Per-Stream Jitter Analysis in CBR ATM Multiplexors

Aleksandr Privalov, Khosrow Sohraby

Abstract— **Constant Bit Rate (CBR) traffic is expected to be a major source of traffic in high-speed networks. Such sources may have stringent delay and loss requirements and in many cases, they should be delivered exactly as they were generated.**

A simple delay priority scheme will bound the cell delay and jitter for CBR streams, so that in the network switches, CBR traffic will only compete with other CBR traffic in the networks. In this paper, we will consider a multiplexor in such an environment. We provide an exact analysis of the jitter process in the homogeneous case. In this case, we obtain the complete characterization of the jitter process showing the inaccuracies of the existing results.

Our results indicate that jitter variance is bounded and never exceeds the constant $\frac{2}{3}$ slot. It is also shown that the per-stream successive cell inter-departures times are negatively correlated with the lag 1 correlation of $-\frac{1}{2}$. Higher order correlation coefficients are shown to be zero. Simple asymptotic results on per-stream behavior are also provided when the number of CBR streams is considered large.

In the heterogeneous case, we bound the jitter distribution and moments. Simple results are provided for the computation of the bound on the jitter variance for any mix of CBR streams in this case. It is shown that streams with a low rate (large period) do experience little jitter variance. However, the jitter variance for the high-rate streams could be quite substantial.

Keywords—**Jitter, cell delay variation, cell based networks, ATM, periodic arrivals, statistical multiplexing, asymptotic analysis**

I. Introduction

Broadband Integrated Services Digital Networks (B-ISDN) will transport diverse classes of services such as data, voice, image, and video. ATM (Asynchronous Transfer Mode) is being standardized as the transport mechanism to integrate and support such services in a single network. ATM is a *connection-oriented* packet-switched network where packets are segmented into fixed size cells which are statistically multiplexed over the high speed links. Statistical multiplexing of the cells generated from diverse traffic sources with possibly very different characteristics on an ATM link introduces considerable flexibility and potential saving in the allocation of network resources. However, if the network is not properly designed, dimensioned and controlled, extensive cell delay, cell loss and *cell delay variation* (CDV) or *jitter* may deteriorate the performance of the network to an unacceptable level.

The success of the ATM networks will be heavily dependent upon being able to provide the required QoS (Quality of Service) to the classes of traffic supported by the network. An important class of traffic to be supported by the ATM networks are those for which a timing relation should be maintained between source and destination. These services are classified as Class A and Class B in the ITU-T Recommendation I.362. Class A services are connection-oriented constant bit-rate (CBR) like DS1 and $n\times 64$ kbps circuit emulation. This class is supported by AAL1. Class B which is supported by AAL2 denotes the variable bit-rate (VBR) services like packet video or audio. As in Class A, these services are connection-oriented and require a timing relation between source and destination. We note that Class C services are connection-oriented, but do not require timing (e.g., Frame Relay and X.25).

Class A (CBR) and B (VBR) services with timing relation between source and destination are expected to comprise a major portion of the total traffic on the network generated from the multimedia sources. However, as such sources traverse multiple nodes in the network and compete for the network resources with other traffic, they lose their original pattern. Jitter is defined as the alteration of the original pattern of the cell arrival process at the multiplexing stages of the network. A play-out delay (or "elastic buffer") is used for the reconstruction of the stream into its original pattern before it is delivered to the final destination. The duration of the play-out delay is determined by the jitter process and the QoS. Important performance measures for classes A and B are delay, loss and jitter. For class C, delay and loss are important measures.

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In this paper we provide the analysis of jitter incurred to *individual* periodic cell streams going through a node in an ATM network. It should be noted that the periodic arrival processes are not only limited to periodic sources (e.g., real-time speech or video, output of a peak rate enforcer), it is also due to window flow control protocols which have a periodic cycle equal to the connection round trip time [8].

In this work, we consider nodes which support a number of CBR sources. Most of the existing teletraffic analysis in computer and communication networks cannot handle periodic cell streams which are generated from the CBR traffic. This is largely due to the fact that we deal with a combinatorial problem and not necessarily a traditional "queueing" problem.

The problem of characterizing the superposition of periodic processes is an important problem which dates back to 1953 [4]. There has been number of works in attempting to provide the queueing analysis of such processes. References [5], [1], [24], [2] are examples of infinite buffer and [15] consider the finite buffer case.

In the jitter analysis, the existing work concentrates on a single *tagged* stream and a *random* background traffic modeling the superposition of all the other traffic competing for resources with the tagged stream in a node. The background traffic is assumed to have a known distribution disregarding the probabilistic nature of individual streams. The tagged stream has been assumed to be originally periodic (i.e., CBR traffic) or having a general renewal process [20], [18].

The problem of jitter calculation has been also addressed by many other authors. Examples are [21], [10], [11], [14], [7], [6], [22], [9], [23], [3], [13], [16]. In [21], [10], [11], [22], [9], the jitter process is studied in the context of peak rate enforcement in ATM networks. In the analysis of the jitter process, as in [17], [19], only a tagged stream is assumed to be periodic.

In [7], [6], [23] and [13], the issues of real-time transmission and bounding the jitter is considered. In [3], the delay and the jitter process is analyzed for a single arrival periodic batch process where a random number of cells arrive periodically.

In [17] and [19], an analysis of the jitter for a single node was given. Simple closed-form results were obtained in the light and heavy traffic. In [18], the multiple node environment was considered where the departure process of a tagged stream from any node in the network was approximated by a renewal process characterized by its inter-arrival time marginal distribution of the cells. It was shown that this approximation was indeed, very good as far as predicting the marginal distribution of the tagged stream from a node was concerned. It was also shown that irrespective of the traffic level, as the number of nodes in the network increases, the marginal density of the departure process of the tagged stream (from the last node in the network) converges to a distribution. In the heavy traffic, a simple functional equation satisfied by this distribution which upperbounds all the moments (e.g., variance) of the jitter process in all ranges of traffic levels and arbitrary number of nodes in the network was provided.

The work in this paper departs sharply from all the existing work in number of directions. First, we consider the case where the *individual* traffic streams (and not only the tagged stream) are periodic. Unlike the existing work, we do not approximate the background traffic with a known and simple distribution. We note that in an environment where CBR traffic is given delay priority, this class of traffic competes for bandwidth in a node with only CBR traffic. In this situation, all the individual streams competing with a tagged periodic stream are in fact periodic.

Second, we do not assume "mini-priority" for the tagged stream since we deal with individual CBR streams and any stream could indeed be a tagged stream! The mini-priority assumption which had been made for simplicity, assumes that, when there is more than one cell arriving in a slot, the cell from the tagged stream enters the buffer first. In this paper, no mini-priority to any CBR stream is given. In the event that multiple cells arrive from different streams in the same time slot, they enter the buffer in a *random* order. In fact, it is not difficult to see that if mini-priority is given to *all* the streams (say the cell from stream i enters the buffer before the cells from stream j if $i < j$), the streams will preserve their original periodic pattern. Although such a behavior is quite desirable, it may be quite difficult to implement it in practice.

In this paper, our main goal is the characterization of the inter-departure process (marginal and joint) of individual streams in both homogeneous and heterogeneous environments. We note, unlike the classical queues, with all arrivals being periodic, we do not have an unstable situation and the queue is always bounded and periodic. In fact, due to the periodicity of the individual streams, most of the results in classical queueing theory may not be applicable. For mathematical tractability, we will pay special attention to the cases where the buffer is fully utilized, i.e., the queue utilization is 100 percent. We note that a fully loaded queue will provide a worst average case analysis of the system. The quality of the bound will be examined in the homogeneous case where an exact analysis is possible for any level of utilization.

This paper provides some new insight in the behavior of ATM multiplexors serving CBR streams. Among many results, it establishes simple bounds on the inter-departure of individual cell streams. It also shows that in the homogeneous case, the variance of the inter-departure process is (upper) bounded by the constant $\frac{2}{3}$. This may indicate that (homogeneous) CBR multiplexors have a tendency to preserve the shape of individual traffic streams. This concludes Section 1 of the paper.

The outline for the remainder of the paper is as follows. In Section 2 we provide the mathematical model and the basic definitions and the problem statement. Section 3 deals with the homogeneous case. In Section 4 we cover the analysis of the heterogeneous multiplexor, where the periodic streams may not have the same period. Some comparisons with the existing models are reported in Section 5. Finally, the numerical results and conclusions are provided in Section 6.

II. Mathematical Model

We assume an ATM environment, where time is slotted and takes non-negative integer values $t = \{0, 1, 2, \cdots\}$. The time interval $[t-1, t)$ is referred to as slot t. We assume that sources produce fixed-length packets (ATM cells) independently of each other. The cells are stored in a loss-free buffer (queue). It is assumed that the departures from the cell buffer take place at the beginning of slots, and the arrivals during a slot. We define:

q_t = queue length (in number of cells) at the end of t^{th} slot

A_t = number of arrivals from all sources in the t^{th} slot

so that we have the following evolution equation

$$q_{t+1} = max(q_t - 1, 0) + A_t, \quad (1)$$

In what follows, we describe the arrival process of individual streams. Without loss of generality, the individual traffic source of interest (tagged stream) is assumed to be periodic with period T and cells arrive in slots $t_n = (n-1)T + 1, \quad n \geq 1$ so that the n^{th} tagged cell is arrived in slot t_n. Other periodical streams are described with respect to the tagged stream. A stream i is fully described by the doublet $\{I_i, T_i\}$ where I_i denotes the (initial) offset random variable denoting the slot number of the first cell arriving from source i, and T_i denotes the source period. We will assume that I_i is an integer-valued random variable uniformally distributed in $[1, T_i]$. It is also assumed that the offset random variables of all sources are mutually independent. The latter assumption establishes the independence of individual periodic streams.

The cell transmission is assumed to be FCFS and one cell per slot is transmitted as long as the buffer is non-empty. The cells arriving in the same time slot enter the buffer randomly. We are concerned with the tagged stream inter-departure times. We let $Q(t_n)$ denote the number of cells seen in the buffer by the n^{th} tagged cell arriving at time t_n. We note that in the event of multiple arrivals at time t_n, $Q(t_n)$ includes those cells entering the buffer ahead of the tagged cell.

As in [17], [18], [19], we define the random variable J_n as the inter-departures of n^{th} and $(n+1)^{st}$ cells. We have

$$J_n = Q(t_{n+1}) - Q(t_n) + T \quad (2)$$

and the centered jitter process

$$\tilde{J}_n = J_n - T. \quad (3)$$

In the next section we provide the analysis. A major part of this manuscript will be devoted in the characterization of the random sequence $J_n, \ n \geq 1$. We find it most convenient to separate the homogeneous case where all the streams have the same period equal to T from the heterogeneous case.

III. Homogeneous CBR Multiplexors

In this section we consider the homogeneous environment. It is assumed that the multiplexor supports $(N+1)$, $0 \leq N \leq T-1$ periodic streams each with period T.

Based on the description of the individual arrival streams, it is easy to see that the superposition process of all the streams is also periodic with period T, so that the total number of cells arriving in any slot i satisfies

$$A_{nT+i} = A_i, \quad (4)$$

In particular, we are interested in the random variable A_1 which represents the total number of arrivals in the arrival slots of the tagged stream. We can decompose this random variable into three parts of (see Fig. 1)

$$A_1 = 1 + b_n^{(1)} + b_n^{(2)}, \tag{5}$$

where 1 accounts for the cell from the tagged stream and $b_n^{(1)}$ ($b_n^{(2)}$) denotes the number of cells entering the buffer before (after) the cell from the tagged stream.

In the next proposition, we provide the jitter distribution. We will need the following two discrete distribution functions

$$B_k(p, N) = \binom{N}{k} p^k (1-p)^{N-k}, \quad k \in [0, N] \tag{6}$$

and

$$f_k(j) = \begin{cases} \frac{1}{k+1} - \frac{|j|}{(k+1)^2}, \text{ for } \ |j| \leq k \\ 0, \quad \text{otherwise} \end{cases}, \tag{7}$$

where $B_k(p, N)$ is the probability mass function of a binomial distribution with parameters p and N, and $f_k(j)$ is the probability mass distribution of a discrete triangular random variable centered at zero in the range $[-k, k]$.

Proposition 1: If the initial queue length $q_0 = 0$, then for $t \geq T$, q_t is periodic with period T and for $n > 1$, the per-stream jitter process satisfies ($\tilde{J}_n = J_n - T$)

$$\Pr\{\tilde{J}_n = j\} = \sum_{k=|j|}^{N} B_k(\frac{1}{T}, N) f_k(j), \quad |\, j \,| \leq N, \tag{8}$$

$$var(J_n) = \frac{N(3T + N - 1)}{6T^2} \leq \frac{2}{3}. \tag{9}$$

When $N = T - 1$ (fully utilized system) and $q_0 \geq T - 1$, above results are true for $n \geq 1$.

Proof: The periodicity of q_n is already reported in [2], [12] based on the assumption of $q_0 = 0$. For this case, we have [12]

$$q_t = \begin{cases} \max_{0 \leq i < t}(\sum_{j=t-i}^{t} A_j - i), \text{when } t < T \\ \max_{0 \leq i < T}(\sum_{j=t-i}^{t} A_j - i), \text{when } t \geq T, \end{cases} \tag{10}$$

since for $t \geq T$, the term $(\sum_{j=t-i}^{t} A_j - i)$ is periodic, so it follows that $q_{nT+i} = q_{mT+i}$ for any $n, m \geq 1$ and $i \geq 0$ in particular we have $q_{t_n} = q_{t_{n+1}}$. For the fully utilized system (i.e., when $N = T - 1$) the queue will not empty during first period and the periodical behavior will take place for all periods, including first (in particular $q_1 = q_{T+1}$). To have this, it is sufficient that $q_0 \geq T - 1$.

Based on the above discussion, at least for $n \geq 2$ we have

$$J_n = T + b_{n+1}^{(1)} - b_n^{(1)}. \tag{11}$$

In what follows, we proceed to prove Eq. (8). Under the assumption of uniformally distributed offset random variables I_i in $[1, T]$, it is clear that the number of arrivals in arrival slots of the tagged stream has a Binomial distribution and we have

$$\Pr\{A_1 = k + 1\} = B_k(\frac{1}{T}, N), \quad k \in \{0, \ldots, N\} \tag{12}$$

It should be also clear that conditioning on A_1, the random variables $b_{n+1}^{(1)}$ and $b_n^{(1)}$ have a (discrete) uniform distribution. We have ($0 \leq i \leq k$)

$$\begin{aligned} & \Pr\{b_n^{(2)} = i \mid A_1 = k+1\} = \\ = \ & \Pr\{b_{n+1}^{(1)} = i \mid A_1 = k+1\} = \\ = \ & \frac{1}{k+1}, \ 0 \leq i \leq k \end{aligned} \tag{13}$$

Since the order of arrivals from all the streams in slot $(n-1)T + 1$ is independent of those in slot $nT + 1$, therefore random variables $b_n^{(1)}$ and $b_{n+1}^{(1)}$ are also *conditionally independent* given A_1. So that

$$\Pr\{b_{n+1}^{(1)} - b_n^{(1)} = j \mid A_1 = k\} = f_k(j) \tag{14}$$

and Eq. (8) follows. The variance of jitter $var(J_n) = var(\tilde{J}_n)$ can be found by direct calculation from Eq. (8). This completes the proof. □

The above proposition provides simple results on the distribution and any moment of the jitter process. As expected, the jitter variance is an increasing function of the number of periodic streams. The maximum jitter variance occurs at $N = T - 1$ which corresponds to the system utilization of 100%. It is interesting to note that this variance never exceeds the constant $\frac{2}{3}$.

Another interesting insight on the operation of the CBR homogeneous multiplexor is that as the number of sources increases, the variance of the jitter process approaches to a constant. This limiting behavior is indicative of the inherent "smoothness" of the departure process of *individual* streams in homogeneous CBR multiplexors. This result should be contrasted to the case where the background traffic does not consist of individual identical periodic processes (see Section 5).

In what follows, we examine the marginal distribution of the jitter process for large T. This case is motivated by high-speed environment where a multiplexor is serving large number of slow (large T) CBR streams, e.g., circuit emulation of CBR packetized voice traffic streams. In the

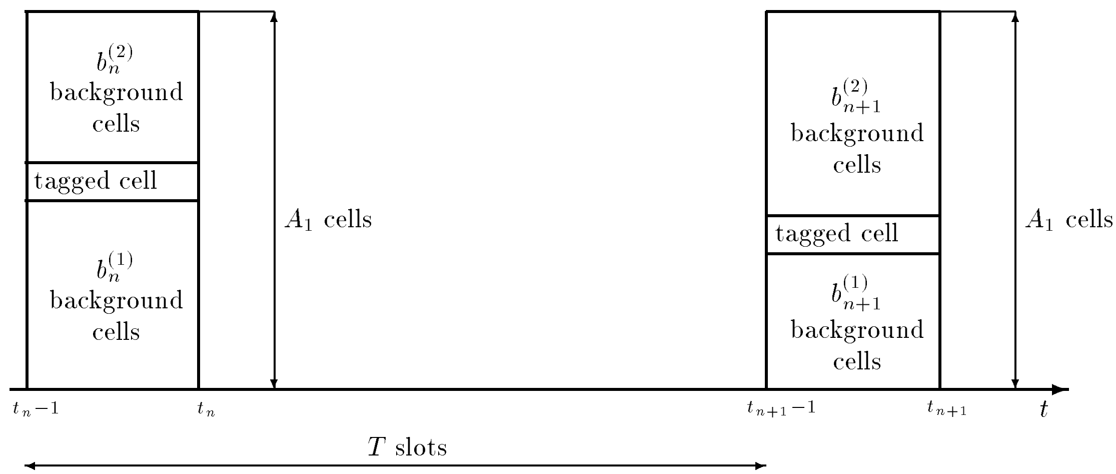

Fig. 1. System Variables for Homogeneous Case

limiting case as $T \to \infty$, we will let $N \to \infty$ too, to keep a constant system utilization of $\rho = (N+1)/T$. Using the Poisson distribution as the limiting behavior of Binomial distribution, we can easily establish this limit. We denote the Poisson distribution function

$$\pi_k(\lambda) = \exp(-\lambda)\frac{\lambda^k}{k!} \tag{15}$$

so that

$$\lim_{T\to\infty} \Pr\{\tilde{J}_n = j\} = = e^{-\rho}\left(\sum_{i=|j|}^{\infty}\frac{\rho^i}{(i+1)!} - |j|\sum_{i=|j|}^{\infty}\frac{\rho^i}{(i+1)(i+1)!}\right) \tag{16}$$

Based on the above result, the probability of zero jitter is simply

$$\lim_{T\to\infty} \Pr\{\tilde{J}_n = 0\} = \frac{1-e^{-\rho}}{\rho}, \tag{17}$$

which is a *decreasing* function of ρ and takes its *minimum* value of $1 - e^{-1} \approx .632$ at $\rho = 1$. Other masses of the limiting behavior of the marginal distribution may be shown to be an *increasing* function of ρ and they take their *maximum* value at $\rho = 1$. The limiting distribution for them can be simplified to an expression with finite summations

$$\lim_{T\to\infty,\rho\to 1} \Pr\{\tilde{J}_n = j\} = = 1 - e^{-1}\sum_{k=0}^{|j|}\frac{1}{k!} - |\ j\ |\left(c - e^{-1}\sum_{k=1}^{|j|}\frac{1}{kk!}\right) \tag{18}$$

where $c = e^{-1}\sum_{k=1}^{\infty}\frac{1}{k!k} = .484829107$ up to nine significant digits.

In the remainder of this section, we consider the joint distribution of successive inter-departure times. We will need the probability generating function of an integer-valued uniform random variable in $[0, k]$ which is given by

$$u(z) = \frac{1 - z^{k+1}}{(k+1)(1-z)}. \tag{19}$$

Now define the joint probability generating function of m successive jitters random variables $\tilde{J}_{n+i}$, $0 \le i \le m$ as

$$J(z_1, z_2, \ldots, z_{m+1}) = \mathbf{E}(\prod_{i=0}^{m} {z_i}^{\tilde{J}_{n+i}}), \tag{20}$$

and we have the following proposition summarizing the correlation structure of successive jitters.

Proposition 2: The joint probability generating function of $\tilde{J}_{n+i}$, $0 \le i \le m$ is given by

1.
$$J(z_1, z_2, , \ldots, z_{m+1}) = \sum_{k=0}^{N} B_k(\frac{1}{T}, N) \times \times u(\frac{1}{z_1})[\prod_{i=1}^{m} u(\frac{z_i}{z_{i+1}})]u(z_{m+1}), \tag{21}$$

2. The non-consecutive cell jitters are uncorrelated (i.e., random variables $\tilde{J}_n$ and $\tilde{J}_{n+m}$ are uncorrelated for $m \ge 2$) and their joint distribution is given by
$$\Pr\{\tilde{J}_{n+m} = j, \tilde{J}_n = i\} = = \begin{cases} \sum_{k=0}^{N} B_k(\frac{1}{T}, N) f_k(j) f_k(i), & m \ge 2 \\ \sum_{k=0}^{N} B_k(\frac{1}{T}, N)\frac{1}{k+1} f_k(i+j), & \\ \quad i, j \ge 0 \text{ or } i, j \le 0, & m = 1 \end{cases} \tag{22}$$

3. $\tilde{J}_n$ and $\tilde{J}_{n+1}$ (successive cell jitters of any tagged stream) are negatively correlated. The normalized correlation of $\tilde{J}_n$ and $\tilde{J}_{n+1}$ is $-\frac{1}{2}$ *independently* of T and number of background streams as long as $1 \le N \le T - 1$.

Proof: First we note that $\tilde{J}_{n+i} = b^{(1)}_{n+i+1} - b^{(1)}_{n+i}$, so following our approach in the determination of the marginal distribution, we condition on the random variable $A_{nT+1} = A_1$. However, since the order of cell arrivals to the queue at the arrival slots of the tagged stream (slots $nT+1,\ n \geq 0$) are independent of each other, it is easy to see that the random variables $b^{(1)}_{n+i}$ are conditionally independent and we have

$$\Pr\{b^{(1)}_{n+i} = j \mid A_1 = k+1\} = \frac{1}{k}, \quad 0 \leq j \leq k \tag{23}$$

Now using the fact that A_1 has a Binomial distribution, Part 1 and 2 follow easily.

To prove Part 3, we note that $\mathbf{E}(\tilde{J}_n) = 0$ and the normalized lag 1 coefficient is given by

$$\gamma_1 = \frac{\mathbf{E}(\tilde{J}_n \tilde{J}_{n+1})}{var(\tilde{J}_n)} \tag{24}$$

Now, by specializing the expression given by Eq. (21) for $m = 1$, routine differentiation of $J(z_1, z_2)$ provides the claimed result of $\gamma_1 = -\frac{1}{2}$. □

IV. Heterogeneous CBR Multiplexors

In this section, we present the per-stream jitter analysis of a general heterogeneous CBR multiplexor. Due to the complexity of the analysis, a fully utilized system is only considered so that the multiplexor is serving a sufficient number of periodic streams resulting a total utilization of one. This assumption will be crucial in the simplification of the analysis (see below) and will bound the jitter for any heterogeneous CBR multiplexor. Consider a situation where the total utilization is less than one, we can always add a periodical stream to the system to attain a total utilization of one. Now, it is easy to argue that the new system will always result in a worse jitter as compared to the original system.

As in the homogeneous case, we assume that our tagged stream has a period of T slots. The multiplexor supports N other background traffic streams where stream i has period T_i. We are not making any restriction on the values of T_i except that the total utilization ρ is assumed to be unity so that

$$\rho = \frac{1}{T} + \sum_{i=1}^{N} \frac{1}{T_i} = 1. \tag{25}$$

Before we proceed further, we identify three regions between the arrival slots of the tagged stream. Region 1 (Region 3) consists of only one slot t_n (t_{n+1}) and Region 2 denotes the $T-1$ slots in open range (t_n, t_{n+1}) (slots between the arriving slots of the tagged stream). $K_i^{(r)},\ 1 \leq i \leq 3,\ \ 1 \leq r \leq N$ denotes the number of cells arriving from background stream r in region i. We also define the joint pgf

$$h_r(z_1, z_2, z_3) = \mathbf{E}({z_1}^{K_1^{(r)}} {z_2}^{K_2^{(r)}} {z_3}^{K_3^{(r)}}) \tag{26}$$

and we let K_i denote the total number of cells from all background streams in region i. We also let

$$H(z_1, z_2, z_3) = \mathbf{E}({z_1}^{K_1} {z_2}^{K_2} {z_3}^{K_3}) \tag{27}$$

and finally, the following representation of T_r is used as it compares with T, the period of the tagged stream

$$T = m_r T_r + j_r, \quad 0 \leq j_r \leq T_r - 1, \tag{28}$$

where m_r denotes the number of T_r periods fitting in the period of T and resulting in j_r remainder slots.

For clarity, the subscript n is dropped in the definitions. Figure 2, provide a pictorial representation of various system variables.

The key in the simplification in the heterogeneous case is that if the initial queue length q_0 is sufficiently large, then the queue never empties and the "max" operator in the evolution Eq. (1) does not play any role. Now, based on this fact, the evolution equation Eq. (2) becomes a simple first order difference equation and we easily relate the queue length at different slot times. In particular, we get

$$J_n = Q(t_{n+1}) - Q(t_n) + T = 1 + b^{(2)}_n + b^{(1)}_{n+1} + K_2 \tag{29}$$

where the random variable $b^{(1)}_{n+1}$ $(b^{(2)}_n)$ represent the number of cells entering the queue before (after) the tagged cell arriving in slots t_{n+1} (t_n). We note $b^{(2)}_n$ and $b^{(1)}_{n+1}$ are uniformally distributed in $[0, K_1]$ and $[0, K_3]$, respectively. Denote $U[0, K]$ as an integer-valued uniform random variable in range of $[0, K]$, we have

$$J_n = 1 + U[0, K_1] + K_2 + U[0, K_3] \tag{30}$$

Proposition 3: We have the following results for a heterogeneous CBR multiplexor:

1.

$$h_r(z_1, z_2, z_3) = \begin{cases} z_2^{m_r}\left(\frac{z_1}{T_r} + \frac{(j_r-1)z_2}{T_r} + \frac{z_3}{T_r} + \frac{T_r - j_r - 1}{T_r}\right), & \text{if } j_r \neq 0, \\ z_2^{(m_r-1)}\left(\frac{z_1 z_3}{T_r} + \frac{(T_r-1)z_2}{T_r}\right), & \text{if } j_r = 0 \end{cases} \tag{31}$$

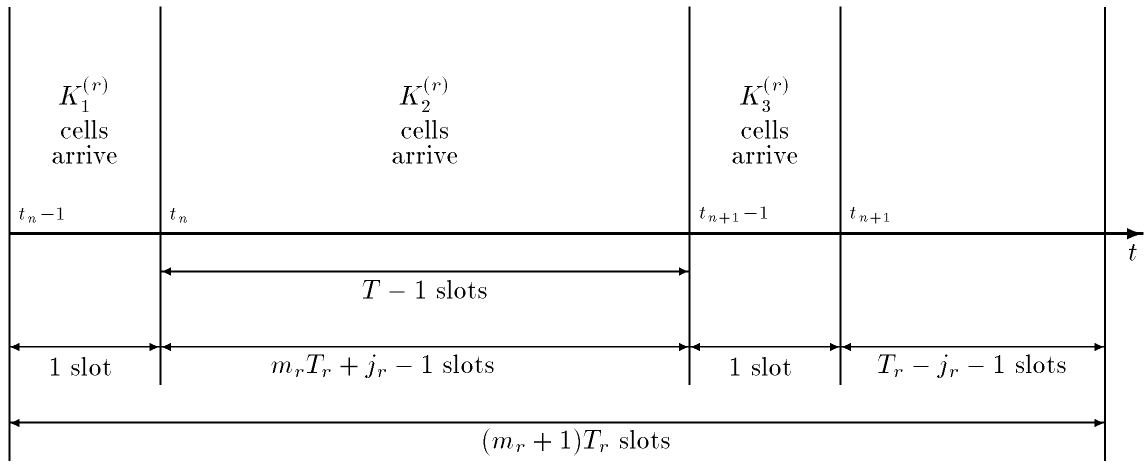

Fig. 2. System variables and notations for background stream with period T_r

and

$$H(z_1, z_2, z_3) = \prod_{r=1}^{N} h_r(z_1, z_2, z_3). \quad (32)$$

2. The probability generating function of J_n is given by

$$J(z) = \frac{z}{(1-z)^2} \int_z^1 \int_z^1 H(z_1, z, z_3)\ dz_1 dz_3 \quad (33)$$

3. Variance of J_n is given by

$$var(J_n) = \left(2T^2 + \frac{1}{3}\right) \sum_{r=1}^{N} \sum_{j=r+1}^{N} \frac{1}{T_r T_j} +$$
$$+ \sum_{r=1}^{N} ({m_r}^2 - m_r + \beta_r) - (T-1)(T-2) \quad (34)$$

where

$$\beta_r = \begin{cases} \frac{2 m_r j_r}{T_r}, \text{ if } j_r \neq 0 \\ \frac{1}{2T_r}, \text{ if } j_r = 0 \end{cases} \quad (35)$$

4. Maximal value of J_n is

$$\max(J_n) = 1 + N + \sum_{r=1}^{N} m_r \quad (36)$$

Proof: First, we concentrate on a *single* stream of period T_r. Assume that for this stream $j_r \neq 0$. Since one cell from this stream arrives with equal probability $\frac{1}{T_r}$ in slots $[t_n, t_n + T_r - 1]$, it is obvious that

$$\Pr\{K_1^{(r)} = i_1, K_2^{(r)} = m_r + i_2, K_3^{(r)} = i_3\} =$$
$$= \begin{cases} \frac{T_r - j_r - 1}{T_r}, & \text{for } i_1 = i_2 = i_3 = 0 \\ \frac{1}{T_r}, & \text{for } i_1 = 1,\ i_2 = i_3 = 0 \\ \frac{j_r - 1}{T_r}, & \text{for } i_1 = 0,\ i_2 = 1,\ i_3 = 0 \\ \frac{1}{T_r} & \text{for } i_1 = 1,\ i_2 = 0,\ i_3 = 1 \end{cases} \quad (37)$$

so that $h_r(z_1, z_2, z_3)$ for $j_r \neq 0$ follows immediately. For the case of $j_r = 0$, we have (obviously $m_r \geq 1$ for this case)

$$\Pr\{K_1^{(r)} = i_1, K_2^{(r)} = m_r - 1 + i_2, K_3^{(r)} = i_3\} =$$
$$= \begin{cases} \frac{1}{T_r}, & \text{for } i_1 = 1,\ i_2 = 0,\ i_3 = 1 \\ 1 - \frac{1}{T_r}, & \text{for } i_1 = 0,\ i_2 = 1,\ i_3 = 0 \end{cases} \quad (38)$$

so $h_r(z_1, z_2, z_3)$ for $j_r = 0$ follows.

The product form of $H(z_1, z_2, z_3)$ follows immediately because of the independence of individual streams.

To find $J(z)$, we note that if K is an integer-valued random variable taking non-negative integers and has a pgf $f(z)$, then the p.g.f. of $U[0, K]$ is

$$\mathbf{E}(z^{U[0,K]}) = \frac{1}{1-z} \int_z^1 f(z) dz \quad (39)$$

which easily follows by conditioning on K. Taking advantage of this fact in Eq. (32), will immediately give Eq. (33).

To find $var(J_n) = J''(1) + T - T^2$, we note that

$$(1-z)^2\ J(z) = z \int_z^1 \int_z^1 H(z_1, z, z_3)\ dz_1 dz_3 = F(z) \quad (40)$$

so that $J''(1) = \frac{1}{12} F^{(iv)}(1)$. Using standard results of differentiation of integrals, we get

$$J''(1) = \Big(\frac{\partial}{\partial z_1} + 2\frac{\partial}{\partial z_2} + \frac{\partial}{\partial z_3} + \frac{1}{3}\frac{\partial^2}{\partial {z_1}^2} + \frac{\partial^2}{\partial {z_2}^2} +$$
$$+ \frac{1}{3}\frac{\partial^2}{\partial {z_1}^2} + \frac{\partial^2}{\partial z_1 \partial z_2} + \frac{\partial^2}{\partial z_2 \partial z_3} + \frac{1}{2}\frac{\partial^2}{\partial z_1 \partial z_3}\Big) H(1,1,1) \quad (41)$$

Using the expression for $H(z_1, z_2, z_3)$, after some tedious manipulations, we get Eq. (34).

The last item is obvious from the fact the that maximum number of background cells arriving from one individual source during T slots is $m_r + 1$. This can be easily shown to be true for $j_r = 0$. □

Significant simplification in the expression (34) may result if we consider the practical situation where we have

R *classes* of traffic streams. Class k is characterized by the doublet (N_k, T_k) where N_k denotes the total number of streams in this class each having period T_k. We denote $\rho_k = N_k/T_k$ to be the total utilization of all the streams in class k. Assuming that tagged stream belongs to class k and using the fact that $\sum_{k=1}^{R} \rho_k = 1$, it is possible to show that $var(J_n^{(k)})$, the jitter variance for any stream in class k is given by

$$var(J_n^{(k)}) = \frac{1}{6}\left(1 + \frac{2}{T_k^2} - \frac{5}{T_k} - \sum_{r=1}^{R} \frac{\rho_r}{T_r}\right) + \sum_{r=1}^{R} \tilde{\beta}_r, \quad (42)$$

where

$$\tilde{\beta}_r = \begin{cases} \rho_r/2, \text{if} j_r = 0 \\ j_r \rho_r (1 - j_r/T_r), \text{if} j_r \neq 0 \end{cases} \quad (43)$$

and as before, the representation $T_k = m_r T_r + j_r$, $1 \leq r \leq R$ is used for T_k.

Similarly, we can find higher order statistics of the jitter process. To find the joint pgf of J_n and J_{n+1} we identify five regions. Region 1, Region 3 and Region 5 consists of only one slot t_n, t_{n+1} and t_{n+2} respectively, Region 2 denotes the $T-1$ slots in open range (t_n, t_{n+1}), and finally Region 4 represents the $T-1$ slots in open range (t_{n+1}, t_{n+2}). We also let $K_i^{(r)}$, $1 \leq i \leq 5$, $1 \leq r \leq N$ denote the number of cells arriving from background stream r in region i. Define the joint pgf

$$h_r(z_1, z_2, z_3, z_4, z_5) = \mathbf{E}(\prod_{i=1}^{5} z_i^{K_i^{(r)}}) \quad (44)$$

and we let K_i denote the total number of cells from all background streams in region i. We can show $h_r(z_1, z_2, z_3, z_4, z_5)$ is given by

$$\begin{cases} (z_2 z_4)^{m_r - 1}\left(\frac{z_1 z_3 z_5}{T_r} + \frac{(T_r - 1) z_2 z_4}{T_r}\right), \text{if } j_r = 0 \\ (z_2 z_4)^{m_r}\left(\frac{z_1 z_5}{T_r} + \frac{z_3}{T_r} + \frac{(j_r - 1) z_2}{T_r} + \frac{(j_r - 1) z_4}{T_r}\right), \text{if } j_r = T_r/2 \\ (z_2 z_4)^{m_r}\left(\frac{z_1}{T_r} + \frac{(j_r - 1) z_2}{T_r} + \frac{z_3}{T_r} + \frac{(j_r - 1) z_4}{T_r} + \frac{z_5}{T_r} + \right. \\ \quad \left. + \frac{T_r - 2 j_r - 1}{T_r}\right), \text{if } 0 < j_r < T_r/2 \\ (z_2 z_4)^{m_r}\left(\frac{z_1 z_4}{T_r} + \frac{(2 j_r - T_r - 1) z_2 z_4}{T_r} + \frac{z_2 z_5}{T_r} + \frac{(T_r - j_r - 1) z_2}{T_r} + \right. \\ \quad \left. + \frac{z_3}{T_r} + \frac{(T_r - j_r - 1) z_4}{T_r}\right), \text{if } T_r/2 < j_r < T_r \end{cases} \quad (45)$$

and

$$H(z_1, z_2, z_3, z_4, z_5) = \mathbf{E}(\prod_{i=1}^{5} z_i^{K_i}) = \prod_{r=1}^{N} h_r(z_1, z_2, z_3, z_4, z_5) \quad (46)$$

and

$$J(z_1, z_2) = \frac{z_1 z_2}{(1 - z_1)(z_2 - z_1)(1 - z_2)} \times \\ \times \int_{z_1}^{1} dy_1 \int_{z_1}^{z_2} dy_2 \int_{z_2}^{1} H(y_1, z_1, y_2, z_2, y_3) dy_3 \quad (47)$$

so that successive differentiations of $(z_1 - 1)(z_2 - 1)(z_2 - z_1)J(z_1, z_2)$ at $z_1 = z_2 = 1$ will provide joint moments of J_n and J_{n+1}. Similar to the variance calculations, the joint moments are expressible to derivatives of the function H.

V. Comparisons With Existing Results

In this section, we make comparisons with the existing results. The performance measure of interest will be the jitter variance in the homogeneous environment since its exact form is quite simple (see Eq. 8). We also assume a fully loaded system (total utilization of unity), due to the availability of closed form expressions in this case.

The first comparison we perform will be with *random* background traffic. The cell arrivals in successive slots are assumed *i.i.d* with p.g.f $B(z)$, i.e., probability of k background cells arriving in any slot is b_k and $B(z) = \sum_{k=0}^{\infty} b_k z^k$. It is assumed that $B'(1) = 1 - \frac{1}{T}$ so that the total utilization including that of the periodic tagged stream adds up to unity.

Extending the simple analysis for the mini-priority case in [17], [18], [19] (fully loaded system is assumed) to the case of random priority, we can easily find the p.g.f of jitter to be

$$J(z) = z\,[B(z)]^{\;T-1}\left[\frac{1}{1-z}\int_{z}^{1} B(y)dy\right]^2 \quad (48)$$

After simple calculations, we get

$$var = \frac{(T-1)(3T-1)}{2T^2} + \frac{3T-1}{3} B''(1) \quad (49)$$

which simplifies to

$$var = \frac{(T-1)(3T-1)}{6T^2} + \frac{3T-1}{3} {\sigma_b}^2, \quad (50)$$

where ${\sigma_b}^2$ represent the variance of the batch size of the background traffic.

The simple result above clearly shows that inaccuracy of approximating the superposition of the identical periodic traffic by *any* i.i.d batch process. The batch background approximation predicts a jitter variance which grows linearly with the period T and batch size variance ${\sigma_b}^2$. The result in the exact model, predict an upper bound (constant $\frac{2}{3}$) as reported in Eq. 8.

Another interesting comparison we report here is when the periodic arrival streams are not independent from each other and in the worst case situation are *completely synchronized.* This might happen in certain applications. In this case, every T slots, we have a deterministic batch of T

slots arriving. Assuming our usual random priority assignment and fully loaded system, we can show that

$$J(z) = z \left[\frac{1 - z^T}{T(1 - z)} \right]^2, \qquad (51)$$

so that the jitter variance is given by

$$var = \frac{(T-1)(T+1)}{6}, \qquad (52)$$

showing a variance growing with *square* of the period T which could grossly overestimate the exact variance. It also shows that such synchronizations could result in considerably large jitter variance.

VI. Numerical Results and Conclusions

In this section, we present the numerical results and conclusions. Tables 1-3 depict the results for the homogeneous case. In Table 1, the marginal density of the centered jitter $\tilde{J}_n$ and jitter variance are presented. A period of $T = 32$ slots is considered. The number of background streams N takes different values resulting in different total utilization ρ. As expected, the probability of zero (centered) jitter is minimized at maximum utilization of $\rho = 1$ and any probability of non-zero jitter is maximized at this utilization.

Table 2 depicts the marginal density of jitter at the $\rho = 1$ as the period T changes. It is interesting to note that as T increases, the jitter distribution approaches to its asymptotic value very quickly. The asymptotic behavior (as $T \to \infty$) at $\rho = 1$ provides a simple bound on the jitter distribution in the homogeneous case, since independently of the multiplexor utilization and the rate, it provides a simple result on the worst case jitter behavior. It is also evident even as $T \to \infty$, the jitter distribution goes to zero very fast. For finite T, the maximum value of (centered) jitter is N, the number of background streams.

In Table 3, we depict the conditional distribution of $\tilde{J}_{n+m}$ given $\tilde{J}_n = 0$ for various values of T. The results are based on Eq. (22). It is interesting to note that the convergence to the limiting case of $T = \infty$ appears to be quite rapid. Also, a substantial part of the probability mass of the centered jitter is located at zero, indicating that if the inter-departure time of two successive cells belonging to any stream is not perturbed, the likelihood of perturbation in future cell inter-departure times is rather small.

Tables 4 and 5 provide some results in the heterogeneous environment. In Table 4, we assume that the tagged stream has a period of $T = 6$ slots. Jitter distribution and variance are reported. Different columns provide a different mix of background traffic. N_i denotes the number of background periodic streams with period T_i. It is interesting to note that the tagged stream tend to "suffer" more (larger variance in cell inter-departure time distribution) in scenarios in which its period is small as compared to the periods of other background streams. In general, we cannot conclude that the smallest rate streams (largest period) will always experience the largest variance. However, under certain conditions, the stream with the largest period *always* experiences the smallest variance (see the scenario below).

In Table 5 we consider five classes and we report the jitter variance and lag 1 correlation for each class. Different "base" periods $\tilde{T}$ are considered. The total utilization for classes 1 and 5 are $\frac{7}{30}$, and $\frac{5}{30}$, respectively and Classes 2, 3 and 4 each have a total utilization of $\frac{1}{5}$. The period of class i is chosen to be $T_i = \frac{\tilde{T}}{i}$. The number of streams in class i is then determined by $N_i = \rho_i T_i$, resulting in a total utilization of ρ_i for class i. It is interesting to note that the stream with slowest rate (largest period) experiences the least variance and the lag 1 correlation for this stream coincides with the homogeneous case. In fact, based on the Eq. (42), it is easy to see that if all the periods are fraction of a given base period, then the streams with the largest period experiences the smallest variance never exceeding the constant $\frac{2}{3}$. The lag 1 correlation for this stream is also the same as the homogeneous case, namely the constant $-\frac{1}{2}$.

It is also interesting to note that as the base period $\tilde{T}$ increases, the variance of all the streams (except the one with the largest period) increases almost linearly with the base period $\tilde{T}$. However, the correlation coefficient γ_1 appears to be converging as $\tilde{T} \to \infty$. In general, we did not observe a direct relationship between the jitter variance and its lag 1 correlation coefficient in the heterogeneous case.

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TABLE I

The distribution of centered jitter in homogeneous case ($\rho < 1$, T=32)

i	$Pr\{\tilde{J}_n = i\}$ $\rho = 0.25$ (N=7)	$\rho = 0.5$ (N=15)	$\rho = 0.75$ (N=23)	$\rho = 1$ (N=31)
0	0.8972	0.796579	0.711005	0.637945
± 1	0.049272	0.092152	0.123749	0.146581
± 2	0.002065	0.008844	0.018297	0.028952
± 3	$6.13*10^{-5}$	$6.72*10^{-4}$	0.002212	0.004756
± 4	$1.25*10^{-6}$	$4.05*10^{-5}$	$2.20*10^{-4}$	$6.54*10^{-4}$
± 5	$1.66*10^{-8}$	$1.96*10^{-6}$	$1.82*10^{-5}$	$7.61*10^{-5}$
± 6	$1.30*10^{-10}$	$7.62*10^{-8}$	$1.26*10^{-6}$	$7.58*10^{-6}$
Jitter Variance				
	0.116211	0.268555	0.441732	0.635742

TABLE II

The distribution of centered jitter in homogeneous case ($\rho = 1$)

	$Pr\{\tilde{J}_n = i\}$				
i	T=2	T=3	T=10	T=30	T=∞
0	0.75	0.703704	0.651322	0.638338	0.632121
±1	0.125	0.135802	0.144802	0.146532	0.147291
±2	0	0.012346	0.025703	0.028857	0.0303418
±3	0	0	0.003459	0.004716	0.00536198
±4	0	0	$3.47*10^{-4}$	$6.44*10^{-4}$	$8.20*10^{-4}$
±5	0	0	$2.56*10^{-5}$	$7.41*10^{-5}$	$1.10*10^{-4}$
±6	0	0	$1.34*10^{-6}$	$7.29*10^{-6}$	$1.31*10^{-5}$
Jitter Variance					
	0.2500	0.37037	0.5700	0.633704	0.666667

TABLE III

Conditional distribution of centered jitter and jitter variance ($\rho = 1$)

	$Pr\{\tilde{J}_{n+m} = i \mid \tilde{J}_n = 0,\}$				
i	T=2	T=3	T=10	T=30	T=∞
0	0.833333	0.807018	0.777679	0.770448	0.766988
± 1	0.083333	0.090643	0.097545	0.099055	0.099753
± 2	0	0.005848	0.012232	0.013766	0.014494
± 3	0	0	0.001272	0.001740	0.001982
± 4	0	0	$1.04*10^{-4}$	$1.93*10^{-4}$	$2.47*10^{-4}$
± 5	0	0	$6.45*10^{-6}$	$1.88*10^{-5}$	$2.79*10^{-5}$
± 6	0	0	$2.92*10^{-7}$	$1.59*10^{-6}$	$2.87*10^{-6}$
Jitter Variance					
	0.166667	0.22807	0.319524	0.346819	0.360659

TABLE IV

Jitter distribution and variance (T=6, $\rho = 1$):

(A) $T_1 = 3,\ N_1 = 2;\ T_2 = 6,\ N_2 = 1;$

(B) $T_1 = 8,\ N_1 = 4;\ T_2 = 6,\ N_2 = 2;$ (C) $T_1 = 12,\ N_1 = 10;$

(D) $T_1 = 15,\ N_1 = 5;\ T_2 = 2,\ N_2 = 1;$ (E) $T_1 = 18,\ N_1 = 15;$

	$Pr\{J_n = i\}$				
i	(A)	(B)	(C)	(D)	(E)
1	0	$3.15*10^{-5}$	0.001191	0	0.002571
2	0	$8.69*10^{-4}$	0.011007	0	0.018391
3	0.001157	0.010390	0.046558	0.011805	0.062005
4	0.020833	0.063090	0.118732	0.094757	0.130732
5	0.151620	0.210241	0.202251	0.245310	0.192792
6	0.652778	0.378149	0.240519	0.312922	0.210648
7	0.151620	0.304595	0.202251	0.223191	0.176153
8	0.020833	0.031276	0.118732	0.090720	0.114792
9	0.001157	0.001359	0.046558	0.019572	0.058764
10	0	0	0.011007	0.001723	0.023627
11	0	0	0.001191	0	0.007398
Jitter Variance					
	0.490741	1.01273	2.60417	1.52037	3.44136

TABLE V

Variance and γ_1 for heterogeneous case

period of tagged stream		$\tilde{T}$=60	$\tilde{T}$=120	$\tilde{T}$=240
$T = T_1$	var	0.644907	0.655764	0.661209
	γ_1	-1/2	-1/2	-1/2
$T = T_2$	var	5.331296	10.348889	20.357755
	γ_1	-0.940793	-0.968649	-0.983845
$T = T_3$	var	5.773426	11.353171	22.476533
	γ_1	-1/2	-1/2	-1/2
$T = T_4$	var	5.454630	10.735278	21.250879
	γ_1	-0.513750	-0.518630	-0.521175
$T = T_5$	var	5.294907	10.451875	20.710792
	γ_1	-0.447119	-0.446421	-0.445922

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