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A Constant Gain Kalman Filter Approach to track Maneuvering Targets

Ashwin Yadav[1], Peeyush Awasthi[2], Naren Naik[2], M. R. Ananthasayanam[3]

1. Communication Engineering Department, Faculty of Electronics, Military College of Electronics and Mechanical Engineers, Trimulgherry, Secunderabad-500015 .
2. Department of Electrical Engineering, Indian Institute of Technology Kanpur-208016.
3. Retired Professor, Department of Aerospace Engineering, Indian Institute of Science, Bangalore-560012.

***Abstract*—Tracking of maneuvering targets is an important area of research with applications in both the military and civilian domains. One of the most fundamental and widely used approaches to target tracking is the Kalman filter. In presence of unknown noise statistics there are difficulties in the Kalman filter yielding acceptable results. In the Kalman filter operation for state variable models with near constant noise and system parameters, it is well known that after the initial transient the gain tends to a steady state value. Hence working directly with Kalman gains it is possible to obtain good tracking results dispensing with the use of the usual covariances. The present work applies an innovations based cost function minimization approach to the target tracking problem of maneuvering targets, in order to obtain the constant Kalman gain. Our numerical studies show that the constant gain Kalman filter gives good performance compared to the standard Kalman filter. This is a significant finding in that the constant gain Kalman filter circumvents or in other words trades the gains with the filter statistics which are more difficult to obtain. The problems associated with using a Kalman filter for tracking a maneuvering target with unknown system and measurement noise statistics can be circumvented by using the constant gain approach which seeks to work only with the gains instead of the state and measurement noise covariances. The approach is applied to a variety of standard maneuvering target models.**

I. INTRODUCTION

Tracking a maneuvering target is a challenge because of uncertainty regarding the use of specific models to a type of target, or in other words specifically determining the precise model for the target being tracked. A maneuvering target depending on the type of maneuver can range from a civillian transport plane to a fighter jet. In all cases tracking for either navigation or interception will be required. One of the most widely used target tracking algorithms is the Kalman filter (KF) [1]. However the KF solution is a formal solution in the sense that it is optimal only when the noise statistics in the form of the state and measurement noise covariances (Q and R respectively) as well as the initial state error covariance (P_0) , is available a priori. In case of maneuvering targets the knowledge of the precise values of these covariances is paramount in achieving accurate tracking results. Tuning of these parameters is important to achieve good performance of the filter algorithm.

Tuning is a still not a well researched field though some studies have been made such as the innovations adaptive estimation (IAE) based method by Mehra [2] who showed its use in correlation and covariance matching techniques. Myers and Tapley [3] formalized this method in an effective manner to provide a mechanism for online adaptive tuning for Q and R. More recent studies provide a combination of the innovation based IAE and adaptive faded Kalman filter (AFKF) in a hybrid scheme proposed in [4]. Another alternative is [5] which makes use of the IAE and proposes a cost function approach. Gemson, Ananthasayanam proposed a scheme for an adaptive extended Kalman filter in [6] to obtain P_0, Q and R using the minimization of the cost function based on innovation.

Constant gain Kalman filters (CGKF) have been analyzed in [7-9]. However these do not completely circumvent the use of filter statistics P_0, Q and R which may not be optimal or near optimal for deriving the constant Kalman gains. A simple cost function minimization based CGKF approach has been suggested by Anil Kumar et al [10] in a problem concerned with prediction of re-entry of risk objects wherein they have used a genetic algorithm (GA) based minimization of an innovation cost function to compute an optimal constant gain matrix. In previous work by the author and colleagues [11] a similar cost function approach as in [6,10] was applied to the target tracking in Wireless sensor Network (WSN) domain. What is further known as a fundamental observation is that the KF gain stabilizes to a constant value after some point of time during the filter (algorithm) operation under conditions that the covariance matrices R, Qdo not change subsequently. So the conceptual change involved is that one now works with the Kalman gain rather than the error and noise covariances (P, Q, R). It is typically observed that the filter estimates obtained are more robust to variation in gain as against variation in the error and noise covariances.Various maneuvering target models have been surveyed by Rong Li and Jilkov in [12] and Bar Shalom in [13] which provides a firm base and motivation for applying the constant gain approach described above to these models. Thus the present work endeavours to apply the constant gain approach to following types of models :- Non Maneuvering or Discrete White Noise Acceleration (DWNA) model, Moderately maneuvering or Discrete Weiner Process Acceleration (DWPA) model and Highly maneuvering to include Jerk, Coordinated Turn model (CT) with known turn rate and CT with unknown turn rate, models. Thus they total number of models forming part of the simulation study are five in number.

The main contribution of this paper is the following 1) The application of an innovations cost function based CGKF to a problem of tracking maneuvering targets. The results are compared to those obtained from a reference KF (where noise covariances are assumed known) and further 2) To the best knowledge of the authors such a comparative analysis of the KF/CGKF as applied to variety of maneuvering target models, has not been carried out till date.

The paper is organized as follows. Section II describes the state variable (SV) model corresponding to the various maneuvering target models which defines the problem and the type of target being tracked. Section III gives a brief theory of the CGKF. Section IV contains the numerical studies and results. Section V provides a summary and discussion of the results.

II.STATE VARIABLE MODEL

A) Problem Description and Scope

The objective is to track five categories of maneuvering targets successfully to include DWNA,DWPA,Jerk model,coordinated turn (CT) with known value of angular velocity w and coordinated turn model with unknown value of angular velocity w as described in [12,13]. These models can alternatively be classified as mentioned above in the Introduction, into the following categories:- Non maneuvering (DWNA), moderately maneuvering (DWPA), highly maneuvering (jerk, both CT models). The aim of the present work is to conduct a comparative analysis between the standard/reference KF versus the CGKF.

B) State Variable Model:DWNA

A three dimensional model for the target tracking problem is described as follows

State Equation:

$$X_{t+1} = AX_t + Bw_t \tag{1}$$

where state vector is : $X_t = \begin{pmatrix} x(t) & y(t) & z(t) & \dot{x(t)} & \dot{y(t)} & \dot{z(t)} \end{pmatrix}^T$, *state transition matrix* $A = \begin{pmatrix} I & C_1 \\ 0 & I \end{pmatrix}$, $B = \begin{pmatrix} C_1 \\ D \end{pmatrix}$ *where* $C_1 = \triangle t I$, $D = (\triangle t^2/2)I$ *and* w_t *represents system noise. Here we have considered the state vector to include X, Y and Z coordinates of the target as well as the speed in the three coordinates. :*

Measurement Equation:

$$Y_t = CX_t + n_t \tag{2}$$

: where Y_t is the measurement vector, $C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$,
n_t is measurement noise which is Gaussian

C) State Variable Model:DWPA

A three dimensional model for the target tracking problem is described as follows

State Equation:

$$X_{t+1} = AX_t + Bw_t \tag{3}$$

:

where state vector is: $X_t = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \\ \dot{x(t)} \\ \dot{y(t)} \\ \dot{z(t)} \\ \ddot{x(t)} \\ \ddot{y(t)} \\ \ddot{z(t)} \end{pmatrix}$, state transition matrix $A = \begin{pmatrix} I & C_1 & D \\ 0 & I & C_1 \\ 0 & 0 & I \end{pmatrix}$, $B = \begin{pmatrix} D \\ C_1 \\ I \end{pmatrix}$ where C_1,D are as described earlier. and w_t represents system noise which is Gaussian. Here we have considered the state vector to include X, Y and Z coordinates of the target as well as the speed and acceleration in the three coordinates.

Measurement Equation:

$$Y_t = CX_t + n_t \tag{4}$$

: where Y_t is the measurement vector, $C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$, n_t is measurement noise which is Gaussian

D) State Variable Model:Jerk

A three dimensional model for the target tracking problem is described as follows

State Equation:

$$X_{t+1} = AX_t + Bw_t \tag{5}$$

:

where state vector is $X_t = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \\ \dot{x(t)} \\ \dot{y(t)} \\ \dot{z(t)} \\ \ddot{x(t)} \\ \ddot{y(t)} \\ \ddot{z(t)} \end{pmatrix}$, *:*

state transition matrix $A = \begin{pmatrix} I & C_1 & D \\ 0 & I & C_1 \\ 0 & 0 & I \end{pmatrix}$, $B = \begin{pmatrix} E \\ C_1 \\ I \end{pmatrix}$ *where* $E = (\triangle t^6/3)I$, D,C_1 *are as described earlier, and* w_t *represents system noise which is Gaussian. Here again we have considered the state vector to include X, Y and Z coordinates of the target as well as the speed and acceleration in the three coordinates. The only difference is in the* B *matrix which factors in the jerk as a derivative of the acceleration. :*

Measurement Equation:

$$Y_t = CX_t + n_t \tag{6}$$

: where Y_t is the measurement vector,
$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$,
n_t is measurement noise which is Gaussian

E) State Variable Model:CT with known w

A two dimensional model for the target tracking problem (maneuver in horizontal 2D plane) is described as follows

State Equation:

$$X_{t+1} = AX_t + Bw_t \tag{7}$$

: where state vector is $X_t = \begin{pmatrix} x(t) & \dot{x(t)} & y(t) & \dot{y(t)} \end{pmatrix}^T$, state transition matrix $A = \begin{pmatrix} A_1 & -A_2 \\ A_2 & A_1 \end{pmatrix}$ $B = \begin{pmatrix} B_1 & B_2 \\ B_2 & B_1 \end{pmatrix}$
where $A_1 = \begin{pmatrix} 1 & Sin(w\triangle t)/w \\ 0 & Cos(w\triangle t) \end{pmatrix}$,$A_2 = \begin{pmatrix} 0 & (1-Cos(w\triangle t))/w \\ 0 & Sin(w\triangle t) \end{pmatrix}$, $B_1 = \begin{pmatrix} \triangle t^2/2 & 0 \\ \triangle t & 0 \end{pmatrix}$,$B_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and w_t represents system noise which is Gaussian. Here we have considered the state vector to include X and Y coordinates of the target as well as the speed in the two coordinates.

Measurement Equation:

$$Y_t = CX_t + n_t \tag{8}$$

: where Y_t is the measurement vector,
$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, n_t is measurement noise which is Gaussian

F) State Variable Model:CT with unknown w

A two dimensional model for the target tracking problem (maneuver in horizontal 2D plane) is described as follows

State Equation:

$$X_{t+1} = AX_t + Bw_t \tag{9}$$

: where state vector is
$X_t = \begin{pmatrix} x(t) & \dot{x(t)} & y(t) & \dot{y(t)} & w(t) \end{pmatrix}^T$, state transition matrix $A = \begin{pmatrix} A_1 & -A_2 & B_2 \\ A_2 & A_1 & B_2 \\ B_3 & B_3 & 1 \end{pmatrix}$,

$B = \begin{pmatrix} \triangle t^2/2 & 0 & 0 \\ \triangle t & 0 & 0 \\ 0 & \triangle t^2/2 & 0 \\ 0 & 0 & \triangle t \\ 0 & 0 & \triangle t \end{pmatrix}$ where $B_3 = \begin{pmatrix} 0 & 0 \end{pmatrix}$,A_1,A_2,B_2,B_3 are as described above and w_t represents system noise which is Gaussian. Here we have considered the state vector to include X and Y coordinates of the target as well as the speed in the two coordinates. Note that the problem is non linear and an Extended KF or Extended CGKF (termed henceforth as Constant Gain Extended KF or CGEKF) is employed.

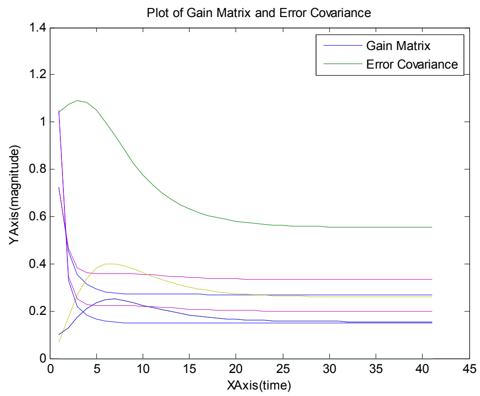

Fig. 1. Gain K vs error covariance matrix P

Measurement Equation:

$$Y_t = CX_t + n_t \tag{10}$$

: where Y_t is the measurement vector, $C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, n_t is measurement noise which is Gaussian. Note that the measurement model is linear

III. CONSTANT GAIN KALMAN FILTER

A) Motivation

A detailed description covering the background of the CGKF and its foundation is covered in [11], but the same is restated here for better understanding in the current context. It is observed as illustrated in Figure 1 that the gain matrix stabilizes to a constant value and this coincides with a similar plot for the state error covariance P. From the Figure 1. following can be inferred

1) The gain K stabilizes to a steady state value at around given point of time which also coincides with a similar plot for error covariance P . This implies that the target is being tracked well at the time when the gain K has stabilized to a steady state value, since at this point error covariance P, also settles to its steady state value 2) Hence predetermination of and subsequent use of the gain K right from the start apart from the transients, should logically yield comparable tracking results . These are the reasons that motivate the use of the CGKF approach in tracking the target, and lay a foundation for it use.

B) The Estimation Scheme

The CGKF employs a cost function minimization approach in order to predetermine the optimum value of the constant gain K. The cost function is

$$J(K, \mathcal{R}) = \frac{1}{N}\sum_{t=1}^{N}(v_t^T \mathcal{R} v_t + log(|\mathcal{R}|)) \tag{11}$$

$$v_t = Y_t - C\bar{X}_t \tag{12}$$

where $\mathcal{R}$ represents the covariance of the innovations. In order to solve the optimization problem we can either use local gradient based methods (such as Newton type schemes) or global schemes such as a GA [14]. As part of the optimization problem the unknown parameters namely K, and R are initialised using $rand$ function in MATLAB code for the GA. Being a poulation based method, the program using ga function of MATLAB proceeds to solve the optimization problem in order to obatin an optimum value of K, henceforth denoted by K^*. As the filter tracks the target the gain K is seen to stabilize to a value given by the solution of the above problem. Once the optimization problem has been solved, the optimum K^* is employed in a simple two step process (viz a viz the 5 step approach employed in the standard KF) and this is essentially the core of the CGKF

Predict :

$$\overline{X}_{t+1} = A\hat{X}_{t+1} \tag{13}$$

:

Update:

$$\hat{X}_{t+1} = \overline{X}_{t+1} + K^* v_t \tag{14}$$

: where v_t is as described in 12 above.We observe that the typically expensive covariance time update step is not needed in the constant gain approach. So in summary, the advantage gained in use of the CGKF is very clearly, that precise values of the state error covariance P_o (initial), system noise covariance Q and measurement noise covariance R are not required to be known and moreover their use is circumvented here. We merely work with the optimum value of the gain K^*, which has been obtained by solving the optimization problem as given in 12 above. This is relevant in view of the fact that during estimation or track reconstruction stage of the KF, the only purpose of the P, Q and R matrices is in determining the value of gain K at each step. Hence once the optimum value K^*, has been determined there is no use of employing P, Q and R matrices. **Even though the constant gains have been obtained by using a reference case other cases which are even moderately different from the reference case, the constant gains derived have been known to work quite well [10]**.

IV) NUMERICAL STUDIES AND RESULTS

The two dimensional numerical studies have been carried out on a set of forty data points for all models

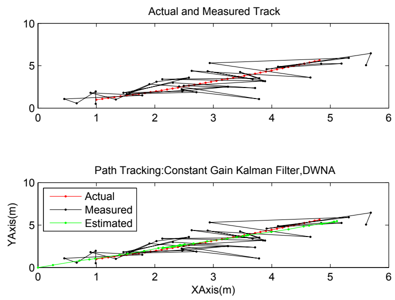

Fig. 2. Constant gain Kalman filter:DWNA

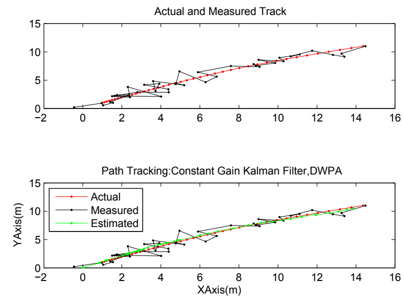

Fig. 3. Constant gain Kalman filter:DWPA

except CT (with known and unknown w both) where it has been carried out on a set of seventy data points in order to generate a smooth trajectory. The following system and measurement covariances matrices are used to generate the simulated track $Q = .05I$, $R = .5I$ for all models except CT (with unknown w) where the values are $Q = .03I$, $R = .3I$, choice of the initial value of w in the CT model has been obtained by via standard fighter aircraft (eg F-16) data available on the internet [15] . This value of w has been set to .5, which corresponds to approx 28.6 degrees/s (which happens to be the maximum instantaneous turn rate of any current generation fighter aircraft). Error metric used is Percentage Fit Error (PFE) defined as $PFE = \frac{|X_t - \hat{X}_t|}{|X_t|} \times 100$ which represents the difference between the estimated and actual track. Here the PFE is based on sum total PFE obtained along X and Y coordinates respectively. The error metric shown in tables is the average value computed over 500 runs while the plots correspond to one specific run wherein results are presented in the form of 2D plots of the simulated target trajectory, simulated measurements and the estimated track against time. Due to the constarint of space typical plots for only CGKF are shown while those corresponding to the reference KF are omitted for the mentioned reason. Figures. 2-6 and Table. I clearly show that the performance of the CGKF is comparable to the reference KF which uses complete knowledge of system parameters unlike the CGKF which works with only the constant gain. Note that the fifth model (CT with unknown w) is based on application of a CGEKF since this is a nonlinear model. This provides the necessary justification in using the CGKF for the target tracking problem in SA mode as well.

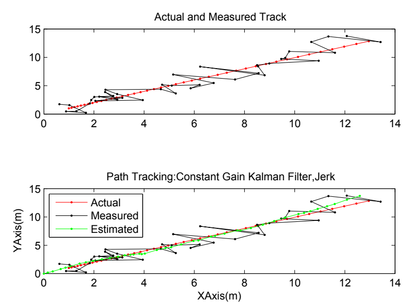

Fig. 4. Constant gain Kalman filter:Jerk

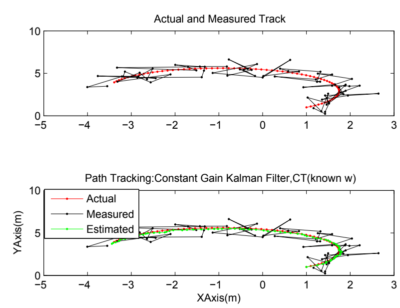

Fig. 5. Constant gain Kalman filter:CT(known w)

	KF/EKF*	**CGKF/CGEKF***
DWNA	22.4%	16.5%
DWPA	14.7%	13.2%
Jerk	12.6%	10.5%
CT(known w)	17.9%	14.8%
CT(unknown w)*	15.8%	12.7%

TABLE I
PFE COMPARISON ,***:-NON LINEAR CASE**

V. CONCLUSION

To the best of the knowledge of the authors, these are the only studies of a constant gain Kalman filter applied to tracking maneuvering target models.The

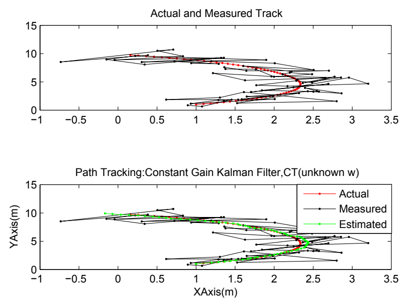

Fig. 6. Constant gain extended Kalman filter:CT(unknown w)

results obtained show the CGKF/CGEKF performing comparably with a reference KF/EKF for all models. This is a significant result since the CGKF/CGEKF circumvents, or in other words trades the gains with the filter statistics which are more difficult to obtain. The present results prove that the CGKF/CGEKF is successful in target tracking applications wherein the constant gain approach overcomes uncertainty regarding noise statistics that exist in the framework of the problem. In the current work it has been employed for tracking of five types of maneuvering targets.

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