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# Term Structure Economics from A to B

by Joseph G. Haubrich

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## Introduction

Most people understand that the term "interest rates" is plural and acknowledge the difference between the rates on a savings account, overnight federal funds rate, and 10-year Treasury bonds. Of the many differences one can point to, such as risk, issuer, or denomination, among the most basic and most important factors for determining the interest rate is the maturity, or length, of the bond. In this case, a surprisingly small amount of economics can yield some valuable insights into the relationship between interest rates on bonds of various maturities, or what is more often called the term structure of interest rates.

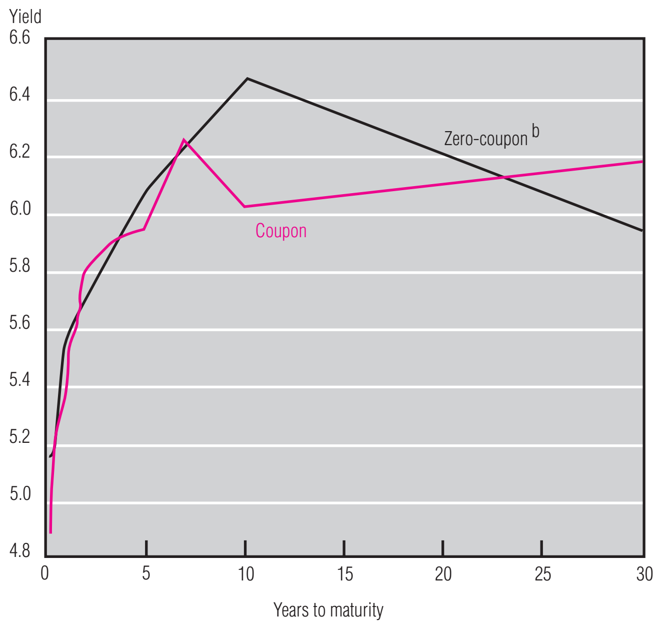
Economics tells us that at the most basic level, interest rates are a price that borrowers pay investors for moving purchasing power from the present to the future. This price obviously has both real and nominal components —the future value of the money you invest will depend on how high inflation is in the meantime. The price also reflects aspects of risk. Because you're uncertain exactly what you'll need for retirement, you're uncertain about how much consumption you should transfer into the future. Real variables, inflation, and uncertainty interact in rather complex ways, and some common perspectives ignore factors that play a key role in determining interest rates. A careful look with an economist's eye can sort out these different effects.

### Term Structure versus Yield Curve

Two closely related but distinct terms are often used interchangeably. If we are interested in how interest rates vary with maturity, it is useful to look at the *yield curve*, which plots the yield to maturity of different bonds against maturity. The problem is that most Treasury bonds are coupon bonds, paying a fixed amount semi-annually. For the purist then, the yield on a five-year T-bond is really an average of the five-year interest rate on the principal and many shorter rates on the coupon payments. One solution is to look at yields on zero-coupon bonds, which have no coupons. Figure 1 shows the recent yield curves for coupon and zero-coupon bonds. Some liquidity and tax differences between coupons and zeroes lead many to prefer to estimate the pure interest rates, known as the *term structure* (of interest rates) from coupon bonds. See McCulloch, Huston, and Kwon (1993) and Dhillon and Lasser (1998) for a discussion of this. So, while the term structure is the more useful theoretical concept, the yield curve is easier to observe.

FIGURE 1

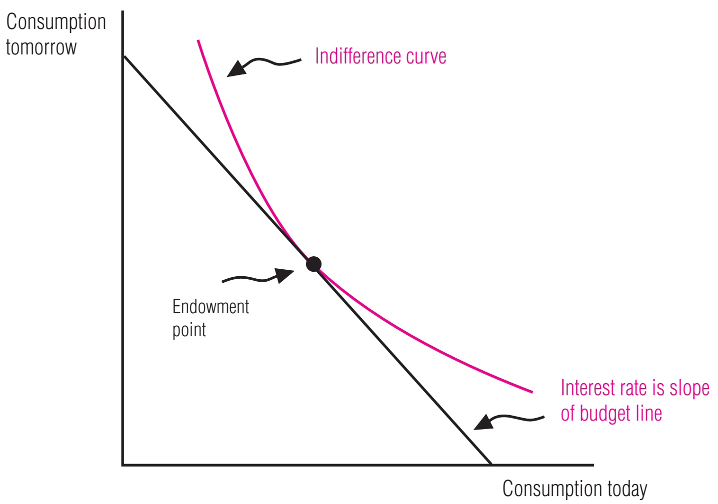
**Yield Curve for October 5, 1999[a]**




a. All instruments are Treasury constant-maturity series.
b. For each maturity, the yield is the average of yields on zero-coupon Treasury bonds with that maturity, as of October 5, 1999.
SOURCE: *Wall Street Journal,* October 5, 1999, p. H15.

FIGURE 2

**Endowment, Preferences, and Interest Rates**




SOURCE: Author's calculations.

## I. Real Term Structure

To understand the interplay of factors that determine interest rates, it is easier to begin by ignoring the problem of inflation and think of real bonds. Given that a dollar tomorrow will buy just as much beef, beer, or baby-sitting time as a dollar will today, we further simplify and talk about bonds in terms of abstract consumption units (although, for the sake of concreteness, it sometimes helps to think of it as ice cream).

The economic logic behind interest rates represents an application of supply and demand. The interest rate serves as the price expressing the trade-off of consuming today versus consuming tomorrow. It adjusts to equate the supply of savings with the demand for savings. Even at this general level, we can note that an increase in the demand for savings will increase interest rates. If we specialize further, we can answer more specific questions, such as how recessions or economic growth affect interest rates.

The first step is to aggregate everyone in the economy into a single representative agent and to consider the choice problem of this agent.[1] The second step is to consider an endowment economy without production. The consumption good just drops from the trees. The last step is to assume no storage possibilities. In other words, bonds are in "zero net supply," so that when someone is borrowing, someone is lending. Any individual can save or borrow by using a "consumption loan," say, giving up one unit of consumption today for some units tomorrow, but the economy as a whole cannot.

Thus, in a very simple two-period case, in equilibrium the interest rate will adjust to make the representative agent content to hold her endowment. In figure 2, this is seen as the line tangent to the agent's indifference curve at the endowment point. The basic idea behind this simple case—where preferences and the amount of consumption today and consumption tomorrow determine the interest rate—extends to more complicated cases with uncertainty and many time periods.

1 Of course, different investors may have different preferences. Wang (1996) considers this case.

## Many Periods: A More Formal Approach

Extending this analysis to many periods and to uncertainty about future consumption requires a more formal, mathematical approach. This section sets up such a model.

There is a single representative agent with preferences

$$(1) \quad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $E_0$ denotes expectations as of period 0, $\beta$ denotes the discount factor, and $u(c_t)$ denotes the utility of consumption in period $t$. The agent faces a budget constraint,

$$(2) \quad c_t + B_{1t} + B_{2t} \le d_t + B_{1t-1} R_{1t-1} + B_{2t-2} R_{2t-2},$$

where $B_{jt}, j=1,2$ is the amount of a bond of length $j$ bought in period $t$. These bonds are perfectly safe, and at the beginning of period $t$ investors know the gross rates of return $R_{1t}$ and $R_{2t}$. The endowment, or dividend, for a period is denoted $d_t$.

The agent seeks to arrange consumption to maximize utility, subject to the budget constraint, so a natural way to solve the problem is to substitute (2) into (1) and obtain the first-order conditions.[2]

$$J = E_0 \sum_{t=0}^{\infty} \beta^t u(d_t + B_{1t-1} R_{1t-1} + B_{2t-2} R_{2t-2} - B_{1t} - B_{2t}),$$

$$\frac{\partial J}{\partial B_{1t}} = 0 = E_0[-\beta^t u'(d_t + B_{1t-1} R_{1t-1} + B_{2t-2} R_{2t-2} - B_{1t} - B_{2t})$$
$$+ \beta^{t+1} R_{1t} u'(d_{t+1} + B_{1t} R_{1t} + B_{2t} R_{2t} - B_{1t} - B_{2t})]$$

and

$$\frac{\partial J}{\partial B_{2t}} = 0 = E_0[-\beta^t u'(d_t + B_{1t-1} R_{1t-1} + B_{2t-2} R_{2t-2} - B_{1t} - B_{2t})$$
$$+ \beta^{t+2} R_{2t} u'(d_{t+2} + B_{1t+1} R_{1t+1} + B_{2t+1} R_{2t+1} - B_{1t+1} - B_{2t+1})].$$

We can simplify this in two ways. First, we use (2) again to get consumption back into the equations. Next, we take the perspective of time period $t$, where $R_{1t}$, $R_{2t}$, and $c_t$ are known, which allows us to drop some of the expectations operators. We get

$$(3) \quad u'(c_t) = \beta E_t[R_{1t} u'(c_{t+1})]$$

and

$$(4) \quad u'(c_t) = \beta^2 E_t[R_{2t} u'(c_{t+2})].$$

These have an intuitive explanation. The left-hand side is the marginal utility of consuming one unit less in period $t$, that is, what you give up (in utility terms) if you invest. The right-hand side tells you what you gain: the discounted expected marginal utility of an extra $R_{1t}$ units of consumption in a future period. The agent equates marginal cost and marginal benefits, leading to equations (3) and (4).

To focus on the interest rates, it is useful to rearrange (3) and (4) as

$$(5) \quad \frac{1}{R_{1t}} = \beta E_t[\frac{u'(c_{t+1})}{u'(c_t)}]$$

and

$$(6) \quad \frac{1}{R_{2t}} = \beta^2 E_t[\frac{u'(c_{t+2})}{u'(c_t)}].$$

The left-hand sides of (5) and (6) are the current (date $t$) prices of a bond that pays one unit of consumption one period or two periods in the future. The lower the price, that is, the *less* you pay for such a bond, the *higher* the interest rate: Bond prices and rates move in opposite directions.

Even here we are not quite finished. Both $R_{1t}$ and $R_{2t}$ are *gross* returns, and $R_{2t}$ in particular is a two-period gross return. For example, if the interest rate is 10 percent, $R_{1t}$ is 1.10 and $R_{2t}$ is 1.21. Because we want to compare the returns on bonds of different maturities, however, we need to standardize the returns—if one period is a year, we would want to annualize the returns. To transform $R_{2t}$ into a one-period return we can take the square root.[3] The annualized return on the long (that is, two-period) bond is then

$$L_t = \sqrt{R_{2t}}.$$

This simplified model, expressed by equations (5) and (6), is the basis of an analysis that can give us a lot of insight into the term structure.

**2** Although the budget constraint assumes that the representative agent holds only one- and two-period bonds, the equilibrium interest rates on these bonds will be the same even if the agent can hold bonds of other maturities.

**3** This makes sense in the discrete time framework. In some cases, it is more convenient to take logarithms. See Campbell, Lo, and MacKinlay (1997), chapter 1.

## The Expectations Hypothesis and Beyond

What do equations (5) and (6) tell us about the term structure? A good place to start is the simple case of no uncertainty, where the consumer knows everything today—all future interest rates and all future consumption endowments. Then we can rewrite (6) as

$$(7)\qquad \frac{1}{R_{2t}} = \beta^2\left[\frac{u'(c_{t+1})}{u'(c_t)}\frac{u'(c_{t+2})}{u'(c_{t+1})}\right] = \frac{1}{R_{1t}} \bullet \frac{1}{R_{1t+1}}.$$

Put differently, $L_t = \sqrt{R_{1t}R_{1t+1}}$. The long-term rate is the average between today's short-term rate and tomorrow's short-term rate. The rational investor has two ways of moving consumption from $t$ into $t+2$: invest in a long bond with per-period return $L_t$, or roll over a short-term bond, getting rate $R_{1t}$ at the start and $R_{1t+1}$ next period. The two ways of investing must have the same return; otherwise, the investor moves her savings from the low-return investment to the high-return investment. So, in the case of perfect certainty, the interest rates of long- and short-term bonds will adjust to keep today's long rate an average of today's and tomorrow's short rate.

Equation (7) is often seen in a slightly modified form, which, although not exactly correct, is often useful when high precision is not necessary. This approximation to (7) takes the form

$$L_t - 1 = \frac{(R_{1t}-1) + (R_{1t+1}-1)}{2}.$$

For example, if interest rates are 3 percent today and 7 percent tomorrow, the long-term rate should be 5 percent. This is not quite exact, as $L_t = \sqrt{(1.03)(1.07)} = 1.0498$; but, for many purposes, it is close enough.

Perfect certainty, as anyone who watches the stock market can attest, is a rather unrealistic assumption. One common way to incorporate uncertainty is to replace unknown future rates in (7) by their expectation. Thus,

$$(7a)\qquad L_t = \sqrt{R_{1t}E_tR_{1t+1}}.$$

The long-term rate is an average of current and expected future short-term rates. This is often termed *the expectations hypothesis of the term structure*. A useful approach, it is not derived from (5) and (6), and it ignores the risk effect of uncertain interest rates.[4]

A more correct treatment with uncertainty comes from a closer look at equations (5) and (6). Rewrite (6) as

$$(8)\qquad \frac{1}{R_{2t}} = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\beta\frac{u'(c_{t+2})}{u'(c_{t+1})}\right]$$

or

$$(9)\qquad \frac{1}{R_{2t}} = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}E_{t+1}\beta\frac{u'(c_{t+2})}{u'(c_{t+1})}\right]$$

or

$$(10)\qquad \frac{1}{R_{2t}} = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\frac{1}{R_{1t+1}}\right].$$

To split out the risk terms, we use the standard formula

$$(11)\qquad E(XY) = E(X)\,E(Y) + \operatorname{cov}(X,Y),$$

where cov stands for the covariance of $X$ and $Y$. Using (11), (10) becomes

$$(12)\qquad \frac{1}{R_{2t}} = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\right]E_t\left[\frac{1}{R_{1t+1}}\right] + \operatorname{cov}\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}, \beta\frac{u'(c_{t+2})}{u'(c_{t+1})}\right].$$

A little more work yields

$$(13)\qquad \frac{1}{R_{2t}} = \frac{1}{R_{1t}}E_t\left[\frac{1}{R_{1t+1}}\right] + \operatorname{cov}\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}, \beta\frac{u'(c_{t+2})}{u'(c_{t+1})}\right].$$

A lot of insight about the effect of uncertainty comes from comparing (13), the correct model with uncertainty, with (7), the correct model with perfect certainty, and (7a), the expectations hypothesis. The correct interest rate differs from the simple expectations hypothesis in two ways. The first is a Jensen's inequality term that would arise even with risk-neutral investors. The second way is a risk premium that arises precisely because investors are not risk neutral.

4 The extent to which the expectations hypothesis is a good approximation to the data is a much discussed issue. See Campbell, Lo, and MacKinlay (1997), section 10.2.

The Jensen's inequality term arises because $E_t[\frac{1}{R_{1t+1}}]$ does not equal $\frac{1}{E_t R_{1t+1}}$; indeed, $E_t\,[\frac{1}{R_{1t+1}}] \geq \frac{1}{E_t R_{1t+1}}$. The reason is that a given change in the interest rate has more effect on the price of a bond when rates are low than when rates are high.[5] The difference can be large. Hearkening back to the simple numerical example above, suppose short-term rates stand at 3 percent today and are expected to be 7 percent tomorrow, but have an even chance of being either at 3 percent or 11 percent. Using the Jensen's inequality part of (13) (ignoring the covariance term) the (annualized) yield on the long-term bond is

$$L_t = 1/\sqrt{(1.03)/[0.5\frac{1}{1.03} + 0.5\frac{1}{1.11}]} = 1.0487.$$

Thus, correctly considering uncertainty leads to an interest rate of 4.87 percent—a bit below the 4.98 percent suggested by the simple expectations hypothesis. You might not notice this on your savings account, but if you were a pension fund investing millions of dollars, it would add up.

This example highlights another key feature of the model: The interest rate can change significantly even if expected rates stay constant. If future rates become more or less uncertain, rates will change today. The numerical example showed this quite clearly: If future short-term interest rates were known with certainty to be 5 percent, then the long-term rate would be 4.98 percent. When those future rates became uncertain, the long-term rate fell to 4.87 percent.

The second way the model in (13) differs from the expectations hypothesis is that interest rates also have a risk premium. In focussing on the Jensen inequality term, we've ignored the covariance terms—in a sense, we've said that the world got riskier, but nobody cared. And we've also ignored the underlying link with consumption—when the whole point of the exercise is to stop taking interest rates as given and consider their underlying determinants. Casual inspection of (13) suggests that this general investigation might get quite complicated, as we have a covariance term involving nonlinear functions of consumption in *three* time periods. In this case, discretion is the better part of valor, and it makes sense to examine some simplified versions of the general problem.

■ **5** For an excellent discussion of this point, see Litterman, Scheinkman, and Weiss (1991).

■ **6** For a more general version of this approach, see Campbell (1986), and Campbell, Lo, and MacKinlay (1997), chapter 11. See also Sargent (1987), section 3.5.

## A Specialized Example

By making a number of special assumptions, we can get to a series of explicit equations that make it easy to look at the effects of various factors on the term structure. First, specialize to log utility,[6] so that $u(c) = \log(c)$.

Recalling that in equilibrium, consumption must equal the dividend endowment for the representative agent, equations (5) and (6) reduce to

$$\frac{1}{R_{1t}} = \beta E_t(\frac{d_t}{d_{t+1}})$$

and

$$\frac{1}{R_{2t}} = \beta^2 E_t(\frac{d_t}{d_{t+2}}).$$

We further specialize by specifying a particular stochastic process for the dividends: We base it on an AR(1) process, of the form $\log d_{t+1} = g + \rho \log d_t + \Theta_{t+1}$, where $\Theta_{t+1}$ is a sequence of independent and identically distributed random variables. Adding a time trend (and normalizing the growth rate $g$, the process for dividends is given by:

$$(14) \quad \log d_{t+1} = gt + \sum_{k=0}^{\infty} \rho^k \Theta_{t-k}.$$

We further assume that the $\Theta_{t+1}$ terms are distributed log-normally. This lets us invoke the useful substitution that if $X$ is distributed log-normally,

$$\log E(X) = E[\log(X)] + \tfrac{1}{2} VAR[\log(X)].$$

It also helps if we change the definition of interest rate slightly. We have been thinking about rates on a discrete time basis; if the yearly interest rate $R_{1t}$ is 1.05, an investment of \$1 returns \$1.05 at the end of the year, and it is natural to say that the interest rate is 5 percent. When we start using logs, however, it is more convenient to consider continuously compounded rates of return, leading to the definitions

$$r_t = \log R_{1t}$$

and

$$l_t = \log L_t = \log \sqrt{R_{2t}}.$$

The difference between the two definitions is often small: log(1.05)=0.0488.

Taking (14) as the dividend process, these various assumptions allow equations (5) and (6) to take a relatively convenient, if not exactly simple, form:

$$(15)\quad r_{1t} = \log\frac{1}{\beta} + g + (\rho - 1)\sum_{k=0}^{\infty} \rho^k \Theta_{t-k} - \tfrac{1}{2}\sigma_\Theta^2$$

and

$$(16)\quad l_t = \log\frac{1}{\beta} + g + \tfrac{1}{2}(\rho^2 - 1)\sum_{k=0}^{\infty} \rho^k \Theta_{t-k} - \tfrac{1}{4}(1+\rho^2)\sigma_\Theta^2.$$

## What Moves the Term Structure?

Equations (15) and (16) provide a reference point for illustrating term-structure economics. A variety of factors will move interest rates and the term structure. These include the value of today's shock $\Theta_t$, the persistence of the endowment shocks $\rho$, the growth trend $g$, the variance of the endowment shocks $\sigma_\Theta^2$, and the time preference parameter $\beta$. Of these, the most interesting are $\Theta_t$, $g$, and $\sigma_\Theta^2$.

How do interest rates react to the endowment shock $\Theta_t$? Simple calculus shows that

$$\frac{\partial r_{1t}}{\partial \Theta_t} = (\rho - 1)$$

and

$$\frac{\partial l_t}{\partial \Theta_t} = \tfrac{1}{2}(\rho^2 - 1).$$

A positive shock today lowers interest rates as long as $\rho<1$. Income today is relatively high, so people want to save the extra income; consequently, they drive up the price of bonds and correspondingly drive down the interest rate. In addition, the term structure steepens because short rates fall more than long rates. This is because with $\rho<1$, the effect of the shock dies off, so that if income is high today, it is also expected to be higher than average next period, but not quite so high. The size of the effect, and thus the incentive to save, diminishes, leading to a smaller increase in long rates.

A somewhat different picture emerges if $\rho>1$. Then, an increase today means an even bigger increase tomorrow, depressing the incentive to save and increasing rates.[7] If $\rho=1$, then the shock has no effect on interest rates—income is expected to go up exactly as much in the next period, so there is no change in the demand for saving.

This intuition follows through to the case of changes to the growth rate of endowments, $g$. In that case, $\frac{\partial r_{1t}}{\partial g}=1$, and $\frac{\partial l_t}{\partial g}=1$. Growing dividends means that future dividends are expected to be greater than current dividends (similar to the case for $\rho>1$). An increase in the growth rate means that future dividends will be increasingly greater than current dividends, leading to a lessening of the desire to save today. This lower demand for savings, and thus for bonds, decreases bond prices and increases interest rates. An increase in the growth rate of dividends increases both short- and long-term interest rates one for one.

Changing the stochastic process of the dividends will also change the term structure. Consider the effects of an increase in the variance of the shocks to income, $\sigma_\Theta^2$. In this case,

$$\frac{\partial r_{1t}}{\partial \sigma_\Theta^2} = -\tfrac{1}{2}$$

and

$$\frac{\partial l_t}{\partial \sigma_\Theta^2} = -\tfrac{1}{4}(1+\rho^2).$$

The increased uncertainty lowers both short- and long-term rates. The basic intuition is that as uncertainty increases, investors wish to save more "for a rainy day." The increased demand for saving drives down interest rates.[8] The yield curve steepens as long as $\rho<1$, because if shocks die out, an increased variance is less important the further out it is, and the demand for savings responds correspondingly less. Notice that though an increase in uncertainty leads to a steeper term structure, this happens not because long rates rise, but because they do not fall as far as short rates. In some sense, the increase in uncertainty is proportionally not so bad for the long term as for the short term, and thus has less of an impact on long-bond prices. This result must be interpreted carefully, however, because with a log-normal distribution, changing the variance of shocks also

■ **7** With $\rho<1$, some delicate issues arise about the existence of solutions to equation (1). For a discussion, see Campbell (1986) or Labadie (1994).

■ **8** Not every utility function displays such behavior, so the result is not completely general. See Zeldes (1989) for a good discussion.

changes the mean of the distribution. Increasing the variance here does not induce a mean-preserving spread.

## II. Nominal Term Structure

In the real world, the vast majority of bonds pay off in dollars—not in gold, sides of beef, or Internet-connect time. Some bonds are indexed for inflation, but, in the United States at least, most are not. This means that bonds do not have a certain payoff in consumption terms—you don't know for sure what $1,000 will be worth in 10 years—and bond pricing must take inflation risk into account.[9]

Fortunately, the analysis of section I can accommodate the shift to nominal interest rates relatively easily. Start by considering the *nominal* return on a bond, $R^{\$}_{1t}$, and note that to convert the dollars into consumption units and get a real return, we must consider inflation $\Pi_{t+1}$.[10] The nominal return on a bond is constant, so we can get a revised version of equation (5) as

$$(17)\qquad \frac{1}{R^{\$}_{1t}} = \beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\frac{1}{\Pi_{t+1}}\right]$$

or

$$(18)\qquad R^{\$}_{1t} = \frac{1}{\beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\frac{1}{\Pi_{t+1}}\right]}$$

$$= \frac{1}{\frac{1}{R_{1t}}E\frac{1}{(\Pi_{t+1})} + \beta\,\mathrm{cov}\left(\frac{u'(c_{t+1})}{u'(c_t)}, \frac{1}{\Pi_{t+1}}\right)}.$$

Equation (18) has a classic simplification due to Irving Fisher. Note that if consumption and inflation are perfectly certain (that is, there is no uncertainty), (18) reduces to $R^{\$}_{1t} = R_{1t} \bullet \Pi_{t+1}$. Shifting the perspective to rates, $R^{\$}_{1t} - 1 \approx R_{1t} - 1 + \Pi_{t+1}$. That is, a nominal interest rate of 5 percent may be broken into a real interest rate of 3 percent and an inflation rate of 2 percent. Notice that even with perfect certainty, this approximation does not hold for high interest rates: While 5 percent is a good approximation to (1.03)(1.02)=1.0506, 50 percent is not such a good approximation to (1.30)(1.20)=1.56.

With uncertainty, the simplification becomes an even worse approximation. As illustrated earlier, uncertainty has two components. One is the Jensen's inequality term. The other is the risk premium, the covariance between the real interest rate (or consumption) and inflation. Notice that this term can be positive or negative. Uncertainty about inflation may move interest rates up or down. This may seem counterintuitive, but it makes sense. For example, if inflation covaries positively with consumption growth, a nominal bond acts as a sort of insurance. If we get lucky next period, and have a high income, we regret having saved a lot—but a high inflation rate reduces the value of our savings. If we are unlucky, and income is low next period, we wish we had saved more—but a low inflation rate increases the value of our savings. Positive covariance, though, is probably not the most important case. A variety of studies find that inflation is negatively correlated with consumption growth (or, equivalently, real interest rates; see Pennacchi [1991]), so that inflation risk in fact increases interest rates. The risk premium is positive.

Looking at longer rates merely compounds the effect of uncertainty. Thus we have

$$(19)\qquad \frac{1}{R^{\$}_{2t}} = \beta^2 E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\frac{1}{\Pi_{t+1}}\frac{u'(c_{t+2})}{u'(c_{t+1})}\frac{1}{\Pi_{t+2}}\right].$$

Longer-term rates depend on how the real economy will evolve, how the price level will move, and the interactions between the two. Of course, simplifying assumptions can make (19) easier to interpret, but its more challenging form is probably more useful. What will higher consumption growth do to interest rates? Trick question—we don't really know until we have decided what will happen to inflation, and how inflation will react to the higher growth.

## III. Conclusion

The Roman poet Horace once remarked that getting rid of folly was the beginning of wisdom. Something similar might be said of the term structure. Understanding the simplifications involved in averaging current and future interest rates or in subtracting off expected

■ **9** For several approaches to adding inflation to a term structure model, see Sun (1992), Campbell, Lo, and MacKinlay (1997), section 11.2.1, Labadie (1994), and den Haan (1995). Sargent (1987) provides an in-depth view of monetary economies.

■ **10** More precisely, let $P^{\$}_{1t}$ be the dollar price of a pure discount bond in time $t$ with one period left to maturity. That is, the bond will pay $1 in period $t$+1. Let the price level ($/unit of consumption good) be $Q_t$. Then the real return is $R^{\$}_{1t} = \frac{1}{P^{\$}_{1t}} \bullet \frac{Q_t}{Q_{t+1}}$.

## Returns and Compounding

This article mostly uses the *simple net return* as a measure of the interest rate, defined as $r_t = \frac{P_{t+1}}{P_t} - 1$. Exactly what this rate is depends on the length of the period, although in the financial press, returns are usually annualized and expressed as if the return were for one year. Academic work often uses continuously compounded returns, $r_t = \log(\frac{P_{t+1}}{P_t})$, because they simplify calculations. Using continuously compounded rates, equation (7) becomes

$$e^{-2l_t} = \beta^2 [\frac{u'(c_{t+1})}{u'(c_t)} \frac{u'(c_{t+2})}{u'(c_t)}] = e^{-r_{1t}} \bullet e^{-r_{1t+1}}.$$

This implies that $l_t = \frac{1}{2}(r_{1t} + r_{1t+1})$, making the long rate an exact average of the current and future short rates. Similarly, using continuously compounded rates for (18) would give an exact Fisher equation, $r_{1t}^{\$} = r_{1t} + \pi_{t+1}$.

inflation is one benefit of looking at the deeper theory of interest rates. Another benefit arises from a better understanding of how uncertainty influences interest rates.

By suggesting that current long-term interest rates are an average of current and expected short-term rates, the expectations hypothesis captures an important truth. But it is not the whole truth. We have seen how changes in the uncertainty surrounding future rates may change the term structure, even if expected rates stay the same.

The effects of uncertainty are more varied, and often more subtle, than many people realize. An increased uncertainty about future interest rates has an effect on the Jensen's inequality factor that tends to lower long-term interest rates today. An increased uncertainty about future consumption has an effect on the risk premium that tends to lower interest rates today, as people save for a rainy day, but it steepens the term structure. An increased uncertainty about inflation will increase nominal interest rates, at least if inflation and consumption covary negatively. The effects on longer rates are more complicated.

The real world is undoubtedly more complex than the model of interest rates considered here. Like a map, which can never show every detail, our model can highlight important and dangerous areas of that rather mysterious area known as the term structure. This can lead to better decisions, be they on the part of particular investors or of monetary policymakers. Examining the underlying economic theory becomes the first step in understanding the interplay between real and nominal risk factors, where they come from, and how changes in those factors matter.

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# Depositor-Preference Laws and the Cost of Debt Capital

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## Introduction

The subsidy inherent in the current deposit-insurance system creates perverse incentives for risk taking by insured depository institutions (Kane [1985]). The thrift debacle and its attendant financial and political costs have exposed the dangers of combining virtually unlimited federal deposit guarantees and regulatory discretion. Federal deposit guarantees, the too-big-to-let-fail doctrine, and capital forbearance programs have effectively limited markets' ability to discipline troubled institutions. On the other hand, principal-agent conflicts have often produced government regulatory policies designed to forestall disciplinary actions against troubled banks and thrifts (Kane [1989]; Thomson [1992]).

Armed with increased awareness of the role played by regulatory forbearance in the thrift debacle and in record losses from the 1980s' bank closings, Congress passed the Federal Deposit Insurance Corporation Improvement Act of 1991.[1] The FDICIA contains four important reforms: First, it requires prompt corrective action for undercapitalized banks and for those considered problem institutions by their primary federal regulator.[2] Second, the FDICIA limits Federal Reserve discount-window loans to troubled depositories.[3] Third, the FDICIA now requires the Federal Deposit Insurance Corporation to charge insured institutions a risk-related deposit-insurance premium. Finally, the FDICIA replaces the too-big-to-let-fail doctrine with the systemic-risk exception, which codifies the terms and conditions under which the FDIC can bail out uninsured claimants of failed depositories.[4]

■ 1 DeGennaro and Thomson (1996) show that capital forbearance increased the total taxpayer bill in the thrift debacle more than 500 percent.

■ 2 Carnell (1993) notes that the FDICIA does not remove regulatory discretion but progressively limits it as an institution slides toward insolvency.

■ 3 Todd (1993) argues that these discount-window provisions are designed to prevent the Federal Reserve from propping up insolvent banks through improper solvency-based loans.

■ 4 Carnell (1993) contends that abuse of the exception can be limited by FDICIA provisions requiring written authorization from the Federal Reserve Chairman and the Secretary of the Treasury for financing systemic-risk losses by a special assessment on banks' total liabilities (total deposits).

Shortly after enacting the FDICIA, Congress added another potentially important measure to limit the FDIC's (and hence taxpayers') exposure. The Omnibus Budget Reconciliation Act of 1993 created a national depositor-preference law, changing the priority of depositors' (and thus the FDIC's) claims on the assets of failed banks by making other senior claimants subordinate to depositors.[5] In other words, Congress implemented depositor preference in an effort to reduce the FDIC's losses by changing the capital structure of banks.

This paper analyzes the impact of depositor-preference laws on banks' cost of debt capital and on the value of FDIC deposit guarantees.[6] We extend the single-period-cash-flow version of the capital-asset pricing model, presented by Chen (1978) and modified by Osterberg and Thomson (1990, 1991), to include depositor preference. In this model, the value of a firm is the present value of its future cash flows. The values of a firm's debt and equity are the present values of these claims on the firm's cash flows. Riskless cash flows are discounted at the risk-free rate of interest. Risky cash flows are converted to certainty-equivalent cash flows by deducting a risk premium from the expected cash flow. In this model, the risk premium is simply the market price of risk, multiplied by the covariance of the risky cash flow with the market portfolio.

In section I of this paper, we present the results of a single-period analysis of a bank that has both uninsured and insured deposits and subordinated debt, as derived in Osterberg and Thomson (1991). Section II extends our 1991 analysis to include the intended impact of depositor-preference laws. In section III, we investigate the laws' effects on the value of debt capital and deposit guarantees when general creditors behave strategically. We present conclusions and policy implications in section IV.

## I. Banks' Cost of Capital and the Value of Deposit Insurance: No Depositor Preference

The following assumptions are used throughout this paper: 1) the risk-free rate of interest is constant; 2) capital markets are perfectly competitive; 3) expectations are homogeneous respecting the probability distributions of the yields on risky assets; 4) investors are risk averse and seek to maximize the utility of terminal wealth; 5) there are no taxes or bankruptcy costs; 6) all debt instruments are discount instruments, so the total promised payment to depositors and subordinated debtholders includes both principal and interest; and 7) the deposit-insurance premium is paid at the end of the period.[7]

In this section, we present results from Osterberg and Thomson (1990) for a bank with insured deposits and uninsured deposits, extended to include general creditors. The FDIC charges a fixed premium of $\rho$ on each dollar of insured deposits. The total liability claims against the bank, $D$, is the sum of the end-of-period promised payments to the uninsured depositors, $B_u$, insured depositors, $B_i$, general creditors, $G$, and the FDIC, $z(\rho B_i)$. We assume that the FDIC underprices its deposit guarantees on average, and that in the absence of regulatory taxes (Buser, Chen, and Kane [1981]), the FDIC provides a subsidy that reduces banks' cost of capital and increases banks' value.

Under these assumptions, the end-of-period cash flows to insured depositors, $Y_{bi}$, clearly equal the promised payments to insured depositors, $B_i$, in every state. Therefore, whatever a bank's capital structure may be, the value, expected return, and cost of one dollar of insured deposits are defined as $V_{bi} = R^{-1}B_i$, $E(R_{bi}) = r$, and $r+\rho$, respectively.

■ 5 Title III of the Omnibus Budget Reconciliation Act of 1993 instituted depositor preference for all insured depository institutions by amending Section 11(d)(11) of the Federal Deposit Insurance Corporation Act [12 U.S.C. 1821(d)(11)]. At the time when national depositor preference was enacted, 29 states had similar laws covering state-chartered banks, and 18 had depositor-preference statutes covering state-chartered thrift institutions.

■ 6 For empirical studies of the impact of depositor-preference laws, see Hirschhorn and Zervos (1990), Osterberg (1996), and Osterberg and Thomson (1998).

■ 7 For simplicity, we assume that the deposit-insurance premium is an end-of-period claim on the bank. This is equivalent to assuming that the premium is subordinate to $B_i$ and that, in effect, the bank receives coverage while not necessarily paying the full premium. However, although this assumption affects how the deposit-insurance subsidy enters into the expressions in this paper and the actual size of the subsidy, it does not qualitatively affect the results.

**BOX 1**

### Definition of Notation

$B_i$ = Total promised payment to insured depositors.

$B_u$ = Total promised payment to uninsured depositors.

$G$ = Total promised payment to general creditors.

$\rho$ = Deposit-insurance premium per dollar of insured deposits.[8]

$z$ = Total promised payment to the FDIC ($\rho B_i$).

$B$ = Total promised payment to depositors and the FDIC ($B_i + B_u + z$).

$S$ = Total promised payment to subordinated debtholders.

$D$ = Total promised payment ($B_i + B_u + G + S + z$).

$Y_{bi}$, $Y_{bu}$, $Y_G$, $Y_s$, $Y_e$, and $Y_{FDIC}$ = End-of-period cash flows to insured depositors, uninsured depositors, general creditors, subordinated debtholders, stockholders, and the FDIC.

$V_{bi}$, $V_{bu}$, $V_G$, $V_s$, $V_e$, and $V_{FDIC}$ = Values of insured deposits, uninsured deposits, general-creditor claims, subordinated debt, bank equity, and the FDIC's claim.

$V_f$ = Value of the bank.

$E(R_{bi})$, $E(R_{bu})$, $E(R_G)$, $E(R_s)$, and $E(R_e)$ = Expected rates of return on insured and uninsured deposits, general-creditor claims, subordinated debt, and equity.

$r$ = Risk-free rate ($R = 1 + r$).

$X$ = End-of-period gross return on bank assets.

$F(X)$ = Cumulative probability-distribution function for $X$.

$\lambda$ = Market risk premium.

$COV(X, R_m)$ = Systematic or nondiversifiable risk.

$R_m$ = Return on the market portfolio.

$CEQ(X)$ = Certainty equivalence of $X\,[E(X) - \lambda COV(X, R_m)]$.

## Uninsured Depositors

End-of-period cash flows to uninsured depositors depend on the promised payment to the uninsured depositors and on total promised payments minus subordinated debt:

$$Y_{bu} = \begin{cases} B_u & \text{if } X > D-S = B_i + B_u + G + z, \\ B_u X/(D-S) & \text{if } D-S > X > 0, \text{ and} \\ 0 & \text{if } 0 > X. \end{cases}$$

While total promised payments to debtholders and the FDIC equal $D$, the effective bankruptcy threshold for uninsured depositors is $D$ less the claims of subordinated debtholders. The value of and the required rate of return on uninsured deposits are

$$(1)\quad V_{bu} = R^{-1}\{B_u[1-F(D-S)] + [B_u/(D-S)]\,CEQ_0^{D-S}(X)\}$$

and

$$(2)\quad E(R_{bu}) = \frac{1-F(D-S) + [1/(D-S)]\,E_0^{D-S}(X)}{V_{bu}} - 1.0.$$

Equation (2) shows that the cost of debt (uninsured-deposit) capital is a function of the bank's systematic risk, as measured by $\lambda COV(X, R_m)$; total promised payments to depositors and the FDIC, $(D-S)$; the probability that losses will exceed the level of subordinated debt, $F(D-S)$; and the risk-free rate of return. Osterberg and Thomson (1990, 1991) show that when the FDIC misprices its guarantees, the cost of uninsured deposit capital also depends on the deposit mix, because underpriced (overpriced) deposit guarantees lower (raise) the effective bankruptcy threshold for senior claims, $F(D-S)$, as well as the bankruptcy threshold, $F(D)$. Furthermore, underpriced (overpriced) deposit guarantees increase (decrease) the claims of uninsured depositors relative to senior claims, $B_u/(D-S)$, and relative to total claims, $B_u/D$. The size of this effect is a function of the FDIC's pricing error per dollar of insured deposits and of the weight of insured deposits in the senior creditor pool.

## General Creditors

General creditors have the same priority of claim as uninsured depositors; consequently, they will have similar end-of-period cash flows.

$$Y_G = \begin{cases} G & \text{if } X > D-S = B_i + B_u + G + z, \\ GX/(D-S) & \text{if } D-S > X > 0, \text{ and} \\ 0 & \text{if } 0 > X. \end{cases}$$

As before, total promised payments equal $D$, and the effective bankruptcy threshold is $D-S$. The value of and the required rate of return on senior nondeposit debt are

$$(3)\quad V_G = R^{-1}\{G[1-F(D-S)] + [G/(D-S)]\,CEQ_0^{D-S}(X)\}$$

and

$$(4)\quad E(R_G) = \frac{1-F(D-S) + [1/(D-S)]\,E_0^{D-S}(X)}{V_G} - 1.0.$$

8 For simplicity, we express the premium as a function of insured deposits. The results of interest are not materially affected by adopting the more realistic assumption that premiums are levied on total domestic deposits, insured and uninsured.

Equation (4) shows that the cost of nondeposit debt (general-credit) capital is a function of the same factors as uninsured deposits, including the bank's systematic risk, $\lambda COV(X, R_m)$, total promised payments to senior creditors and the FDIC, $(D-S)$, the probability that losses will exceed the level of subordinated debt, $F(D-S)$, and the risk-free rate of return. It also depends on the size of the deposit-insurance subsidy.

## Subordinated Debtholders

The end-of-period expected cash flows accruing to subordinated debtholders are

$$Y_s = \begin{cases} S & \text{if } X>D, \\ X+S-D & \text{if } D>X>D-S, \text{ and} \\ 0 & \text{if } D-S>X. \end{cases}$$

The value of the subordinated debt and the required rate of return on subordinated debt capital are

$$(5) \quad V_S = R^{-1}\{S[1-F(D-S)]-D[F(D)-F(D-S)] + CEQ_{D-S}^{D}(X)\}$$

and

$$(6) \quad E(R_s) = \frac{S[1-F(D-S)]-D[F(D)-F(D-S)]}{V_S} + \frac{E_{D-S}^{D}(X)}{V_S} - 1.0.$$

Equations (5) and (6) show that the cost and value of subordinated debt capital depend on the probability of bankruptcy, $F(D)$, the face value of the subordinated debt, $S$, total promised payments, $D$, and the probability that senior claimants will not be repaid in full, $F(D-S)$. Note that the last two terms in equation (6) represent the claims of subordinated debtholders in states where they are the residual claimants.

## Equityholders

The end-of-period cash flows accruing to stockholders are

$$Y_e = \begin{cases} X-D & \text{if } X>D, \text{ and} \\ 0 & \text{if } D>X. \end{cases}$$

The value of equity and the expected return to stockholders are

$$(7) \quad V_e = R^{-1}\{CEQ_D(X)-D[1-F(D)]\}$$

and

$$(8) \quad E(R_e) = \frac{E_D(X)-D[1-F(D)]}{V_e} - 1.0.$$

## The FDIC's Claim

The net value of deposit insurance is the value of the FDIC's claim on the bank, that is, the value of the FDIC's premium less the value of its deposit guarantee. In the absence of depositor-preference laws, the end-of-period cash flows to the FDIC and the value of its position are

$$Y_{FDIC} = \begin{cases} z & \text{if } X>D, -S, \\ (B_i+z)X/(D-S)-B_i & \text{if } D-S>X>0, \text{ and} \\ -B_i & \text{if } 0>X, \text{ so that} \end{cases}$$

$$(9) \quad V_{FDIC} = R^{-1}\{z[1-F(D-S)] + \frac{B_i+z}{D-S} CEQ_0^{D-S}(X) - B_i F(D-S)\}.$$

Equation (9) shows that the net value of deposit insurance is a function of the composition of the senior claims, the bank's systematic risk, the presence of junior debt claims in the bank's capital structure, the risk-free rate of return, the effective probability of bankruptcy, $F(D-S)$, the level of promised payments to insured depositors, and the deposit-insurance premium. In fact, equation (9) can be interpreted as showing that the equity-like buffer provided by subordinated debt affects the value of the FDIC's position by changing the probability that put options corresponding to the FDIC guarantee will be "in the money" at the end of the period. Equation (9) also demonstrates that if deposit insurance is to be priced fairly, $V_{FDIC} = 0$, the premium will be influenced by the degree to which the bank funds itself with claims junior to insured deposits.

Osterberg and Thomson (1990) show that the value of the uninsured bank is $R^{-1}CEQ_0(X)$. The value of the insured bank, $V_f$, equals the uninsured bank's value minus equation (9), which is the value of the FDIC's claim.

$$(10)\quad V_f = R^{-1}\{CEQ_0(X) + B_i F(D-S) - [(B_i+z)/(D-S)]CEQ_0^{D-S}(X) - z[1-F(D-S)]\}.$$

Equation (10) shows that the structure of a bank's debt (in terms of payment priority) affects the value of the bank only through the net value of deposit insurance to the bank. To see this, note that $B_i F(D-S) - [(B_i+z)/(D-S)]CEQ_0^{D-S}(X)$ is the value of FDIC guarantees, and $z[1-F(D-S)]$ is the value of the FDIC premium. If deposit insurance is correctly priced (that is, the value of its guarantee equals the value of its premium), then the structure of a bank's liability claims does not affect the bank's value.

## II. Banks' Cost of Capital and the Value of the Insurance Fund: Depositor Preference

In this section, we rederive the results to incorporate depositor preference, which subordinates the claims of general creditors to those of uninsured depositors and of the FDIC. As in section I, we assume that the FDIC charges a flat-rate insurance premium of $\rho$ on each dollar of insured deposits, and that on average the FDIC underprices its deposit guarantees.[9] To simplify the analysis, we assume that depositor preference does not change total liability claims against the bank, $D$.[10] Under this assumption, depositor-preference laws have no impact on claims that are junior to deposits and general-creditor claims.

### Uninsured Depositors

The end-of-period cash flows to uninsured depositors depend on the promised payment to uninsured depositors and on the total level of promised payments minus subordinated debt and the now-subordinated claims of general creditors:

$$Y_{bu} = \begin{cases} B_u & \text{if } X > B = B_i + B_u + z, \\ B_u X/B & \text{if } B > X > 0, \text{ and} \\ 0 & \text{if } 0 > X. \end{cases}$$

While total promised payments to debtholders and the FDIC equal $K$, the effective bankruptcy threshold for uninsured depositors is $B(=D-G-S)$. The value of—and the required rate of return on—uninsured deposits are

$$(11)\quad V_{bu} = R^{-1}\{B_u[1-F(B)] + (B_u/B)CEQ_0^B(X)\},$$

and

$$(12)\quad E(R_{bu}) = \frac{1-F(B)+(1/B)E_0^B(X)}{V_{bu}} - 1.0.$$

From the standpoint of uninsured deposit capital, depositor-preference laws have the same impact as a requirement that banks issue subordinated debt. That is, when uninsured depositors and the FDIC have claims in bankruptcy that are senior to those of general creditors, the effective bankruptcy threshold for uninsured depositors is lowered from $D-S$ to $D-G-S$. For uninsured depositors (and, as we shall see, for the FDIC), the pecking order of more junior claims is irrelevant to the value of their own.

To assess depositor preference's impact on the value of uninsured deposits, we control for possible changes in total promised payments by normalizing their expected cash flows by the level of uninsured deposits, and compare uninsured deposits in banks with and without depositor-preference laws. We then separate the expected cash flow to an uninsured deposit (with a par value of one dollar) in the presence of depositor-preference debt into two instruments. One is identical to the uninsured deposit in section I; the other has the following end-of-period payoffs and value:

■ **9** The results are qualitatively the same if the FDIC charges a variable-rate premium, so long as the deposit guarantees are mispriced.

■ **10** The results for uninsured depositors, FDIC, and general-creditor claims are qualitatively the same if we assume that depositor-preference laws change the level of total promised payments (see Osterberg and Thomson [1991, 1994]).

$$\Delta Y_{bu} = \begin{cases} 0 & \text{if } X > D-S, \\ 1 - X/(D-S) & \text{if } D-S > X > B, \\ X/B - X/(D-S) & \text{if } B > X > 0, \text{ and} \\ 0 & \text{if } 0 > X, \text{ so that} \end{cases}$$

(13) $$\Delta Y_{bu} = R^{-1}[F(D-S) - F(B) + \frac{(D-S-B)}{B(D-S)} CEQ_0^B(x) - \frac{1}{D-S} CEQ_B^{D-S}(x)] > 0.$$

Equation (13) is positive; note that the first term in the brackets is strictly greater than the third term. Moreover, since by definition $D-S>B$, the middle term is also positive. Therefore, depositor preference must increase the value of a dollar of uninsured deposits.

## General Creditors

Under depositor preference, general-creditor claims are junior to those of depositors and the FDIC but senior to those of subordinated creditors; hence, end-of-period cash flows to general creditors are

$$Y_G = \begin{cases} G & \text{if } X > D-S = B_i + B_u + G + z, \\ X-B & \text{if } D-S > X > B, \text{ and} \\ 0 & \text{if } B > X. \end{cases}$$

The total promised payments to debtholders and to the FDIC equal $D$, and the effective bankruptcy threshold is $D-S$. The value of and the required rate of return on general-creditor claims are

(14) $$V_G = R^{-1}\{G[1-F(D-S)] - B[F(D-S) - F(B)] + CEQ_B^{D-S}(X)\},$$

and

(15) $$E(R_G) = \frac{G[1-F(D-S) - B[F(D-S) - F(B)]}{V_G} + \frac{E_B^{D-S}(X)}{V_G} - 1.0.$$

Equations (14) and (15) show that non-deposit debt (general credit) behaves like subordinated debt (equations [5] and [6]), except that subordinated debt protects general creditors from loss. The value of general-creditor claims depends on the effective bankruptcy threshold, $F(D-S)$, the face value of their claims, $G$, total promised payments to senior claimants, $B$, and the probability that senior claimants will not be repaid in full, $F(B)$. Note that when earnings fall between $B$ and $D-S$, general creditors are the residual claimants, and theirs will behave like an equity claim.

Following the procedure used in the previous section, we construct the replicating portfolio for a general-creditor claim (with a par value of one dollar) under depositor preference. With depositor preference, the expected cash flow to such a claim is divided into one part that is identical to the general-creditor claim in section I, and a second that has the following end-of-period payoffs and value:

$$\Delta Y_G = \begin{cases} 0 & \text{if } X > D-S, \\ (X-B)/G - X/(D-S) & \text{if } D-S > X > B, \\ -X/(D-S) & \text{if } B > X > 0, \text{ and} \\ 0 & \text{if } 0 > X, \text{ so that} \end{cases}$$

(16) $$\Delta V_G = R^{-1}[\frac{CEQ_B^{D-S}(X) - B[F(D-S) - F(B)]}{G} - \frac{CEQ_0^{D-S}(X)}{D-S}] < 0.$$

Equation (16) is unambiguously negative. That is, depositor preference decreases the value of a general-creditor claim.

## The FDIC's Claim

As before, the net value of deposit insurance is simply the value of the FDIC's claim on the bank. Under depositor preference, the end-of-period cash flows to the FDIC and the value of its position are

$$Y_{FDIC} = \begin{cases} z & \text{if } X > B, \\ (B_i + z)X/B - B_i & \text{if } B > X > 0, \text{ and} \\ -B_i & \text{if } 0 > X, \text{ so that} \end{cases}$$

$$(17)\quad V_{FDIC} = R^{-1}\{z[1-F(B)] + \frac{B_i+z}{B} CEQ_0^B(X) - B_i F(B)\}.$$

As with uninsured deposits, the impact of a depositor-preference law is indistinguishable from a subordinated-debt requirement. Depositor preference affects the net value of the FDIC's claim by changing the senior claimants' probability of loss and by altering the weight of the FDIC in the pool of senior claims.

The change in the value of the FDIC guarantee on a one-dollar-par-value deposit is the value of a security with the following cash flows (where $\rho = z/B_i$):

$$\Delta Y_{FDIC} = \begin{cases} 0 & \text{if } X > D-S, \\ \rho-(1+\rho)X/(D-S)+1 & \text{if } D-S > X > B, \\ (1+\rho)X/B-(1+\rho)X/(D-S) & \text{if } B > X > 0, \text{ and} \\ 0 & \text{if } 0 > X, \text{ so that} \end{cases}$$

$$(18)\quad V_{FDIC} = \frac{1+\rho}{R}[F(D-S)-F(B) - \frac{1}{D-S} CEQ_B^{D-S}(X) + \frac{D-S-B}{B(D-S)} CEQ_0^B(X)] > 0.$$

Equation (18) is positive; to see this, note that the first term in the brackets is strictly greater than the second term. Since we assume that on average the FDIC underprices its guarantees, its claim on the bank is negative; hence, the size of the FDIC subsidy is smaller under depositor preference.

Finally, depositor preference affects the value of the bank entirely through its effect on the net value of deposit insurance.

$$(19)\quad V_f = R^{-1}\{CEQ_0(X) - z[1-F(B)] + \frac{B_i+z}{B} CEQ_0^B(X) - B_i F(B)\}.$$

Thus, if deposit insurance is always correctly priced (that is, if its net value to the bank is zero), depositor preference has no impact on bank value. But it does change the fair value of deposit insurance and so must be accounted for when setting the premium.

## III. Banks' Cost of Debt Capital and the Value of Deposit Guarantees: Depositor Preference when General Creditors Behave Strategically

The results in section II assume that general creditors do not respond to the subordination of their claims under depositor preference. However, in practice, general creditors of insured depositories will respond to changes in the priority of their claims and the higher risk that results. At the very least, general creditors will charge the depository institution a higher rate of interest to compensate for the increased risk of loss. As nondeposit funds become more expensive relative to deposits, institutions will lessen their funding in nondeposit markets, thus reducing the loss buffer afforded to uninsured depositors and the FDIC by nondeposit creditors.

Senior nondeposit creditors might also respond to depositor preference by reducing the average maturity of their claims. This response increases creditors' ability to "run" on the depository institution if its condition deteriorates. In fact, financially distressed institutions may find it difficult or impossible to issue unsecured nondeposit claims. This response by nondeposit creditors to depositor preference has two implications. First, if nondeposit creditors can effectively exit a troubled institution before it is closed, little or no loss cushion will be afforded to uninsured depositors and the FDIC. Second, the failure of nondeposit creditors to renew their claims could trigger a liquidity crisis that causes the institution to be closed.[11]

The third option for unsecured general creditors is to take collateral against their claim. By becoming secured creditors, they will have transformed their claim into one that is senior (to the extent of the collateral) to deposit claims. This, in turn, will have two effects on the claims of uninsured depositors and the FDIC. First, the loss buffer afforded by general-creditor claims is reduced. Second (and more importantly), the general asset pool available to pay unsecured claims is also reduced. If enough general-creditor claims take collateral, the total loss exposure of the FDIC and uninsured depositors could increase.

■ **11** The decision to close a bank is based on one of two measures of solvency: the incapacity to pay obligations as they mature or book-value, balance-sheet insolvency. Inability to renew nondeposit credits could trigger insolvency under the maturing-obligations test (see Thomson [1992]).

## Structural Arbitrage

The static nature of our model does not allow us to study the dynamic reaction of general creditors to depositor-preference laws directly. However, we can examine the implications of structural arbitrage by general creditors through its impact on the cash flows accruing to each class of claimant. Under the assumption that general creditors effectively collateralize their claims on the bank, we can show the unintended effect of depositor-preference laws on the cost of capital for banks and on the FDIC's claim.

As in section I, we assume that the FDIC charges a flat-rate insurance premium of $\rho$ on each dollar of insured deposits and that, on average, the FDIC underprices its deposit guarantees. The total liability claims against the bank, $D$, are the sum of the end-of-period promised payments to uninsured depositors, $B_u$, insured depositors, $B_i$, general creditors, $G$, subordinated debtholders, $S$, and the FDIC, $z\,(=\rho\, B_i)$. As in the previous section, we assume that total claims, $D$, are not affected by depositor preference and general creditors' responses to it.

## Uninsured Depositors

The end-of-period cash flows to uninsured depositors depend on the promised payment to uninsured depositors, the total level of promised payments minus subordinated debt and claims, and the claims of general creditors:

$$Y_{bu} = \begin{cases} B_u & \text{if } X > D-S, \\ B_u(X-G)/(D-S) & \text{if } D-S > X > G, \text{ and} \\ 0 & \text{if } 0 > X. \end{cases}$$

While the total promised payments to debtholders and the FDIC equal $D$, the effective bankruptcy threshold for uninsured depositors is $(D-S)$. The value of and the required rate of return on uninsured deposits are

$$(20)\quad V_{bu} = R^{-1}\left(B_u[1-F(D-S)+\frac{B_u}{D-S}\{CEQ_G^{D-S}(X)-G[F(D-S)-F(G)]\}\right),$$

and

$$(21)\quad E_{bu} = \frac{1-F(D-S)}{V_{bu}} + \frac{\frac{1}{D-S}\{E_G^{D-S}(X)-G[F(D-S)-F(G)]\}}{V_{bu}} - 1.0.$$

From the standpoint of uninsured deposit capital, general creditors' strategic behavior has rendered depositor claims junior to their own.

To isolate the de facto impact of depositor preference on the value of uninsured deposits in this case, we control for possible changes in total promised payments by normalizing expected cash flows at the level of uninsured deposits, and compare uninsured deposits in banks in the presence and absence of depositor-preference laws. We then separate the expected cash flow to an uninsured deposit (with a par value of one dollar) in the presence of depositor-preference debt into two instruments: one that is identical to the uninsured deposit in section I, and a second that has the following end-of-period payoffs and value:

$$\Delta Y_{bu} = \begin{cases} 0 & \text{if } X > D-S, \\ -G/(D-S) & \text{if } D-S > X > G, \text{ and} \\ -X/(D-S) & \text{if } G > X > 0, \text{ so that} \end{cases}$$

$$(22)\quad \Delta V_{bu} = [R(D-S)]^{-1}\{-G[F(D-S)-CEQ_0^G(X)]\} < 0.$$

Equation (22) is unambiguously negative. Hence, a potential unintended effect of depositor-preference laws is to reduce the value of uninsured depositor claims on the bank.

## General Creditors

The intended effect of depositor preference is to make general-creditor claims junior to those of depositors and the FDIC, but senior to those of subordinated creditors. However, the de facto effect of depositor preference may be to make general-creditor claims senior to all others. Under this scenario, the end-of-period cash flows to general creditors are

$$\Delta Y_G = \begin{cases} G & \text{if } X > G, \\ X & \text{if } G > X > 0, \text{ and} \\ 0 & \text{if } 0 > X. \end{cases}$$

The total promised payments to debtholders and to the FDIC equal $K$, and the effective bankruptcy threshold for general creditors is $D-B-S=G$. The value of and the required rate of return on general creditor claims are

(23) $V_G = R^{-1}\{G[1-F(G)] + CEQ_0^G(X)\}$,

and

(24) $E(R_G) = \dfrac{G[1-F(G)] + E_0^G(X)}{V_G} - 1.0.$

Equations (23) and (24) show that the value and return on general-creditor claims depend only on the level and variability of cash flows and the size of $G$. The presence and structure of other claims on the bank do not affect the valuation of such claims because we have assumed that general creditors have de facto secured the most senior claim on the bank. Following the procedure used in the previous sections, we construct the replicating portfolio for a general-creditor claim (with a par value of one dollar) under depositor preference. The expected cash flow to a such a claim is divided into one part that is identical to the general-creditors claim in section I, and another that has the following end-of-period payoffs and value:

$$\Delta Y_G = \begin{cases} 0 & \text{if } X > D-S, \\ 1 - X/(D-S) & \text{if } D-S > X > G, \text{ and} \\ X/G - X/(D-S) & \text{if } G > X > 0, \text{ so that} \end{cases}$$

(25) $\Delta V_G = R^{-1}\{[F(D-S) - F(G)] - [\dfrac{CEQ_0^{D-S}(X)}{D-S} - \dfrac{CEQ_0^G(X)}{G}]\} > 0.$

Whether the value of general-creditor claims increases or decreases depends, in this case, on whether the difference between the first two bracketed terms in (25) is larger than the difference between the second two bracketed terms.

## The FDIC's Claim

As before, the net value of deposit insurance is the value of the FDIC's claim on the bank. Under depositor preference, end-of-period cash flows to the FDIC and the value of its position are

$$Y_{FDIC} = \begin{cases} z & \text{if } X > D-S, \\ (B_i + z - G) X/(D-S) - B_i & \text{if } D-S > X > G, \text{ and} \\ -Bi & \text{if } G > X, \text{ so that} \end{cases}$$

(26) $V_{FDIC} = R^{-1}\{z[1-F(D-S)] + \dfrac{B_i + z - G}{D-S} CEQ_G^{D-S}(X) - B_i F(D-S)\}.$

As with uninsured deposits, depositor-preference law's impact on the FDIC's claim on the bank depends on the degree to which general creditors engage in structural arbitrage. The change in the value of the FDIC's guarantee on a one-dollar-par-value deposit is the value of a security that has the following cash flows (where $\rho = z/B_i$):

$$\Delta Y_{FDIC} = \begin{cases} 0 & \text{if } X > D-S, \\ GX/(B_i(D-S) & \text{if } D-S > X > G, \\ -(1+\rho)X/(D-S) & \text{if } G > X > 0, \text{ and} \\ 0 & \text{if } 0 > X, \text{ so that} \end{cases}$$

(27) $V_{FDIC} = [R(D-S)]^{-1}[-\dfrac{G}{B_i} CEQ_G^{D-S}(X) - (1+\rho) CEQ_0^G(X)] < 0.$

Equation (27) is clearly negative. Hence, a possible unintended outcome of the national depositor-preference law is to reduce the value of the FDIC's claim on the bank—that is, to increase the value of the FDIC's guarantees.

Finally, depositor preference influences the value of the bank entirely through its effect on the net value of deposit insurance:

$$(28)\quad V_f = R^{-1}[CEQ_0(X) - z[1-F(D-S)] - \frac{B_i+z-G}{D-S}CEQ_G^{D-S}(X) + B_iF(D-S)].$$

As in the previous case, if deposit insurance is always priced correctly (that is, if its net value to the bank is zero), it has no impact on bank value. However, depositor preference does change the fair value of deposit insurance and so must be accounted for when setting the premium.

## IV. Conclusions

Using the cash-flow version of the capital-asset pricing model, we show how depositor-preference laws affect the value and pricing of claims on insured banks. The intended effect of depositor preference is to change the bank's capital structure in a way that increases the value of uninsured deposit claims and reduces the size of the FDIC subsidy. Under the assumptions in this paper, all general creditors would see the value of their one-dollar-par claims reduced, to the benefit of the FDIC and uninsured depositors. Under less restrictive assumptions, other claimants junior to depositors would also see the value of their claims reduced. Overall, the intended effect of a depositor-preference statute would be the same as that of a mandatory subordinated debt requirement.

Depositor-preference laws, however, have another possible effect. Unlike subordinated-debt holders, general creditors can act to offset the statutory junior status of their claims.[12] In its most extreme form, structural arbitrage by general creditors can de facto render depositor and FDIC claims junior to those of general creditors. Hence, the national depositor-preference law may actually decrease the value of depositor and FDIC claims—that is, it may increase the value of deposit guarantees. Ultimately, whether this unintended effect of depositor-preference law will dominate is an empirical issue.

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■ **12** For example, holders of general-creditor claims could conceivably restructure their claims by taking collateral, thereby improving their position relative to depositors and the FDIC. Hirschhorn and Zervos (1990) find that for thrifts in states with depositor-preference laws, general creditors are more likely to be collateralized; hence, in those states these laws give depositors little protection.

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# Household Production and Development

by Stephen L. Parente, Richard Rogerson, and Randall Wright

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## Introduction

**D**ifferences in standards of living across countries are large. For example, Summers and Heston (1991) indicate that income per worker is around 30 times higher in the richest countries than it is in the poorest countries.

Why are differences in living standards so big? One position is that some countries have relatively low income levels due to their relatively low stocks of capital; this is particularly true if we interpret capital generally to include human and other intangible capital (see Mankiw, Romer, and Weil [1992]). Of course, this raises the question, why do some countries have such low capital stocks in the first place? One suggestion is that these countries are burdened with policies that distort agents' incentives to accumulate capital, policies that will be referred to here as *barriers to capital accumulation.*

This paper analyzes the effects of such barriers quantitatively. Compared to previous studies that have analyzed the effects of such policies on relative levels of income in the neoclassical growth model,[1] the key difference in this study is that we explicitly incorporate nonmarket activity—that is, *household production*—into the analysis. We argue that distinguishing between economic activity in market and nonmarket sectors may go a long way toward understanding international differences in capital and income.

The essence of our argument is as follows: First, the nature of the development experiences that we describe leads us to explain differences in per-worker income levels across countries (rather than differences in growth rates). The question, therefore, is how much of the observed differences in income levels can be attributed to empirically realistic barriers to capital accumulation? It is well known that the standard neoclassical growth model accounts for very few of these differences. However, the effects of such policies can be significantly larger when home production is included in the model, at least for certain parameter values (in particular, values that imply that capital is less important in nonmarket than in market production, such that nonmarket- and market-produced goods are relatively close substitutes). For example, in a standard model without home

1 For example, Parente and Prescott (1994) and Chari, Kehoe, and McGrattan (1996).

production, the distortionary policy must be about 100 times larger in one country for it to have one-tenth the income of another country, while in one (admittedly extreme) version of our home-production model, the distortionary policy need be only about three times bigger.

In models with household production, agents are generally more willing to shift resources out of market activity in response to policy distortions. Intuitively, policies that affect capital accumulation may also influence the mix of economic activity in market and nonmarket sectors, and so policy distortions can have significant effects. In the standard model without home production, the policy distortion required to generate a given income difference is so large because the time agents spend working in the market does not depend on the size of the distortion (given functional forms consistent with balanced growth). Hence, in that model, cross-country differences in output per worker are entirely attributed to differences in capital per worker. In the home-production model, although these same policies may not affect total hours worked, they generally do affect how hours are allocated between the market and nonmarket sectors. As individuals change their allocation of time spent in market work and in home work, differences in output per person will be due to both differences in capital and in market hours per worker.

As Parente and Prescott (1994) and Chari, Kehoe, and McGrattan (1996) have noted, an augmented neoclassical growth model without household production—but with capital broadly defined to include tangible and intangible capital—can go quite far in accounting for differences in income with reasonably sized barriers, *if* one is willing to assume that total capital's share in the production function is large. Such models, however, imply a large amount of unmeasured capital and investment. Household production is a complementary extension of the neoclassical model, in that a sizable fraction of the observed differences in income across countries can be accounted for in a model without intangible capital. If we include both intangible capital and home production, the amount of intangible capital and, hence, the role assigned to unmeasured investment, will be smaller.

The model with intangible capital and the model with home production both entail unmeasured output in the economy. However, in the home-production model, this unmeasured output takes the form of consumption rather than investment. Furthermore, the model without home production predicts the same fraction of unmeasured output in rich and poor countries, while the home-production model predicts a greater fraction of unmeasured output in poor countries. Hence, the home-production model predicts that the true differences in output, and especially in consumption, are smaller than those reported in the National Income and Product Accounts because nonmarket production and consumption are relatively more important in poor countries. This helps us to understand how individuals can survive on the amount of consumption reported in the official data in the poorest countries—an issue raised, for example, by Lucas (1988). It also allows us to compute the welfare implications of policy differences and, thus, of output differences in a way that explicitly recognizes nonmarket activity and its importance in poorer countries.

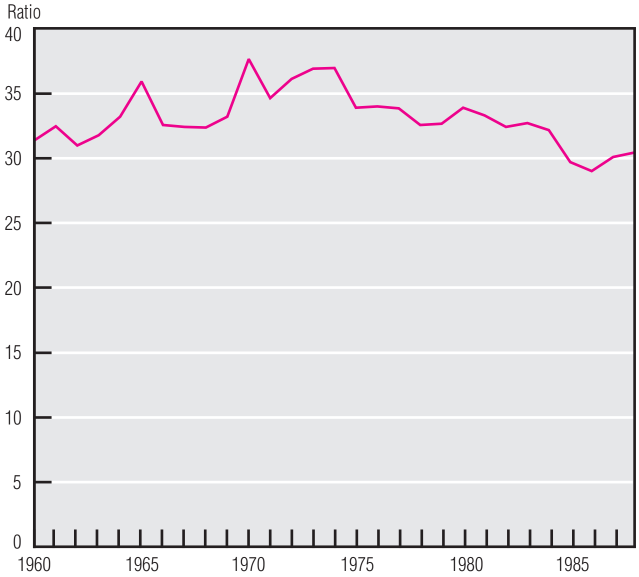
It may be worth mentioning at this point an analogy between the approach to development economics adopted here and modern business-cycle theory. In that literature, one attempts to identify and measure impulses (the underlying sources of fluctuations, such as technology shocks, changes in monetary policy, and so on) and then study the extent to which these impulses are amplified or propagated by different economic models. For example, one might ask, what fraction of observed business-cycle fluctuations can be accounted for with a given impulse and model? In this paper, we take as a maintained (if presumably counterfactual) hypothesis that countries differ only with respect to their barriers to capital accumulation. We establish reasonable magnitudes for these barriers, and then we ask, what fraction of the observed income differences can we account for? The point of this analysis is that the answer to the question changes once one recognizes that much economic activity takes place outside the formal market. Likewise, answers to several questions in business-cycle theory change once one incorporates home production into otherwise standard models.[2]

The rest of the paper is organized as follows: Section I reviews some basic development facts. Section II documents how the standard neoclassical growth model fails to account for these facts, given empirically plausible parameter values. We show how this model, augmented to include a second form of capital, can account for these facts, but also can predict a large amount of unmeasured capital and investment. Section III introduces home production into the

■ 2 See Greenwood and Hercowitz (1992), Behabib, Rogerson, and Wright (1992), and McGrattan, Rogerson, and Wright (1997), for example, for applications of home production in business-cycle theory.

FIGURE 1

**Relative Income, Richest Countries to Poorest Countries**




SOURCE: Summers and Heston (1991).

basic neoclassical model (without the second form of capital) and reports the quantitative impact of size differences in the barrier for several parameterizations of the model on observable variables and on welfare. Section IV integrates the model with two types of capital and with household production. Section V considers evidence supporting the view that home production is relatively important in less-developed economies.[3] Section VI contains some brief concluding remarks.

## I. Key Development Facts

In this section we briefly present some key development facts (more detailed discussions can be found in Parente and Prescott [1993, 1994]) which dictate our choices of questions and modeling strategies. Some researchers have concluded that a theory of relative income levels, as opposed to a theory of growth-rate differentials, is appropriate for understanding the pattern of economic development. Since an exogenous growth model of relative income levels is the framework of this paper, we motivate our choice by describing the relevant data.

Let $y_t$ measure gross domestic product (GDP) per worker in a country at date $t$ divided by GDP per worker in the United States at date $t$, computed at world prices. For the 102 countries in Summers and Heston (1991) with at least one million in population for which the data is complete between 1960–88, figure 1 plots the ratio of the average $y_t$ in the five richest countries to the average $y_t$ in the five poorest countries. First, notice that the GDP disparity is big—the richest five countries are about 30 times richer than the poorest five. Second, observe that this disparity has not increased. The ratio remains essentially the same over the period. (The standard deviation of the income distribution increases some, from about 1.25 to 1.50, with some of the mass spreading from the center to the tails.) In addition, while the rich got richer, so did the poor: with rare exceptions, all countries grew, suggesting no absolute poverty trap. The average annual growth rate in the sample is 1.9 percent.

Although it cannot be seen in figure 1, individual countries have moved within the distribution, suggesting no relative poverty trap: there have been both miracles and disasters.

Let us take as a base $\tilde{y}$, set equal to 10 percent of per capita GDP in the United States in 1985. Figure 2 plots the year in which each country achieved this level against the number of years it took that country to double its per capita output (that is, to go from $\tilde{y}$ to $2\tilde{y}$). Countries that achieve an income level of $\tilde{y}$ relatively early will take a longer period of time to double their income, while countries that achieve an income level of $\tilde{y}$ later can double their income much more rapidly. (This does not depend crucially on the choice of the base, and a similar pattern emerges for other values of $\tilde{y}$.) Hence, while the frontier is growing at a given rate, if a country lags significantly behind, it is possible to make rapid advances toward the frontier. This suggests that some countries have a policy or a set of institutions, perhaps, that keeps their income relatively low, but they are capable of catching up somewhat if the policy is eliminated or ameliorated.

■ 3 The idea that household production may be important to understanding economic development is not new. Kuznets (1960), for example, noted that nonmarket activities are more important in relatively poor nations. Eisner (1994) attributes the difference between the true and reported outputs to home production. Previous attempts to model household production in the context of economic development include Hymer and Resnick (1969) and Locay (1992), but these are not quantitative, dynamic, general-equilibrium models. Easterly (1993) studies an endogenous growth model that can be interpreted as having a formal and informal sector, although it could just as well be interpreted as a one-sector model with two types of capital.

FIGURE 2

## Years to Grow from $2,000 to $4,000 Per Capita Income versus Year Reached $2,000


a. Measured in 1990 U.S. dollars.
SOURCE: Parente and Prescott (forthcoming 2000).

These facts influence our choice of questions and models. In an endogenous growth model, differences in policies translate into differences in growth rates, but the data indicate that growth-rate differences are not permanent, as income levels across countries do not diverge over the postwar period. Even if we allow policies and (therefore) growth rates to change over time, an endogenous growth model cannot clearly explain the fact that countries that achieve a given base income later are able to double their income more quickly. Therefore, it is reasonable to adopt an exogenous growth model—that is, to assume that countries grow at the same average rate—and to ask what produces the observed differences in relative income levels.[4] In this model, countries that are behind the frontier because of a particular policy can indeed move up rapidly within the income distribution once the policy is removed. It is also desirable to consider policies for which we have some quantitative information. This will lead us to model barriers to capital accumulation in a particular way, as we discuss below.

## II. Background

The starting point of our analysis is the standard one-sector growth model. Assume an infinitely lived representative agent with preferences given by

$$(1) \quad \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \alpha \log(1-n^t)\right],$$

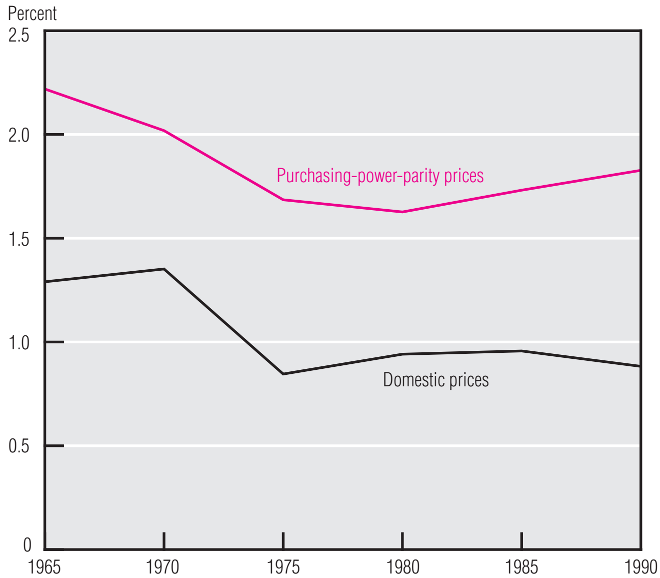
where $c_t$ denotes consumption, $n_t$ denotes time spent working at date $t$, and $\beta \in (0,1)$ denotes the discount factor. The representative agent is endowed with one unit of time in each period and $k_0$ units of capital at $t=0$. A constant-returns-to-scale production function uses capital and labor to produce output

$$(2) \quad y_t = A\, k_t^{\theta} \left[(1+\gamma)^t n^t\right]^{1-\theta},$$

4 Exogenous growth, incidentally, does not mean that a country can realize increases in output without undertaking any action. Parente and Prescott (1994) show that the equilibrium behavior of a model in which firms choose whether to adopt better technologies over time and in which the stock of knowledge that firms can adopt increases exogenously, is equivalent to the neoclassical growth model augmented with a second form of capital.

FIGURE 3

**Relative Investment Shares, Rich vs. Poor Countries**




SOURCES: Summers and Heston (1991), International Monetary Fund (1994)

where exogenous, labor-augmenting technological change occurs at rate $\gamma$ per period. Output in each period is divided between consumption and investment,

$$(3) \quad c_t + x_t \leq y_t.$$

Capital accumulation is represented by

$$(4) \quad k_{t+1} = (1-\delta)k_t + x_t,$$

where $\delta \in (0,1)$ is the depreciation rate.

One approach to studying the implications of the neoclassical growth model for development is to view each country as a closed economy, described by the same preferences and technology, and to look for policies that differ across countries and that may affect relative levels of output. Since the key economic decision in the model is the consumption–savings decision, it is natural to look for policies that distort incentives for agents to accumulate capital. There are many candidate factors, ranging from taxation and regulation to fear of confiscation. Two policies that have been studied in the literature are capital income taxation and policies affecting the relative price of capital goods, which we refer to as barriers to capital accumulation. From the perspective of modeling relative income, these two policies have similar effects. However, there are reasons to believe that barriers that distort the relative price of investment goods may be a more promising route.

If cross-country income differences are explained by differences in tax rates, then per capita income and taxes should be negatively correlated; however, the data do not show such a relationship (see Easterly and Rebelo [1993]). If one assumes that barriers to capital accumulation are the source of income differences, then there should be a negative correlation between per capita income and the price of investment relative to consumption goods. Jones (1994) documents differences in the price of equipment relative to consumption goods across countries and does indeed find a strong negative correlation between this variable and per capita output.

Since the two policies have different implications for prices, they have different implications for the investment–income ratio. Empirically, investment shares measured using domestic prices display no correlation with per capita output, whereas investment shares computed using purchasing-power-parity prices show a positive correlation with output per capita. Figure 3 plots the ratio of investment share in rich countries to the investment share in poor countries, computed with both domestic prices and with purchasing-power-parity prices. We argue that models with barriers that distort the relative price of capital are better able to match this observation.

We focus on policies that affect the price of investment relative to consumption goods, parameterized by changing capital accumulation (equation [4]) to

$$(5) \quad k_{t+1} = (1-\delta)k_t + \frac{x_t}{\pi},$$

where $\pi$ is the size of the barrier. If $\pi = 1$, then one unit of consumption can be turned into one unit of capital, while in a country with $\pi > 1$, each unit of consumption invested yields only $(1/\pi < 1)$ units of capital. Thus, $\pi$ is the relative price of investment goods. Jones' (1994)

TABLE 1

**Differences in Relative Income, $y^*/y^*_{US}$**

| | $\theta$=1/4 | $\theta$=1/3 | $\theta$=1/2 | $\theta$=2/3 | $\theta$=3/4 |
|---|---|---|---|---|---|
| $\pi$ = 2 | 0.79 | 0.71 | 0.50 | 0.25 | 0.13 |
| $\pi$ = 3 | 0.69 | 0.58 | 0.33 | 0.11 | 0.04 |
| $\pi$ = 4 | 0.63 | 0.50 | 0.25 | 0.06 | 0.02 |
| $\pi$ = 10 | 0.46 | 0.32 | 0.10 | 0.01 | 0.001 |

SOURCE: Authors' calculations.

evidence not only gives us reason to believe that differences in barriers may be the source of income differences, it also allows us to establish reasonable estimates of the magnitude of $\pi$. Jones reports a range of equipment relative to consumption goods prices between 1 and 4 across countries, with the United States normalized to 1.[5]

Pursuing the business-cycle analogy, we now have a more or less quantifiable impulse for the phenomenon in which we are interested, and we can now think about asking, how much of the observed income disparity can be accounted for by variations in $\pi$? Although we do not believe that policies distorting the prices of investment goods are the *only* factor accounting for cross-country income differences, it is still of interest to examine the effects that can be generated as a function of $\pi$, given a particular model.

Given $\pi$, the unique equilibrium in the model has the property that, starting from any $k_0>0$, we converge to a balanced growth path, along which output, consumption, investment, capital, and wages all grow at the same rate $\gamma$, while the rate of return to capital and hours worked are constant.[6] For the most part, we will focus on balanced growth paths, and in particular on the relative level of balanced-growth-path output across economies with different values of $\pi$ (although we will also take into account transitions from one balanced growth path to another when we analyze welfare). With our functional forms, it is straightforward to characterize the balanced growth path as a function of $\pi$. First, the fraction of time devoted to work is independent of $\pi$. Second, given that $\pi$=1 in the United States, the relative capital stock and output in an economy with a barrier of size $\pi$ will be

$$(6) \qquad \frac{k^*}{k^*_{US}} = \pi^{-1/(1-\Theta)} \text{ and } \frac{y^*}{y^*_{US}} = \pi^{-\Theta/(1-\Theta)}.$$

Does this yield a good theory of development? To answer this question, one must say something about parameter values. Table 1 reports the relative output differences generated by differences in barriers from $\pi$=2 to $\pi$=10 for various values of $\Theta$. If $k$ is interpreted as physical capital, we are led to consider a value of $\Theta$ between 1/4 and 1/3. For these parameter values, the model accounts for very little of the observed differences in per capita income; for example, with $\Theta$ = 1/3, U.S. output is only twice as high as output in an economy with $\pi$ = 4, and only three times as high as an economy with $\pi$ = 10. Recall that the data indicate that U.S. output per worker is 30 times higher than output per worker in the poorest countries. To generate output differences of 30 with $\Theta$ =1/3, we would need a barrier of $\pi$ = 900. This model is off by orders of magnitude.

Table 1 provides results for higher values of $\Theta$, which, as one can see, allow the model to account for much larger differences in relative income. For example, if $\Theta$=2/3, we can generate output differences of 30 with a barrier of about $\pi$=5.5. To rationalize such higher values of $\Theta$, several researchers (including Mankiw, Romer, and Weil [1992], Parente and Prescott [1994], and Chari, Kehoe, and McGrattan [1996])

■ 5 Restuccia and Urrutia (1996) find a range closer to 12. Like Jones (1994), they compute the ratio of the price level of investment to the price level of consumption goods, taken from the Penn World Tables; unlike Jones, they consider all investment goods, rather than a subcategory, and use both benchmarked and unbenchmarked countries. However, the big difference seems to be due to revisions of the price data in the Penn World Tables between PWT and PWT5.6. In another sense, the range of 4 may be too large. According to PWT5.6, the prices of consumption goods vary much more across countries than prices of investment goods (that is, it is not that computers are expensive in poor countries, but that haircuts are cheap). It appears that differences in the price of consumption goods account for about half of the variation in these relative prices, suggesting a barrier closer to 2.

■ 6 Note that, even though this economy has a distortionary policy, one can characterize a competitive equilibrium by solving an augmented social planner's problem, where the planner faces the law of motion for $k_t$ that includes the barrier $k_t = (1-\delta)k_t + x_t/\pi_t$.

TABLE 2

## Model with Unmeasured Capital

| $\theta_k$ | $\theta_z$ | $\frac{x_z}{c+x_k}$ | $\frac{x_k}{c+x_k}$ | $\frac{k}{z}$ |
|---|---|---|---|---|
| 0.10 | 0.57 | 0.76 | 0.13 | 0.18 |
| 0.20 | 0.47 | 0.55 | 0.24 | 0.43 |
| 0.30 | 0.37 | 0.39 | 0.32 | 0.82 |
| 0.40 | 0.27 | 0.26 | 0.38 | 1.50 |
| 0.50 | 0.17 | 0.15 | 0.44 | 3.00 |
| 0.60 | 0.07 | 0.05 | 0.48 | 9.00 |

SOURCE: Authors' calculations.

have discussed expanding the notion of capital beyond physical capital to broader notions, including human and organizational capital. However, some issues must be faced if one goes this route. First, the values of $\pi$ that we infer from empirical work are based on data for physical capital, and it is unclear to what extent barriers of this size apply to other types of investment. For example, in the case of human capital, a substantial component of accumulation takes place in the formal education sector, which is heavily subsidized in many countries, especially in poor countries. Second, since current national-income-accounting procedures do not recognize capital other than physical capital, and investments in intangible capital (with the exception of some education expenditures) are not measured, assuming high values of $\Theta$ implies that a large amount of capital and investment will go unmeasured.

To illustrate this point explicitly, we modify the previous model to include an intangible capital good, $z$. Let investment in physical and in intangible capital be denoted by $x_k$ and $x_z$, respectively, so that we have

(7) $$c_t+x_{kt}+x_{zt} \leq y_t = Ak_t^{\Theta_k} z_t^{\Theta_z}[(1+\gamma)^t n_t]^{1-\Theta_k-\Theta_z}.$$

The empirical counterpart of $y_t$ is no longer output as reported in the National Income and Product Accounts, due to the unmeasured investment; rather, output in the national product accounts corresponds to $(c_t+x_{kt})$ in the model. We assume the two capital stocks evolve according to:

(8) $$k_{t+1}=(1-\delta_k)k_t+x_{kt}/\pi$$

and

(9) $$z_{t+1}=(1-\delta_z)z_t+x_{zt}/\pi.$$

It is a straightforward generalization of the standard model to characterize the balanced growth path for this model as a function of $\pi$.

Table 2 presents summary statistics for several combinations of $\Theta_k$ and $\Theta_z$ that sum to 2/3, meaning that the $\pi$ needed to generate the observed international income differences is 5.5.[7] The third column reports unmeasured investment as a fraction of measured output; the fourth column reports measured investment as a fraction of measured output; and the last column reports the ratio of the two capital stocks. If $\Theta_k$=0.40, for example, unmeasured investment is only 26 percent of measured output. However, such a value for $\Theta_k$ implies that measured investment equals 38 percent of measured output—nearly twice as high as in the U.S. data. To match the measured investment–output ratio, $\Theta_k$ cannot be greater than 0.20; for such values of $\Theta_k$, the implied value of unmeasured investment exceeds half of measured output, and the unmeasured capital stock is well over double the measured capital stock.[8]

This discussion is not meant to suggest that models with higher capital shares are necessarily inconsistent with the data; after all, it is tautological to say that we do not have measures of unmeasured investments.

We believe, however, it does suggest that it may be worthwhile to consider other approaches.

**7** Other parameters are calibrated as follows: $\delta_k = \delta_z = 0.06$; $\gamma$ is set to achieve 2 percent growth per year; $\beta$ is set to achieve a real interest rate of 4.5 percent; the preference parameter $\alpha$ is set so that the fraction of time spent working is 1/3.

**8** Chari, Kehoe, and McGrattan (1996) set $\Theta_k$=1/3, making their model inconsistent with the ratio of measured investment to measured output. Mankiw, Romer, and Weil (1992) likewise set $\Theta_k$=1/3, but their model is not inconsistent with this observation, since they calibrate other parameters to match the measured investment–output ratio; however, this implies a real rate of interest in excess of 10 percent. Parente and Prescott (1994) calibrate to a real rate of return of 4.5 percent and a measured investment-output ratio of 20 percent, implying that $\Theta_k$ = 0.19 and that unmeasured investment is 41 percent of measured output. This is lower than reported in table 2 because Parente and Prescott's calibration implies a lower depreciation rate on intangible capital of 3.5 percent. Their ratio of unmeasured to measured capital, however, is still quite large.

The remainder of this paper outlines an alternative but complementary framework—the home-production model. This model helps to account for cross-country income differences, but at the same time has implications differing from the previous models. For example, while both approaches imply unmeasured output, the model with intangible capital and without home production emphasizes unmeasured investment, whereas the home-production model emphasizes unmeasured consumption. Although it is not apparent from table 2, an important point for future reference is that the model with intangible capital and without home production implies that unmeasured investment as a fraction of measured output is independent of $\pi$.[9] Thus, high- and low-distortion economies have the same ratios of unmeasured investment to measured output, and so the difference in measured output across countries accurately reflects the difference in total (measured plus unmeasured) output. This will not be true for the home-production model.

## III. The Home-Production Model

Following Becker's (1965) line of thought, we now extend the standard model to allow for nonmarket, or household, production. Some researchers in macroeconomics have found this useful in accounting for high-frequency aspects of the data (for example, Benhabib, Rogerson, and Wright [1991] or Greenwood and Hercowitz [1991]). One reason is that home-production models provide additional margins of adjustment relative to models without home production. Thus, whereas in the standard model there are only two uses for output—consumption and investment—in a home-production model there are three—consumption, investment in market capital, and investment in nonmarket capital. Likewise, in the standard model there are only two uses for time—leisure and work—but in a home-production model there are three—leisure, market work, and nonmarket work. In particular, a reduction in return-to-market activity reduces time spent in market work in the standard model only to the extent that it increases leisure. In a home-production model, agents can adjust their market hours by altering the mix of market work and home activity, even if they do not change their leisure-time allocation.[10]

Generalizing the previous model, preferences are now given by

$$(10)\quad \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \alpha \log(1-n_t)\right],$$

where $c_t$ is an aggregate of market and nonmarket consumption,

$$(11)\quad c_t = \left[\mu c_{mt}^{\varepsilon} + (1-\mu) c_{nt}^{\varepsilon}\right]^{1/\varepsilon},$$

and $n_t$ represents the sum of time spent working in the market and at home,

$$(12)\quad n_t = n_{mt} + n_{nt}.$$

The market-production function is unchanged,

$$(13)\quad y_t = A k_{mt}^{\Theta} \left[(1+\gamma)^t n_{mt}\right]^{1-\Theta}.$$

However, a home-production function is now given by

$$(14)\quad c_{nt} = A k_{nt}^{\phi} \left[(1+\gamma)^t n_{nt}\right]^{1-\phi}.$$

Exogenous technological change occurs at the same rate in the home and market sectors, implying that from any initial condition, the model converges to a balanced growth path along which $y_t$, $c_{mt}$, $c_{nt}$, $k_{mt}$, and $k_{nt}$ all grow at the rate $\gamma$, while $n_{mt}$ and $n_{nt}$ remain constant.

■ **9** For unmeasured investment, note that along the balanced growth path, $x_z = \pi(\gamma + \delta_z)z$, where all variables have been transformed into stationary equivalents by dividing the date $t$ by $(1+\gamma)^t$. For the transformed variables, it follows that

$$\frac{x_z}{c+x_k} = \frac{x_z}{y-x_z} = \frac{\pi(\gamma+\delta_z)z}{k^{\Theta_k} z^{\Theta_z} - \pi(\gamma+\delta_z)z}.$$

Using the first-order condition for profit maximization, we have

$$\pi(i+\delta_z) = \Theta_z\, k^{\Theta_k} z^{\Theta_z - 1},$$

where $i$ is the interest rate, and so it follows that $x_z/(c+x_k)$ is independent of $\pi$.

■ **10** It should be noted that models with home production cannot explain observations on market variables that could not be explained, *in principle*, by a model without home production (see Benhabib, Rogerson, and Wright [1991]). That is, one can always choose preferences in a model without home production that perfectly mimic the market outcomes of a given model with home production. However, the implied choice of preferences might generally be viewed as nonstandard. For instance, to be consistent with balanced growth and different amounts of market work for different barriers, the implied choice of preferences in a model without home production would be time dependent; therefore, even *if* we only want to look at data on market variables, an advantage of the home production framework is that it permits one to consider a richer class of specifications without sacrificing time-independent preferences or balanced growth. In fact, there are data on how time is allocated across activities, including home production, and these models provide a structure for interpreting this data.

TABLE 3

**Values of $\pi$ That Will Generate $y^*/y^*_{US} = 1/10$**

| | $\phi \approx 0$ | $\phi = 0.05$ | $\phi = 0.10$ | $\phi = 0.20$ | $\phi = 0.33$ |
|---|---|---|---|---|---|
| $\varepsilon \approx 0$ | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| $\varepsilon = 0.2$ | 59.3 | 65.1 | 71.2 | 83.7 | 100.0 |
| $\varepsilon = 0.4$ | 27.6 | 34.1 | 42.2 | 63.1 | 100.0 |
| $\varepsilon = 0.6$ | 10.3 | 13.9 | 19.4 | 38.9 | 100.0 |
| $\varepsilon = 0.8$ | 3.3 | 4.4 | 6.5 | 16.4 | 100.0 |

SOURCE: Authors' calculations.

TABLE 4

**Values of $\phi$ and $\varepsilon$ That Will Generate $y^*/y^*_{US} = 1/10$**

| $\phi$ | $\varepsilon$ | True output ratio Domestic prices | Welfare cost U.S. prices | Welfare cost Steady states | Welfare cost Dynamics |
|---|---|---|---|---|---|
| 0.001 | 0.77 | 0.42 | 0.52 | 1.30 | 1.18 |
| 0.05 | 0.82 | 0.43 | 0.52 | 1.32 | 1.21 |
| 0.10 | 0.87 | 0.44 | 0.52 | 1.42 | 1.24 |
| 0.15 | 0.98 | 0.45 | 0.52 | 1.43 | 1.25 |

SOURCE: Authors' calculations.

An important distinction between the market and home sectors is that, by assumption, capital goods can be produced only in the market sector. That is, market output is still divided between consumption and investment in the two types of capital,

(15) $$c_{mt} + x_{mt} + x_{nt} \leq y_t \,,$$

while all home-produced output is consumed. At this stage, we allow distortionary policies to differ for the two types of investment goods. Thus, the two capital stocks evolve according to

(16) $$k_{mt+1} = (1-\delta_m)\,k_{mt} + x_{mt}/\pi_m$$

and

$$k_{nt+1} = (1-\delta_n)\,k_{nt} + x_{nt}/\pi_n.$$

We assume that capital is not mobile across sectors, though this assumption does not actually affect balanced-growth-path analysis.

The degree to which individuals respond to economic distortions typically depends on the other opportunities they face. Home production is an alternative to market production, and hence, it may be relevant to evaluating the way market activity responds to distortions. This intuition suggests that three parameters are especially relevant in determining the impact of the distortions on which we are focusing: 1) the elasticity of substitution between home and market consumption, $\varepsilon$; 2) the share of capital in the home sector, $\phi$; and 3) the relative size of the barriers in the two sectors, $\pi_m$ and $\pi_n$. For home production to produce a greater response of market output to a given investment barrier, the model requires that individuals be willing to substitute home consumption for market consumption; such willingness is obviously affected by $\varepsilon$. Additionally, even if individuals are willing to substitute between market- and home-produced goods, the distortions on investment must create an incentive to do so. For this to occur, either home activity must be less capital intensive than market activity, or the distortion to the price of capital used in the nonmarket sector must be smaller than the distortion to the price of capital used in the market sector.

We begin with the case in which the barriers on the two investments are the same, $\pi_m = \pi_n = \pi$. Table 3 displays the value of $\pi$ that is required to decrease market output by a factor of 10 relative to the $\pi = 1$ case. Several combinations of values for the two key parameters, $\phi$ and $\varepsilon$, are considered, and in each case $\Theta$ is set to 1/3.[11]

As a benchmark, it can be shown formally that if $\phi = \varepsilon = 0$, the home-production economy yields predictions for market variables that are identical to those for the basic model. In other words, for these parameter settings, home

11 The other parameters are set as follows: $\delta_m = \delta_m = 0.06$, $\gamma = 0.02$, $\beta = 0.98$, and preference parameters are set to generate $n_m = 0.33$ and $n_n = 0.28$ when $\pi = 1$.

**TABLE 5**

## Equilibrium Relative to Undistorted Economy, $\pi_m = \pi_n$

| | $\varepsilon$=0.0001 | $\varepsilon$=0.2 | $\varepsilon$=0.4 | $\varepsilon$=0.6 | $\varepsilon$=0.8 |
|---|---|---|---|---|---|
| $y$ | 0.50 | 0.47 | 0.41 | 0.32 | 0.12 |
| $k_m$ | 0.13 | 0.12 | 0.10 | 0.08 | 0.03 |
| $c_m$ | 0.50 | 0.47 | 0.41 | 0.30 | 0.09 |
| $c_n$ | 0.90 | 0.97 | 1.09 | 1.31 | 1.74 |
| $x/y$ | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| $y/n_m$ | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 |
| $k_m/n_m$ | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 |
| $r$ | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 |
| $n_m$ | 1.00 | 0.94 | 0.83 | 0.62 | 0.22 |
| $n_n$ | 1.00 | 1.08 | 1.21 | 1.46 | 1.94 |

SOURCE: Authors' calculations.

production does not matter. Hence, the cell corresponding to $\Theta = \varepsilon \approx 0$ indicates that in the standard model without home production, given $\Theta = 1/3$, we require $\pi = 100$ in order to generate an income differential of 10.

Not surprisingly, the results in table 3 accord well with the above intuition, in that a higher elasticity of substitution between the two consumption goods or a lower capital intensity in home production implies that a lower barrier is needed to achieve a given reduction in market output. However, the quantitative results are quite striking: if the home technology uses very little capital and the two consumption goods are close substitutes, we can reduce the required barrier from $\pi = 100$ in the standard model to $\pi = 3.3$ in our model.[12]

Table 4 presents similar information in a different way: we fix $\pi = 4$, and for various values of $\phi$, we report the value of $\varepsilon$ needed to yield $y^*/y^*_{US} = 1/10$ (other parameters are as in table 3). The results confirm our intuition that when capital's share in the home is bigger, individuals must be more willing to substitute between the two goods to generate the same results. We also report several other statistics, including the ratio of total output in the two economies, with total output computed two ways: market-produced output plus home-produced output weighted by its domestic shadow price; and market-produced output plus home-produced output weighted by its shadow price in the undistorted economy. Although the measured market output ratio is 1/10, the true output ratio by either measure is closer to 1/2, because home production plays a relatively more important role in the distorted economy.[13] Recall from table 1 that when $\pi = 4$ and $\Theta = 1/3$, the model without home production implies $y^*/y^*_{US} = 1/2$. Thus, while the home-production model generates much larger ratios of measured outputs, it generates similar ratios of true outputs.

Because the true output ratio is so different from the measured output ratio, it seems interesting to ask about welfare. Table 4 reports the welfare cost of the barrier in terms of additional consumption required for an individual who is indifferent to having or not having the distortion (for example, a number of 1.25 means that an agent needs 25 percent more of both market and home consumption to be as well off with the barrier as he would be without it). We compute these welfare measures first by simply comparing steady states—in which case, the result tells us how much we need to pay an agent to induce him to *not* move from a distorted economy to an undistorted economy already on its balanced growth path—and also by taking into account transition paths, in which case the result tells us how much we need to pay an agent to induce him to not remove the distortions where he lives. The table reports that agents must be paid 30 percent–40 percent of their consumption to not move, and 18 percent–25 percent to not remove the barrier. As a

■ **12** While it is not clear that one wants to use the same parameter values in a development context that one uses for studying U.S. business cycles, we mention here the values of the two key parameters found in some of the related literature. Estimates of $\varepsilon$ from micro data in Rupert, Rogerson, and Wright (1995) and from macro data in McGrattan, Rogerson, and Wright (1997) both yield values close to $\varepsilon = 0.4$. Business-cycle models with home production have a range of values for $\phi$; for example, in Benhabib, Rogerson, and Wright (1992), $\phi = 0.1$, while in Greenwood and Hercowitz (1991) and in McGrattan, Rogerson, and Wright (1997) $\phi$ is much bigger. The issue is what one wants to match: lower $\phi$ generates values for $k_n/y$ consistent with measuring home capital in terms of consumer durables but not residential structures, while higher values are needed to match $k_n/y$ if housing is to be included.

■ **13** The true output ratio is slightly bigger when it is computed using the price of home-produced output in the undistorted economy because home-produced goods are relatively more scarce and hence more expensive in the undistorted economy.

**TABLE 6**

**Equilibrium Relative to Undistorted Economy, $\pi_m > \pi_n$**

| | $\varepsilon$=0.0001 | $\varepsilon$=0.2 | $\varepsilon$=0.4 | $\varepsilon$=0.6 | $\varepsilon$=0.8 |
|---|---|---|---|---|---|
| $y$ | 0.50 | 0.48 | 0.45 | 0.40 | 0.30 |
| $k_m$ | 0.13 | 0.12 | 0.11 | 0.10 | 0.07 |
| $c_m$ | 0.50 | 0.46 | 0.41 | 0.31 | 0.12 |
| $c_n$ | 0.80 | 0.83 | 0.88 | 0.98 | 1.17 |
| $x/y$ | 1.00 | 1.08 | 1.13 | 1.27 | 1.72 |
| $y/n_m$ | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 |
| $k_m/n_m$ | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 |
| $r$ | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 |
| $n_m$ | 1.00 | 0.95 | 0.90 | 0.62 | 0.59 |
| $n_n$ | 1.00 | 1.04 | 1.11 | 1.46 | 1.47 |

SOURCE: Authors' calculations.

benchmark, in the model without home production, if $\pi = 4$, which yields $y^*/y^*_{US} = 1/2$, we need to pay agents 40 percent of their consumption to not remove the barrier.

Compared to the model without home production, the model with home production generates much larger differences in measured output, comparable differences in true output, and smaller welfare differences. What is behind these results? To answer this question, it is instructive to look at a larger set of statistics describing the equilibrium. For $\phi = 0.05$, $\pi = 4$, and various values of $\varepsilon$, table 5 reports the ratio of several variables in the distorted and the undistorted economies (with the other parameters set as above). The first column serves as a benchmark: with $\varepsilon \approx 0$, the model is very close to the model without home production, while the other columns illustrate what happens as individuals become more willing to substitute between home and market goods.

Several features are worth noting. First, as $\varepsilon$ increases, market activity as measured by $y$, $c_m$, and $n_m$ falls, while household activity as measured by $c_n$ and $n_n$ increases. Second, $x/y$ is unaffected by the size of $\pi$. Hence, the investment-to-output ratio is not lower in poorer countries if it is measured using domestic prices, although it is lower if measured using U.S. prices (because the U.S. price of capital is 1, while the domestic price is $\pi$). This matches well with the data, which shows a relationship across countries between output and investment, measured using world prices but not domestic prices (recall figure 3). Third, although the rental rate of capital, $r$, is four times greater in the distorted economy, this is independent of $\varepsilon$; therefore, larger differences in $y$ across countries do not imply larger differences in $r$, as is true in the model without home production. Fourth, while increases in $\varepsilon$ reduce market output, they do not affect the average productivity of labor in the market: $y/n_m$ is half as large in the distorted economy as it is in the undistorted economy, independent of $\varepsilon$. The key factor behind this result is that when $k_m$ falls, so does $n_m$, so that $k_m/n_m$ stays constant when $\varepsilon$ increases. Also notice that as $n_m$ decreases, $n_n$ increases, so that total time working remains roughly constant.

The above results are for a relatively low value of capital's share in home production, $\phi = 0.05$. As $\phi$ increases, the overall effect of adding home production decreases, because barriers to capital accumulation divert resources from market to nonmarket activity to a greater extent when nonmarket activity is relatively less capital intensive.[14] Indeed, if the home and market technologies have the same capital share, $\phi = \Theta$, there is no difference between the models with and without home production in terms of the barrier required to generate a factor-10 difference in output (see table 3). However, this is true only if we assume the *same* barriers to market and nonmarket capital accumulation, $\pi_m = \pi_n$, which is not necessarily the most interesting case.

Table 6 reports the ratios of variables in one economy with $\pi_m = 4$ and $\pi_n = 1$, and another economy with $\pi_m = \pi_n = 1$, assuming equal capital shares in the home and in the market, $\phi = \Theta = 1/3$ (other parameters are set as above). The distortion affects only market capital accumulation. Even with $\Theta$, output in the distorted

■ **14** Also, for larger values of $\Theta$, the investment share measured using domestic prices may be increasing in $\varepsilon$, since the capital used in the home sector is produced in the market sector but does not produce measured output. This somewhat reduces the model's ability to match the fact discussed above, that there is a relationship between output and savings measured using world prices but not using domestic prices; however, the magnitude of this effect is relatively small for $\phi$ less than 0.10 and $\varepsilon$ less than 0.60.

**TABLE 7**

**Parameters in the Integrated Model That Generate $y^*/y^*_{US} = 1/10$**

| $\Theta_z$ | $\Theta_k$ | $\varepsilon$ | True output ratio Domestic prices | True output ratio U.S. prices | Welfare cost Steady states | $\frac{x_z}{c+x_k}$ |
|---|---|---|---|---|---|---|
| 0 | — | — | 0.42 | 0.52 | 1.30 | 0 |
| 0.1 | 0.25 | 0.75 | 0.40 | 0.49 | 1.33 | 0.08 |
| 0.2 | 0.22 | 0.66 | 0.30 | 0.38 | 1.52 | 0.18 |
| 0.3 | 0.20 | 0.50 | 0.20 | 0.26 | 1.93 | 0.30 |
| 0.4 | 0.19 | 0.25 | 0.12 | 0.16 | 2.99 | 0.44 |

SOURCE: Authors' calculations.

economy falls with $\varepsilon$, although not by very much. Asymmetric barriers do not have a significant effect when $\phi = \Theta$, even with large values of $\varepsilon$. Although agents want to increase nonmarket activity, when nonmarket activity is capital intensive, they cannot reduce the size of the market sector by much because we assume that household capital must be produced in the market. While asymmetric barriers imply a reallocation from market to home production, the effect will not be significant if household production is very capital intensive and nonmarket capital can be produced only in the market.

## IV. An Integrated Model

We have discussed how two extended versions of the standard neoclassical growth model can account for cross-country income differences, based on reasonable differences in policies or institutions that act as barriers to capital accumulation: the model augmented to include intangible capital, and the model augmented to include household production. These approaches are not mutually exclusive, however, and in this section we briefly discuss the implications of including both intangible capital and household production in the same model. We will not present the equations explicitly, since it should be clear how one would combine the two; we simply report the results.

Table 7 presents information for an integrated structure similar to that of table 4 for the home-production model without intangible capital. Here, we vary intangible capital's share, $\Theta_z$, then choose $\Theta_k$ to match the ratio of measured investment to measured output, and choose $\varepsilon$ to generate $y^*/y^*_{US} = 1/10$ with $\pi = 4$. For each $\Theta_z$, the table reports $\Theta_k$, $\varepsilon$, and several other statistics. For the sake of illustration, we set $\phi$ near zero. As the importance of intangible capital increases (larger $\Theta_z$), less importance must be assigned to home production (lower $\varepsilon$) to generate $y^*/y^*_{US} = 1/10$. Conversely, the more importance one is willing to assign to household production, the less one must rely on intangible capital and, hence, the less unmeasured investment one has to accept. For example, suppose $\varepsilon = 1/2$, which is not far from estimates for the United States. In this case, $\Theta_z = 0.3$ generates a ratio of measured outputs of $y^*/y^*_{US} = 1/10$ and implies that unmeasured investment is 30 percent of measured output. It also implies that the ratio of true output is between 0.20 and 0.26, depending on how home-produced output is priced. Finally, the barrier of $\pi = 4$ entails a large welfare cost: an agent would have to receive an additional 93 percent of his consumption to induce him not to move to an undistorted economy.

## V. Evidence

We have demonstrated that, compared to the model without home production, the model with home production can generate much larger differences in measured output, comparable differences in true output, and smaller differences in welfare. The home-production model also has some implications that differ from standard models, namely, that hours of market work will be lower in poor economies. In this section, we discuss some evidence relating to these implications.

The prediction that individuals in poorer economies devote less time to market work is straightforward; however, it is not easy to test using conventional data sources, because these countries do not measure hours of market work in a systematic fashion. The International Labor Office publishes statistics on participation rates for a large number of countries, but this is clearly different from hours of market work. In fact, participation rates have very little meaning in the poorest countries, where more than 80 percent of the population may be engaged in agriculture, much of which is subsistence farming (and thus better characterized as home rather than market production). Therefore, one must be somewhat resourceful in evaluating this prediction of the model.

Mueller (1984) uses the *Rural Income Distribution Survey* in Botswana. These data cover agricultural workers and are constructed from time diaries in which interviewers asked respondents to account for their time during the previous day. Interviews took place five times during the year on various days of the week. The survey included roughly 4,600 individuals best described as subsistence farmers. The data present percentages of total time devoted to several activities, with 12 hours per day as the base. Time is allocated to each of the following activities: crop husbandry, animal husbandry, wage labor, trading/vending /processing, hunting/gathering, repairing/new building, fetching water, child care, housework, schooling, leisure, and a few miscellaneous categories. The findings are striking. For males, only about 10 percent of working time is accounted for by wage labor, and for females, the figure is closer to 2 percent. Kirkpatrick (1978) finds similar time allocations in a survey of the rural sector in Melanesia. In 84 hours per week of daylight, the average adult in five Melanesian villages spent 19.7 hours on all phases of agriculture. A good portion of the remaining waking hours were spent spinning, weaving, gathering and processing fuels and food, metalworking, dressing and tanning leather, manufacturing and repairing tools, and fence repairing, to name a few. Transportation, recreational, religious, financial, and insurance services are also provided in the household sector.

One measurement issue that must be addressed is that many poor countries make some attempt to impute in their GNP accounts amounts to cover own-use agriculture and, in some cases, house building. Blades (1975) discusses these issues in detail and finds that in some countries, the imputed value of subsistence activities is as high as 40 percent of base-level GNP, although the average for the African countries in his sample is 10 percent–15 percent. However, it is important to note that virtually no attempt is made to impute values for any services that may be produced in the nonmarket sector, and it seems likely that this is a sizable omission. For instance, it seems more than likely that care for children and the elderly, or financial and social services, just to name two examples, are provided to a relatively greater extent outside the formal market in poorer countries.

The poorest countries in the world also have a great deal of economic activity which could be classified as illegal or informal, and while this type of activity may not fit perfectly into the explicit home-production model analyzed above, it is in the same spirit. MacGaffey et al. (1991) report an estimate of total output in Zaire which includes black-market goods and services as well as goods and services produced for self-consumption that is three times larger than output in the official national accounts. They estimate the size of the black-market economy alone for other African nations between two-thirds and one-third of reported output. Important sources of these estimates are household consumption surveys. These surveys show that households consume much more than they earn in wages and salaries. MacGaffey et al. summarize a 1986 survey for households in Kinshasa, Zaire, showing that households consumed more than twice as much as they reportedly earned in wages and salaries.

Young (1994) documents large increases in market participation rates for workers in his study of the economic miracles of East Asia: Taiwan, Singapore, South Korea, and Hong Kong (see also Pack [1988]). Over the period 1966–90, the participation rate increased from 0.38 to 0.49 in Hong Kong; 0.27 to 0.51 in Singapore; 0.27 to 0.36 South Korea; and 0.28 to 0.37 in Taiwan. Data for hours per worker are not reported, but Young claims that only in Hong Kong have hours per worker declined. Such increases are consistent with the home-production model. It should be noted, however, that even in a model in which hours are independent of distortions along the balanced growth path, hours will typically change along the transition from one balanced growth path to another, and so more time must elapse before these cases constitute definitive evidence.

One more piece of evidence is contained in Kuznets (1960) concerning a related but distinct implication of the analysis in the previous section. In models in which balanced-growth-path hours of market work are not affected by the policies under consideration, differences in income are proportional to differences in average productivity in the market sector. As we noted following table 5, however, the home-production model predicts that differences in the average productivity of market labor are much smaller than differences in income. Kuznets presents data for a sample of 33 countries and finds that differences in the average product of labor in manufacturing are smaller across rich and poor countries than are differences in incomes. In contrast, differences in the average product of labor in agriculture are greater than differences in income. One interpretation of this finding that is consistent with our model is that productivity in agriculture in poor countries is biased downward by systematic overmeasurement of labor input. This follows from the tendency for all rural workers to be counted as agricultural workers, without any attempt to measure hours devoted to agriculture. If, in fact, substantial amounts of time are devoted to nonmarket activities, the average product of labor will be understated. Mueller (1984), for example, reports that workers on commercial farms work more hours per day than workers on noncommercial farms. Livingstone (1981) reports that the *Survey of Rural Workers* in Kenya shows rural workers devote only about half as much time to agriculture per week than the standard workweek in industry.

In summary, while there are some serious measurement issues that deserve further study, and while we would obviously like to have more and better data, it seems that the information we do have supports explicitly incorporating household production into models of economic development.

## VI. Conclusions

Many economists have suggested that an important difference between rich and poor countries is the fraction of economic activity in formal versus informal markets. Most recent work that tries to account for income differences across countries abstracts from this feature. In this paper, we have argued that explicitly incorporating this feature into an otherwise standard model may enhance the model's ability to account for the data. In our model, policies which decrease the incentive to accumulate capital also lead to a substitution of economic activity away from the market sector and into the household-production sector. Our analysis suggests that this channel may be quantitatively important.

An implication of our analysis is that poor countries are relatively not as poor as published measures of income would indicate. However, we are not arguing that poor countries are just like rich countries except that less economic activity is measured: even when home production plays a big role, we found that poorer countries are still substantially worse off in terms of welfare.

To the extent that the output of goods and services outside formal markets is poorly measured, it is difficult to find direct evidence to support our framework. However, the model has some predictions that are consistent with empirical findings. For example, Young (1994) reports that labor participation rates increased substantially in each of the four "Asian tigers" during their periods of high growth. Also, Kuznets (1960) reports that international productivity differences are greatest in agricultural and least in manufacturing, which is consistent with our approach, if one views measurement of labor input in manufacturing to be higher quality. Finally, direct evidence from time diaries in sub-Saharan Africa supports the finding that individuals spend a relatively small fraction of their time in formal market activities. While more empirical work needs to be done to corroborate these findings, we conclude that it may be important to explicitly model the nonmarket sector in the context of studying economic development.

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