}$$

Remark 3: In general, there are three approaches for constructing fuzzy models: 1) acquirement from experts; 2) identification (fuzzy modeling) using input–output data [14]; and 3) derivation from given nonlinear system equations [16]. This paper focuses on the third approach. This approach utilizes the idea of "sector nonlinearity," "local approximation," or a combination to construct fuzzy models.

The main motivation for proposing the FE is to estimate the plant states all the time, including the sampling instant and during two sequential effective packets because the packets may drop out or disorder. The FE-based approach can effectively reduce the number of data packets transmitted, attenuate the influence of modeling errors and external disturbances on the fuzzy system (8), and provide continuous control signals for the robust stability of the overall closed-loop system in the network-based environment. The FE consists of r fuzzy rules and shares the same fuzzy premises as those of the plant rules. In our control scheme, $x(t_k)$ and the states of the FE are used to calculate the control input. Specifically, in every rule's consequence of the FE, two additional terms are involved, namely, a disturbance attenuation term and an estimator gain term. The ith rule of the FE is shown as follows:

FE rule i :

IF $x_1(t_k)$ is $F_{i1}, \ldots,$ and $x_s(t_k)$ is F_{is}

THEN $\dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i\left(u(t) + v(t)\right) + L_i\left(x(t_k) - \hat{x}(t_k)\right)$.

Then, the inferred FE is given by

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i\left(x(t_k)\right)\left[A_i\hat{x}(t) + B_i\left(u(t) + v(t)\right)\right.$$
$$\left. + L_i\left(x(t_k) - \hat{x}(t_k)\right)\right], \qquad t \in [t_k + \tau_k, t_{k+q} + \tau_{k+q}) \quad (9)$$

where $\hat{x}(t)$ is the state vector of the FE, $L = \sum_{i=1}^{r} h_i(x(t_k))L_i$ is estimator gain matrix

$$v(t) = K_v\left(x(t_k) - \hat{x}(t_k)\right) \quad (10)$$

is used to attenuate the influence of modeling errors and external disturbances on the system, where K_v is the disturbance attenuation gain matrix described in Theorem 1 in the next section.

Next, we define the control law as

$$u(t) = u_{\mathrm{f}}(t) - v(t) \quad (11)$$

where $v(t)$ is the disturbance attenuation term described by (10) and $u_{\mathrm{f}}(t)$ is employed as a fuzzy control input, which is defined by the following fuzzy rules:

Control rule i :

IF $x_1(t_k)$ is $F_{i1}, \ldots,$ and $x_s(t_k)$ is F_{is}

THEN $u_{\mathrm{f}}(t) = K_i\hat{x}(t)$, for $i = 1, 2, \ldots, r$.

Hence, the inferred fuzzy controller from the FE is given by

$$u_{\mathrm{f}}(t) = \sum_{i=1}^{r} h_i\left(x(t_k)\right)K_i\hat{x}(t) \quad (12)$$

where $K_i (i = 1, 2, \ldots, r)$ is the fuzzy control gain matrix.

Substituting (11) and (12) into (9) yields the overall FE with closed-loop control as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(x(t_k)\right)h_j\left(x(t_k)\right)\left[(A_i + B_iK_j)\hat{x}(t)\right.$$
$$\left. + L_i\left(x(t_k) - \hat{x}(t_k)\right)\right], \quad t \in [t_k + \tau_k, t_{k+q} + \tau_{k+q}). \quad (13)$$

Remark 4: The packet is transmitted at the instant $t_k (k = 0, 1, 2, \ldots)$, which contains the sensor data of the plant state vector $x(t_k)$. In the following, we use $x(t_k)$ to denote a piecewise constant function after the ZOH, which can be sent to the FE at instant $t_k + \tau_k$ and keep the value until the next packet arrives. If the next packet arrives at $t_{k+q} + \tau_{k+q}$, $x(t_k)$ will keep the value in the interval $t \in [t_k + \tau_k, t_{k+q} + \tau_{k+q})$.

For the purpose of analyzing the performance of the FE-NCS, we need to introduce the estimation error vector as

$$e(t) = x(t) - \hat{x}(t). \quad (14)$$

Obviously, the estimation error vector at instant t_k is $e(t_k) = x(t_k) - \hat{x}(t_k)$.

Remark 5: From (13), it is clear that $A_i + B_iK_j$ characterizes the dynamics of the FE. Since $e(t_k)$ is only available information about the estimation error during the intersampling period, the estimator gain matrix L weights for $e(t_k)$ in order to improve the estimation precision to the FE. $v(t)$ is used to attenuate the influence of $\omega(t)$, namely, to attenuate the influence of bounded modeling errors and external disturbances. Here, $u(t)$ is an overall control input vector for the plant, which has (piecewise) continuous signals.

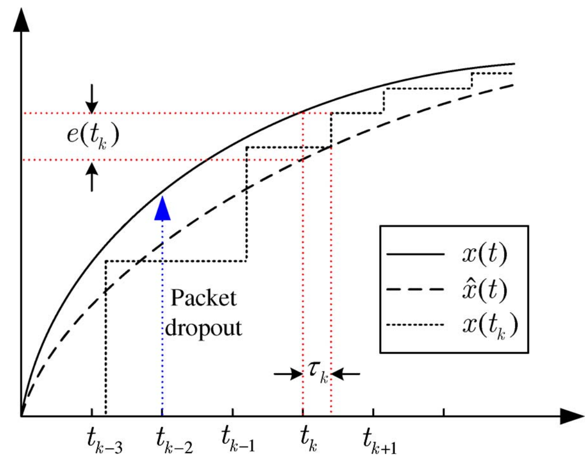

Fig. 3. Relationship among system states, network-induced delay, and packet dropout.

The relationship among system states, network-induced delay, and packet dropout is shown in Fig. 3, where $x(t)$ is the state of the plant, $\hat{x}(t)$ is the state of the FE that is used to estimate the state $x(t)$. When the sampling data $x(t_{k-2})$ is lost, $x(t_{k-3})$ will keep the value until the next sampling data $x(t_{k-1})$ arrives.

By differentiating (14), one has

$$\begin{aligned}\dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t)\\ &= \sum_{i=1}^{r} h_i\left(x(t_k)\right)\left[A_ie(t) - B_iv(t) - L_ie(t_k)\right] + \omega(t)\\ &= \sum_{i=1}^{r} h_i\left(x(t_k)\right)\left[A_ie(t) - (L_i + B_iK_v)e(t_k)\right] + \omega(t),\\ &\qquad t \in [t_k + \tau_k, t_{k+q} + \tau_{k+q}).\end{aligned} \quad (15)$$

In order to attenuate the influence of the modeling errors and external disturbances on the fuzzy system (8), we introduce H_∞ performance index [2], related to an augmented vector $z(t)$

$$\int_{t_0}^{\infty} z^{\mathrm{T}}(t)z(t)dt \le \gamma^2 \int_{t_0}^{\infty} \omega^{\mathrm{T}}(t)\omega(t)dt \quad (16)$$

where $z^{\mathrm{T}}(t) = [\hat{x}^{\mathrm{T}}(t), e^{\mathrm{T}}(t)]$, $\gamma > 0$ denotes prescribed attenuation level, and $t_0 \ge 0$ is initial instant.

Remark 6: The physical meaning of (16) is that the effect of any $\omega(t)$ on $z(t)$ has to be attenuated below a desired level γ from the viewpoint of energy. No matter what $\omega(t)$ is, the L_2 gain from $\omega(t)$ to $z(t)$ has to be equal to or less than a prescribed value γ^2. Moreover, $z(t)$ indicates $x(t)$. The effect of $\omega(t)$ on $x(t)$ will be discussed in Theorem 2 in the next section.

In the following section, we discuss the design method of a fuzzy robust controller for the FE-NCS, which obtains the gain matrices of $u_f(t)$ and $v(t)$ together by solving a set of LMIs. For simplicity, the following notations are used: $h_i = h_i(x(t_k))$, $x = x(t)$, $\hat{x} = \hat{x}(t)$, $e = e(t)$, $u = u(t)$, $z = z(t)$, and $\omega = \omega(t)$.

III. Fuzzy Robust Controller Design

Before presenting our results, the following lemmas are introduced.

Lemma 1 ([19]): Let Q be any of a $n \times n$ matrix. We have for any constant $\alpha > 0$ and any matrix $T > 0$ that

$$2x^{\mathrm{T}}Qy \le \alpha x^{\mathrm{T}}QT^{-1}Q^{\mathrm{T}}x + \frac{1}{\alpha}y^{\mathrm{T}}Ty \tag{17}$$

holds for all $x, y \in \mathbb{R}^n$.

Lemma 2 ([5]): For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}$, $M > 0$, scalar $\alpha > 0$, vector function $\xi : [0, \alpha] \to \mathbb{R}^n$, such that the integrations in the following are well defined, then:

$$\alpha \int_0^{\alpha} \xi^{\mathrm{T}}(\beta)M\xi(\beta)d\beta \ge \left(\int_0^{\alpha} \xi(\beta)d\beta\right)^{\mathrm{T}} M \left(\int_0^{\alpha} \xi(\beta)d\beta\right). \tag{18}$$

Consequently, the following results are obtained.

Theorem 1: For the system (13) and (15), if there exist matrices $P_1 > 0$, $P_2 > 0$, $T_1 > 0$, $T_2 > 0$, and matrices $\mathfrak{R}$, $\mathfrak{S}_i$, $\mathfrak{T}_j$, U, W, Y_l, for given scalars $\delta > 0$ and $\epsilon_l (l = 1, \dots, 6)$, the following LMIs hold:

$$\begin{bmatrix} \frac{\Xi ij + \Xi ji}{2} & \delta\bar{Y}_1 & \delta\bar{Y}_2 \\ * & -\delta T_1 & 0 \\ * & * & -\delta T_2 \end{bmatrix} < 0, \qquad 1 \le i \le j \le r \tag{19}$$

then the H_∞ performance in (16) is guaranteed for a prescribed γ with the control law (11) in the FE-NCS, where

$$\bar{Y}_1^{\mathrm{T}} = [Y_1^{\mathrm{T}} \quad Y_2^{\mathrm{T}} \quad Y_3^T \quad 0 \quad 0 \quad 0 \quad 0]$$
$$\bar{Y}_2^{\mathrm{T}} = [0 \quad 0 \quad 0 \quad Y_4^{\mathrm{T}} \quad Y_5^{\mathrm{T}} \quad Y_6^{\mathrm{T}} \quad 0]$$
$$\Xi_{ij} = \begin{bmatrix} \Pi & \tilde{\Pi}_1 \\ * & \tilde{\Pi}_2 \end{bmatrix}$$
$$\tilde{\Pi}_1^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & -\epsilon_4 I & -\epsilon_5 I & -\epsilon_6 I \\ U^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & W^{\mathrm{T}} & 0 & 0 \end{bmatrix}$$
$$\tilde{\Pi}_2 = \mathrm{diag}(-\gamma^2 I, -I, -I)$$
$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & \Pi_{15} & 0 \\ * & \Pi_{22} & \Pi_{23} & 0 & \Pi_{25} & 0 \\ * & * & \Pi_{33} & 0 & \Pi_{35} & 0 \\ * & * & * & \Pi_{44} & \Pi_{45} & \Pi_{46} \\ * & * & * & * & \Pi_{55} & \Pi_{56} \\ * & * & * & * & * & \Pi_{66} \end{bmatrix}$$
$$\Pi_{11} = Y_1 + Y_1^{\mathrm{T}} - \epsilon_1(A_iU^{\mathrm{T}} + B_i\mathfrak{T}_j) - \epsilon_1(A_iU^{\mathrm{T}} + B_i\mathfrak{T}_j)^{\mathrm{T}}$$
$$\Pi_{12} = -Y_1 + Y_2^{\mathrm{T}} - \epsilon_2(A_iU^{\mathrm{T}} + B_i\mathfrak{T}_j)^{\mathrm{T}}$$
$$\Pi_{13} = P_1 + Y_3^{\mathrm{T}} + \epsilon_1 U^{\mathrm{T}} - \epsilon_3(A_iU^{\mathrm{T}} + B_i\mathfrak{T}_j)^{\mathrm{T}}$$
$$\Pi_{15} = -\epsilon_1\mathfrak{S}_i, \qquad \Pi_{22} = -Y_2 - Y_2^{\mathrm{T}}$$
$$\Pi_{23} = -Y_3^{\mathrm{T}} + \epsilon_2 U^{\mathrm{T}}, \qquad \Pi_{25} = -\epsilon_2\mathfrak{S}_i$$
$$\Pi_{33} = \delta T_1 + \epsilon_3 U + \epsilon_3 U^{\mathrm{T}}, \qquad \Pi_{35} = -\epsilon_3\mathfrak{S}_i$$
$$\Pi_{44} = Y_4 + Y_4^{\mathrm{T}} - \epsilon_4 A_i W^{\mathrm{T}} - \epsilon_4 W A_i^{\mathrm{T}}$$
$$\Pi_{45} = -Y_4 + Y_5^{\mathrm{T}} + \epsilon_4(\mathfrak{S}_i + B_i\mathfrak{R}) - \epsilon_5 W A_i^{\mathrm{T}}$$
$$\Pi_{46} = P_2 + Y_6^{\mathrm{T}} + \epsilon_4 W^{\mathrm{T}} - \epsilon_6 W A_i^{\mathrm{T}}$$
$$\Pi_{55} = -Y_5 - Y_5^{\mathrm{T}} + \epsilon_5(\mathfrak{S}_i + B_i\mathfrak{R}) + \epsilon_5(\mathfrak{S}_i + B_i\mathfrak{R})^{\mathrm{T}}$$
$$\Pi_{56} = -Y_6^{\mathrm{T}} + \epsilon_5 W^{\mathrm{T}} + \epsilon_6(\mathfrak{S}_i + B_i\mathfrak{R})^{\mathrm{T}}$$
$$\Pi_{66} = \delta T_2 + \epsilon_6 W + \epsilon_6 W^{\mathrm{T}}. \tag{20}$$

Moreover, the disturbance attenuation gain matrix can be obtained as $K_v = \mathfrak{R}W^{-\mathrm{T}}$, the estimator gain matrix as $L_i = \mathfrak{S}_i W^{-\mathrm{T}}$, and the fuzzy control gain matrix as $K_j = \mathfrak{T}_j U^{-\mathrm{T}}$.

Proof: Consider a Lyapunov–Krasovskii functional as

$$V(t) = \hat{x}^{\mathrm{T}}P_1\hat{x} + e^{\mathrm{T}}P_2 e + \int_{t-\delta}^{t}\int_{s}^{t} \dot{\hat{x}}^{\mathrm{T}}(v)T_1\dot{\hat{x}}(v)dvds + \int_{t-\delta}^{t}\int_{s}^{t} \dot{e}^{\mathrm{T}}(v)T_2\dot{e}(v)dvds \tag{21}$$

where $P_1 > 0$, $P_2 > 0$, $T_1 > 0$, and $T_2 > 0$, δ in the integrals implicates both network-induced delay and packet dropout problems as defined in (1).

It can be seen that the following equations hold for any nonsingular matrices Y_l and Z_l $(l = 1, \dots, 6)$ of appropriate dimensions:

$$\Gamma_1 = \left(\hat{x}^{\mathrm{T}}Y_1 + \hat{x}^{\mathrm{T}}(t_k)Y_2 + \dot{\hat{x}}^{\mathrm{T}}Y_3\right) \times \left(\hat{x} - \hat{x}(t_k) - \int_{t_k}^{t} \dot{\hat{x}}(s)ds\right) = 0 \tag{22}$$

$$\Gamma_2 = \left(\hat{x}^{\mathrm{T}}Z_1 + \hat{x}^{\mathrm{T}}(t_k)Z_2 + \dot{\hat{x}}^{\mathrm{T}}Z_3\right) \times \left[\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left(-(A_i + B_iK_j)\hat{x} - L_i e(t_k)\right) + \dot{\hat{x}}\right] = 0 \tag{23}$$

$$\Gamma_3 = \left(e^{\mathrm{T}}Y_4 + e^{\mathrm{T}}(t_k)Y_5 + \dot{e}^{\mathrm{T}}Y_6\right) \times \left(e - e(t_k) - \int_{t_k}^{t} \dot{e}(s)ds\right) = 0 \tag{24}$$

$$\Gamma_4 = \left(e^{\mathrm{T}}Z_4 + e^{\mathrm{T}}(t_k)Z_5 + \dot{e}^{\mathrm{T}}Z_6\right) \times \left[\sum_{i=1}^{r} h_i\left(-A_i e + (L_i + B_iK_v)e(t_k)\right) - \omega + \dot{e}\right] = 0. \tag{25}$$

Considering (22)–(25), the corresponding time derivative of $V(t)$, for $t \in [t_k + \tau_k, t_{k+q} + \tau_{k+q})$, is given by

$$\begin{aligned}\dot{V}(t) =& 2\hat{x}^{\mathrm{T}} P_1 \dot{\hat{x}} + 2e^{\mathrm{T}} P_2 \dot{e} + \delta \dot{\hat{x}}^{\mathrm{T}} T_1 \dot{\hat{x}} \\ &- \int_{t-\delta}^{t} \dot{\hat{x}}^{\mathrm{T}}(s) T_1 \dot{\hat{x}}(s) ds + \delta \dot{e}^{\mathrm{T}} T_2 \dot{e} \\ &- \int_{t-\delta}^{t} \dot{e}^{\mathrm{T}}(s) T_2 \dot{e}(s) ds + \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4. \end{aligned} \quad (26)$$

Since t is defined in $[t_k + \tau_k, t_{k+q} + \tau_{k+q})$ and (1) holds, we have $t - t_k \le t - \delta$. Then, it is concluded that the following inequality:

$$\int_{t_k}^{t} \dot{\hat{x}}^{\mathrm{T}}(s) T_1 \dot{\hat{x}}(s) ds \le \int_{t-\delta}^{t} \dot{\hat{x}}^{\mathrm{T}}(s) T_1 \dot{\hat{x}}(s) ds \quad (27)$$

which is convenient to eliminate the terms about t_k from (26).

From Lemmas 1 and 2, one can obtain

$$\begin{aligned} &-2\left(\hat{x}^{\mathrm{T}} Y_1 + \hat{x}^{\mathrm{T}}(t_k) Y_2 + \dot{\hat{x}}^{\mathrm{T}} Y_3\right) \int_{t_k}^{t} \dot{\hat{x}}(s) ds \\ &\le \delta \Lambda^{\mathrm{T}} \bar{Y}_1 T_1^{-1} \bar{Y}_1^{\mathrm{T}} \Lambda + \frac{1}{\delta} \left(\int_{t_k}^{t} \dot{\hat{x}}(s) ds\right)^{\mathrm{T}} T_1 \left(\int_{t_k}^{t} \dot{\hat{x}}(s) ds\right) \\ &\le \delta \Lambda^{\mathrm{T}} \bar{Y}_1 T_1^{-1} \bar{Y}_1^{\mathrm{T}} \Lambda + \int_{t-\delta}^{t} \dot{\hat{x}}^{\mathrm{T}}(s) T_1 \dot{\hat{x}}(s) ds \end{aligned} \quad (28)$$

where $\Lambda^{\mathrm{T}} = [\hat{x}^{\mathrm{T}}\ \hat{x}^{\mathrm{T}}(t_k)\ \dot{\hat{x}}^{\mathrm{T}}\ e^{\mathrm{T}}\ e^{\mathrm{T}}(t_k)\ \dot{e}^{\mathrm{T}}\ \omega^{\mathrm{T}}]$.

Similarly, we get

$$\int_{t_k}^{t} \dot{e}^{\mathrm{T}}(s) T_2 \dot{e}(s) ds \le \int_{t-\delta}^{t} \dot{e}^{\mathrm{T}}(s) T_2 \dot{e}(s) ds \quad (29)$$

and

$$\begin{aligned} &-2\left(e^{\mathrm{T}} Y_4 + e^{\mathrm{T}}(t_k) Y_5 + \dot{e}^{\mathrm{T}} Y_6\right) \int_{t_k}^{t} \dot{e}(s) ds \\ &\le \delta \Lambda^{\mathrm{T}} \bar{Y}_2 T_2^{-1} \bar{Y}_2^{\mathrm{T}} \Lambda + \int_{t-\delta}^{t} \dot{e}^{\mathrm{T}}(s) T_2 \dot{e}(s) ds. \end{aligned} \quad (30)$$

Using the inequalities (27)–(30), the derivative of $V(t)$, for $t \in [t_k + \tau_k, t_{k+q} + \tau_{k+q})$, can be presented as follows:

$$\begin{aligned} \dot{V}(t) \le& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Lambda^{\mathrm{T}} \left(\Psi_{ij} + \delta \bar{Y}_1 T_1^{-1} \bar{Y}_1^{\mathrm{T}} + \delta \bar{Y}_2 T_2^{-1} \bar{Y}_2^{\mathrm{T}}\right) \Lambda \\ &- z^{\mathrm{T}} z + \gamma^2 \omega^{\mathrm{T}} \omega \\ =& \sum_{i=1}^{r} h_i^2 \Lambda^{\mathrm{T}} \left(\Psi_{ii} + \delta \bar{Y}_1 T_1^{-1} \bar{Y}_1^{\mathrm{T}} + \delta \bar{Y}_2 T_2^{-1} \bar{Y}_2^{\mathrm{T}}\right) \Lambda \\ &+ 2\sum_{i=1}^{r-1}\sum_{i<j}^{r} h_i h_j \Lambda^{\mathrm{T}} \\ &\times \left(\frac{\Psi_{ij} + \Psi_{ji}}{2} + \delta \bar{Y}_1 T_1^{-1} \bar{Y}_1^{\mathrm{T}} + \delta \bar{Y}_2 T_2^{-1} \bar{Y}_2^{\mathrm{T}}\right) \\ &\times \Lambda - z^{\mathrm{T}} z + \gamma^2 \omega^{\mathrm{T}} \omega \end{aligned} \quad (31)$$

where

$$\Psi_{ij} = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & 0 & \Upsilon_{15} & 0 & 0 \\ * & \Upsilon_{22} & \Upsilon_{23} & 0 & \Upsilon_{25} & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & \Upsilon_{35} & 0 & 0 \\ * & * & * & \Upsilon_{44} & \Upsilon_{45} & \Upsilon_{46} & -Z_4 \\ * & * & * & * & \Upsilon_{55} & \Upsilon_{56} & -Z_5 \\ * & * & * & * & * & \Upsilon_{66} & -Z_6 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\begin{aligned} \Upsilon_{11} &= Y_1 + Y_1^{\mathrm{T}} - Z_1(A_i + B_i K_j) - (A_i + B_i K_j)^{\mathrm{T}} Z_1^{\mathrm{T}} + I \\ \Upsilon_{12} &= -Y_1 + Y_2^{\mathrm{T}} - (A_i + B_i K_j)^{\mathrm{T}} Z_2^{\mathrm{T}} \\ \Upsilon_{13} &= P_1 + Y_3^{\mathrm{T}} + Z_1 - (A_i + B_i K_j)^{\mathrm{T}} Z_3^{\mathrm{T}} \\ \Upsilon_{15} &= -Z_1 L_i \quad \Upsilon_{22} = -Y_2 - Y_2^{\mathrm{T}} \\ \Upsilon_{23} &= -Y_3^{\mathrm{T}} + Z_2 \quad \Upsilon_{25} = -Z_2 L_i \\ \Upsilon_{33} &= \delta T_1 + Z_3 + Z_3^{\mathrm{T}} \quad \Upsilon_{35} = -Z_3 L_i \\ \Upsilon_{44} &= Y_4 + Y_4^{\mathrm{T}} - Z_4 A_i - A_i^{\mathrm{T}} Z_4^{\mathrm{T}} + I \\ \Upsilon_{45} &= -Y_4 + Y_5^{\mathrm{T}} + Z_4(L_i + B_i K_v) - A_i^{\mathrm{T}} Z_5^{\mathrm{T}} \\ \Upsilon_{46} &= P_2 + Y_6^{\mathrm{T}} + Z_4 - A_i^{\mathrm{T}} Z_6^{\mathrm{T}} \\ \Upsilon_{55} &= -Y_5 - Y_5^{\mathrm{T}} + Z_5(L_i + B_i K_v) + (L_i + B_i K_v)^{\mathrm{T}} Z_5^{\mathrm{T}} \\ \Upsilon_{56} &= -Y_6^{\mathrm{T}} + Z_5 + (L_i + B_i K_v)^{\mathrm{T}} Z_6^{\mathrm{T}} \\ \Upsilon_{66} &= \delta T_2 + Z_6 + Z_6^{\mathrm{T}}. \end{aligned} \quad (32)$$

If $((\Psi_{ij} + \Psi_{ji})/2) + \delta \bar{Y}_1 T_1^{-1} \bar{Y}_1^{\mathrm{T}} + \delta \bar{Y}_2 T_2^{-1} \bar{Y}_2^{\mathrm{T}} < 0$ holds for any $1 \le i \le j \le r$, we can obtain

$$\begin{bmatrix} \frac{\Psi ij + \Psi ji}{2} & \delta \bar{Y}_1 & \delta \bar{Y}_2 \\ * & -\delta T_1 & 0 \\ * & * & -\delta T_2 \end{bmatrix} < 0, \qquad 1 \le i \le j \le r. \quad (33)$$

Using the Schur complement [1] implies

$$\dot{V}(t) \le -z^{\mathrm{T}} z + \gamma^2 \omega^{\mathrm{T}} \omega \quad (34)$$

for $t \in [t_k + \tau_k, t_{k+q} + \tau_{k+q})$.

Sets $Z_1 = \epsilon_1 U^{-1}$, $Z_2 = \epsilon_2 U^{-1}$, $Z_3 = \epsilon_3 U^{-1}$, $Z_4 = \epsilon_4 W^{-1}$, $Z_5 = \epsilon_5 W^{-1}$, and $Z_6 = \epsilon_6 W^{-1}$. Thus, $\Xi_{ij} < 0$ implies that U and W are nonsingular since Π_{33} and Π_{66} in (20) must be negative definite. Then, pre, postmultipling both sides of (33) with $\mathrm{diag}(U, U, U, W, W, W, I, U, W)$ and its transpose, respectively, setting $\mathfrak{R} = K_v W^{\mathrm{T}}$, $\mathfrak{S}_i = L_i W^{\mathrm{T}}$, $\mathfrak{T}_j = K_j U^{\mathrm{T}}$, and replacing $U P_1 U^{\mathrm{T}}$, $U T_1 U^{\mathrm{T}}$, $U \bar{Y}_1 U^{\mathrm{T}}$, $W P_2 W^{\mathrm{T}}$, $W T_2 W^{\mathrm{T}}$, $W \bar{Y}_2 W^{\mathrm{T}}$ with P_1, T_1, $\bar{Y}_1$, P_2, T_2, $\bar{Y}_2$, respectively, we can obtain (19) directly by using the Schur complement again.

Integrating both sides of (34) from $t_k+\tau_k$ to $t\in[t_k+\tau_k, t_{k+q}+\tau_{k+q})$, one has

$$V(t)-V(t_k+\tau_k)\le -\int_{t_k+\tau_k}^{t} z^{\mathrm T} z dt + \int_{t_k+\tau_k}^{t} \gamma^2\omega^{\mathrm T}\omega dt. \quad (35)$$

Since $V(t)$ is continuous in $t\in[t_0,\infty)$, it can be seen that

$$V(t)-V(t_0)\le -\int_{t_0}^{t} z^{\mathrm T} z dt + \int_{t_0}^{t} \gamma^2\omega^{\mathrm T}\omega dt. \quad (36)$$

Letting $t\to\infty$ and under zero initial condition, we have

$$\int_{t_0}^{\infty} z^{\mathrm T} z dt \le \int_{t_0}^{\infty} \gamma^2\omega^{\mathrm T}\omega dt. \quad (37)$$

Since $V(t)>0$, $\forall t>t_0$, the above inequality implies that the H_∞ performance in (16) can be satisfied.

Therefore, if (19) is satisfied, the control law (11) can stabilize the FE-NCS, and the estimation errors e and the states of the FE $\hat{x}$ are bounded with the H_∞ performance in (16) for a prescribed γ. This completes the proof. ■

Remark 7: In this paper, $\dot{V}(t)=\limsup_{\rho\to 0^+}(1/\rho)[V(t+\rho)-V(t)]$ [6], and the zero initial condition is specified that $z_{t_0}(t)=\phi(t)=0, \forall t\in[-\delta,0]$, where $z_t(\cdot)$, for a given $t\ge t_0$, denotes the restriction of $z(\cdot)$ to the interval $[t-\delta,t]$ being translated to $[-\delta,0]$. The Lyapunov–Krasovskii in (21) is usually used to analyze retarded functional differential equations, which satisfies the Krasovskii stability theory. The interested reader may refer to [3] for the details.

Remark 8: The optimal values of the tuning parameters $\epsilon_l (l=1,\ldots,6)$ that were introduced in Theorem 1 can be found as follows. We choose the cost function $\mathrm{t_{min}}$, which is obtained by solving the feasibility problem using Matlab's LMI Toolbox [The MathWorks (1995, Version 1.0.8)]. If the cost function $\mathrm{t_{min}}$ is negative, there exists a feasible solution to the set of LMIs under consideration. Then, a genetic algorithm can be used to search the combinations of ϵ_l $(l=1,\ldots,6)$ with the cost function $\mathrm{t_{min}}$ for the given $\delta>0$. We use **gatool** and Direct Search Toolbox [The MathWorks (2004, Version 1.0.1)] to search the optimal combination of ϵ_l $(l=1,\ldots,6)$. If all the resulting minimum values of the cost function $\mathrm{t_{min}}$ are negative, the tuning parameters can be obtained.

According to Theorem 1, we can derive the following theorem for the closed-loop system.

Theorem 2: For the nonlinear system (4), if the control law is given by (11), then all states of the closed-loop system are bounded and the following H_∞ performance is guaranteed:

$$\int_{t_0}^{\infty} x^{\mathrm T} x dt \le 3\gamma^2 \int_{t_0}^{\infty} \omega^{\mathrm T}\omega dt. \quad (38)$$

Proof: Because of $x=\hat{x}+e$, we can obtain the following inequality:

$$\left(\int_{t_0}^{\infty} x^{\mathrm T} x dt\right)^{\frac12} \le \left(\int_{t_0}^{\infty} \hat{x}^{\mathrm T}\hat{x} dt\right)^{\frac12} + \left(\int_{t_0}^{\infty} e^{\mathrm T} e dt\right)^{\frac12}.$$

From Theorem 1, it can be concluded that the inequality (38) holds. This completes the proof. ■

Remark 9: It is worthy of pointing out for a given $\delta>0$, the following convex optimization problem can be obtained for the stability of (4) by solving (19) with minimize γ:

$$\begin{aligned}&\text{minimize } \gamma\\ &s.t\ P_1>0, P_2>0, T_1>0, T_2>0, (19).\end{aligned} \quad (39)$$

Summarizing the above discussions, the following design procedures for the FE-NCS are listed.

Design procedures

Step 1) Select a sufficiently small real number $\varepsilon>0$ and fuzzy membership functions.
Step 2) Construct fuzzy plant rules (5).
Step 3) Construct the FE (9).
Step 4) Choose an initial $\delta=\delta_0$ according to the current network burden.
Step 5) Search the tuning parameters $\epsilon_l (l=1,\ldots,6)$ by **gatool** based on Theorem 1 and Remark 8.
Step 6) Solve the convex optimization problem in (39) to obtain $\gamma_{\min}$, K_j, L_i, and K_v.
Step 7) Set $\delta=\delta+\varepsilon$ and repeat Step 5), until K_j, L_i, K_v cannot be found.
Step 8) Construct the fuzzy controller according to (10) and (12).

IV. Simulation Examples

To illustrate the FE-based approach, we present two examples: 1) a mass-spring system that can be expressed precisely by a T–S fuzzy system if not considering external disturbances and 2) an inverted pendulum on a cart that is a classical nonlinear plant.

A. Example 1: Mass Spring

Consider the following nonlinear mass-spring system [21]:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -0.01x_1-0.67x_1^3+d(t)+u\end{aligned} \quad (40)$$

where $x_1\in[-1,1]$ and $d(t)=0.2\sin(2\pi t)\exp(-0.1t)$ is the external disturbance.

Choose fuzzy membership function as $\mu_1(x_1)=1-x_1^2$ and $\mu_2(x_1)=1-\mu_1(x_1)$. The following fuzzy model is used to model the nonlinear system:

Rule 1 :

IF $x_1(t_k)$ is μ_1, THEN $\dot{x}=A_1x+B_1u$

Rule 2 :

IF $x_1(t_k)$ is μ_2, THEN $\dot{x}=A_2x+B_2u$ (41)

where $A_1=\begin{bmatrix}0 & 1\\ -0.01 & 0\end{bmatrix}$, $A_2=\begin{bmatrix}0 & 1\\ -0.68 & 0\end{bmatrix}$, $B_1=B_2=\begin{bmatrix}0\\1\end{bmatrix}$.

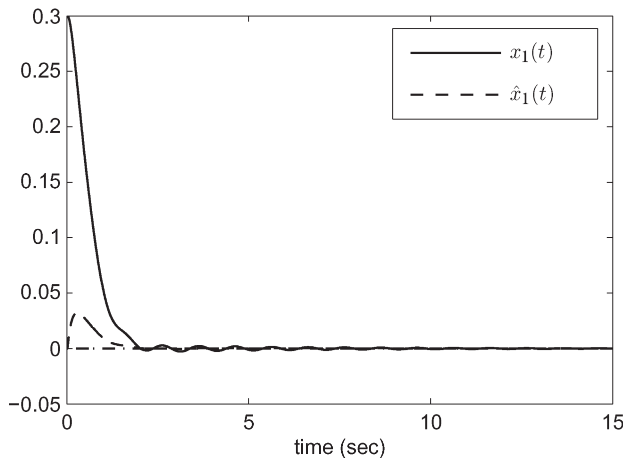

Fig. 4. Trajectories of $x_1, \hat{x}_1$ with $h = 0.02$ s, $\tau_k \in [0, 0.018]$ s, $r = 20\%$, and $x(0) = [0.3, 0]^{\mathrm{T}}$.

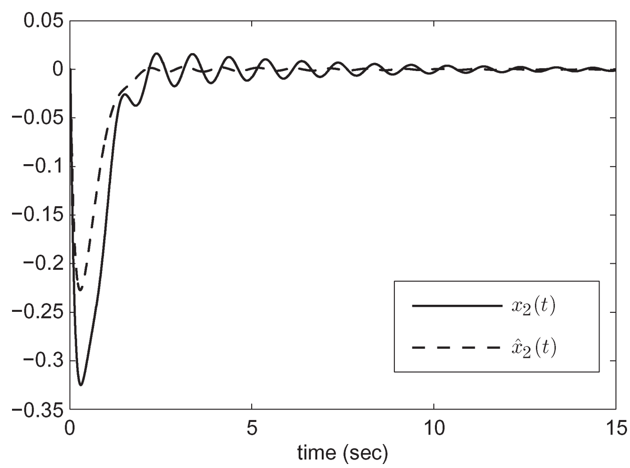

Fig. 5. Trajectories of $x_2, \hat{x}_2$ with $h = 0.02$ s, $\tau_k \in [0, 0.018]$ s, $r = 20\%$, and $x(0) = [0.3, 0]^{\mathrm{T}}$.

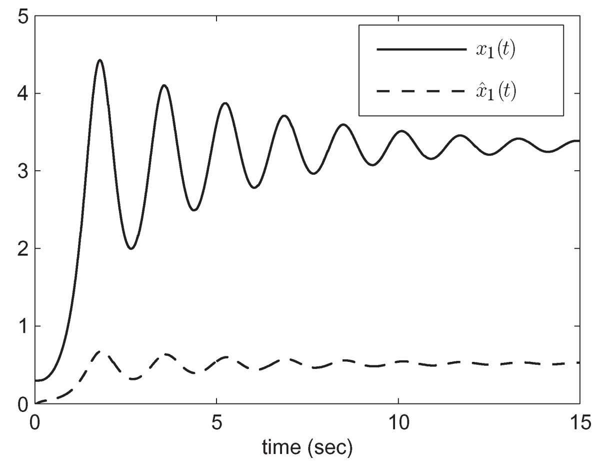

Fig. 6. Trajectories of states without v.

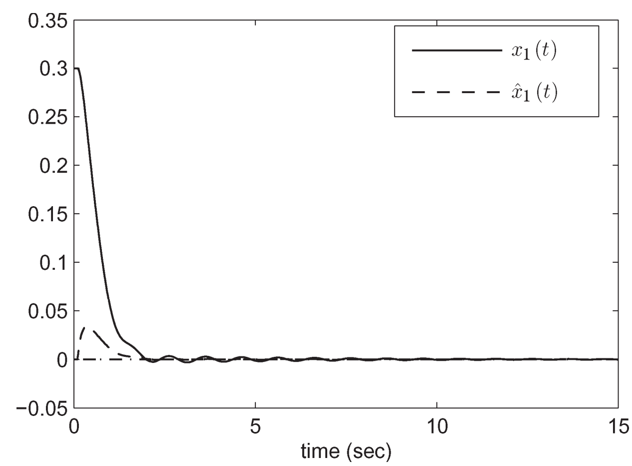

Fig. 7. Trajectories of $x_1, \hat{x}_1$ with $h = 0.1$ s, $\tau_k \in [0, 0.09]$ s, $r = 20\%$, and $x(0) = [0.3, 0]^{\mathrm{T}}$.

We select $\delta = 0.2$ s. Then, applying Theorem 1 and Remark 8, a feasible combination of ϵ_l, $(l = 1, \ldots, 6)$, can be obtained as: $\epsilon_1 = 0.2033$, $\epsilon_2 = 0.0062$, $\epsilon_3 = 0.0074$, $\epsilon_4 = 0.0099$, $\epsilon_5 = 0.0013$, $\epsilon_6 = 0.0027$, with $\mathrm{t}_{\min} = -5.7968 \times 10^{-8}$.

By solving the optimization problem (39), we obtain that $\gamma_{\min} = 0.9495$, $K_1 = [-1.2967 \quad -0.1843] \times 10^4$, $K_2 = [-1.2961 \quad -0.1843] \times 10^4$, $K_v = [9.8430 \quad 4.4213]$, $L_1 = \begin{bmatrix} 1.1451 & 0.1155 \\ -6.9880 & -0.7720 \end{bmatrix}$, and $L_2 = \begin{bmatrix} 1.1482 & 0.1219 \\ -7.6538 & -0.7663 \end{bmatrix}$.

Next, under the same initial value $x(0) = [0.3, 0]^{\mathrm{T}}$, $\hat{x}(0) = [0, 0]^{\mathrm{T}}$, we show the simulation results with different sampling periods and packet dropout rates. The network-induced delay τ_k is randomly varying with an unknown distribution under the condition (1).

Case I: $h = 0.02$ s, $\tau_k \in [0, 0.018]$ s, packet dropout rate $r = 20\%$.

The state trajectories of (40) are shown in Figs. 4 and 5. The system can be stabilized well. The fuzzy model (41) cannot express the mass spring system (40) precisely because of external disturbance ω. In order to verify the validity of the disturbance attenuation term $v(t)$, we force $K_v = [0, 0]$. The trajectories of x_1 and $\hat{x}_1$ are shown in Fig. 6. The system cannot arrive at the equilibrium point within 15 s. Therefore, the term v is necessary for the FE-NCS.

Case II: $h = 0.1$ s, $\tau_k \in [0, 0.09]$ s, packet drop rate $r = 20\%$.

The state trajectories of (40) are shown in Fig. 7. Comparing *CASE I* and *CASE II*, they are not obviously different, but the communication burden of *CASE II* is much less than that of *CASE I*.

Case III: $h = 0.1$ s, $\tau_k = 0$ s.

In this case, we try to explore how the packet dropout affects the stability of the FE-NCS. For $\delta = 0.2$ s and $h = 0.1$ s, the maximum allowable packet dropout rate is $r_{\max} = 50\%$. Therefore, we simulate packet dropout phenomena with $r = 40\%$ and $r = 70\%$ to verify our results, respectively. The simulation results are shown in Figs. 8 and 9. Comparing Fig. 7 with Fig. 8, the performance of the system does not degrade much. However, the system is unstable in Fig. 9. Therefore, the maximum allowable control interval δ can be used to reflect both the network-induced delay and packet dropout problems. Moreover, it is important to choose a proper sampling period h to satisfy $r_{\max} > r$ based on δ when packets dropout occurs inevitably in real network condition.

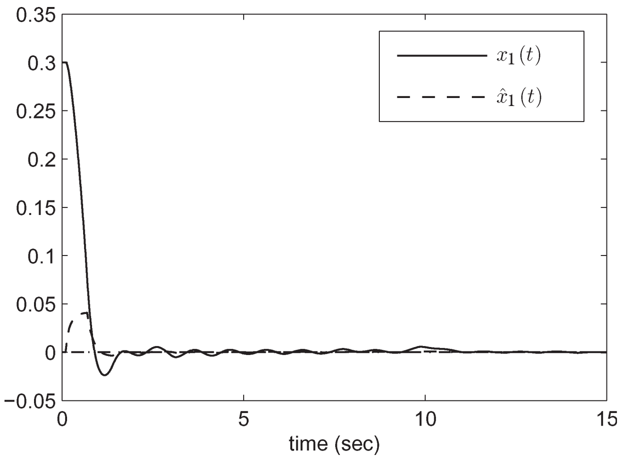

Fig. 8. Trajectories of x_1, $\hat{x}_1$ with $h = 0.1$ s, $\tau_k = 0$ s, $r = 40\%$, and $x(0) = [0.3, 0]^{\mathrm{T}}$.

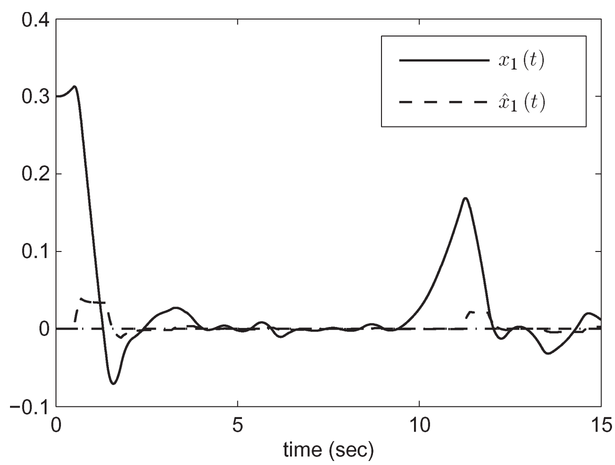

Fig. 9. Trajectories of x_1, $\hat{x}_1$ with $h = 0.1$ s, $\tau_k = 0$ s, $r = 70\%$, and $x(0) = [0.3, 0]^{\mathrm{T}}$.

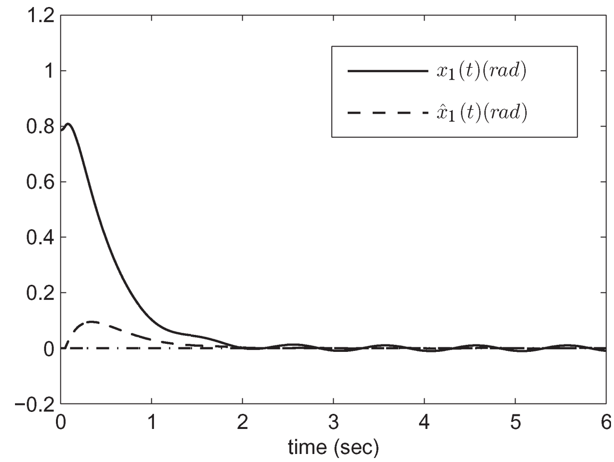

Fig. 10. Trajectories of x_1, $\hat{x}_1$ with $h = 0.05$ s, $\tau_k \in [0, 0.045]$ s, $r = 0\%$, and $x(0) = [\pi/4, 0]^{\mathrm{T}}$.

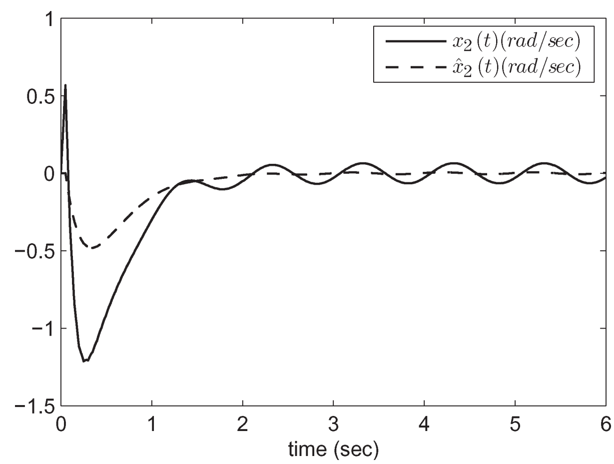

Fig. 11. Trajectories of x_2, $\hat{x}_2$ with $h = 0.05$ s, $\tau_k \in [0, 0.045]$ s, $r = 0\%$, and $x(0) = [\pi/4, 0]^{\mathrm{T}}$.

B. Example 2: Inverted Pendulum on a Cart

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} + d(t) \quad (42)$$

where x_1 denotes the angle (in radians) of the pendulum from the vertical and x_2 is the angular velocity, $d(t) = 0.5\sin(2\pi t)$ is the external disturbance, $g = 9.8$ m/s^2 is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and u is the force applied to the cart (in newtons), and $a = 1/(m+M)$. We choose $m = 2.0$ kg, $M = 8.0$ kg, $2l = 1.0$ m here. In [16], the system is approximated by following two rules:

Rule 1 :

IF $x_1(t_k)$ is about 0, THEN $\dot{x} = A_1x + B_1u$

Rule 2 :

IF $x_1(t_k)$ is about $\pm\pi/2$, THEN $\dot{x} = A_2x + B_2u$

where $A_1 = \begin{bmatrix} 0 & 1 \\ g/(4l/3 - aml) & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ -a/(4l/3 - aml) \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 \\ 2g/\pi(4l/3 - aml\beta^2) & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ -a\beta/(4l/3 - aml\beta^2) \end{bmatrix}$, and $\beta = \cos(88^\circ)$.

Choose the fuzzy membership function as $\mu_1(x_1) = (0.5\pi - |x_1|)/0.5\pi$, and $\mu_2(x_1) = 1 - \mu_1(x_1)$.

We select $\delta = 0.1$ s. Then, applying Theorem 1 and Remark 8, a feasible combination of ϵ_l, $(l = 1, \ldots, 6)$, can be obtained as follows $\epsilon_1 = 0.9288$, $\epsilon_2 = 0.0084$, $\epsilon_3 = 0.0027$, $\epsilon_4 = 0.0097$, $\epsilon_5 = 0.0078$, $\epsilon_6 = 0.0040$, with $\mathrm{t_{min}} = -1.1154 \times 10^{-7}$.

By solving the optimization problem (39), we obtain that $\gamma_{\min} = 0.9632$, $K_1 = [12.359 \quad 2.4812] \times 10^3$, $K_2 = [23.976 \quad 4.71] \times 10^4$, $K_v = [-157.97 \quad -57.666]$, $L_1 = \begin{bmatrix} 0.998 & 0.0214 \\ -4.432 & -2.738 \end{bmatrix}$, and $L_2 = \begin{bmatrix} 0.998 & 0.0213 \\ 14.822 & 7.422 \end{bmatrix}$.

Figs. 10 and 11 show the inverted pendulum responses with $h = 0.05$ s, $\tau_k \in [0, 0.045]$ s, $r = 0\%$, and the initial condition $x(0) = [\pi/4, 0]^{\mathrm{T}}$, $\hat{x}(0) = [0, 0]^{\mathrm{T}}$.

The inverted pendulum system has been studied in several references, such as [4], where the typical sampling period is less than 0.02 s. Since the FE can estimate the plant state effectively

TABLE I
CONTROL PERFORMANCE EVALUATION WITH DIFFERENT r

No.	h	τ_{max}	r_{max}	r	ISE	Performance Evaluation
1	0.02	0.018	75%	0	0.7985	Good
2	0.02	0.018	75%	20%	0.8052	Good
3	0.02	0.018	75%	40%	0.8660	Good
4	0.02	0.018	75%	60%	0.9522	Good
5	0.02	0.018	75%	75%	3.3182	Nomoral
6	0.02	0.018	75%	80%	1.0852×10^6	Unstable

in a network-based environment, the sampling period can be prolonged under condition (1), which is verified by the above simulation results. Note that the robustness and stability of the NCS can be guaranteed by Theorems 1 and 2 although we adopt the longer sampling period.

In order to show the relationship among control performance, packet dropout rate, and network-induced delay, we define the integral square error (ISE) performance index as

$$\text{ISE} = \int_{t_0}^{t_{\mathrm{f}}} x^{\mathrm{T}}(t)x(t)dt. \tag{43}$$

The performance evaluation results are shown in Table I with $h = 0.02$ s, $\tau_k \in [0, 0.018]$ s, and different r. By (2) and (3), $q_{\max} = int[(\delta - \tau_{\max})/h] = int[(0.1 - 0.018)/0.02] = 4$, and $r_{\max} = (q_{\max} - 1)/q_{\max} = 75\%$.

From Table I, we see that if the packet dropout rate $r < r_{\max}$, the control performance is good. Moreover, when $r = 80\% > r_{\max}$, the system dynamics are unstable. The results validate the proposed control scheme. However, it should be noted that our result is only a sufficient condition. If condition (1) is satisfied, the FE-NCS can be stabilized by the controller. Otherwise, we cannot draw the conclusion on the stability of the system based on the proposed method.

In all examples, it should be pointed out that different choices of membership functions may lead to different degrees of approximate accuracy. In this paper, we have collected the influence of different choices of membership functions on the system into modeling errors Δf and Δg. It should be noted that our results on controller gain matrices such as K_j do not depend on the information of membership functions, but on the number of fuzzy rules as in (19), which implies that the robustness is enough to compensate for different choices of membership functions. We should point out that the dynamical behavior of the closed-loop system is different when choosing different membership functions. Therefore, we believe that our result can be applied widely if enough fuzzy rules are used under different membership functions.

V. CONCLUSION

In this paper, a fuzzy H_∞ control scheme for a class of nonlinear NCSs via the FE has been proposed. The FE is designed to estimate the states of a nonlinear plant via limited sampling information. Both the network-induced delay and packet dropout rate are considered in a uniform framework. The disturbance attenuation term is designed to attenuate the influence of modeling errors and external disturbances on the system.

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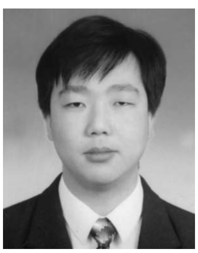

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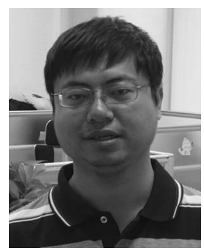

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