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How Efficient is Naive Portfolio Diversification? An Educational Note

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Abstract

Standard textbooks of Investment/Financial Management will tell you that although portfolio diversification can help reduce investment risk without sacrificing the expected rate of return, the benefit of diversification is exhausted with a portfolio size of 10 to 15. Since by then, most of the diversifiable risk is eliminated, leaving only the portion of systematic risk. How valid is this "common" knowledge? What is the exact value of "most" in the above statement? This paper examines the issue on naive (equal weight) diversification and analytically shows that for an infinite population of stocks, a portfolio size of 20 is required to eliminate 95% of the diversifiable risk. However, an addition of 80 stocks (i.e., a size of 100) is required to eliminate an extra 4% (i.e., 99% total) of diversifiable risk. This result depends neither on the investment horizons, sampling periods nor the markets involved.

Keywords: Naive Diversification, Efficiency, Portfolio, Diversifiable risks

1. Introduction

Over the past 40-50 years, portfolio diversification is one of the main modern investment theories that have been developed. It may not be the most important theory being developed, however, it is no doubt that holding portfolios is the most widely accepted investment concept and the most being practised knowledge in real life by general investors. All university business graduates learn that through portfolio diversification, investment risk can be reduced without sacrificing the expected return. This concept can be easily applied without any complex techniques via naive diversification. That is, one can hold a diversified portfolio by randomly select a certain number of stocks and invest equal amount of money in each of them. While this is simple enough, however, standard textbooks of Investment/Financial Management will also tell you that the benefit of diversification is mostly exhausted with a portfolio size of 10 to 15. Since by then, most of the diversifiable risk is eliminated, leaving only the portion of systematic risk.

The existence of systematic risk is the reason why the benefit of portfolio diversification can be exhausted no matter how large is the portfolio size since by definition, systematic risk cannot be eliminated through diversification. All stocks would be affected at the same time by some economy-wide factors. Hence, studying the relationship between the portfolio risk and the portfolio size is important as this will dictate the necessary number of stocks required in naive diversification to obtain the largest benefit. In theory, one should go for holding as many stocks as possible as long as the portfolio's variance keeps on decreasing. But in practice, the additional benefit gained through further risk reduction may not be large enough to offset the extra transaction costs involved. Investors have to make the trade-off between the reduced risks due to more effective diversification versus the additional transaction costs that mean lower returns from adding more stocks to their portfolios.

If individual investors indeed can obtain most of the benefits of diversification by holding small-size portfolios, say 10-15 stocks as suggested by most Investment/Financial Management textbooks, effective diversification can be accessed easily and directly. Then those unit trust and fund managers would need great effort to justify their existence and the high management fees charged through either superior selectivity or good market timing. So, it becomes interesting and important to answer these questions: How valid is this "common" knowledge? What is the exact meaning of "most" in the statement that most of the diversifiable risk is eliminated? Can we obtain an exact relationship between the portfolio's variance and the increasing portfolio size? Does it matter on the effectiveness of diversification with an infinite population or a finite population of stocks? These questions are interesting because they have never been clearly and directly addressed in the current textbooks of Investment and Financial Management despite of its direct relevance to general investors. They are also important in the sense that they have major implications for the business of unit trusts and fund houses and for the behaviour of general investors. This paper aims to examine these issues.

The rest of this paper is organised as follows: Section 2 summaries the textbooks' recommendations; Section 3 presents an analytical relationship between portfolio size and risk, and provides an answer to "most" in the above statement; the differences between infinite and finite populations on our results are discussed in Section 4; Section 5 gives a remark on portfolio diversification and concludes the paper.

2. Textbooks' Recommendations

Before reviewing the textbooks' recommendations, let us first describe the standard approach in studying the relationship between risk and portfolio size through naive diversification. For empirical analysis on a selected market, the population of stocks (population size) is first defined. For example, the 500 stocks of the S&P 500 Index in U.S. or it can be the total number of national stock market indices if international diversification is being studied. A stock is selected randomly from the population and its risk is measured by the variance (or standard deviation) calculated from the series of stock returns. Another stock is then selected from the population to form a portfolio of size 2 with the first stock. The portfolio's variance is calculated by assigning equal weight to these two stocks. Stock 3, stock 4, and so on are selected randomly from the population in sequence without replacement. At each time, equally weighted portfolios are formed and the portfolios' variances are calculated. Hence, a series of portfolios with sizes ranging from 1 to say, 100 are obtained. The whole process is then repeated for many times, say 100 times. This means that we have obtained 100 portfolio's variances for each individual portfolio size. For each individual portfolio size, the average portfolio's variance is calculated. By plotting the average portfolio's variance against the portfolio size, the relationship between risk and the number of stocks in the portfolio is thus obtained.

The above research approach is mentioned in all major Investment/Financial Management textbooks. After this, the optimal number of stocks required in a diversified portfolio is stated out at which the authors claim that most of the benefits of diversification will then be obtained. Table 1 lists the recommendations on this "magic" number from ten Investment textbooks and ten Financial Management textbooks. These textbooks are believed to be the most representative and widely adopted by universities for investment/finance courses. Column 1 shows the authors' names and years of publication (or years of latest edition). Column 2 indicates the page numbers respectively for each textbook where the required information can be found. Column 3 shows the recommended optimal number of stocks in portfolio. Of the ten Investment textbooks, the minimum number is 8 while the largest number is around 40. Most of them recommend a size of 10 to 15. For the ten Financial Management textbooks, the minimum and maximum numbers are 10 and 40 respectively. The most common recommendation is 10 to 20. A closer look at these textbooks' recommendations reviews that most of their recommendations are actually based on some academic journal articles. Column 4 of Table 1 presents the main sources of reference cited in each textbook.

3. Size vs Risk: Analytical Relationship

The general formula for the variance of a portfolio with size n is normally stated as follows:

$$s_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \qquad \textbf{(1)}$$

where w_i and w_j are the investment proportions on assets i and j respectively; r_i and r_j are the returns of assets i and j respectively, and cov(r_i, r_j) represents the covariance between returns of assets i and j. In naive diversification where an equally weighted portfolio is formed, we have w_i = w_j = 1/n and equation (1) can be rewritten as:

$$s_p^2 = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{n}s_i^2 + \sum_{i=1_{i\neq j}}^{n}\sum_{j=1}^{n}\frac{1}{n^2}Cov(r_i, r_j) \qquad \textbf{(2)}$$

In equation (2), we know that there are n variance terms and n(n-1) covariance terms. Let the average variance and average covariance be as follows:

$$\overline{s^2} = \frac{1}{n}\sum_{i=1}^{n}s_i^2 \qquad \textbf{(3)}$$

$$\overline{Cov} = \frac{1}{n(n-1)}\sum_{i=1_{i\neq j}}^{n}\sum_{j=1}^{n}Cov(r_i, r_j) \qquad \textbf{(4)}$$

Then equation (2) can be expressed as

$$s_p^2 = \frac{1}{n}\overline{s^2} + \frac{n-1}{n}\overline{Cov} \qquad \textbf{(5)}$$

From equation (5), it is clear that when n increases, that is, when the portfolio size increases, the first term on the right hand side tends to zero while the second term tends to the average covariance (as *(n-1)/n* tends to 1).

Now suppose N is the population size and equation (5) can be used to describe the variance of the portfolio composed of equal investment in each of the N stocks of the population. For a portfolio with size $n < N$, the same equation can also describe the portfolio variance. However, unlike holding the whole population, we have a total of ${}_NC_n$ possible portfolios with the same portfolio size n. We are interested in the average portfolio variance for size n. For example, for a population of 10 stocks, we have 45 (${}_{10}C_2$) portfolios of size 2, 120 (${}_{10}C_3$) portfolios of size 3, etc. The variance of each portfolio is first calculated and then the average variance is obtained by averaging all variances with the same portfolio size. By using all possible combinations of each portfolio size, the average mean return is guaranteed the same regardless of the portfolio size. In fact, Tang and Choi (1998) employed this methodology to examine empirically the portfolio effect on the standard deviation, skewness and kurtosis of international stock index portfolios. However, the limitation of this methodology is that the population size must be restricted to avoid a huge amount of computational work in all possible combinations of stock portfolios.

Taking all possible combinations of portfolios into consideration is the same as taking the expectation of equation (5). In fact, this is the result mentioned by Elton and Gruber (1977) as equation B1 of Appendix B and therefore, equation (5) can be regarded as the correct formula for the average variance of a portfolio with n stocks regardless of what the population size is. The only modification is that now the average variance and average covariance will be calculated from all stocks in the population. To make it more specific, the relationship can be re-stated as follows:

$$\overline{\sigma_n^2} = \frac{1}{n}\overline{\sigma^2} + \frac{n-1}{n}\overline{Cov} \qquad \textbf{(6)}$$

where n = the number of stocks in the portfolio, $n = 1, 2, \ldots\ldots, N$

N = the number of stocks in the population

$\overline{\sigma_n^2}$ = the average portfolio variance with portfolio size n

$\overline{\sigma^2}$ = the average variance of all stocks in the population

$\overline{Cov}$ = the average covariance of all stocks in the population

One implication from our results is that for an infinite population of stocks (i.e., N tends to infinite), when the portfolio size, n increases from 1, the first term in the right hand side of equation (6) will become smaller and smaller and tends to zero, while the second term will become larger and larger and tends to $\overline{Cov}$. Hence, the first term is the diversifiable (non-systematic) part while the second term is the non-diversifiable (systematic) part.

In order to illustrate the above point more clearly and to indicate the efficiency of naive portfolio diversification, we compute the relative average variances of portfolios with different portfolio sizes (see Tang and Choi, 1998). This is accomplished by dividing all average portfolio variances by the average variance of portfolio with size equals to one. From equation (6), when n equals 1, we have $\overline{\sigma_1^2} = \overline{\sigma^2}$. The result is obvious. Hence, dividing equation (6) by $\overline{\sigma^2}$, we have

$$RV = \frac{1}{n} + (1 - \frac{1}{n})X \qquad \textbf{(7)}$$

where RV = relative average portfolio variance, $\overline{\sigma_n^2}/\overline{\sigma^2}$

X = the ratio of average covariance to average variance of all stocks in the population,

$\overline{Cov} / \overline{s^2}$

Rearranging terms in equation (7), we obtain

$$RV = X + (1 - X)\frac{1}{n} \tag{8}$$

Now it is clear that *X* is the relative systematic risk that cannot be eliminated through diversification while *(1 - X)* is the relative non-systematic risk that can be eliminated completely through naive diversification. When n tends to infinite, *(1 - X)* is completely gone, leaving only *X*, the systematic part.

Several implications are drawn from the above result. First, the power of naive diversification on risk reduction is inversely proportional to the portfolio size n. With only a portfolio size of two, half of the diversifiable risk is eliminated on average. With a size of 10, 90% of diversifiable risk is eliminated and 95% of such risk can be eliminated with a portfolio size of 20 on average. Hence, the value "most" in the statement which appears in many financial/investment management textbooks, that most of the diversifiable risk is eliminated with 10-15 stocks in the portfolio can now be answered specifically and directly. Second, the effectiveness of naive diversification on reducing diversifiable risks is independent neither of the sampling periods, investment horizons nor of the markets involved. It does not matter whether the stocks are hold for one month, two months, or one year nor whether we are investing in the U.S., U.K. or the Japanese stock markets or even are investing in international stock markets. The only relevant factor is the portfolio size.

Third, the part on non-systematic risk cannot be completely eliminated unless we have an infinite stock population. However, the marginal benefit of larger portfolio size due to further risk reduction is a decreasing function of n. In fact, for a portfolio size of n, an addition stock to the portfolio will further eliminate $1/(n^2 + n)$ (or $1/n - 1/(n+1)$) of the diversifiable risk, that is, (1 - *X*) in equation (8). Fourth, that previous empirical results which found that the impact of diversification on portfolio risk varies across different stock samples and different periods is because of the variations in the relative systematic risk, that is, *X* in equation (8), the ratio of average covariance to the average variance of all stocks in the population. The power (or effectiveness) of naive diversification on reducing diversifiable risks has not changed.

Figure 1 plots the relative average variance against the portfolio size with three different assumptions on the value of X, the relative systematic risk. We let X equal to 0.75, 0.5 and 0.25 and check the impact on the downward sloping curves. It is clear that the shape is similar to those presented in textbooks (e.g., Figure 9.1 (p.229), Francis, 1991; Figure 5.4 (p.123), Pinches, 1996). However, in our case, we clearly show the exact relationship between portfolio risks and sizes graphically given the value of relative systematic risk. When the relative systematic risk is 0.75, the curve levels off at a portfolio size of 10. However, when the corresponding value is 0.5 (0.25), Figure 1 shows that the curve levels off at a portfolio size of 15 (20). Hence, this explains why recommendations from textbooks say that empirically for a portfolio size of 10 to 15, most of the diversification benefit is exhausted.

4. Finite Population of Stocks

Section 3 presents the analytical relationship between the average portfolio variance and the portfolio size. Equation (8) also implies that the part on non-systematic risk cannot be completely eliminated unless we have an infinite stock population. However, under a normal investment environment, investors can only select stocks within a population of limited size. Furthermore, investors may even want to restrict their potential pools of stocks to a smaller size than the whole population for various reasons. For example, fund managers may have interest only on those blue-chip stocks in each market. Hence, a relevant question is what will be the impact of different population sizes on the number of stocks required to eliminate a certain percentage of the non-systematic risk. A logical prediction is that a smaller number of stocks are required to achieve the same level of risk reduction for a population with smaller size. Is that true? This section aims to give a quantitative answer.

According to equation (8), even when you hold the whole population of stocks in your portfolio, you still cannot completely eliminate all diversifiable risk. The non-systematic part of relative risk that is still remained equals $(1 - X)/N$. In other words, all you can do best is to eliminate $[(N - 1)/N]$ of $(1 - X)$, the maximum diversifiable risk of the whole population of size N that can be diversified away. Similarly, for a portfolio with a size n, $[(n - 1)/n]$ of $(1 - X)$ is reduced through diversification. Hence, we can see that the proportion of the maximum relative diversifiable risk eliminated with a portfolio size n is equal to $[(n - 1)/n]/[(N - 1)/N]$. Here, $(n - 1)/n$ is the part of diversifiable risk of a portfolio with size n where $(N - 1)/N$ is the total (maximum) diversifiable risk of the whole population of size N. Letting this proportion, say a, to vary for different percentages, we can solve the value for n given a particular number of N. That is, the number of stocks required to achieve a certain level of risk reduction for different population sizes can be found precisely.

Table 2 presents the number of stocks required in a portfolio to eliminate a certain percentage of diversifiable risk given different population sizes. Our results confirm that the smaller the population size, the smaller is the required number of stocks. Table 2 shows that if one wants to eliminate only 50% of the diversifiable risk, the population size really does not matter since you still need 2 stocks (for a population of 100 stocks, you need 1.98 stocks on average). However, if one wants to eliminate 95% of the diversifiable risk, the number of stocks required varies greatly across different population sizes. For a population of 1,000 stocks, you need 19.6 stocks on average but you only need 16.8 stocks on average for a population size of 100. If you further restrict your population size to 40, what you need is just 13.6 stocks to achieve the same target.

5. Remarks and Conclusions

Naive diversification is a simple but powerful way to reduce your portfolio's risk effectively without sacrificing the expected rate of return. Business graduates know this result well. However, how true is this fact and what is the impact of portfolio sizes on the efficiency of naive diversification? This paper shows analytically that for an infinite population of stocks, a portfolio size of 20 is required to eliminate 95% of the diversifiable risk. However, an addition of 80 stocks (i.e., a size of 100 stocks) is required to eliminate an extra 4% (i.e., 99% total) of diversifiable risk. This result depends neither on the sampling periods, investment horizons nor the markets involved. For a finite population of stocks, the corresponding portfolio size required is smaller, the smaller the population size. Our findings have seldom been mentioned or discussed in many Finance/Investment textbooks.

Although results presented in this paper are important and highly relevant to all investors, there are some remarks on diversification benefits that we should aware. First, our findings are based on the average portfolio variances for different portfolio sizes. There is no guarantee that one particular portfolio's risk is the same as the average portfolio risk with the same size. Hence, there are additional sample risks in that your portfolio may not be the same as the population average. Because of this additional risk, Newbould and Poon (1993) argued that investors need substantially more than 20 stocks in a portfolio to eliminate diversifiable risk. Second, even though the power of naive diversification on reducing the percentage of diversifiable risk is independent neither of the markets involved, sampling periods nor investment horizons, the actual amount eliminated does vary depending on the ratio of the average covariance to the average stocks variance in different markets. Obviously, transaction costs also matter. The contribution of this paper is in stating out the efficiency of naive diversification, which is almost left out in many university finance/investment textbooks, from an educational point of view.

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Table 1 **Recommendations from Textbooks on the Number of Stocks that can Eliminate Most of the Portfolio's Diversifiable Risk**

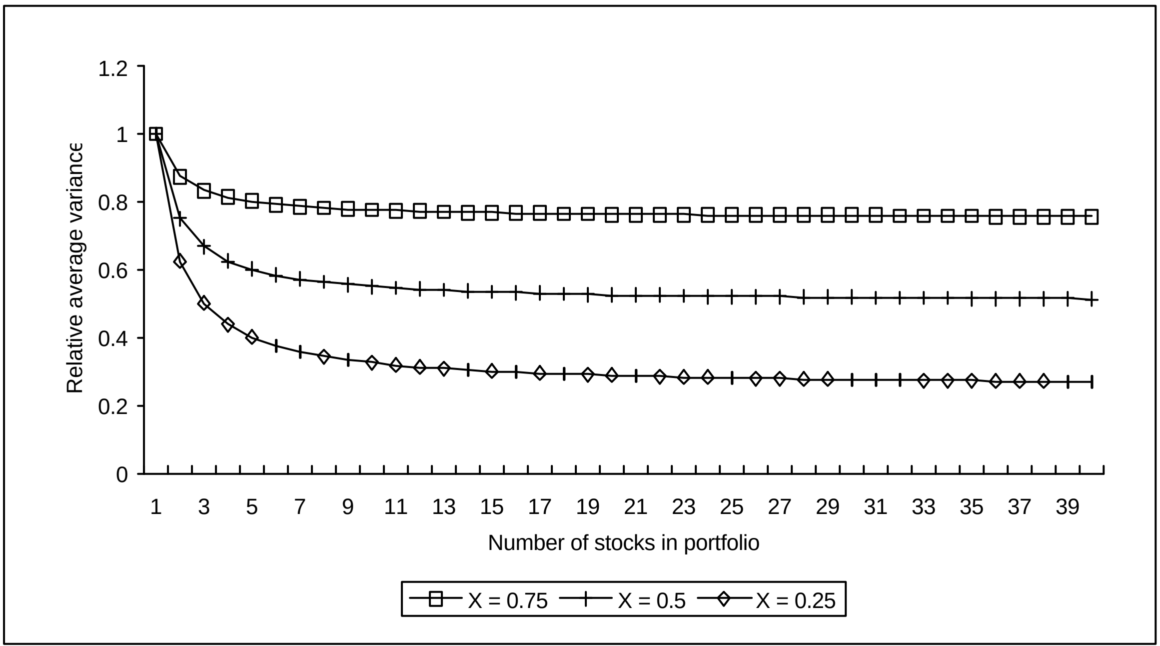
Author(s)	Page	# of stocks	Main sources cited
A. 10 Investment Management Textbooks			
1. Amling (1989)	p.609	10-15	Evans & Archer
2. Bodie, Kane & Marcus (1999)	p.202	20	Elton & Gruber
3. Fabozzi (1995)	p.89	~20	Wagner & Lau
4. Francis (1991)	p.229	10-15	Evans & Archer
5. Gitman & Joehnk (1996)	p.674	8-15, ~40	-
6. Jones (1996)	p.228	10-15, >30	Evans & Archer; Statman
7. Lee, Finnerty & Wort (1990)	p.227	15	Evans & Archer
8. Levy (1996)	p.269	12-18	-
9. Mayo (1993)	p.147	10-15	Evans & Archer
10. Sharpe, Alexander & Bailey (1995)	p.215	~30	-
B. 10 Corporate Finance Textbooks			
1. Arnold (1998)	p.265	10-15	-
2. Brealey & Myers (1996)	p.156	20	-
3. Emery (1998)	p.200	30-40	Statman
4. Emery & Finnerty (1997)	p.219	25-30	-
5. Gitman (2000)	p.256	15-20	Wagner & Lau; Evans & Archer
6. Lee, Finnerty & Norton (1997)	p.319	20-30	Evans & Archer
7. Moyer, McGuigan & Kretlow (1998)	p.203	10-15	Wagner & Lau
8. Pinches (1996)	p.123	20-30	-
9. Rao (1995)	p.138	25-30	Statman
10. Ross, Westerfield & Jordan (2000)	p.394	10-30	Elton & Gruber; Statman

Table 2 The Number of Stocks Required in a Portfolio to Eliminate a Certain Percentage of Diversifiable Risk given Different Population Sizes

	Percentage of diversifiable risk to be eliminated, a							
	50%	75%	90%	93%	95%	97%	98%	99%
Population size, N	*Number of stocks required in the portfolio given the above percentage, n*							
∞	2	4	10	14.2857	20	33.3333	50	100
10000	1.9998	3.9988	9.9910	14.2668	19.9621	33.2259	49.7562	99.0197
8000	1.9998	3.9985	9.9888	14.2620	19.9526	33.1992	49.6956	98.7776
6000	1.9997	3.9980	9.9850	14.2542	19.9369	33.1547	49.5950	98.3768
4000	1.9995	3.9970	9.9776	14.2384	19.9054	33.0660	49.3949	97.5848
2000	1.9990	3.9940	9.9552	14.1914	19.8118	32.8030	48.8043	95.2835
1000	1.9980	3.9880	9.9108	14.0984	19.6271	32.2893	47.6644	90.9918
800	1.9975	3.9851	9.8888	14.0523	19.5360	32.0384	47.1143	88.9878
600	1.9967	3.9801	9.8522	13.9762	19.3861	31.6289	46.2250	85.8369
400	1.9950	3.9702	9.7800	13.8265	19.0931	30.8404	44.5434	80.1603
200	1.9900	3.9409	9.5694	13.3958	18.2648	28.6944	40.1606	66.8896
100	1.9802	3.8835	9.1743	12.6103	16.8067	25.1889	33.5570	50.2513
80	1.9753	3.8554	89888	12.2511	16.1616	23.7389	31.0078	44.6927
60	1.9672	3.8095	8.6957	11.6959	15.1899	21.6606	27.5229	37.7358
40	1.9512	3.7209	8.1633	10.7239	13.5593	18.4332	22.4719	28.7770
20	1.9048	3.4783	6.8966	8.5837	10.2564	12.7389	14.4928	16.8067

Note: The figures in the table are calculated based on the following formula: $[(n - 1)/n]/[(N - 1)/N] = a$. Here, $(n - 1)/n$ is the part of diversifiable risk of a portfolio with size n where $(N - 1)/N$ is the total diversifiable risk of the whole population of size N. The table shows that if one wants to eliminate only 50% of the diversifiable risk, population size does not matter as you still need 2 stocks. However, if one wants to eliminate more than 95% of the diversifiable risk, the number of stocks required varies greatly for different population sizes.

Figure 1 Diversification Benefit: Relative Risk vs Portfolio Size



This graph plots the relative risk (defined as the ratio of average portfolio variance divided by the average variance of all stocks in the population) against the portfolio size, given different values (0.75, 0.5, and 0.25) of the relative systematic risk, X (defined as the average covariance divided by the average variance of all stocks in the population). In all three cases, the curves are decreasing functions of n, the portfolio size. The graph shows that when $X = 0.75$, the curve levels off at a portfolio size of 10 while when $X = 0.5$ (0.25), the curve levels off at a size of 15 (20).