High Resolution 3D Bayesian Image Reconstruction using the microPET Small Animal Scanner

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Abstract. A Bayesian method is described for reconstruction of high resolution 3D images from the microPET small animal scanner. Resolution recovery is achieved by explicitly modeling the depth dependent geometric sensitivity for each voxel in combination with an accurate detector response model that includes factors due to photon pair non-colinearity and inter-crystal scatter and penetration. To reduce storage and computational costs we use a factored matrix in which the detector response is modeled using a sinogram blurring kernel. Maximum *a posteriori* (MAP) images are reconstructed using this model in combination with a Poisson likelihood function and a Gibbs prior on the image. Reconstructions obtained from point source data using the accurate system model demonstrate a potential for near isotropic FWHM resolution of approximately 1.2mm at the center of the field of view compared to approximately 2mm when using an analytic 3D reprojection (3DRP) method with a ramp filter. These results also show the ability of the accurate system model to compensate for resolution loss due to crystal penetration producing nearly constant radial FWHM resolution of 1mm out to a 4mm radius. Studies with a point source in a uniform cylinder indicate that as the resolution of the image is reduced to control noise propagation, the resolution obtained using the accurate system model is superior to that obtained using 3DRP at matched background noise levels. Additional studies using pie phantoms with hot and cold cylinders of diameter 1 - 2.5mm and ^{18}FDG animal studies appear to confirm this observation.

1. Introduction

MicroPET is a high resolution PET scanner designed for imaging small laboratory animals [Cherry et al, 1997]. It consists of a ring of 30 position-sensitive scintillation detectors, each with an 8×8 array of 2mm x 2mm x 10mm lutetium oxyorthosilicate (LSO) crystals coupled via optical fibers to a multi-channel photomultiplier tube. The detector ring diameter of microPET is 172mm with an imaging field of view of 112mm transaxially by 18mm axially. The measured intrinsic detector pair resolution is 1.68mm FWHM (full width at half maximum) and the reconstructed image resolution, as measured using the 3D reprojection method of [Kinahan and Rogers, 1989], is approximately 2.0mm FWHM isotropically at the center of the field of view. The scanner has no septa and operates exclusively in 3D mode. Here we report on the development of a Bayesian reconstruction method for the scanner.

The majority of fully 3D PET studies are reconstructed using algorithms based on the analytic relationship between the source distribution and its sinograms [Colsher, 1980, Kinahan and Rogers, 1989, Defrise et al, 1995]. The resolution of these methods is limited by the implicit assumption that the data correspond to true line integrals through the unknown source distribution. In contrast, iterative reconstruction algorithms that are based on a statistical model are able to maximize resolution recovery by accurate modeling of the system response. Furthermore, these methods can optimize performance in low count

situations through explicit modeling of the statistical variability inherent in photon limited coincidence detection.

Iterative 3D reconstruction represents a daunting computational challenge due to the large of number of lines of response (LORs) collected for each data set. [Kinahan et al, 1996] reduce the dimension of the data by pre-processing using Fourier rebinning before applying the OSEM algorithm. While this leads to substantial cost savings, Fourier rebinning again assumes that the data are measurements of line integrals so that the potential for resolution recovery is lost with this approach. Several other investigators have approached the problem using a combination of sparse matrix structures and in-plane and axial symmetries to reduce computation and storage requirements [Chen et al, 1991, Johnson et al, 1995, Johnson et al, 1997, Ollinger and Goggin, 1996, Terstegge et al, 1996]. These methods model the detection process using geometrical computations based either on the intersection of detection "tubes" with each voxel [Ollinger and Goggin, 1996] or on depth dependent geometric sensitivity calculations based on the solid angles subtended at the detectors by each voxel [Chen et al, 1991, Terstegge et al, 1996]. [Johnson et al, 1995] also use a detection tube intersection model but include a shift-variant weighting to model detector response. Here we build on this work by using a depth dependent solid angle calculation in combination with a spatially variant detector response model. Using a factored matrix form, similar to that described for the 2D case in [Mumcuoglu et al, 1996b], we are able to include this more accurate model at little additional computational cost compared to models that do not include detector response.

Many of the statistically based 3D reconstruction methods are based on the EM [Chen et al, 1991, Johnson et al, 1995] or OSEM [Johnson et al, 1997] algorithms. Both approaches can exhibit high variance behavior at high iteration numbers and are regularized through early termination of the algorithm or by subsequent smoothing of the reconstructed images. Here we use a MAP formulation in which the variance and resolution of the reconstruction are controlled through the regularizing influence of a prior. The MAP methods do not typically exhibit the instabilities at higher iterations encountered using EM and OSEM and hence the choice of stopping point is not critical once effective convergence has been reached. To compute the MAP solution, we use a 3D extension of the pre-conditioned conjugate gradient algorithm described in [Mumcuoglu et al, 1996b] and [Mumcuoglu et al, 1996a].

2. Factored System Model

2.1. The detection probability matrix

The theory of 3D MAP image reconstruction is essentially the same as for the 2D problem, differing primarily in the specifics of the detection probability matrix, P. Even

for microPET, which is small compared to the latest generation of clinical 3D PET systems, the $\boldsymbol{P}$ matrix is huge as shown in Table 1. Sparseness and sinogram symmetry properties can be used to reduce this size to more reasonable proportions as previously described by [Johnson et al, 1995, Chen et al, 1991]. Here we combine the use of these symmetries with the factored system matrix approach that we previously applied to 2D PET [Mumcuoglu et al, 1996b] to develop an accurate system model for which forward and backward projection can be performed efficiently.

ring diameter, mm	172	object size, mm	$100 \times 100 \times 18$
detectors per ring	240	object size, voxels	$128 \times 128 \times 24$
number of rings	8	voxel size, mm	0.75^3
angles per sinogram	120	full size of $\boldsymbol{P}$	280 Gbytes
rays per angle	100	storage size of $\boldsymbol{P}_{Geom}$	16 Mbytes
number of sinograms	64	storage size of $\boldsymbol{P}_{Blur}$	0.02 Mbytes
projections per sinogram	12,000	storage size of $\boldsymbol{P}_{Attn}$ and $\boldsymbol{P}_{Eff}$	1.4 Mbytes
total projection rays	768,000	total storage size of $\boldsymbol{P}$	18 Mbytes

Table 1. 3D Problem Dimensions for the MicroPET system. The projection matrix sizes are based on using 8 bits to store each element of $\mathbf{P}_{geom}$ as described in Section 2.2.

The elements, p_{ij}, of the detection probability matrix $\mathbf{P} \in I\!\!R^{M \times N}$ denote the probability of detecting an emission from pixel site j, $j = 1 \ldots N$ at detector pair i, $i = 1, \ldots M$. In order to reduce the stored size of the $\boldsymbol{P}$ matrix, we factor it as follows:

$$\boldsymbol{P} = \boldsymbol{P}_{det.sens}\boldsymbol{P}_{det.blur}\boldsymbol{P}_{attn}\boldsymbol{P}_{geom}\boldsymbol{P}_{positron} \tag{1}$$

Here $\boldsymbol{P}_{attn} \in I\!\!R^{M \times M}$ is a diagonal matrix containing the attenuation factors. Since there is currently no transmission source for the microPET system, we calculate these factors by re-projecting an estimate of the attenuation image obtained using the region of support estimated from a preliminary emission image and assuming a constant attenuation coefficient of 0.095 cm^{-1}. The diagonal detector normalization matrix $\boldsymbol{P}_{det.sens} \in I\!\!R^{M \times M}$ is measured using a uniform cylindrical source. The data presented here are all for ^{18}F studies in which positron range is sub-millimeter. We therefore ignore these effects and set $\boldsymbol{P}_{positron} \in I\!\!R^{N \times N}$ equal to the identity matrix. In future studies we will investigate the use of positron range blurring model for isotopes other than ^{18}F. Here we concentrate on the two novel aspects of our model, i.e. modeling of geometrical sensitivity in $\boldsymbol{P}_{geom}$ and sinogram blurring factors in $\boldsymbol{P}_{det.blur}$.

2.2. The Geometric Projection Matrix

$\boldsymbol{P}_{Geom} \in \mathbb{R}^{M \times N}$ is the geometric projection matrix with each element (i, j) equal to the probability that a photon pair produced in voxel j reaches the front faces of the detector pair i in the absence of attenuation and assuming perfect photon-pair colinearity. It is calculated from the solid angle spanned by the voxel j to the faces of the detector pair i. After rotating the coordinates, we approximate the 3D solid angle as a product of two angles as illustrated in Fig. 1. The two angles for each voxel are computed, with reference to the regions in Fig. 1, as follows:

$$
\theta = \begin{cases}
\angle d_{21} V d_{22} & \text{voxel in region I} \\
\pi - \angle d_{11} V d_{21} & \text{voxel in region II} \\
\angle d_{11} V d_{12} & \text{voxel in region III} \\
\pi - \angle d_{12} V d_{22} & \text{voxel in region IV}
\end{cases}
\tag{2}
$$

where V denotes the position of the center of each voxel. To improve the accuracy of this model, each voxel is divided into 64 subvoxels and the angle from the center of each subvoxel is computed. The final $p_{geom}(i, j)$ is then computed as the average

$$
p_{geom}(i, j) = \frac{1}{64\pi^2} \sum_{k=1}^{64} \theta_{x'-y'}(k) \theta_{x'-z'}(k).
\tag{3}
$$

In Fig. 2, we show the variation in $p_{geom}(i, j)$ for voxels lying along the center of a projection tube for a single detector pair. This figure shows significant depth dependent sensitivity for this particular LOR, however we note that these pixels contribute to adjacent LORs in such a way that the overall sensitivity to each voxel for each sinogram is approximately uniform.

$\boldsymbol{P}_{Geom}$ is very sparse and has redundancies of which we can take advantage. By choosing the voxel size in the z direction to be an integer fraction of the ring distance, there are the following symmetries in the $\boldsymbol{P}_{Geom}$ matrix [Johnson et al, 1995] [Chen et al, 1991]: in-plane rotation symmetries, resulting from rotating the projection rays by $\theta = 90°, 180°$ and $270°$, and a $\theta = 45°$ reflection symmetry. This provides a total factor of 8 reduction. Axial reflection symmetry provides an additional factor of 2 reduction for ring differences other than zero. The parallel symmetry of sinograms with a common ring difference R_d provides a reduction by a factor of $(N_r - R_d)$ where N_r is the number of rings in the system. Combining these, the total reduction factor from the symmetry operations is approximately 64 for microPET. Therefore, we need only store the non-zero components of the base-symmetry LORs, which amount to 12440 LORs for microPET. Further savings in storage and computation can be realized by using an automated indexing scheme. This is achieved by storing a base pixel index and run length in the y direction for each value of x and z. By choosing the base symmetry lines of response so that they tend to run at small angles to the y direction, this scheme produces close to an additional factor of two saving in storage. Each element of the matrix is stored as a single 8-bit integer which is
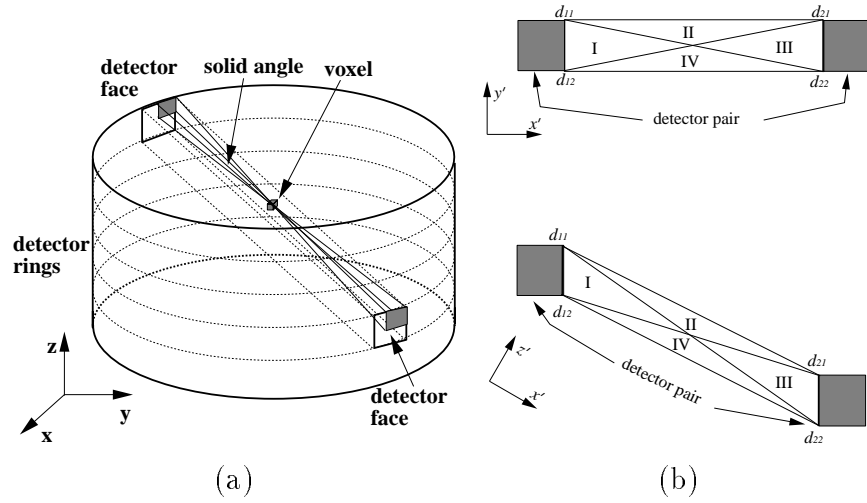
Figure 1. Geometry for computing $p_{geom}(i,j)$. The coordinates (x',y',z') are a rotation of the original coordinate first w.r.t. the z axis and then the y' axis, so that the x' axis is parallel to the line that joins the centers of two detector faces. (a) illustration of the solid angle subtended at the detectors for a single voxel. (b) cross sections in (x',y') and (x',z') planes.
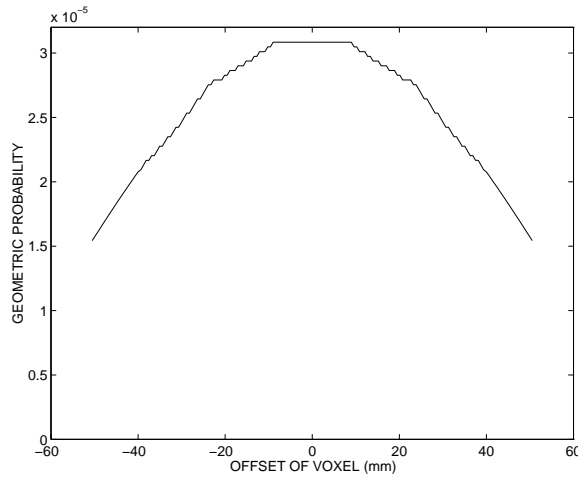
Figure 2. Values of $p_{geom}(i,j)$ for voxels lying along the centers of a detector tube corresponding to a single detector pair. The detector pair shown are at the center of the field of view with a ring difference of one. The field of view is 100 mm and the distance between the two detectors is 172 mm.

computed as $255 * p_{geom}(i,j)/p_{max}$. The constant p_{max} is the maximum solid angle in the system computed at the point midway along the line joining the centers of the two detectors forming the shortest transaxial LOR.

2.3. The Sinogram Blurring Matrix

$\boldsymbol{P}_{Blur}$ is the sinogram blurring matrix used to model photon pair non-colinearity, inter-crystal scatter and penetration. In principal, uncertainties in the angular separation of the photon pair should be included in $\boldsymbol{P}_{Geom}$, however, this will reduce its sparseness. We therefore lump this factor with the detector blurring effects by assuming that it is approximately depth independent. Similarly, we assume that the effects of inter-crystal scatter and detector penetration are independent of the distance of the positron annihilation point from the detectors.

In principle, a 3D sinogram blurring model should be used to model the radial, angular and inter-sinogram blurring. In our current implementation we have assumed that these blurring effects can be confined to a single sinogram and use a 2D blurring model. Furthermore, because the axial acceptance angle of microPET is small ($\leq 7.5°$), we assume that the blurring kernels are identical for sinograms for all ring differences. We treat each crystal as a separate detector and therefore ignore effects associated with the location of each detector within the 8×8 blocks used in the microPET system. We then have a rotational symmetry in the blurring kernels due to rotational invariance of the detector geometry [Mumcuoglu et al, 1996b]. We note that although this model cannot account for variations in the blurring kernels that may occur due to block effects (e.g. photons scattered between a pair of adjacent detectors in two adjacent blocks will be modeled in the same way as scatter between adjacent pairs in the same block), the spatially variant sensitivity due to block structure is included in $\boldsymbol{P}_{det.sens}$. As a result of the rotational symmetry, we need only compute and store the blurring kernels for the projection rays for a single projection angle, which saves both computational time and storage size; the computational cost of performing the sinogram blurring is only a few percent of that for computing the geometric projection. The blurring factors were computed using the Monte Carlo code described in [Mumcuoglu et al, 1996b]. Statistical modeling of non-colinearity, crystal penetration and inter-crystal scatter in the LSO detectors was used to produce the blurring of the sinogram element under consideration into the neighboring elements. Fig. 3 shows the blurring kernels for two different LORs; note the significant crystal penetration for off-center detector pairs due to the small crystal size.

To illustrate the size saving from factoring out the detector blur as compared to incorporating it in the the geometric matrix, as is the case in [Johnson et al, 1995], we counted the number of nonzero elements in $\boldsymbol{P}_{Geom}$ and in $\boldsymbol{P}_{Blur}\boldsymbol{P}_{Geom}$. To make the comparison fair, the resulting elements of $\boldsymbol{P}_{Blur}\boldsymbol{P}_{Geom}$ were quantized to the same number

of levels as we use in $\boldsymbol{P}_{Geom}$. Because of rotation symmetry, only the columns corresponding to the voxels in the plane $y = 0$ need to be counted. The number of nonzero elements corresponding to voxels with the same radial offset were averaged together in the z-direction and the result is shown in Fig. 4. This shows a savings factor of approximately three from using the factored matrix. Further relative savings would be realized if a full 3D blurring model were used.

ray-1	0	ray+1	ray+2	ray+3	ray+4	ray+5
0	0	0	0	0	0.0016	0
0	0.0103	0.0028	0.0432	0.0056	0.0077	0.0019
0.0062	**0.1169**	0.1686	0.2459	0.0600	0.0140	0.0019
0.0063	0.0099	0.1711	0.0449	0.0604	0.0079	0.0018
0	0	0.0025	0	0.0052	0.0014	0.0022

(a)

ray-3	ray-2	ray-1	0	ray+1	ray+2	ray+3
0	0	0	0	0	0	0
0.0014	0.0092	0	0.0058	0	0.0081	0.0017
0	0.0055	0.0651	**0.6778**	0.0618	0.0060	0
0	0.0100	0.0658	0.0054	0.0630	0.0093	0
0.0018	0	0	0	0	0	0.0022

(b)

Figure 3. Sinogram blurring kernels for the sinogram components indicated by the bold characters corresponding to (a) the 20th and (b) the 50th out of 100 projection rays. These kernels are shown for the sinogram stored in interleaved format.

3. Image Reconstruction

The factored system model described above was used in a statistical image reconstruction framework. The standard Poisson likelihood function was used for the data and a Gibbs prior with a Huber potential used to model the 3D image. The image was reconstructed by maximizing the log posterior density:

$$\hat{\boldsymbol{x}}(Y) = \arg\max_{\boldsymbol{x}} L(Y|\boldsymbol{x}) - \beta\phi(\boldsymbol{x}) \tag{4}$$

$$= \arg\max \sum_i [-\hat{Y}_i + Y_i \log(\hat{Y}_i)] - \beta \sum_j \sum_{\substack{k \in \mathcal{N}_j \\ k > j}} \kappa_{jk} V(x_j - x_k), \tag{5}$$

where Y are the measurements, β is the hyperparameter of the Gibbs prior, and $\hat{Y}$ is the data associated with image $\boldsymbol{x}$, i.e. $\hat{Y} = \boldsymbol{P}\boldsymbol{x}$. We have not included scatter or randoms in this
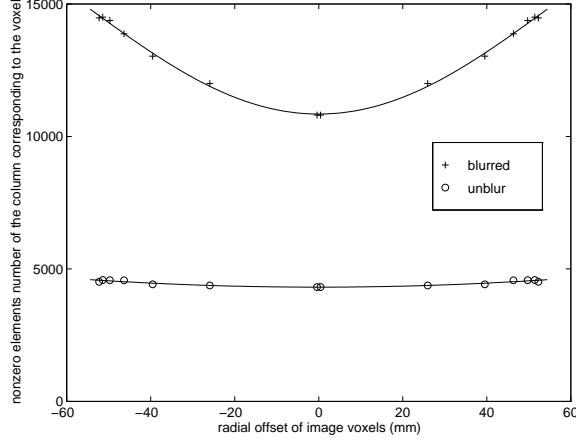
Figure 4. The average number of nonzero elements of different columns in P_{Geom} and $P_{Blur}P_{Geom}$ corresponding to voxels at different radial offsets. The total numbers of nonzero elements in the two matrices were calculated using a radially weighted sum of the two curves to give an average ratio of 3:1 between $P_{Blur}P_{Geom}$ and P_{Geom}.

model. In small animals the total activity is small compared to that for human studies so that scatter and randoms fractions are much lower in the microPET system and are currently ignored in our work.

The Huber potential function $V(x_j - x_k)$ is

$$V(x_j - x_k) = \begin{cases} \frac{1}{2\delta_h}(x_j - x_k)^2, & \text{if } |x_j - x_k| \leq \delta_h \\ |x_j - x_k| - \frac{\delta_h}{2}, & \text{otherwise} \end{cases} \tag{6}$$

where δ_h is a small constant. In the results presented below, δ_h was chosen to be very small, ie. 1% or less of the maximum reconstructed image intensity. The neighborhood $\mathcal{N}_j$ we used here is the second order (26-voxel) neighborhood with κ_{jk} equal to the reciprocal of the distance between the two voxels.

Using the factored matrix approach we gain substantial savings in storage and computational requirements. To fully realize this saving we must consider all data and all pixels at each iteration. Therefore the preconditioned conjugate gradient (PCG) approach [Mumcuoglu et al, 1996a] and EM algorithms are more suitable for 3D reconstruction, when using the factored matrix, than either coordinate-wise methods [Bouman and Sauer, 1996, Fessler, 1994, Sauer and Bouman, 1993] or ordered subset methods [Browne and De Pierro, 1996, Hudson and Larkin, 1994]. In the case of coordinate-wise ascent, one complete iteration through the image will require the re-application of the blurring kernels each time a different pixel forward projects to a particular sinogram element; this results in substantial increases in computation cost. In the case

of the ordered subset methods, efficiency is lost because computation of the sinogram subset associated with the current image estimate requires forward projection into sinogram elements neighboring the current subset. This is because the sinogram subsets are coupled through the blurring kernels. However, by approximating the blurring kernels as a 1D radial-only blurring function, we can use OSEM with the factored system model without loss in computational efficiency.

In all of the results shown below we used 20 iterations of the PCG method since the images were observed to change very slowly beyond this point. The procedure was initialized using a constant image scaled so that the total counts resulting from forward projection of this image equaled the total observed counts. For comparative purposes, we have shown images reconstructed using the reprojection (3DRP) method of [Kinahan and Rogers, 1989]. While it may also be possible to improve the performance of the 3DRP method by pre-filtering the data to compensate for the detector response [Huesman et al, 1989] [Liang, 1994] or using a Wiener filter based approach [Shao et al, 1994][Fessler, 1994], these methods have not yet been widely studied in the 3D literature. In contrast, the standard 3DRP method used here serves as something of a gold standard in fully 3D reconstruction .

The computational cost of one iteration of the PCG algorithm is about 3 mins on a 167MHz Ultrasparc processor. Therefore, a $24\times128\times128$ voxel image requiring 20 iterations can be reconstructed within 60 minutes and a $24\times64\times64$ voxel image within 25 mins. For comparison, the 3DRP method we used takes 15 minutes to reconstruct a $15\times128\times128$ voxel image (although we note that this program has not been optimized to minimize computation time).

4. Experimental results

4.1. Point Source Measurements

Our factored system model was tested using experimental measurements of a point source. A 0.5 mm diameter, 0.5 mCi ^{22}Na point source was scanned at different positions in the field of view. Each data set contained approximately 3,000 events per sinogram. We first compared the sinogram profile measured with that predicted using the factored system model. The results are shown in Fig. 5, with and without the inclusion of $\boldsymbol{P}_{Blur}$. The figure shows that by using the blurring kernels, the asymmetry and peak shift due to crystal penetration are successfully followed. However, as we show below, errors in these blurring factors appear to be at least partially responsible for artifacts in the pie phantom studies. We are currently improving the Monte Carlo model of the detectors to correct this problem.

The point source data were reconstructed to investigate the potential for resolution recovery using our factored system model. The Poisson likelihood with positivity constraint can create artificially high resolution for a point source in zero background; resolution was
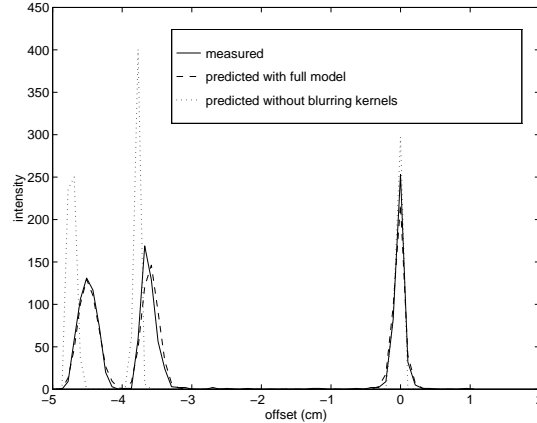
Figure 5. Profiles of the measured and calculated sinograms for the point source at different positions in the field of view.

therefore measured instead using a quadratically penalized weighted least squares (PWLS) method [Fessler, 1994] without a positivity constraint. Profiles were taken through the point source images and the resolution determined by measuring the FWHM. Fig. 6 shows plots of the radial and tangential components of the transaxial resolution, and also the axial resolution, compared to results obtained using the 3D reprojection method with a ramp filter with a cut off at the Nyquist frequency, i.e. half of the spatial sampling frequency for the interleaved sinogram. These clearly show that improvements in resolution can be achieved by using the accurate system model when compared to reconstruction obtained using either a simplified system model (no detector response modeling) or the 3DRP method with a ramp filter. The most dramatic improvement in resolution from the use of the blurring kernels occurs in the radial direction where we see a resolution of approximately 1mm FWHM out to a 4cm radius. The axial resolution is not improved by the 2D blurring kernels because we do not currently model axial blur. Some caution is necessary in interpreting these results. The FWHM resolution does not reflect the behavior of the point spread functions (PSFs) below the half-maximum level and in fact in some locations we noted some lengthening of the tails of the PSF. Furthermore the estimators that we are using are non-linear in the data so that resolution of point sources does not extrapolate directly to more distributed sources.

These point studies illustrate the potential gain that can be realized by accurately modeling the data. However, this gain is only useful if it can be achieved without large levels of noise amplification. We therefore also studied the trade-off between background noise levels and resolution using a point source in a uniform background. It is not possible to collect data for the point source in a uniform background because the point source is encased in a plastic sphere. Instead, we collected data from a uniform cylinder and added

to this a separately acquired point source data set. By positioning the point source and the uniform cylinder at the center of the field of view, the attenuation that would have occurred if the point source was actually inside the uniform cylinder would be constant along each LOR passing through the point source. Consequently, the combined data set should be virtually identical to that which would have been collected from a point source in a uniform background. We reconstructed images from the combined data set using the MAP method with different values of the hyperparameter β and the 3DRP method with a ramp filter with different cut-off frequencies. Fig. 7 shows the resulting contrast recovery vs. background variance plots for the two methods. These plots show that at matched noise levels we can achieve superior contrast using the MAP method with the factored system model than when using 3DRP.

(a) (b) (c)
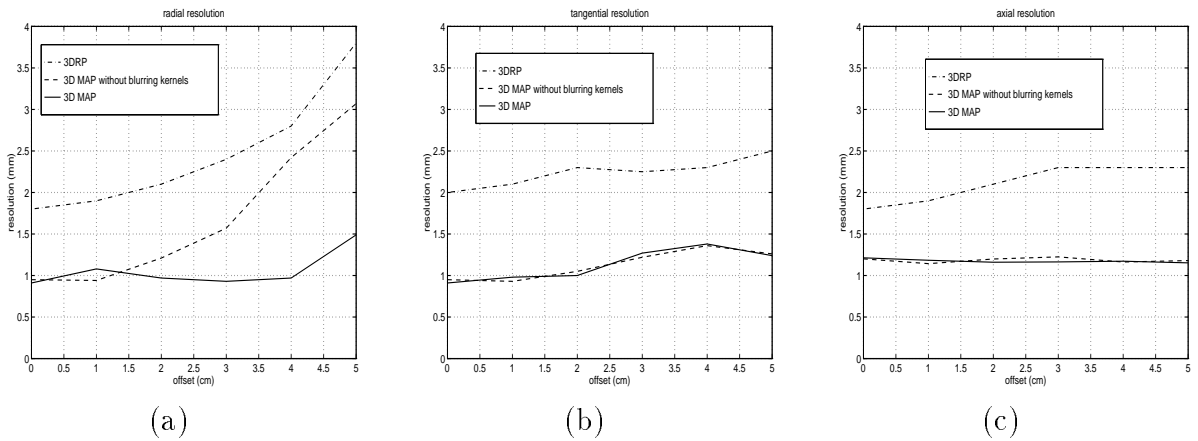
Figure 6. FWHM resolution of the point source image using 3DRP and PWLS, with and without modeling of the sinogram blur: (a) radial resolution, (b) tangential resolution, (c) axial resolution.

4.2. Hot and Cold Phantom Studies

Specially constructed hot and cold pie phantoms were also scanned and reconstructed as shown in Fig. 8. These images show overall improvements in contrast in the MAP images compared to 3DRP and again we see little degradation in performance as the phantom is moved towards the edge of the field of view when using the factored matrix model. There are some differences in the MAP results between the centered and off-centered phantoms due to the small voxels (0.4mm x 0.4mm) used for the centered phantom compared to the larger voxel size (0.85mm x 0.85mm) for the off centered phantom. The different sizes were used because the projection matrix for the small voxels becomes huge for the large field
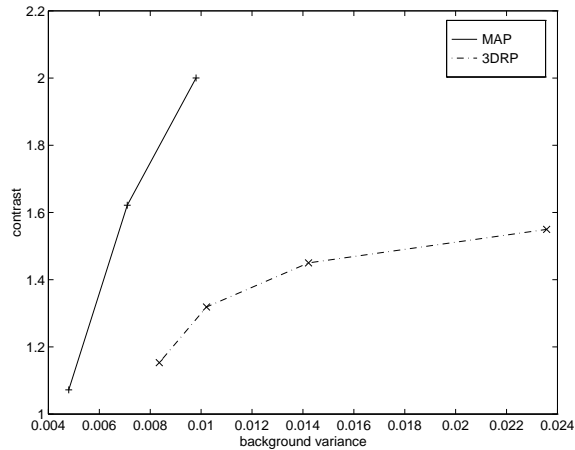
Figure 7. Contrast recovery for the point source vs. spatial variance of the cylinder for different smoothing parameters for the MAP and 3DRP methods.

of view required for the off centered phantom - note that the symmetries that are used to efficiently store the matrix preclude computing and storing the projection matrix for a small off-centered source region. The dark area at the top of the centered cold spot phantom is due to an accumulation of air bubbles.

There is a noticeable ring artifact at the edge of the cold phantom. The amplitude of this artifact is reduced when the MAP method is applied to simulated data, indicating that it is, in part, due to errors in the blurring kernels. However, the fact that the artifact remains when applied to simulated data and when using different iterative reconstruction methods (OSEM, least squares and penalized weighted least squares) is an indication that artifacts also arise from ill-conditioning in the accurate system model. The ill-conditioning gives rise to a Gibbs-like oscillation in the vicinity of sharp boundaries which are particularly prominent for near circularly symmetric objects such as the cold spot phantom. Similar effects have been reported elsewhere, e.g. [Snyder et al, 1987]. The artifact can be reduced or eliminated by increasing the degree of smoothing, but this will be at the expense of an overall loss in resolution.

4.3. Animal Studies

[18]FDG data were collected from a 3 month old baby vervet monkey scanned using the microPET scanner after injection of 2.2mCi of FDG. The total counts were about 1 million per sinogram for a collection time of 40 mins. Fig. 9 shows the image reconstructed by the 3D MAP method in comparison to the 3D reprojection method. The reconstructed field of

| hot centered | hot off-centered | cold centered | cold off-centered |
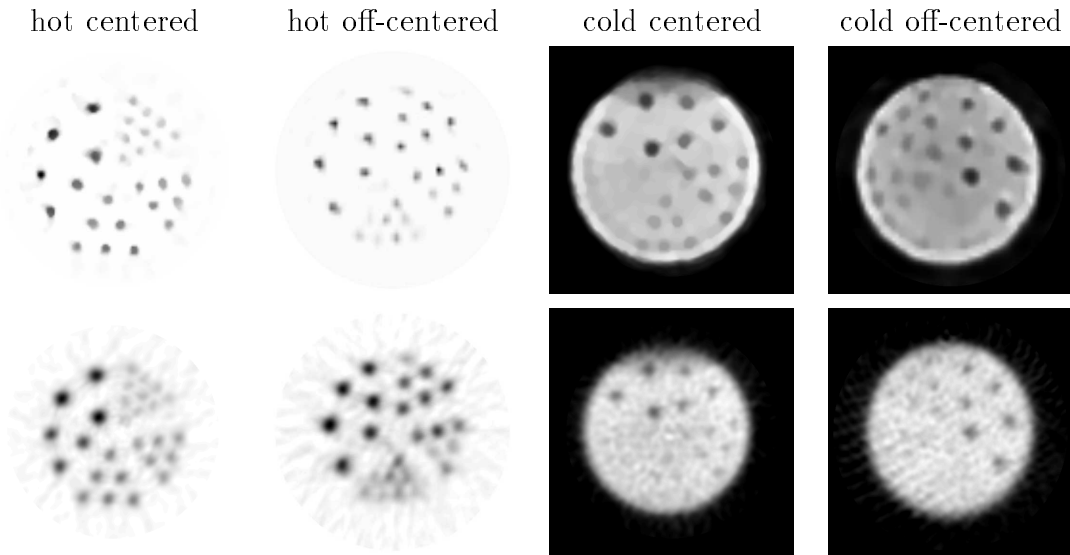
Figure 8. Reconstructions of hot and cold resolution phantoms at the center and edge of the field of view with cylindrical regions of sizes 1.0, 1.25, 1.5, 2.0, 2.5 mm diameter. Top row: MAP reconstructions; Bottom row: 3DRP reconstructions. Hot phantom: ∼32M total counts; cold phantom: ∼100M total counts. The phantom occupied the entire axial field of view.

view in these figures is a circle of diameter 8cm with the maximum diameter of the brain approximately 6cm. These images appear to confirm the resolution enhancement observed in the point source and pie phantom studies. The oblique stripes in the 3DRP reconstructions are probably due to imperfect detector normalization caused by time varying sensitivities. However, these artifacts are not seen in MAP reconstruction indicating a potential robustness of MAP to small errors in normalization factors.

5. Conclusions

We have described a fully 3D MAP reconstruction method for the high resolution microPET animal scanner. We have shown that we can model and deconvolve the system response within this framework to achieve uniform transaxial resolution of 1mm FWHM for objects up to a 4cm diameter, and a resolution of about 1.2mm up to an 8cm diameter. These values reflect the maximum achievable resolution; when using noisy data, some loss in resolution is produced by increasing the hyperparameter β to control noise propagation. However, uniform cylinder plus point source studies show that the MAP method can achieve higher resolution than the 3D reprojection method at matched noise levels. These observations are also confirmed by the hot and cold spot phantom studies. The appearance of ring artifacts in the cylindrical cold spot phantom indicates that the improvement in resolution is gained at

(a)   (b)
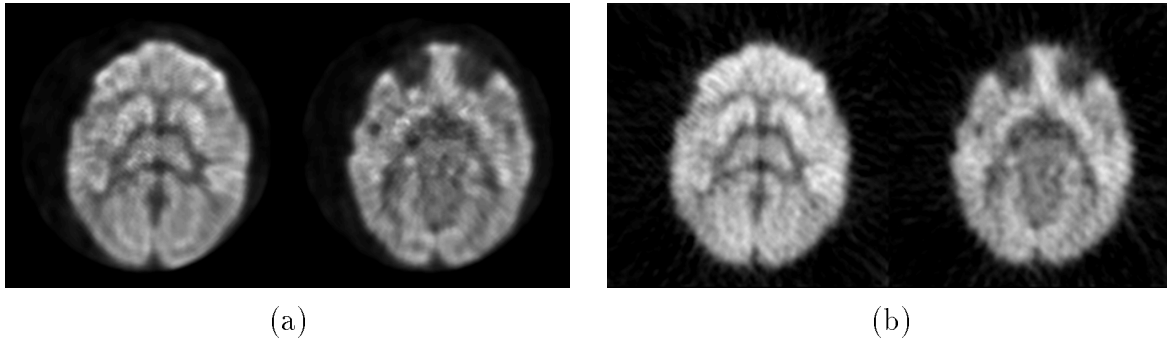
Figure 9. Baby monkey brain images reconstructed using (a) the 3D MAP method with full system model ($\beta = 2 \times 10^6$); (b) the 3D reprojection method with ramp filter (cut-off frequency =0.8 Nyquist frequency).

the expense of increased ill-conditioning in the system which results in increased sensitivity to modeling and numerical errors as well as noise. These artifacts can be reduced or supressed by increasing the hyerparameter β but at the expense of reduced resolution.

Through optimization or storage and computational costs, reasonable reconstruction times can be achieved which should allow the routine use of this method in research studies. The methodology described here is also directly extendible to 3D human PET scanners.

Acknowledgments

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