

# Multiuser Switched Diversity Transmission

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**Abstract**—In this paper, a set of multiuser access schemes are proposed based on switched diversity algorithms originally devised to select between antennas in a spatial diversity system. Instead of relying on feedback from all the users in a multiuser communication system to identify the best user on a time-slot basis (having the best channel quality among all the users), the proposed multiuser access schemes are performed in a sequential manner, looking not for the best user but for an acceptable user. A user qualifies as an acceptable user and is selected by the base station when the reported channel quality is above a predefined switching threshold. The proposed schemes result in a lower average spectral efficiency (ASE) than using the optimal selective diversity scheme, but a gain is obtained by reducing the feedback load. Numerical results that quantify the trade-off between ASE and average feedback load (AFL) are presented, showing that the AFL can be reduced significantly compared to the optimal selective diversity scheme without experiencing a big performance loss. In addition, it is argued that the proposed multiuser access schemes can be quite attractive also from a fairness perspective.

## I. INTRODUCTION

In a traditional spatial diversity system, the diversity gain arises from independent signal paths received by multiple antennas. In a multiuser communication system, multiuser diversity (independent fading channels across different users) can be exploited to maximize the average throughput by always serving the user with the strongest channel [1][2]. A traditional way of performing this task in a time division multiplexed (TDM) system is to let the base station (BS) probe all the users and select the user which reports the best channel quality at any given time-slot. The selected user is given access to the channel to either upload or download information, and the average spectral efficiency (ASE) of the system can be maximized by transmitting with the highest possible rate supported by the selected channel.

The key observation utilized in this paper is that algorithms originally devised to select between antennas in a spatial diversity combiner also may be applied as multiuser access schemes, since multiuser diversity may be looked upon as spatial diversity, in which the antennas of the spatial diversity combiner (acting as a BS) have been replaced by users (each having a single antenna). Hence, a multiuser access scheme based on always serving the user with the strongest channel is equivalent to the selection combining (SC) scheme in a spatial diversity system [3, Section 9.7]. It yields the best ASE for a certain target bit-error-rate (BER), but it comes

at the expense of a high feedback load. Indeed, defining feedback load  $N_e$  as the number of estimated paths/users per time-slot before channel access [4], the feedback load of the multiuser access scheme based on the SC algorithm will be deterministic and equal to  $K$  (the number of users connected to the BS). In an attempt to simplify the selection procedure and reduce the feedback load, a set of *switched* multiuser access schemes are proposed. These access schemes are based on switched diversity algorithms originally devised to select between antennas in a spatial diversity system, and the basic principle is to look for an *acceptable* user instead of the best user, i.e. a user with a channel quality above a predefined switching threshold.

In short, a switched multiuser access scheme works as follows: the BS starts probing the users in a sequential fashion, requesting the signal-to-noise ratio (SNR) of the first user and then comparing it to the switching threshold. If the SNR is below the threshold, the BS moves on to the second user. This user SNR probing/checking process continues until (i) either one user is above the threshold (this user is selected for the subsequent transmission time) (ii) all  $K$  users have been examined and all have failed to exceed the switching threshold, in which either the last examined user is selected (for simplicity), the best user among all the probed users is selected, or one waits a period longer than the channel coherence time and starts a new sequential search.

Using a sequential search to identify an acceptable user can contribute to increase fairness in a multiuser system, since the sequence in which to probe the users can be made different from one time-slot to the next. When independent and identically distributed (i.i.d.) channels are assumed, the users will be competing for the channel on equal terms. In this case, on average, all  $K$  users will have accessed the channel after  $K$  time-slots.

## II. SYSTEM AND CHANNEL MODEL

A TDM system is considered, where only one user has channel access per time-slot (for uplink or downlink). A single time-slot is divided into a guard time and an information transmission time. During the guard time, the base station selects the user who will have access to the channel in the subsequent transmission time. The guard time is assumed fixed and equal to the amount of time necessary to probe all the users. The time duration of a single time-slot is assumed

roughly equal to the channel coherence time, and the data burst is assumed to experience the same fading conditions as the preceding guard period (block fading). For simplicity, i.i.d. Rayleigh fading channels across the different users are assumed, and the individual users and the base station are all equipped with just a single antenna. Finally, perfect channel state information is assumed available at both the BS and the users.

A rate-adaptive coding scheme using a set of  $N$  multi-dimensional trellis codes originally designed for additive white Gaussian noise (AWGN) channels is assumed utilized on each selected link to ensure a high ASE of the system [5]. In this paper,  $N = 8$  different codes based on quadrature amplitude modulation (QAM) signal constellations of growing size  $\{M_n\}_{n=1}^N = \{4, 8, 16, 32, 64, 128, 256, 512\}$  are utilized. Rate adaptation is performed by splitting the SNR range into  $N+1$  fading regions (bins), and the separate fading regions are defined by the SNR thresholds  $0 < \gamma_1 < \gamma_2 < \dots < \gamma_N < \gamma_{N+1} = \infty$ . Code  $n$  with spectral efficiency  $R_n$  [bits/s/Hz] is used for transmission if the SNR  $\gamma$  of the selected channel/user is reported to be within the fading region  $\gamma_n \leq \gamma < \gamma_{n+1}$ . The lower limit  $\gamma_n$  of each fading region is equal to the lowest SNR which guarantees that a predefined target BER ( $\text{BER}_0 = 10^{-4}$ ) is achieved by code  $n$ .

### III. ASE AND BER ANALYSIS

The ASE of the system is obtained as a sum of the spectral efficiencies  $\{R_n\}_{n=1}^N = \{1.5, 2.5, \dots, 8.5\}$  for the individual codes, weighted by the probability  $P_n$  that code  $n$  is used<sup>1</sup>:

$$\text{ASE} = \sum_{n=1}^N R_n \cdot P_n, \quad (1)$$

where

$$P_n = \int_{\gamma_n}^{\gamma_{n+1}} p_{\gamma_{\text{BS}}}(\gamma) d\gamma, \quad (2)$$

The function  $p_{\gamma_{\text{BS}}}(\gamma)$  denotes the probability density function (PDF) of the output SNR at the BS. The shape of this PDF will depend on the mode of operation of the selected multiuser access scheme.

The BER, when averaged over all codes and SNRs, is given as the average number of bits in error divided by the average number of bits transmitted [5][6]:

$$\overline{\text{BER}} = \frac{\sum_{n=1}^N R_n \cdot \overline{\text{BER}}_n}{\sum_{n=1}^N R_n \cdot P_n}, \quad (3)$$

where  $\overline{\text{BER}}_n$  is the average BER experienced when code  $n$  is applied. An expression for  $\overline{\text{BER}}_n$  is obtained by utilizing the exponential approximation  $\text{BER}_n = a_n \cdot e^{-b_n \gamma / M_n}$  for the BER-SNR relationship for varying  $\gamma$  [5], thus

$$\overline{\text{BER}}_n = \int_{\gamma_n}^{\gamma_{n+1}} a_n \cdot e^{-\frac{b_n \gamma}{M_n}} p_{\gamma_{\text{BS}}}(\gamma) d\gamma, \quad (4)$$

<sup>1</sup>In practice, the ASE of all the access schemes presented in this paper should be reduced by a factor  $\frac{T_i}{T_g + T_i}$ , where  $T_g$  is the guard time interval, and  $T_i$  is the information transmission time interval. The factor is omitted, since we emphasize on the relative differences in ASE.

where  $a_n$  and  $b_n$  are code-dependent constants found by least-square fitting to simulated data on AWGN channels. The expression for  $\text{BER}_n$  is invertible, so the smallest SNR required to achieve  $\text{BER}_0$  can be identified as  $\gamma_n = (M_n/b_n) \ln(a_n/\text{BER}_0)$ .

### IV. MULTIUSER ACCESS SCHEMES

In the following subsections, ASE and average BER expressions for a set of multiuser access schemes are presented. The schemes are listed in terms of increasing complexity. For later reference, the PDF and CDF<sup>2</sup> of the output SNR of a single-input single-output Rayleigh fading channel is equal to and denoted as  $p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}$  and  $P_\gamma(\gamma) = 1 - e^{-\gamma/\bar{\gamma}}$ , respectively. The symbol  $\bar{\gamma}$  denotes the average SNR, equal on all channels.

#### A. Scan-and-wait transmission (SWT)

The SWT multiuser access scheme is based on a multi-branch scan-and-wait combining scheme [4] and it works as follows: during the guard time interval, a sequential search is initiated by the BS, requesting the SNR of each user and comparing it to a switching threshold  $\gamma_T$ . The SNR probing/checking process continues until either one user is above  $\gamma_T$  (this user is selected for the subsequent transmission time) or all  $K$  users have been examined and all have failed to exceed  $\gamma_T$ . In the latter case, the BS simply waits a period longer than the channel coherence time (deliberate outage) before it starts a new sequential search. This procedure can be repeated indefinitely until a user with an acceptable SNR is found. With this approach, the output SNR can be described by the following PDF [4, Eq.(6)]:

$$p_{\gamma_{\text{SWT}}}(\gamma) = \begin{cases} \frac{p_\gamma(\gamma)}{1 - P_\gamma(\gamma_T)} & \gamma \geq \gamma_T \\ 0 & \text{otherwise} \end{cases}. \quad (5)$$

Even though the PDF in (5) is correct, it represents the PDF the output SNR when the waiting procedure already is included, since the probability of not exceeding  $\gamma_T$  is zero. In order to better relate the results of the SWT scheme to the other access schemes presented in this paper, the PDF reflecting the output SNR *per time-slot* is needed. Such a PDF can be obtained by assuming that the last examined user is selected (for simplicity) if all  $K$  users have failed to exceed  $\gamma_T$ . However, the selected user is not allowed to transmit anything in the subsequent transmission time. The operation of selecting a user at the end will only ensure that the probability of not exceeding  $\gamma_T$  will be different from zero. Since no users are allowed to transmit information if all have failed to exceed  $\gamma_T$ , the correct ASE and BER expressions are obtained by letting  $\{R_n\}_{n=1}^N = 0$  (no transmission) when  $\gamma < \gamma_T$ . Under these conditions, the following PDF may be utilized [7, Eq.(35)]:

$$p_{\gamma_{\text{SWT}}}^T(\gamma) = \begin{cases} \sum_{k=0}^{K-1} [P_\gamma(\gamma_T)]^k p_\gamma(\gamma) & \gamma \geq \gamma_T \\ [P_\gamma(\gamma_T)]^{K-1} p_\gamma(\gamma) & \gamma < \gamma_T \end{cases}. \quad (6)$$

<sup>2</sup>Cumulative distribution function.

Replacing  $p_{\gamma_{\text{BS}}}(\gamma)$  with  $p_{\gamma_{\text{SWT}}}^T(\gamma)$  in (2) and letting  $p = P_\gamma(\gamma_T)$  for notational simplicity, the following ASE is obtained:

$$\text{ASE}_{\text{SWT}} = \sum_{k=0}^{K-1} p^k \left[ R_q \cdot e^{-\gamma_T/\bar{\gamma}} + \sum_{n=q+1}^N \alpha_n \cdot e^{-\gamma_n/\bar{\gamma}} \right], \quad (7)$$

where  $\alpha_n = (R_n - R_{n-1})$ . Note that  $\text{ASE}_{\text{SWT}}$  is a function of the index  $q \in [1, 2, \dots, N]$ , which denotes the fading region in which  $\gamma_T$  is placed. For fixed  $K$  and  $\bar{\gamma}$ ,  $\text{ASE}_{\text{SWT}}$  will have  $N$  separate solutions corresponding to  $\gamma_T$  residing within each of the  $N$  separate fading regions<sup>3</sup>. Defining  $\gamma_T$  (for fixed  $K$  and  $\bar{\gamma}$ ) as the switching threshold that maximizes the ASE, it is obtained as

$$\gamma_T = \arg \max_{\gamma_1 \leq \gamma \leq \gamma_N} (\text{ASE}_{\text{SWT}}). \quad (8)$$

Defining the set  $\mathcal{X}_n = \{\gamma \in \mathbb{R} : \gamma_n \leq \gamma < \gamma_{n+1}\}$  and using the indicator function

$$\mathcal{I}_{\mathcal{X}_n}(\gamma_T) = \begin{cases} 1 & \text{if } \gamma_T \in \mathcal{X}_n \\ 0 & \text{if } \gamma_T \notin \mathcal{X}_n \end{cases}, \quad (9)$$

the optimal switching threshold is residing within the fading region

$$q = \arg \max_{n \in [1, 2, \dots, N]} (\mathcal{I}_{\mathcal{X}_n}(\gamma_T)). \quad (10)$$

Replacing  $p_{\gamma_{\text{BS}}}(\gamma)$  with  $p_{\gamma_{\text{SWT}}}^T(\gamma)$  in (4), the average BER may be written as in (11). No effort of minimizing  $\overline{\text{BER}}_{\text{SWT}}$  (or any other BER expressions in this paper) as a function of  $\gamma_T$  has been done, since the BER performance already is perceived as acceptable when the predefined target  $\text{BER}_0$  is met.

### B. Switch-and-examine transmission (SET)

The SET scheme is based on a multibranch switch-and-examine combining scheme [7]. It is equal to the SWT scheme except for the following: the last probed user is allowed to transmit information if all  $K$  users have failed to exceed  $\gamma_T$  and the reported SNR is above  $\gamma_1$  (the lowest SNR needed to meet the target  $\text{BER}_0$ ). Thus, in practice, the switching threshold for the SET scheme is relaxed to  $\gamma_T = \gamma_1$  for the last probed user in order to reduce the outage probability. This leads to an improvement in the ASE of the SET scheme compared to the SWT scheme. For the SET scheme, the PDF of the output SNR is equal to [7, Eq.(35)]

$$p_{\gamma_{\text{SET}}}(\gamma) = \begin{cases} \sum_{k=0}^{K-1} [P_\gamma(\gamma_T)]^k p_\gamma(\gamma) & \gamma \geq \gamma_T \\ [P_\gamma(\gamma_T)]^{K-1} p_\gamma(\gamma) & \gamma < \gamma_T \end{cases}, \quad (12)$$

which is equal to  $p_{\gamma_{\text{SWT}}}^T(\gamma)$ . Replacing  $p_{\gamma_{\text{BS}}}(\gamma)$  with  $p_{\gamma_{\text{SET}}}(\gamma)$  in (2), the ASE may be written as

$$\text{ASE}_{\text{SET}} = p^{K-1} S_1^A + (R_q v_q + S_{q+1}^A) \sum_{k=0}^{K-2} p^k, \quad (13)$$

where  $S_1^A = \sum_{n=1}^N R_n \delta_n$ ,  $S_{q+1}^A = \sum_{n=q+1}^N R_n \delta_n$ ,  $\delta_n = (e^{-\gamma_n/\bar{\gamma}} - e^{-\gamma_{n+1}/\bar{\gamma}})$ , and  $v_q = (e^{-\gamma_T/\bar{\gamma}} - e^{-\gamma_{q+1}/\bar{\gamma}})$ . By

<sup>3</sup> $\gamma_T$  may not reside inside the interval range  $0 \leq \gamma_T < \gamma_1$ , since this may result in an outage situation even though a user is above the threshold.

replacing  $p_{\gamma_{\text{BS}}}(\gamma)$  with  $p_{\gamma_{\text{SET}}}(\gamma)$  in (4), the following solution for the average BER is obtained:

$$\overline{\text{BER}}_{\text{SET}} = \frac{p^{K-1} S_1^B + (R_q \Upsilon_q + S_{q+1}^B) \sum_{k=0}^{K-2} p^k}{\text{ASE}_{\text{SET}}}, \quad (14)$$

where  $S_1^B = \sum_{n=1}^N R_n \Delta_n$ ,  $S_{q+1}^B = \sum_{n=q+1}^N R_n \Delta_n$ ,  $\Delta_n = \frac{a_n}{\mu_n \bar{\gamma}} (e^{-\mu_n \gamma_n} - e^{-\mu_n \gamma_{n+1}})$ ,  $\mu_n = \frac{b_n \bar{\gamma} + M_n}{M_n \bar{\gamma}}$ , and  $\Upsilon_q = \frac{a_n}{\mu_n \bar{\gamma}} (e^{-\mu_q \gamma_T} - e^{-\mu_q \gamma_{q+1}})$ .

### C. SET with post-selection (SETps)

This is the same selection procedure as SET, except that if no acceptable link has been found, the best one of all the probed users exceeding  $\gamma_1$  is selected at the end instead of just picking the last one for simplicity [8]. For this scheme, the PDF of the output SNR is given by [8]

$$p_{\gamma_{\text{SETps}}}(\gamma) = \begin{cases} \sum_{k=0}^{K-1} [P_\gamma(\gamma_T)]^k p_\gamma(\gamma) & \gamma \geq \gamma_T \\ K [P_\gamma(\gamma)]^{K-1} p_\gamma(\gamma) & \gamma < \gamma_T \end{cases}. \quad (15)$$

Upon replacing  $p_{\gamma_{\text{BS}}}(\gamma)$  with  $p_{\gamma_{\text{SETps}}}(\gamma)$  in (2), the following ASE is obtained:

$$\begin{aligned} \text{ASE}_{\text{SETps}} &= p^K R_q - R_1 [P_\gamma(\gamma_1)]^K - \sum_{n=2}^q [P_\gamma(\gamma_n)]^K \alpha_n \\ &+ (R_q v_q + S_{q+1}^A) \sum_{k=0}^{K-1} p^k, \end{aligned} \quad (16)$$

Replacing  $p_{\gamma_{\text{BS}}}(\gamma)$  with  $p_{\gamma_{\text{SETps}}}(\gamma)$  in (4) and using binomial expansion of  $P_\gamma(\gamma)$ , the average BER may be written as

$$\overline{\text{BER}}_{\text{SETps}} = \frac{\frac{K}{\bar{\gamma}} \cdot \Sigma + (R_q \Upsilon_q + S_{q+1}^B) \sum_{k=0}^{K-1} p^k}{\text{ASE}_{\text{SETps}}}, \quad (17)$$

where

$$\Sigma = \left( \sum_{n=1}^{q-1} R_n a_n \Lambda(w_n, \gamma_n, \gamma_{n+1}) + R_q a_q \Lambda(w_q, \gamma_q, \gamma_T) \right).$$

In the definition of  $\Sigma$ , the following notation has been introduced:  $\Lambda(a, b, c) = \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{(-1)^k}{a} (e^{-ab} - e^{-ac})$  and  $w_n = \frac{b_n \bar{\gamma} + (k+1) M_n}{M_n \bar{\gamma}}$ .

### D. Selection combining transmission (SCT)

The SCT scheme is the benchmark scheme with which all the proposed switched based access schemes are compared. Within each guard time interval, all  $K$  users are probed and the BS selects the user which reports the highest SNR. With this mode of operation, the PDF of the output SNR is equal to [9, Eq.(5.85)]

$$p_{\gamma_{\text{SCT}}}(\gamma) = K [P_\gamma(\gamma)]^{K-1} p_\gamma(\gamma). \quad (18)$$

Replacing  $p_{\gamma_{\text{BS}}}(\gamma)$  with  $p_{\gamma_{\text{SCT}}}(\gamma)$  in (2), the ASE may be expressed as

$$\text{ASE}_{\text{SCT}} = \sum_{n=1}^N R_n \cdot ([P_\gamma(\gamma_{n+1})]^K - [P_\gamma(\gamma_n)]^K). \quad (19)$$

$$\overline{\text{BER}}_{\text{SWT}} = \frac{\sum_{k=0}^{K-1} p^k \left[ \frac{R_q a_q}{\mu_q \bar{\gamma}} \cdot e^{-\mu_q \gamma_T} + \sum_{n=q+1}^N \frac{R_n a_n}{\mu_n \bar{\gamma}} \cdot e^{-\mu_n \gamma_n} - \sum_{n=q}^{N-1} \frac{R_n a_n}{\mu_n \bar{\gamma}} \cdot e^{-\mu_n \gamma_{n+1}} \right]}{\text{ASE}_{\text{SWT}}} \quad (11)$$

Likewise, replacing  $p_{\gamma_{\text{BS}}}(\gamma)$  with  $p_{\gamma_{\text{SCT}}}(\gamma)$  in (4), the average BER for the SCT transmission scheme can be written as

$$\overline{\text{BER}}_{\text{SCT}} = \frac{K \sum_{n=1}^N R_n a_n \Omega_n}{\text{ASE}_{\text{SCT}}}, \quad (20)$$

where  $\Omega_n = B_{P_{\gamma}(\gamma_{n+1})}(K, \beta_n) - B_{P_{\gamma}(\gamma_n)}(K, \beta_n)$ ,  $\beta_n = 1 + \frac{b_n \bar{\gamma}}{M_n}$ , and  $B_z(x, y)$  denotes the incomplete beta function<sup>4</sup>.

#### V. AVERAGE FEEDBACK LOAD

The SCT scheme yields the best ASE of the multiuser access schemes presented in Section IV, but it comes at the expense of a high and deterministic feedback load. For the proposed switched multiuser access schemes on the other hand, the feedback load  $N_e$  will no longer be deterministic, but can be modelled as a discrete random variable (RV) described by the probability mass function (PMF):

$$P[N_e = k] = \begin{cases} p^{k-1} \cdot (1-p) & k = 1, \dots, K-1 \\ p^{K-1} & k = K \\ 0 & \text{otherwise} \end{cases}. \quad (21)$$

Using this PMF, the *average* feedback load (AFL)  $\overline{N}_e \triangleq \mu_{N_e}$  per time-slot is equal to<sup>5</sup>

$$\mu_{N_e} = \frac{1-p^K}{1-p}. \quad (22)$$

The variance can be written as

$$\sigma_{N_e}^2 = \frac{p - (2K-1)p^K + (2K-1)p^{K+1} - p^{2K}}{(1-p)^2}. \quad (23)$$

#### A. Trade-off between ASE performance and AFL

In the following,  $\gamma_T$  is defined (for fixed  $K$  and  $\bar{\gamma}$ ) as the switching threshold that maximizes the ASE subject to a possible AFL constraint. With no constraint,  $\gamma_T$  is identified within the set  $\mathcal{X} = \{\gamma \in \mathbb{R} : \gamma_1 \leq \gamma \leq \gamma_N\}$  (see (8)). The set  $\mathcal{X}$  is upper bounded by  $\gamma_N$ , since this represents the lowest SNR threshold needed to select the highest rate of the rate-adaptive scheme. There is no point of increasing  $\gamma_T$  beyond  $\gamma_N$ , since this will only increase the AFL with no additional gain in ASE.

<sup>4</sup> $B_z(x, y) = \int_0^z u^{x-1} (1-u)^{y-1} du$ .

<sup>5</sup>For the SWT scheme, the result in (21) differs from [4, Eq.(31)], where  $\mu_{N_e} = 1/(1-p)$ . In [4, Eq.(31)], the result is based on finding the average number per channel access within a time-frame longer than a single time-slot, whereas the result in (21) is valid per time-slot. For the SET scheme, the result in (21) also differs from [4, Eq.(36)], where  $\mu_{N_e} = (1-p^{K-1})/(1-p)$ . The difference is based on whether the last probed user is counted or not. In this paper, it is assumed that for all the proposed switched access schemes, the last user is examined if all the previous  $K-1$  have failed to exceed  $\gamma_T$ . However, the result in [4, Eq.(36)] is based on the traditional SEC combining scheme [7], where it is assumed that the last antenna is selected without comparing it to the threshold  $\gamma_T$ . If the last user is not examined but selected automatically, it will not contribute to increase the feedback load. Hence, both expressions are indeed correct.

When an AFL constraint is introduced,  $\gamma_T$  must be identified within a new set  $\mathcal{X}_{\text{AFL}} = \{\gamma \in \mathbb{R} : \gamma_1 \leq \gamma \leq \gamma^*\}$ , where  $\gamma^* \leq \gamma_N$ . The (possible) size reduction of the cardinality  $|\mathcal{X}_{\text{AFL}}| \leq |\mathcal{X}|$  will be a function of the strongness of the imposed AFL constraint. The ASE performance will become suboptimal when  $\gamma_T$  obtained with unconstrained optimization no longer is available in  $\mathcal{X}_{\text{AFL}}$ . Hence, there is a trade-off between ASE performance and AFL.

As an example, the constraint  $\mu_{N_e} \leq \alpha K$  ( $0 < \alpha \leq 1$ ) is introduced in this paper. This constraint will ensure that the AFL do not exceed a certain percentage of the number of users connected to the BS. Since the condition  $(1-p^K)/(1-p) \leq \alpha K$  cannot be solved in closed-form, the following condition must be satisfied:  $p^K - \alpha K p + \alpha K - 1 \geq 0$ . To minimize the AFL, and since the function  $p^K - \alpha K p + \alpha K - 1$  is converging to zero for large  $\gamma_T$  ( $p$  converging to 1), the upper bound  $\gamma^*$  is selected as the minimum SNR which makes  $p^K - \alpha K p + \alpha K - 1 = 0$ . Hence,  $\mathcal{X}_{\text{AFL}} = \{\gamma \in \mathbb{R} : (\gamma_1 \leq \gamma \leq \gamma^*) \cap (\gamma \leq \gamma_N)\}$ , where  $\gamma^* = \arg \min_{\gamma_1 \leq \gamma_T < \infty} (p^K - \alpha K p + \alpha K - 1 = 0)$ .

#### VI. AVERAGE WAITING TIME

A specific feature of the SWT scheme is that no information is transmitted if none of the  $K$  users are above  $\gamma_T$ . Thus, it is of interest to know some statistics of the number of coherence times  $N_c$  the BS has to wait before an acceptable user is found. The number  $N_c$  will be a discrete RV, with PMF:

$$P[N_c = t] = p^{Kt} (1-p^K), \quad (24)$$

for  $t = 0, 1, \dots$ . Using this PMF, the average waiting time (AWT)  $\overline{N}_c \triangleq \mu_{N_c}$  and variance of  $N_c$  are equal to  $\mu_{N_c} = p^K/(1-p^K)$  and  $\sigma_{N_c}^2 = p^K/(1-p^K)^2$ , respectively.

#### VII. NUMERICAL RESULTS

In Figure 1, ASE for all the presented access schemes are depicted for different average SNR levels. The results are based on selecting the optimal thresholds in a maximum ASE sense, subject to no restrictions on the AFL (unconstrained optimization). In the following, results are limited to the case when  $\bar{\gamma} = 15$  dB due to space limitations and for the sake of clarity<sup>6</sup>. More results will be available in [10].

As depicted in Figure 2, when maximizing the ASE subject to no AFL constraints, the optimal thresholds of the SETps scheme will be high. This will contribute to increase the probability that all the users are examined, which enables the post-selection procedure. As a consequence, the AFL of the

<sup>6</sup>Average BER results are also omitted due to space limitations and since the actual shape of the curves are not important as long as they meet the target  $\text{BER}_0 = 10^{-4}$ . Perfect channel state information and error-free feedback from the users are assumed in this paper, and in this type of scenario, all the presented schemes will achieve the target  $\text{BER}_0$  whenever data are transmitted.

SETps scheme will be deterministic and equal to  $K$  for low and medium average SNR levels. At high SNR ( $\bar{\gamma}$  approaching  $\gamma_T$ ), the AFL will be less than  $K$ , since it is very likely that an acceptable user is found before all the users are examined. For the SET and SWT schemes, the optimal thresholds maximizing the ASE are in general lower than for the SETps scheme, basically to avoid that all users are examined (reducing the risk of an outage). Due to lower optimal thresholds, an acceptable user is identified more quickly, which effectively reduces the AFL. In Figure 3, the AFL corresponding to the ASE results at  $\bar{\gamma} = 15$  dB in Figure 1 is depicted. From the results presented in Figure 1 and 3, it can be deduced that the reduced AFL of the SET and SWT schemes do not translate into a big performance loss in ASE compared to the SCT scheme. Hence, the additional gain offered by the SCT scheme by always identifying the best user is limited.

In Figure 3, the (unmarked) solid line illustrates the upper bound of the AFL constraint  $\mu_{N_e} \leq 0.3K$ . The result of imposing this constraint is depicted in Figure 4. The constraint will immediately limit the AFL for the SETps scheme (for all  $K$ ). The SET and SWT schemes however are largely unaffected, since the optimal AFL curves of the two schemes obtained with unconstrained optimization are not in conflict with the constraint, except when  $K \leq 20$  and  $K > 47$  (SET scheme). In Figure 5, 6, and 7, it is depicted how the ASE curves in Figure 1 ( $\bar{\gamma} = 15$  dB) are affected when the AFL constraint  $\mu_{N_e} \leq \alpha K$  is imposed on the SETps, SET, and SWT schemes, respectively<sup>7</sup>.

In Figure 8, the AWT of the SWT scheme is depicted. It can be seen that the AWT is very low, even for unconstrained optimization (AWT = 0 for simplicity when the imposed AFL constraints cannot be met). Basically, this means that the SWT scheme hardly waits when  $\bar{\gamma} = 15$  dB.

## VIII. CONCLUSIONS

Several switched multiuser access schemes have been proposed for systems operating in a TDM mode. The new access schemes are aimed to reduce the AFL in multiuser systems relying on feedback to maximize the ASE. Numerical results quantifying the trade-off between ASE and AFL have been presented, showing that the AFL can be reduced significantly compared to the optimal SCT scheme without experiencing a big performance loss in ASE. The proposed access schemes are quite attractive also from a fairness perspective.

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<sup>7</sup>The ASE equals zero (for simplicity) when the imposed constraints cannot be met.

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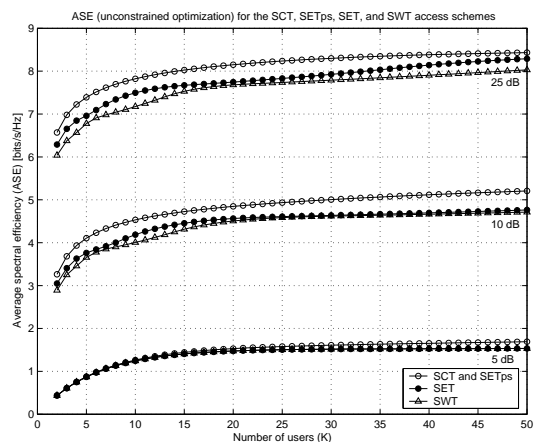


Fig. 1. ASE (unconstrained optimization) for the SCT, SETps, SET, and SWT access schemes when the multiuser system is assumed to be operating on i.i.d. Rayleigh fading channels with  $\bar{\gamma} = [5, 15, 25]$  dB.

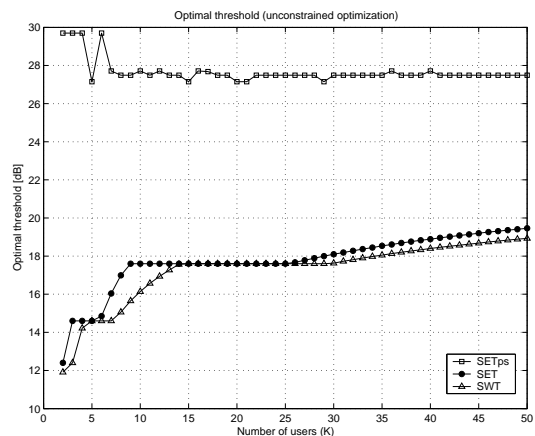


Fig. 2. Optimal thresholds  $\gamma_T$  maximizing the ASE subject to no AFL constraints (unconstrained optimization). The multiuser system is assumed to be operating on i.i.d. Rayleigh fading channels with  $\bar{\gamma} = 15$  dB.

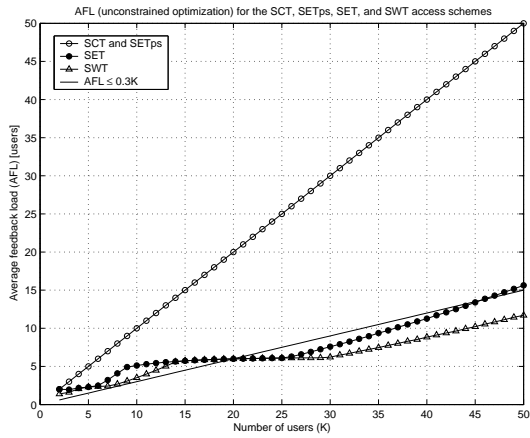


Fig. 3. AFL (unconstrained optimization) for the SCT, SETps, SET, and SWT access schemes. For reference purposes, the solid line visualizes the (linear) upper bound for the constraint  $AFL \leq 0.3K$ . The multiuser system is assumed to be operating on i.i.d. Rayleigh fading channels with  $\bar{\gamma} = 15$  dB.

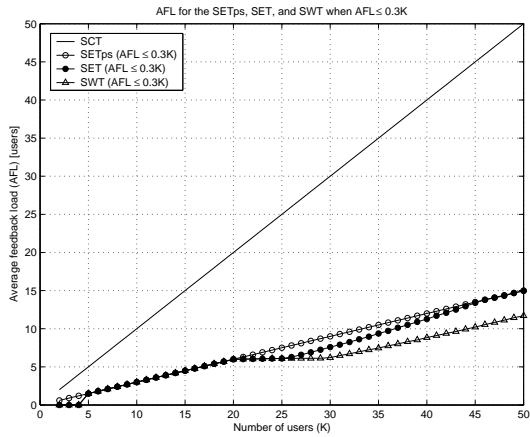


Fig. 4. AFL for the SCT, SETps, SET, and SWT access schemes when  $AFL \leq 0.3K$ . When the constraint cannot be met,  $AFL = 0$  for simplicity. The multiuser system is assumed to be operating on i.i.d. Rayleigh fading channels with  $\bar{\gamma} = 15$  dB.

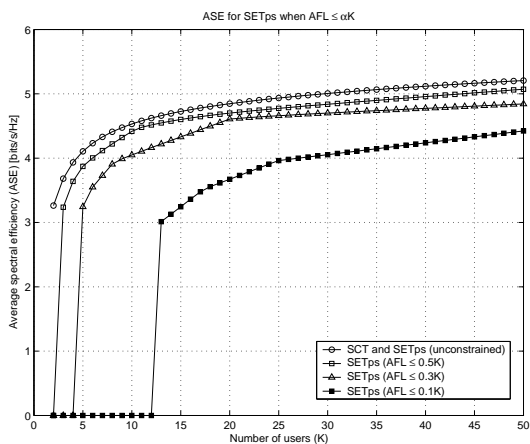


Fig. 5. ASE realized by the SETps access scheme when the AFL is upper bounded by  $AFL \leq \alpha K$ . When the constraint cannot be met,  $ASE = 0$  for simplicity. The multiuser system is assumed to be operating on i.i.d. Rayleigh fading channels with  $\bar{\gamma} = 15$  dB.

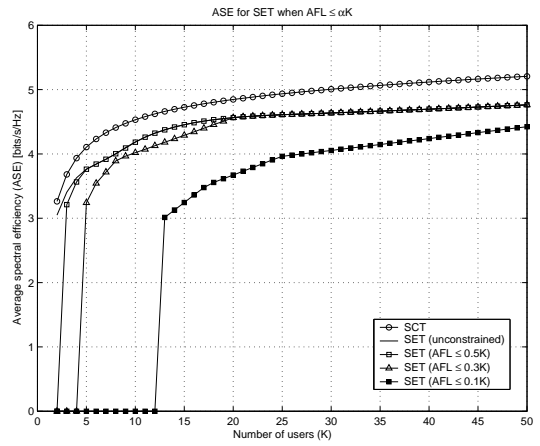


Fig. 6. ASE realized by the SET access scheme when the AFL is upper bounded by  $AFL \leq \alpha K$ . When the constraint cannot be met,  $ASE = 0$  for simplicity. The multiuser system is assumed to be operating on i.i.d. Rayleigh fading channels with  $\bar{\gamma} = 15$  dB.

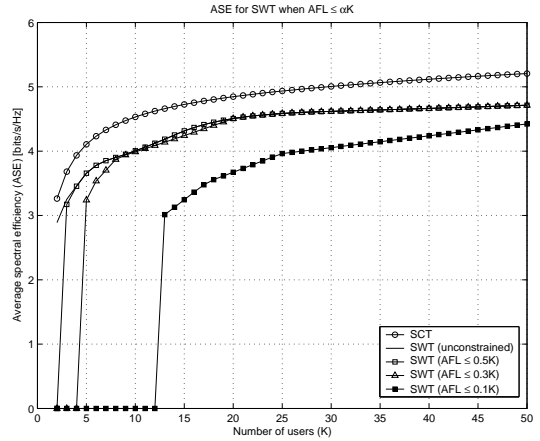


Fig. 7. ASE realized by the SWT access scheme when the AFL is upper bounded by  $AFL \leq \alpha K$ . When the constraint cannot be met,  $ASE = 0$  for simplicity. The multiuser system is assumed to be operating on i.i.d. Rayleigh fading channels with  $\bar{\gamma} = 15$  dB.

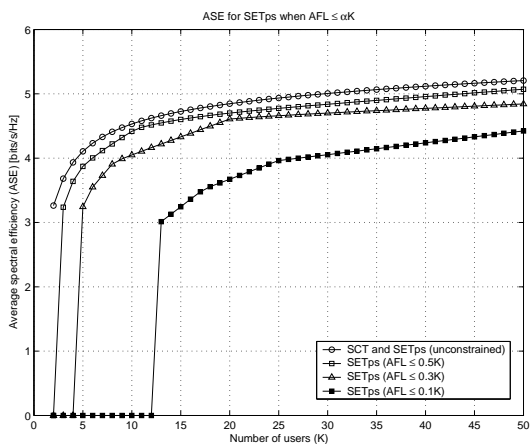


Fig. 8. Average waiting time (AWT) for the SWT access scheme. When the constraint cannot be met,  $AWT = 0$  for simplicity (when  $AFL \leq 0.1K$ ,  $AWT = 0$  for  $K \leq 12$ ). The multiuser system is assumed to be operating on i.i.d. Rayleigh fading channels with  $\bar{\gamma} = 15$  dB.