Decode-and-Forward Two-Way Relaying with Network Coding and Opportunistic Relay Selection

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***Abstract*—In this paper, we study a decode-and-forward two-way relaying network. We propose an opportunistic two-way relaying (O-TR) scheme based on joint network coding and opportunistic relaying. In the proposed scheme, one single "best relay" is selected by MaxMin criterion to perform network coding on two decoded symbols sent from two sources, and then to broadcast the network-coded symbols back to the two sources. The performance of the proposed scheme is analyzed, and verified through Monte Carlo simulations. Results show that the proposed scheme achieves a better performance compared to the fully-distributed space-time two-way relaying (FDST-TR), which has been identified as the best decode-and-forward two-way relaying method so far.**

***Index Terms*—Cooperative communications, decode and forward, network coding, relay selection, two-way relay.**

I. INTRODUCTION

RECENTLY, cooperative relaying technique has attracted an increasing interest in the academia and in the industry. The cooperative relaying not only preserves the benefits inherent from the multiple-input-multiple-output (MIMO) technique, but also possesses a few more remarkable advantages. However, bringing the benefits of the MIMO systems to the relaying systems is not trivial because of other technical challenges such as relaying methods, synchronization, resource allocation, and encoder and decoder design. Distributed space-time coding [1]–[3] is an initiative that tackles these challenges.

At the same time, network coding is a promising method aiming to improve the throughput of communication systems [4]. Motivated by this innovative throughput-boosting method, physical-layer network coding has been proposed to improve the information transmission efficiency in wireless communication systems [5], [6]. Integrating the benefits of relaying and network coding, distributed space-time coding (STC) with modular network coding for two-way relaying systems has been proposed and referred to as the "partial decode-and-forward II (PDF II)" scheme [7]. To reflect the relaying strategy more precisely, in this paper, we name such an relaying scheme as the "fully-distributed space-time two-way relaying (FDST-TR)" scheme. The FDST-TR scheme (i) uses physical-layer network coding; and (ii) decodes the frames and forwards them; and (iii) allows both sources to transmit simultaneously. It has also been shown that the FDST-TR scheme can achieve the full diversity order if the number of symbols in a frame is no less than the number of relays [7]. Further, in order to achieve the optimal performance, the space-time codewords transmitted from different relays have to be orthogonal. Besides, whenever there is a new relay joining the relaying network, all other relays need to change their linear transformation matrices accordingly. Also, the synchronization among all relays becomes more and more difficult when the number of relays is large.

In [8], opportunistic relaying has been proposed as an alternative method for the distributed space-time relaying and is shown to accomplish the full diversity. Opportunistic relaying has been extensively studied for one-way relaying systems and recently some studies have extended the opportunistic relaying to two-way relaying systems [9]–[12]. In [9] and [10], two relay selection criteria have been proposed to optimize the achievable sum-rate in a two-way relaying system. However, no network coding has been considered. In [11] and [12], a two-way relaying system with opportunistic relay selection and amplify-and-forward scheme has been studied in terms of average error rate, outage probability and average sum-rate. It is found that the two-way amplify-and-forward relaying shares the same performance degradation as its one-way counterpart due to noise propagation by the relays.

In this paper, we propose a new decode-and-forward two-way relaying protocol, namely opportunistic two-way relaying (O-TR) method, which is based on modular network coding and opportunistic relay selection. The sources are allowed to transmit simultaneously in the proposed O-TR protocol. However, no distributed space-time coding is needed. Therefore, the requirement that each relay should be assigned an orthogonal precoding matrix is removed. On the other hand, the O-TR protocol requires a relay-selection process which is not needed in distributed space-time coding. Such a relay-selection process, for example, can be implemented by using a distributed algorithm [8] which does not require the channel state information (CSI) of the entire network to be known to all relays. Consequently, instead of selecting a number of relays as in distributed space-time coding, the proposed O-TR protocol selects only one active relay to perform the network coding on the decoded symbols sent by the two sources and to forward the network-coded symbols back to the two sources.

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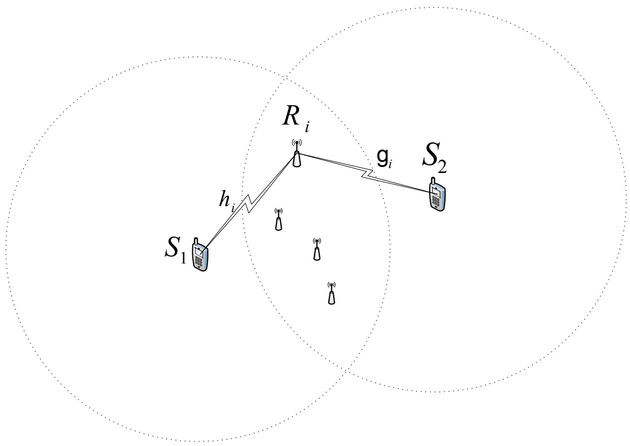


Fig. 1. The system model of a multi-relay system with two sources and two-way communications.

Moreover, the proposed protocol imposes no restriction on the frame length, which results in a more flexible frame design at the sources. We will further show by analysis that the proposed O-TR protocol accomplishes the full diversity.

In Sect. II, the model of a decode-and-forward two-way relaying system is briefly introduced. In Sect. III, the proposed O-TR method is introduced, and the FER performance is analyzed. In addition, an approximated FER is derived when BPSK modulation is used. In Sect. IV, we present our analytical and simulation results.

Notations: $\| \mathbf{x} \|_2 = (\mathbf{x}^H\mathbf{x})^{1/2}$ denotes the l_2 norm or Euclidean norm of the vector $\mathbf{x}$; $(\cdot)^T$ denotes the transposition of a matrix or vector; $(\cdot)^*$ denotes the Hermitian of a matrix or vector; $\Re(x)$ is the real part of the complex variable x; and $\Im(x)$ is the imaginary part of x; $\mathrm{E}_x[f(x)]$ represents the expectation of $f(x)$ over the random variable (RV) x; $\det(\boldsymbol{X})$ is the determinant of the matrix $\boldsymbol{X}$.

II. System Model of Decode-and-Forward Two-Way Relaying

We consider the two-way relaying network shown in Fig. 1, where two source nodes $\mathbb{S}_1$ and $\mathbb{S}_2$ exchange their information with the help of N_R relay nodes $\mathbb{R}_i$ $(i = 1, \ldots, N_R)$. We assume that all the relays lie within the transmission ranges of both $\mathbb{S}_1$ and $\mathbb{S}_2$, but there is no direct link connecting $\mathbb{S}_1$ and $\mathbb{S}_2$. The link between $\mathbb{S}_1$ and $\mathbb{R}_i$ is characterized by the channel coefficient h_i, which is modeled as a complex RV following a complex Gaussian distribution with zero mean and unit variance, i.e., a $\mathcal{CN}(0,1)$ distribution. Meanwhile, the channel coefficient of the $\mathbb{S}_2$-to-$\mathbb{R}_i$ link is denoted by g_i which also follows a $\mathcal{CN}(0,1)$ distribution.

We concentrate on a cooperative communication system in which both $\mathbb{S}_1$ and $\mathbb{S}_2$ employ the same modulation scheme and map each symbol to a corresponding energy-normalized signal $x \in \mathcal{X}$. We assume that the cardinality of the constellation $|\mathcal{X}| = M$, and that each transmission frame consists of N symbols. Also, we denote the minimum distance of the M-ary signal constellation by $d_{\min}$. In each transmission-time unit, $\mathbb{S}_1$ and $\mathbb{S}_2$ will exchange one frame of N symbols, which is fulfilled in two equal-duration time slots. Specifically, in the first time slot, $\mathbb{S}_1$ and $\mathbb{S}_2$ broadcast their signal frames $\mathbf{x}_1 = [x_1(1), \cdots, x_1(N)]^T$ and $\mathbf{x}_2 = [x_2(1), \cdots, x_2(N)]^T$, with powers P_1 and P_2, respectively, to all the relays at the same time. At the ith relay, the received signal frame, denoted by $\mathbf{y}_{\mathbb{R}_i} = [y_{\mathbb{R}_i}(1), \cdots, y_{\mathbb{R}_i}(N)]^T$, is given by

$$\mathbf{y}_{\mathbb{R}_i} = \sqrt{P_1}h_i\mathbf{x}_1 + \sqrt{P_2}g_i\mathbf{x}_2 + \mathbf{n}_i \tag{1}$$

where $\mathbf{n}_i \in \mathbb{C}^N$ is a signal vector consisting of independent complex Gaussian noises with zero mean and variance N_0. The ith relay $(i = 1, \ldots, N_R)$ will decode the received signal frame $\mathbf{y}_{\mathbb{R}_i}$ using a generalized sphere decoder [13]. The detection method is expressed by

$$[\hat{\mathbf{x}}_1^i, \hat{\mathbf{x}}_2^i] = \underset{[\mathbf{s}_1,\mathbf{s}_2]:\mathbf{s}_1,\mathbf{s}_2 \in \mathcal{X}^N}{\operatorname{argmin}} \| \mathbf{y}_{\mathbb{R}_i} - (\sqrt{P_1}h_i\mathbf{s}_1 + \sqrt{P_2}g_i\mathbf{s}_2) \|_2^2 \tag{2}$$

where $\hat{\mathbf{x}}_1^i$ and $\hat{\mathbf{x}}_2^i$ are the decoded signal frames of $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. Afterwards, the ith relay combines the two decoded signal frames into one frame by using modular network coding, which is presented as follows.

Recall that $M = |\mathcal{X}|$ is the cardinality of the signal constellation $\mathcal{X}$. Let $\mathcal{M} = \{0, 1, \ldots, M-1\}$ be a set containing M symbols. Define $\mathcal{A} : \mathcal{M} \mapsto \mathcal{X}$ as a one-to-one mapping with $\mathcal{A}(j) = x_j \in \mathcal{X}$ and $j \in \mathcal{M}$. For a vector $\mathbf{v}$ in which each element is a member of $\mathcal{M}$, $\mathcal{A}$ will map $\mathbf{v}$ on an element-by-element basis onto $\mathcal{X}$. Denote the inverse mapping of $\mathcal{A}$ by $\mathcal{A}^{-1}$, which is also a one-to-one mapping. We can then map the decoded signal frames $\hat{\mathbf{x}}_1^i$ and $\hat{\mathbf{x}}_2^i$ to the decoded symbol vectors, which we denote by $\mathbf{v}_1^i$ and $\mathbf{v}_2^i$, respectively. Therefore, we have $\mathbf{v}_1^i = \mathcal{A}^{-1}(\hat{\mathbf{x}}_1^i)$ and $\mathbf{v}_2^i = \mathcal{A}^{-1}(\hat{\mathbf{x}}_2^i)$. Then the ith relay will determine whether the decoded symbol vectors satisfy the following equation:

$$\mathrm{mod}\{\mathbf{v}_1^i + \mathbf{v}_2^i, M\} = \mathrm{mod}\{\mathbf{v}_1 + \mathbf{v}_2, M\} \tag{3}$$

where $\mathbf{v}_1 = \mathcal{A}^{-1}(\mathbf{x}_1)$, $\mathbf{v}_2 = \mathcal{A}^{-1}(\mathbf{x}_2)$ and $\mathrm{mod}\{\mathbf{x}, M\}$ denotes the modulo-M operation performing on each element in the vector $\mathbf{x}$. If the equation is satisfied, the ith relay is called a *successful relay* [1]. Depending on the protocol being used, one or more relays may broadcast a new signal frame back to $\mathbb{S}_1$ and $\mathbb{S}_2$ in the second time slot. Therefore, the exchange of one pair of information frames between the sources $\mathbb{S}_1$ and $\mathbb{S}_2$ only consumes two time slots, achieving the optimal spectral efficiency for the studied half-duplex two-hop two-way relaying networks.

III. Proposed Opportunistic Two-Way Relaying (O-TR) Method

We propose a protocol based on modular network coding and opportunistic relay selection. In the proposed protocol, each relay will check if the modular condition (3) is satisfied

[1]In this paper, we assume that the condition (3) can be verified, for instance, by a cyclic-redundancy-check (CRC) error-detection code. If $\mathbf{v}_1$ and $\mathbf{v}_2$ are binary vectors, i.e., $M = 2$, the XOR between the CRCs of $\mathbf{v}_1$ and $\mathbf{v}_2$ is exactly the same as the CRC of $\mathrm{mod}\{\mathbf{v}_1 + \mathbf{v}_2, M\}$ [7]. If $M = 2^r$ where r is an integer, the condition (3) can still be verified by mapping the M-ary symbol sequences $\mathbf{v}_1$ and $\mathbf{v}_2$ to binary vectors before performing a CRC detection. However, we have not considered the actual type of error-detection being used, which is outside the scope of this paper. Consequently, the symbols in each frame are considered as independent and identically distributed, and the FER will be evaluated based on the transmitted symbols directly.

after decoding the received signal vectors. Among the *successful relays*, only the "best relay" will be selected and will forward a combined signal frame back to the two sources in the second time slot. If there is no successful relay, none of the relays will be selected. Thus no information frame will be broadcasted back to the sources $\mathbb{S}_1$ and $\mathbb{S}_2$, resulting in a declaration of frame errors at both sources.

Suppose the kth relay is the best relay, which can be selected based on the MaxMin criterion, the maximize-harmonic-mean criterion or any other criteria [8], [14], [15]. We further denote the transmitted signal frame from the kth relay by $\check{\mathbf{t}}_k = \sqrt{N_R P_r}\mathcal{A}(\text{mod}\{\mathbf{v}_1^k + \mathbf{v}_2^k, M\})$ where P_r is the transmission power of each relay. Using (3), the received signal frame at the source $\mathbb{S}_1$ during the second time slot can be expressed as $\check{\mathbf{r}}_1 = h_k\check{\mathbf{t}}_k + \mathbf{w}_1 = \sqrt{N_R P_r} h_k \mathcal{A}(\text{mod}\{\mathbf{v}_1 + \mathbf{v}_2, M\}) + \mathbf{w}_1$. The suboptimal detection method is used and the received signal frame is decoded in favor of

$$\tilde{\mathbf{x}}_2 = \underset{\mathbf{x}=\mathcal{A}(\tilde{\mathbf{v}}_2)}{\text{argmin}} \parallel \check{\mathbf{r}}_1 - \sqrt{N_R P_r} h_k \mathcal{A}(\text{mod}\{\mathbf{v}_1 + \tilde{\mathbf{v}}_2, M\}) \parallel_2^2 . \tag{4}$$

Note that $\mathbf{v}_1$ is a function of the transmitted signal vector $\mathbf{x}_1$ from $\mathbb{S}_1$, and is therefore known to $\mathbb{S}_1$. Unlike the detection for the FDST-TR protocol [7], the detection for the proposed O-TR protocol (4) does not require the summing of different received signal vectors or any matrix transformations. Thus the decoding complexity is lower. Moreover, the average received power at each of the sources is equal to $N_R P_r$ for the proposed O-TR protocol, and is no less than that for the FDST-TR protocol, which has a value of only KP_r ($N_R \geq K \geq 1$)). Therefore, intuitively the O-TR protocol should accomplish a better frame error error (FER) performance than the FDST-TR protocol if the O-TR protocol can achieve the full diversity. In Sect. IV, we will further show the results when the average received powers at each of the sources for the proposed O-TR protocol and for the FDST-TR protocol are identical.

In summary, the benefit of the proposed protocol is threefold. Firstly, there is no restriction on the frame length N, which results in a more flexible frame design at the sources $\mathbb{S}_1$ and $\mathbb{S}_2$. Secondly, no distributed-space-time linear transformation is performed at the relays. Thirdly, the decoding at the sources $\mathbb{S}_1$ and $\mathbb{S}_2$ is made simpler. The drawback of the O-TR protocol compared with the FDST-TR protocol is that a relay-selection algorithm is required. In [8], a distributed algorithm has been proposed to select the relay and the algorithm does not require the channel state information (CSI) of the entire network to be known to all relays.

A. Upper-bound of FER and Diversity Analysis

For simplicity, we assume that the transmission powers of the sources $\mathbb{S}_1$ and $\mathbb{S}_2$, and of the relays are identical, i.e., $P_1 = P_2 = P_r = P$. We also denote the signal-to-noise ratio as $\gamma = P/N_0$, where N_0 is the Gaussian noise variance. Furthermore, we will study the scenario when the MaxMin criterion is used to select the "best relay" [8], [15]. In other words, the i_kth relay will be selected if it can maximize the minimum of the $\mathbb{S}_1$-$\mathbb{R}_i$ channel gain and the $\mathbb{S}_2$-$\mathbb{R}_i$ channel gain for all $i \in \{1, 2, \ldots, N_R\}$, i.e., $i_k = \arg\max_{i\in\{1,2,\ldots,N_R\}}(\min(|h_i|^2, |g_i|^2))$. The approach shown in the following can be applied to cases when another selection criterion is adopted.

1) FER at the relays $\mathbb{R}_i (i = 1, 2, \ldots, N_R)$*:* Considering the ith relay $\mathbb{R}_i$, we rewrite (2) as

$$[\hat{\mathbf{x}}_1^i, \hat{\mathbf{x}}_2^i] = \underset{[\mathbf{s}_1,\mathbf{s}_2]:\mathbf{s}_1,\mathbf{s}_2\in\mathcal{X}^N}{\text{argmin}} \parallel \mathbf{y}_{\mathbb{R}_i} - \sqrt{P}[\mathbf{s}_1\ \mathbf{s}_2][h_i\ g_i]^T \parallel_2^2 . \tag{5}$$

Recall that the signal frames $\mathbf{x}_1$ and $\mathbf{x}_2$ are transmitted from $\mathbb{S}_1$ and $\mathbb{S}_2$, respectively. We let $\boldsymbol{X} = [\mathbf{x}_1\ \mathbf{x}_2]$ and $\boldsymbol{H} = [h_i\ g_i]^T$. Given that $\boldsymbol{X}$ has been transmitted and the estimation is in favor of a particular $\hat{\boldsymbol{X}} = [\hat{\mathbf{x}}_1^i\ \ \hat{\mathbf{x}}_2^i]$, by making use of the method described in Sect. IV of [1], it can be shown that the pairwise error probability (PEP) is bounded above by the Chernoff bound, i.e.,

$$\Pr\{\hat{\boldsymbol{X}} \neq \boldsymbol{X} | \hat{\boldsymbol{X}}, \boldsymbol{X}\} \leq \det{}^{-1}(\boldsymbol{I}_2 + \tfrac{\gamma}{4}(\boldsymbol{X} - \hat{\boldsymbol{X}})^*(\boldsymbol{X} - \hat{\boldsymbol{X}})). \tag{6}$$

Furthermore, applying the analysis that derives (50) in [7] to (6), we obtain $\Pr\{\hat{\boldsymbol{X}} \neq \boldsymbol{X} | \hat{\boldsymbol{X}}, \boldsymbol{X}\} \leq \frac{2}{\gamma d_{\min}^2}$. (Recall that $d_{\min}$ represents the minimum distance of the M-ary signal constellation.) Finally, considering all the cases where $\hat{\boldsymbol{X}} \neq \boldsymbol{X}$ (there are $(M^{2N} - 1)$ such $\hat{\boldsymbol{X}}$) and applying the union bound, we have

$$\begin{aligned}\Pr\{\hat{\boldsymbol{X}} \neq \boldsymbol{X} | \boldsymbol{X}\} &= \sum_{\hat{\boldsymbol{X}}} \Pr\{\hat{\boldsymbol{X}} \neq \boldsymbol{X} | \hat{\boldsymbol{X}}, \boldsymbol{X}\} \\ &\leq \frac{2(M^{2N} - 1)}{\gamma d_{\min}^2} \approx \frac{2M^{2N}}{\gamma d_{\min}^2}.\end{aligned} \tag{7}$$

Consequently, the probability that the modular condition (3) is violated, denoted by p_{relay}, is bounded by

$$\begin{aligned}p_{\text{relay}} &= \Pr\left\{\mathcal{A}(\text{mod}\{\mathbf{v}_1^i + \mathbf{v}_2^i, M\}) \neq \mathcal{A}(\text{mod}\{\mathbf{v}_1 + \mathbf{v}_2, M\})\right\} \\ &\leq \Pr\{\hat{\boldsymbol{X}} \neq \boldsymbol{X} | \boldsymbol{X}\} \leq \frac{2M^{2N}}{\gamma d_{\min}^2}.\end{aligned} \tag{8}$$

2) FER at the sources $\mathbb{S}_1$ *and* $\mathbb{S}_2$*:* Suppose that there are K ($1 \leq K \leq N_R$) relays $\{i_1, \ldots, i_K\} \subset \{1, \ldots, N_R\}$ which can meet the modular condition (3). Assume that the MaxMin criterion is used to select the "best relay", which is denoted as the i_kth relay. In other words, $i_k = \arg\max_{i\in\{i_1,\cdots,i_K\}}(\min(|h_i|^2, |g_i|^2))$. Furthermore, we define $U = \min(|h_{i_k}|^2, |g_{i_k}|^2)$. Since $|h_{i_1}|^2, \ldots, |h_{i_K}|^2$ and $|g_{i_1}|^2, \ldots, |g_{i_K}|^2$ are independent, identical exponential RVs with parameter $\lambda = 1$, the pdf of U is given by [16]

$$\begin{aligned}f_U(u) &= 2K\exp(-2u)[1 - \exp(-2u)]^{K-1} \\ &= 2K\sum_{k=0}^{K-1}\tbinom{K-1}{k}(-1)^k\exp(-2(k+1)u).\end{aligned} \tag{9}$$

At the sources $\mathbb{S}_1$ and $\mathbb{S}_2$, the signal frame received from the selected relay (i_kth relay) are given by $\sqrt{N_R P} h_{i_k}\mathcal{A}(\text{mod}\{\mathbf{v}_1 + \mathbf{v}_2, M\}) + \mathbf{w}_1$ and $\sqrt{N_R P} g_{i_k}\mathcal{A}(\text{mod}\{\mathbf{v}_1 + \mathbf{v}_2, M\}) + \mathbf{w}_2$, respectively, where $\mathbf{w}_2$ is a vector consisting of N independent complex Gaussian noises with zero mean and variance N_0. Denote the decoded symbol frame from $\mathbb{S}_2$ at $\mathbb{S}_1$ by $\tilde{\mathbf{v}}_2$ and the decoded symbol frame from $\mathbb{S}_1$ at $\mathbb{S}_2$ by $\tilde{\mathbf{v}}_1$. Using the decoding mechanism as in (4), the average conditional PEP, which is defined as the average over the conditional PEP $\Pr\{\tilde{\mathbf{v}}_2 \neq \mathbf{v}_2 | \mathbf{v}_2, \tilde{\mathbf{v}}_2, U = u\}$ at the source $\mathbb{S}_1$ and the conditional PEP $\Pr\{\tilde{\mathbf{v}}_1 \neq \mathbf{v}_1 | \mathbf{v}_1, \tilde{\mathbf{v}}_1, U = u\}$ at the source $\mathbb{S}_2$, can be shown equal to (10) at top of next page [7].

$$\begin{aligned}&\tfrac{1}{2}\Pr\{\tilde{\mathbf{v}}_2\neq\mathbf{v}_2|\mathbf{v}_2,\tilde{\mathbf{v}}_2,U=u\}+\tfrac{1}{2}\Pr\{\tilde{\mathbf{v}}_1\neq\mathbf{v}_1|\mathbf{v}_1,\tilde{\mathbf{v}}_1,U=u\}\\&=\tfrac{1}{2}\Pr(\tilde{\mathbf{x}}_2\neq\mathbf{x}_2|\mathbf{x}_2,\tilde{\mathbf{x}}_2,U=u)+\tfrac{1}{2}\Pr(\tilde{\mathbf{x}}_1\neq\mathbf{x}_1|\mathbf{x}_1,\tilde{\mathbf{x}}_1,U=u)\\&\leq\tfrac{1}{2}[e^{-\frac{N_R}{4}\gamma h_{i_k}^*(\tilde{\mathbf{x}}_2-\mathbf{x}_2)^*(\tilde{\mathbf{x}}_2-\mathbf{x}_2)h_{i_k}}+e^{-\frac{N_R}{4}\gamma g_{i_k}^*(\tilde{\mathbf{x}}_1-\mathbf{x}_1)^*(\tilde{\mathbf{x}}_1-\mathbf{x}_1)g_{i_k}}]\\&\leq e^{-\frac{N_R}{4}\gamma\min(|h_{i_k}|^2,|g_{i_k}|^2)d_{\min}^2}=e^{-\frac{N_R}{4}\gamma u d_{\min}^2}.\end{aligned}\tag{10}$$

Combining the results in (9) and (10) and applying [17, Eq(3.312.1)], we obtain

$$\begin{aligned}&\tfrac{1}{2}\Pr\{\tilde{\mathbf{v}}_2\neq\mathbf{v}_2|\mathbf{v}_2,\tilde{\mathbf{v}}_2\}+\tfrac{1}{2}\Pr\{\tilde{\mathbf{v}}_1\neq\mathbf{v}_1|\mathbf{v}_1,\tilde{\mathbf{v}}_1\}\\&\leq\int_0^\infty e^{-\frac{N_R}{4}\gamma d_{\min}^2 u}f_U(u)\,\mathrm{d}u=K\mathbf{B}(\tfrac{N_R}{8}\gamma d_{\min}^2+1,K)\end{aligned}\tag{11}$$

where $\mathbf{B}(\cdot,\cdot)$ is the Beta function [17]. Applying the union bound, the FER at $\mathbb{S}_1$ and $\mathbb{S}_2$ which is defined as the average over the FER $\Pr\{\tilde{\mathbf{v}}_2\neq\mathbf{v}_2\}$ at the source $\mathbb{S}_1$ and the FER $\Pr\{\tilde{\mathbf{v}}_1\neq\mathbf{v}_1\}$ at the source $\mathbb{S}_2$, given that there are K successful relays, is upper-bounded as follows:

$$\begin{aligned}&p_{\text{sources},K}\\&=\tfrac{1}{2}\Pr\{\tilde{\mathbf{v}}_2\neq\mathbf{v}_2\}+\tfrac{1}{2}\Pr\{\tilde{\mathbf{v}}_1\neq\mathbf{v}_1\}\\&=\sum_{\tilde{\mathbf{v}}_1,\tilde{\mathbf{v}}_2}\tfrac{1}{2}\Pr\{\tilde{\mathbf{v}}_2\neq\mathbf{v}_2|\mathbf{v}_2,\tilde{\mathbf{v}}_2\}+\tfrac{1}{2}\Pr\{\tilde{\mathbf{v}}_1\neq\mathbf{v}_1|\mathbf{v}_1,\tilde{\mathbf{v}}_1\}\\&\leq M^N K\mathbf{B}(\tfrac{N_R}{8}\gamma d_{\min}^2+1,K).\end{aligned}\tag{12}$$

Note that if there are no successful relays, i.e., $K=0$, frame errors are declared at both sources and thus $p_{\text{sources},0}=1$.

3) Overall system FER and diversity: Finally, the overall average FER of the two-way relaying system, denoted by $\bar{P}_{\text{FER}}$, is bounded by

$$\begin{aligned}\bar{P}_{\text{FER}}&=\sum_{K=0}^{N_R}\binom{N_R}{K}p_{\text{sources},K}\times(1-p_{\text{relay}})^K\times p_{\text{relay}}^{N_R-K}\\&\leq\sum_{K=0}^{N_R}\binom{N_R}{K}p_{\text{sources},K}\times p_{\text{relay}}^{N_R-K}\\&\leq\left(\frac{2M^{2N}}{\gamma d_{\min}^2}\right)^{N_R}+\sum_{K=1}^{N_R}\binom{N_R}{K}M^N K\\&\qquad\times\mathbf{B}(\tfrac{N_R}{8}\gamma d_{\min}^2+1,K)\left(\frac{2M^{2N}}{\gamma d_{\min}^2}\right)^{N_R-K}\\&=\left(\frac{2M^{2N}}{\gamma d_{\min}^2}\right)^{N_R}+\sum_{K=1}^{N_R}\binom{N_R}{K}M^N K\\&\qquad\times\frac{\Gamma(K)}{(\frac{N_R}{8}\gamma d_{\min}^2+2)^K}\left(\frac{2M^{2N}}{\gamma d_{\min}^2}\right)^{N_R-K}\\&\leq M^N(\gamma d_{\min}^2)^{-N_R}(8+2M^{2N})^{N_R}\end{aligned}\tag{13}$$

where $\Gamma(\cdot)$ is the Gamma function [17]. In deriving the last inequality, we have applied $K\frac{\Gamma(K)}{(\frac{N_R}{8}\gamma d_{\min}^2+2)^K}\leq K\frac{\Gamma(K)}{(\frac{N_R}{8}\gamma d_{\min}^2)^K}\leq(\frac{8}{\gamma d_{\min}^2})^K$. The results in (13) indicate that the upper bound of the average FER is proportional to γ^{-N_R}. Thus, we can conclude that *our proposed O-TR scheme for the two-way relaying system can achieve the full diversity even without employing distributed space-time coding.*

B. FER analysis for binary-phase-shift-keying (BPSK) modulation

Suppose the BPSK modulation is used in the signal transmission. We denote $p_{\mathbb{R}_i|h_i,g_i}$ as the FER at Relay $\mathbb{R}_i$ conditioned on h_i and g_i, i.e., the probability that the condition in (3) fails at Relay $\mathbb{R}_i$. We further define $V_i=\min(|h_i|^2,|g_i|^2)$. Then, $p_{\mathbb{R}_i|h_i,g_i}$ can be approximated by (see Appendix A)

$$\begin{aligned}p_{\mathbb{R}_i|h_i,g_i}&\approx1-\left(1-\mathrm{Q}(\sqrt{2\min(|h_i|^2,|g_i|^2)\gamma})\right)^N\\&=\sum_{n=1}^N\binom{N}{n}(-1)^{n+1}\mathrm{Q}^n\left(\sqrt{2V_i\gamma}\right)\triangleq p_{\mathbb{R}_i|V_i}.\end{aligned}\tag{14}$$

We define $\mathcal{K}=\{i_m\}_{m=1}^K$ as a set of indices where $i_m\in\{1,\ldots,N_R\}$ and $K\leq N_R$. Define "$\mathbb{R}_\mathcal{K}$ successful" as the event that the relays $\{\mathbb{R}_{i_m}\}_{m=1}^K$ are successful while the relays $\{\mathbb{R}_j\}$ with $j\notin\mathcal{K}$ are not successful. The probability for such an event to occur can therefore be approximated by $\Pr(\mathbb{R}_\mathcal{K}\text{ successful})\approx\prod_{i_m\in\mathcal{K}}(1-p_{\mathbb{R}_{i_m}|V_{i_m}})\times\prod_{j\notin\mathcal{K}}p_{\mathbb{R}_j|V_j}$. Furthermore, by applying $f_{V_i}(v)=2\exp(-2v)$ and the approximation (21) in Appendix B to (14), we can approximate the average FER at $\mathbb{R}_i$ as

$$\begin{aligned}p_\mathbb{R}\triangleq p_{\mathbb{R}_i}&=\mathrm{E}_{V_i}[p_{\mathbb{R}_i|V_i}]\\&=\mathrm{E}_{V_i}\left[\sum_{n=1}^N\binom{N}{n}(-1)^{n+1}\mathrm{Q}^n\left(\sqrt{2V_i\gamma}\right)\right]\\&\approx\sum_{n=1}^N\binom{N}{n}(-1)^{n+1}\sum_{\substack{l_1,l_2,\ldots,l_8\\l_1+l_2+\cdots+l_8=n}}2\alpha_n\beta_n(2\gamma)^{\frac{\mu_n}{2}}\\&\qquad\times\Gamma(\tfrac{\mu_n}{2}+1)(n\gamma+2)^{-(\frac{\mu_n}{2}+1)}\end{aligned}\tag{15}$$

where $\alpha_n=n!/(l_1!l_2!\ldots l_{m_a}!)$, $\beta_n=(c_1)^{l_1}\ldots(c_{m_a})^{l_{m_a}}$ and $\mu_n=l_2+2l_3+\cdots+(m_a-1)l_{m_a}$. As in Sect. III-A2, we denote $U=\max_{i_m\in\mathcal{K}}\{\min(|h_{i_m}|^2,|g_{i_m}|^2)\}$. Since only the best relay will transmit the coded signal vector back to the sources in the second time slot, the FER at the sources conditioned on the "$\mathbb{R}_\mathcal{K}$ successful" event is obtained by

$$\begin{aligned}&p_{\text{sources}|\mathbb{R}_\mathcal{K}\text{ successful}}\\&\approx\tfrac{1}{2}(1-(1-\mathrm{Q}(\sqrt{2UN_R\gamma}))^N)\\&=\tfrac{1}{2}\sum_{n=1}^N\binom{N}{n}(-1)^{n+1}\mathrm{Q}^n\left(\sqrt{2UN_R\gamma}\right).\end{aligned}\tag{16}$$

Note that the same approximation shown in Appendix A has been used in the above derivation. Then, the average PEP at the sources over all $\{V_i\}_{i=1}^{N_R}$ when $\{\mathbb{R}_{i_m}\}_{m=1}^K$ are the successful relays, denoted by $p_{\text{sources},\mathbb{R}_\mathcal{K}\text{ successful},K}$, can be found using (17) on the next page. By the definition of U, each V_{i_m} is no greater than U, i.e., $V_{i_m}\leq U$. Thus, the inner expectation in (17) can be written as

$$\begin{aligned}&\mathrm{E}_{\{V_{i_m}\}_{i_m\in\mathcal{K}}}\left[\prod_{i_m\in\mathcal{K}}(1-p_{\mathbb{R}_{i_m}|V_{i_m}})\middle|U=u\right]\\&\approx\mathrm{E}_{\{V_{i_m}\}_{i_m\in\mathcal{K}}}\left[\prod_{i_m\in\mathcal{K}}\left(1-\mathrm{Q}(\sqrt{2v_{i_m}\gamma})\right)^N\middle|U=u\right]\\&\approx\prod_{i_m\in\mathcal{K}}\int_0^u[1-\tfrac{e^{-v_{i_m}\gamma}}{12}-\tfrac{e^{-4v_{i_m}\gamma/3}}{4}][2\exp(-2v_{i_m})]\,\mathrm{d}v_{i_m}\end{aligned}$$

$$\begin{aligned}
p_{\text{sources},\mathbb{R}_{\mathcal{K}}\text{ successful},K} &= \mathrm{E}_{\{V_i\}_{i=1}^{N_R}}\left[p_{\text{sources}|\mathbb{R}_{\mathcal{K}}\text{ successful}} \times \Pr\left(\mathbb{R}_{\mathcal{K}}\text{ successful}\right)\right] \\
&= \mathrm{E}_{\{V_{i_m}\}_{i_m\in\mathcal{K}},\{V_j\}_{j\notin\mathcal{K}}}\left[p_{\text{sources}|\mathbb{R}_{\mathcal{K}}\text{ successful}} \times \prod_{i_m\in\mathcal{K}}(1-p_{\mathbb{R}_{i_m}|V_{i_m}}) \times \prod_{j\notin\mathcal{K}} p_{\mathbb{R}_j|V_j}\right] \\
&= \mathrm{E}_{\{V_{i_m}\}_{i_m\in\mathcal{K}}}\left[p_{\text{sources}|\mathbb{R}_{\mathcal{K}}\text{ successful}} \times \prod_{i_m\in\mathcal{K}}(1-p_{\mathbb{R}_{i_m}|V_{i_m}})\right] \times \prod_{j\notin\mathcal{K}} \mathrm{E}_{V_j}\left[p_{\mathbb{R}_j|V_j}\right] \\
&= \mathrm{E}_{\{V_{i_m}\}_{i_m\in\mathcal{K}}}\left[p_{\text{sources}|\mathbb{R}_{\mathcal{K}}\text{ successful}} \times \prod_{i_m\in\mathcal{K}}(1-p_{\mathbb{R}_{i_m}|V_{i_m}})\right] \times (p_{\mathbb{R}})^{N_R-K} \\
&= \mathrm{E}_U\left[\mathrm{E}_{\{V_{i_m}\}_{i_m\in\mathcal{K}}}\left[p_{\text{sources}|\mathbb{R}_{\mathcal{K}}\text{ successful}} \times \prod_{i_m\in\mathcal{K}}(1-p_{\mathbb{R}_{i_m}|V_{i_m}})\right] \times (p_{\mathbb{R}})^{N_R-K} \mid U=u\right] \\
&= \mathrm{E}_U\left[p_{\text{sources}|\mathbb{R}_{\mathcal{K}}\text{ successful}} \times \mathrm{E}_{\{V_{i_m}\}_{i_m\in\mathcal{K}}}\left[\prod_{i_m\in\mathcal{K}}(1-p_{\mathbb{R}_{i_m}|V_{i_m}}) \mid U=u\right]\right] \times (p_{\mathbb{R}})^{N_R-K}. \qquad (17)
\end{aligned}$$

$$= \Bigg(1 - \tfrac{1}{6(\gamma+2)} - \tfrac{1}{2(4\gamma/3+2)} - e^{-2u} + \tfrac{e^{-(\gamma+2)u}}{6(\gamma+2)} + \tfrac{e^{-(4\gamma/3+2)u}}{2(4\gamma/3+2)}\Bigg)^K. \qquad (18)$$

Here, we have approximated $(1-\mathrm{Q}(\sqrt{2v_{i_m}\gamma}))^N$ by $1-\mathrm{Q}(\sqrt{2v_{i_m}\gamma})$. When $2v_{i_m}\gamma$ is large, $\mathrm{Q}(\sqrt{2v_{i_m}\gamma})$ approaches zero and the approximation becomes more accurate. The same is true when N is small. Further, we approximate $\mathrm{Q}(\sqrt{2v_{i_m}\gamma})$ by $\frac{1}{12}e^{-v_{i_m}\gamma}+\frac{1}{4}e^{-4v_{i_m}\gamma/3}$ [18]. The accuracy of the approximations will be examined when we compare the analytical results with the simulations in Sect. IV. Then, by using (18) and (21), (17) can be expressed as (19). Finally, by taking into account all possible cases of K, i.e., $K = 0, 1, 2, \ldots, N_R$, and the number of possible combinations of $\{i_m\}_{m=1}^K$, the average FER for the O-TR relaying system when BPSK is applied, denoted by $\bar{P}_{\text{FER,BPSK}}$, can be estimated using

$$\begin{aligned}
\bar{P}_{\text{FER,BPSK}} &= \sum_{K=0}^{N_R}\binom{N_R}{K} p_{\text{sources},\mathbb{R}_{\mathcal{K}}\text{ successful},K} \\
&= (p_{\mathbb{R}})^{N_R} + \sum_{K=1}^{N_R}\binom{N_R}{K} p_{\text{sources},\mathbb{R}_{\mathcal{K}}\text{ successful},K}. \qquad (20)
\end{aligned}$$

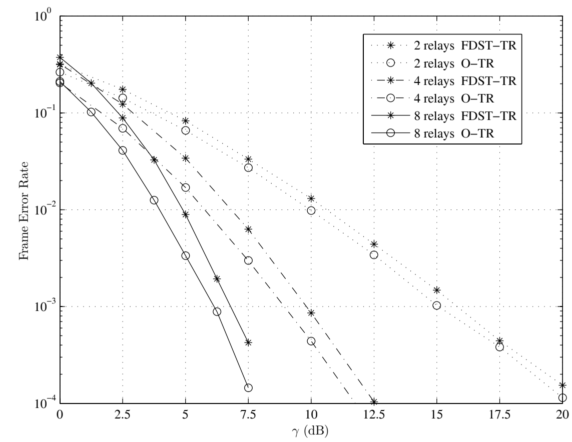


Fig. 2. The frame error rate comparison between the fully-distributed space-time coded two-way relaying method (FDST-TR) and the proposed opportunistic two-way relaying method (O-TR) for three different systems: a 2-relay system (i.e., $N_R = 2$), a 4-relay system (i.e., $N_R = 4$) and an 8-relay system (i.e., $N_R = 8$). The frame length N is set to equal to the number of relays, i.e., $N = N_R$. BPSK modulation is used.

IV. Simulation and Numerical Results

First, we compare the FER performance of our proposed O-TR method with that of the FDST-TR method [7] by simulations. In Fig. 2, we show the simulated FER performance under three system settings: a 2-relay system with frame length $N = 2$, a 4-relay system with $N = 4$ and an 8-relay system with $N = 8$. The curves in Fig. 2 clearly show that the O-TR method can achieve the same full diversity order as the FDST-TR method. In addition, for all the settings being considered, the proposed O-TR method achieves better performance than the FDST-TR method in terms of FER. Note that in the O-TR scheme, the transmissions through the N_R relays can be regarded as a group of N_R independent fading channels. Selecting the best relay using the opportunistic relaying method is equivalent to selecting the best channel among the N_R independent channels. Since each independent fading channel has a diversity of one, the best channel will behave like utilizing the diversities of all the N_R independent fading channels and forming an equivalent fading channel with a diversity of N_R.

In the FDST-TR scheme, for a given SNR, there will be an average of $\mathrm{E}[K]$ successful relays at each broadcasting session. The average transmission power of all the relays is therefore given by $\mathrm{E}[K]P_r$. In our proposed O-TR method, suppose the "best relay" transmits with a power identical to the average power of all the relays in the FDST-TR case, i.e., $\mathrm{E}[K]P_r$, we study the FER under such a scenario. (Note that in practice, it may not be feasible for the "best relay" to transmit with $\mathrm{E}[K]P_r$, which varies as the SNR changes.) In Fig. 3, we plot the FERs for a two-way relaying system with $N_R = 4$. The results indicate that our proposed O-TR method with the "best relay" transmitting with $\mathrm{E}[K]P_r$ still outperforms the FDST-TR scheme. The reason is that the "best relay" spends all the transmission power on the "best channels" while the relays in the FDST-TR scheme spend some of the transmission power on the comparatively "not-so-good" channels. Note also that as the SNR increases, the FER for the O-TR$\{\mathrm{E}[K]P_r\}$

$$
\begin{aligned}
& p_{\text{sources},\mathbb{R}_{\mathcal{K}}\ \text{successful},K} \\
&= \mathrm{E}_U\Big[\tfrac{1}{2}\sum\nolimits_{n=1}^{N}\tbinom{N}{n}(-1)^{n+1}\,\mathrm{Q}^n\left(\sqrt{2uN_R\gamma}\right)\left(1-\tfrac{1}{6(\gamma+2)}-\tfrac{1}{2(4\gamma/3+2)}-e^{-2u}+\tfrac{e^{-(\gamma+2)u}}{6(\gamma+2)}+\tfrac{e^{-(4\gamma/3+2)u}}{2(4\gamma/3+2)}\right)^{K}\Big] \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times (p_{\mathbb{R}})^{N_R-K} \\
&\approx \tfrac{1}{2}\sum\nolimits_{n=1}^{N}\tbinom{N}{n}(-1)^{n+1}\sum_{\substack{l_1,l_2,\ldots,l_8\\ l_1+l_2+\cdots+l_8=n}}\alpha_n\beta_n(2N_R\gamma)^{\frac{\mu_n}{2}}\sum_{k=0}^{K-1}2K\tbinom{K-1}{k}(-1)^k \\
&\quad\times\sum_{\substack{j_1,j_2,j_3,j_4\\ j_1+j_2+j_3+j_4=K}}\tfrac{K!}{j_1!j_2!j_3!j_4!}\left(1-\tfrac{1}{6(\gamma+2)}-\tfrac{1}{2(4\gamma/3+2)}\right)^{j_1}(-1)^{j_2}\left(\tfrac{1}{6(\gamma+2)}\right)^{j_3}\left(\tfrac{1}{2(4\gamma/3+2)}\right)^{j_4} \\
&\qquad\qquad\times\Gamma\left(\tfrac{\mu_n}{2}+1\right)\left[nN_R\gamma+2(k+j_2+1)+(\gamma+2)j_3+(4\gamma/3+2)j_4\right]^{-\left(\frac{\mu_n}{2}+1\right)}\times(p_{\mathbb{R}})^{N_R-K}.
\end{aligned}
\tag{19}
$$

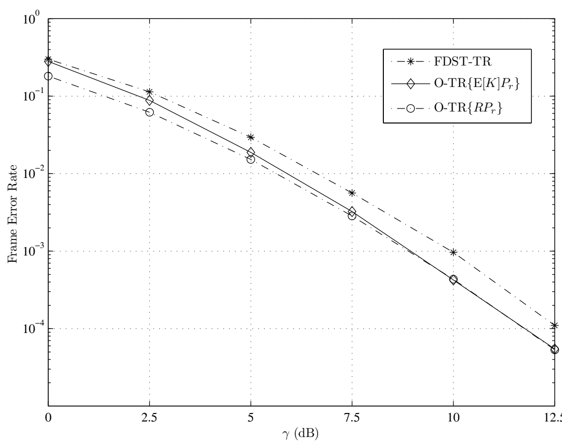


Fig. 3. The frame error rate comparison between the fully-distributed space-time coded two-way relaying method (FDST-TR), the proposed opportunistic two-way relaying method with a relay transmission power of $\mathrm{E}[K]P_r$ (O-TR$\{\mathrm{E}[K]P_r\}$), and the proposed opportunistic two-way relaying method with a relay transmission power of N_RP_r (O-TR$\{N_RP_r\}$) for a 4-relay system (i.e., $N_R = 4$). The frame length N is set to equal to the number of relays, i.e., $N = N_R$. BPSK modulation is used.

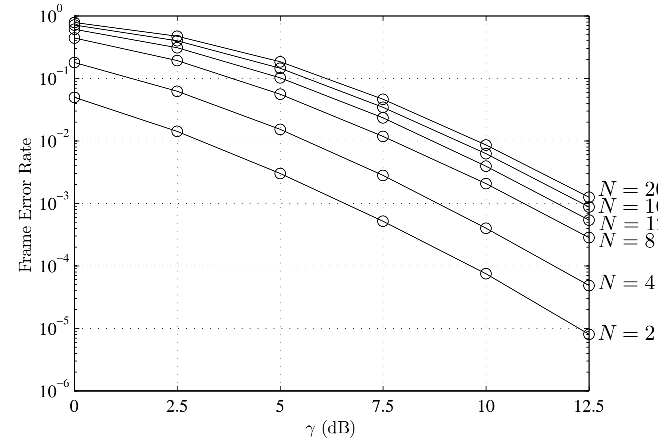


Fig. 4. The frame error rate versus signal-to-noise ratio for a 4-relay system employing the O-TR method and the BPSK modulation scheme. Frame length $N = 2, 4, 8, 12, 16, 20$.

converges to that for the O-TR$\{N_RP_r\}$. It is because when the SNR increases, the number of successful relays increases and approaches N_R. We then examine the impact of the frame length N on the FER performance of the proposed O-TR method. In Fig. 4, we show the simulated FER performance of a 4-relay system under different frame length N. As expected, reducing N improves the FER performance.

Finally, we compare the approximated FER performance (20) with the simulation results in the case of BPSK modulation. The results for a two-relay system and a four-relay system are illustrated in Fig. 5(a) and Fig. 5(b), respectively. The curves indicate that the FER approximation in (20) forms a lower-bound of the actual FER. We can further observe that for the same γ, a smaller value of N gives a smaller absolutely difference between the simulated FER and approximated FER. The same observation occurs when we increase γ while keeping N constant. Such findings are consistent with a more accurate approximation of (18) (which forms part of (20)) when N is small and γ is large.

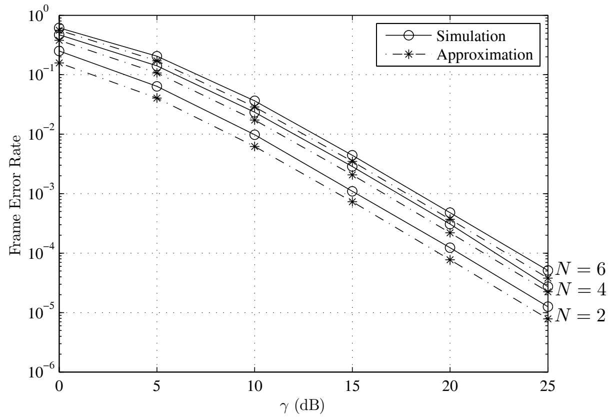
V. Conclusions

In this paper, we have proposed a relaying method for a two-way relaying network, namely opportunistic two-way relaying (O-TR) method. It is based on the modular network coding method and the opportunistic relay selection. We have derived the upper-bound of the frame error rate (FER) of the proposed O-TR method and have shown that the proposed method can accomplish the full diversity order. Simulation results have further shown that the proposed O-TR method outperforms the fully-distributed space-time two-way relaying method in terms of FER.

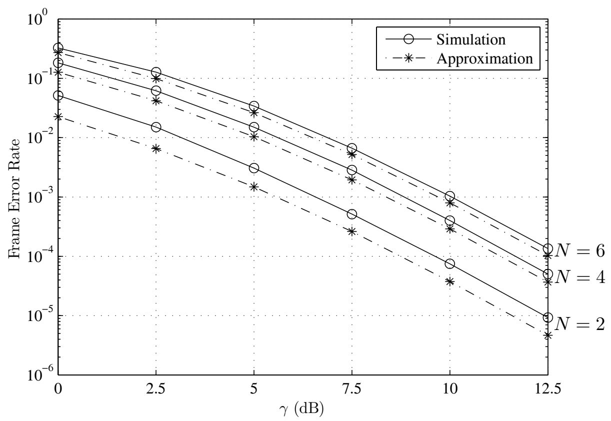
Appendix A: FER at the Relays

At time instance t $(1 \le t \le N)$, the transmitted BPSK signals from $\mathbb{S}_1$ and $\mathbb{S}_2$ are denoted by $x_1(t)$ and $x_2(t)$, respectively. We remove the time index and represent these signals by x_1 and x_2 in order to simplify the notation. At the ith relay, the received signal is expressed as $\begin{bmatrix}\Re(y_{\mathbb{R}_i})\\ \Im(y_{\mathbb{R}_i})\end{bmatrix} = \sqrt{P}\begin{bmatrix}\Re(h_i) & \Re(g_i)\\ \Im(h_i) & \Im(g_i)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\Re(n_i)\\ \Im(n_i)\end{bmatrix}$. The corresponding decoded signal-vector results that made the modular condition (3) invalid are $\left[\begin{smallmatrix}-x_1\\ x_2\end{smallmatrix}\right]$ and $\left[\begin{smallmatrix}x_1\\ -x_2\end{smallmatrix}\right]$. Given that $\left[\begin{smallmatrix}x_1\\ x_2\end{smallmatrix}\right]$ has been transmitted, the probability that the detected signal vector being $\left[\begin{smallmatrix}-x_1\\ x_2\end{smallmatrix}\right]$ can be shown equal to [19, Sect. A.2]

$$
\Pr\left(\left[\begin{smallmatrix}-x_1\\ x_2\end{smallmatrix}\right] \middle| \left[\begin{smallmatrix}x_1\\ x_2\end{smallmatrix}\right]\right) \approx \mathrm{Q}\left(\|\sqrt{P}\left[\begin{smallmatrix}\Re(h_i) & \Re(g_i)\\ \Im(h_i) & \Im(g_i)\end{smallmatrix}\right]\left[\begin{smallmatrix}x_1\\ x_2\end{smallmatrix}\right]\right.
$$



(a)



(b)

Fig. 5. The simulated FER and the approximated FER for an O-TR two-way relaying system. All systems employ BPSK modulation. Frame length $N = 2, 4, 6$. (a) A 2-relay system; (b) a 4-relay system.

$$-\sqrt{P}\left[\begin{smallmatrix}\Re(h_i) & \Re(g_i)\\ \Im(h_i) & \Im(g_i)\end{smallmatrix}\right]\left[\begin{smallmatrix}-x_1\\ x_2\end{smallmatrix}\right] \|_2 /(2\sqrt{1/2})\Big)$$
$$= \mathrm{Q}(\sqrt{2|h_i|^2\gamma})$$

where $\mathrm{Q}(\cdot)$ is the Q-function. In the above derivation, we have made use of the fact that $|x_1| = 1$. Similarly, we have $\Pr\left(\left[\begin{smallmatrix}x_1\\ -x_2\end{smallmatrix}\right] | \left[\begin{smallmatrix}x_1\\ x_2\end{smallmatrix}\right]\right) \approx \mathrm{Q}(\sqrt{2|g_i|^2\gamma})$. Therefore the frame error rate of the signal frame detection at the relay $\mathbb{R}_i$ is calculated by $p_{r_i|h_i,g_i} \approx 1 - \left(1 - \mathrm{Q}(\sqrt{2|h_i|^2\gamma}) - \mathrm{Q}(\sqrt{2|g_i|^2\gamma})\right)^N \approx 1 - \left(1 - \mathrm{Q}(\sqrt{2\min(|h_i|^2, |g_i|^2)\gamma})\right)^N$.

Appendix B: Approximation of Gaussian Q-Function

Using the approximations in [20], we can express the Gaussian Q-function as $\exp(-\frac{x^2}{2})\sum_{m=1}^{m_a} c_m x^{m-1}$ where $c_m = \frac{(-1)^{m+1}A^m}{B\sqrt{\pi}(\sqrt{2})^{m+1}m!}$; and can approximate the n-th power of the Gaussian Q-function by

$$\mathrm{Q}^n(x) \approx \exp(-nx^2/2)(\sum_{m=1}^{m_a} c_m x^{m-1})^n$$
$$= \exp(-nx^2/2) \sum_{l_1,l_2,\ldots,l_{m_a} \text{ s.t. } l_1+l_2+\cdots+l_{m_a}=n} \alpha_n\beta_n x^{\mu_n} \quad (21)$$

where $\alpha_n = n!/(l_1!l_2!\ldots l_{m_a}!)$, $\beta_n = (c_1)^{l_1}\ldots(c_{m_a})^{l_{m_a}}$ and $\mu_n = l_2+2l_3+\cdots+(m_a-1)l_{m_a}$. In our analysis, we employ the following parameters: $m_a = 8$, $A = 1.98$ and $B = 1.135$, which has been found to be a good setting in approximating the Q-function [20].

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