Progressive Second Price Auctions with Elastic Supply for PEV Charging in the Smart Grid

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Abstract—**Recent years have seen increasing deployment of Plug-in Electric Vehicles (PEVs) for personal transportation, which can lead to energy cost savings as well as reduce our carbon footprint. However, the bursty nature of PEV demand implies that the aggregate PEV load can impart significant stress on the distribution grid unless PEV charging is coordinated through efficient control mechanisms. In this paper, we study the energy supplier's problem of selling energy to the PEVs – while buying the same from the generators (market) – through an auction. In this context, we analyze the properties of an *elastic-supply Progressive Second Price (es-PSP)* auction mechanism, which requires each PEV agent to submit a desired energy quantity and a per-unit willingness-to-pay value. We establish that social efficiency is attained at Nash equilibria, and PEV agents acting in self-interest have no incentive to untruthfully declare their willingness-to-pay value for the quantity they choose to declare. We also validate some of our theoretical results through simulations in a specific distribution network scenario.**

I. INTRODUCTION

The deployment of Plug-in Electric Vehicles (PEVs) has been increasing steadily in recent years. PEVs (which include hybrid vehicles or PHEVs) draw their energy in full or part from the electric grid, charging their batteries at times they are "plugged in". Use of PEVs for personal transportation can not only reduce greenhouse gas emissions, but also result in lowering our individual energy bills. Large-scale deployment of PEVs could cause major overloading problems in the distribution grid, however, due to the PEVs typically connecting to the grid and attempting to charge all at the same times. This motivates the need for electricity aggregators or utilities to devise smart charging protocols that would provide incentives to PEV owners (agents) to spread their energy demand over time, and preferably charge at times when the grid is underloaded [1]-[3]. A growing body of literature has attempted to solve the problem of optimal scheduling of PEV charging in the smart grid through different techniques like dynamic programming, sequential quadratic optimization, queuing theory, game theory and other heuristic methods [4]-[12].

Since PEVs can differ in terms of their charging time constraints and energy needs, it is desirable that energy is given to the PEVs based on their needs (or valuations) and constraints, while at the same time minimizing the cost on the electric grid due to the PEV load. This goal can be posed as that of maximizing the *economic surplus* in the grid, where the economic surplus (or *social value*) of a charging solution (schedule) corresponds to the aggregate valuation of the PEV agents for the energy supplied to them, minus the total cost of supplying the energy. It is worth noting that the cost of energy supply (which can partly reflect the generation cost of energy) can differ in time, depending on the aggregate load at that time, and minimizing the total supply cost also tends to flatten the peaks in the total load curve.

The valuations of the PEV agents (users) being private information, there is a need for designing mechanisms that would result in the PEV agents declaring (implicitly or explicitly) their valuation functions truthfully, so that the goal of maximizing the social valuation (or the "social optimality") of the charging schedule can be realized. This can be attained through the classical *Vickrey-Clark-Groves (VCG)* auction mechanism [13] that requires agents participating in the auction to pay according to the *social opportunity cost* that the agent imposes on the system. However the high message complexity of declaring the entire valuation function by the agents is a deterrent in applying the VCG mechanism in practice. These have led researchers to look into mechanisms where the bid message complexity is limited [16], [17], [18], [19], mostly in the communications/networking application context.

The auction mechanism for PEV charging that we analyze in this paper is based on the *Progressive Second Price (PSP)* auction mechanism proposed and analyzed for a single divisible good by Semret and Lazar in [15], [16]. In this mechanism the agents are required to declare a simple two-dimensional bid: the amount of energy it wants to obtain and the per-unit price it is willing to pay for the energy desired. Based on this bid, the aggregator computes the optimal charging schedule for the PEVs. The payments made by the agents are "VCG like", in that they capture the "externality" that the agent causes to the system through its own presence in the auction.

The contributions of our work are summarized as follows. Under the reasonable assumptions that user (PEV agent) valuation functions are increasing and strictly concave, and the supply cost curves are increasing and strictly convex, we show the existence of an *efficient* (or "socially optimal") Nash equilibrium of the game implied by the chosen auction mechanism. Secondly, we show that all Nash equilibria of this game must result in efficient energy allocation. Finally, we show that given a particular bidding strategy of the other agents, and whether the system is in equilibrium or not, a PEV agent acting in self-interest does not have any incentive for untruthfully declaring its willingness-to-pay value for the quantity of energy it wishes to obtain.

Existing literature has analyzed the properties of the PSP

mechanism for a *fixed* amount of resources [16], [17], which we have applied to the PEV charging problem in our recent work [14]. In the current work, however, we consider the case of *elastic* supply (energy resource) which provides a model that is better grounded in practical realities of the PEV charging context. We not only provide parallels of the full suite of results in [16] and a key result in [17] in the PEV charging context, but also obtain stronger results by taking into account the elastic supply model (instead of the fixed resource model). In particular, we show that the energy allocation at *all* Nash equilibria of the auction mechanism is efficient, a result that does not hold for the fixed resource model. We also provide a new and fairly general analysis technique that combines modeling via "ramp functions" and using subgradient optimality conditions for analyzing the PSP mechanism. Recently, Suli *et al.* [24] have studied the PSP auction mechanism for a model and application that is closely related to ours. However, we our model is more general in that it considers heterogeneous charging (time) constraints across PEVs, and we prove two additional, important results (Propositions 2 and 3), which have not been shown in [24]. Furthermore, we also provide a different and general analysis method which will likely be useful in analyzing the PSP auction in broader classes of network models and convex environments. More detail on our contributions with respect to existing work on the analysis of the PSP mechanism is provided in Section II-C.

The remainder of the paper is organized as follows. Section II describes the system model we analyze for the rest of the paper. Section III describes the *es*-PSP (PSP with *elastic supply*) auction mechanism that we analyze, and then provides our theoretical analysis and results. Section IV describes the results of a limited simulation study exploring/validating some of the properties of the *es*-PSP mechanism.

II. SYSTEM MODEL

Consider an auction window comprising of T time slots, denoted by $\mathcal{T} = \{1, 2, ..., T\}$. Let $\mathcal{K} = \{1, 2, ..., K\}$ be the set of all PEVs in the distribution network under consideration. The set of charging constraints (preferences) can differ across PEVs; for PEV k it is given by a set $T_k \subseteq \mathcal{T}$ at which it can charge (i.e., it is connected to the grid). Also, let each PEV $k \in \mathcal{K}$ have a remaining battery capacity of α_k at the start of the auction window. We assume that the non-PEV based inelastic demand is given by D_t for $t = 1, \ldots T$. The cost of supplying electricity in any time slot t, denoted by C_t, is assumed to be an *increasing, strictly convex* function of the total load (sum of PEV load and non-PEV load) in that time slot. The supply cost at time t is thus given by $C_t(D_t + \sum_{k=1}^{K} q_k^t)$, where q_k^t represents the energy allotted to PEV agent k in time slot t. Let the vector $\mathbf{q}_k = (q_k^1, q_k^2, ..., q_k^T)$ represent the charging schedule for PEV k. We call $\mathbf{q} = (\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_K)$ the *schedule vector* for all PEVs in a feasible solution. Let Q_k represent the total energy received by the PEV k over all accessible time slots. We call $\mathbf{Q} = (Q_1, Q_2, ..., Q_K)$ the *allocation vector* for all PEVs. Note that $Q_k = \sum_{t \in T_k} q_k^t$, and therefore $\mathbf{Q}$ can be expressed as $\mathbf{Q} = \mathbf{Mq}$, where $\mathbf{M}$ is an appropriately defined matrix of dimension $K \times KT$ containing

either 0 or 1 as elements. The total energy allocated for PEV charging at time t, denoted by Q^t, is then expressed as $Q^t = D_t + \sum_{k \in \mathcal{K}} q_k^t$.

Every PEV (agent) k is associated with a (privately known) *valuation function* $v_k(\cdot)$ for the energy obtained. We assume $v_k(0) = 0$, and $v_k(Q_k)$ is *increasing, twice differentiable* and *strictly concave* in Q_k in the range $[0, \alpha_k]$.
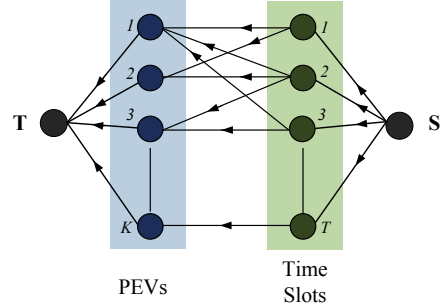

Fig. 1. Graph theoretic model for auction of electricity to PEVs in the different time slots covered by the auction window.

A. Graph-theoretic representation

The PEV charging problem across multiple contiguous time slots can be translated into a graph theoretic setting as shown in Figure 1, where the (possibly incomplete) bipartite graph comprises of two sets of nodes: one representing the set of PEVs $\mathcal{K}$ and the other representing the set of time slots $\mathcal{T}$. Note that the edges correspond to the timing constraints of the PEVs, i.e. an edge (k, t) exists in the bipartite graph iff $t \in T_k$. Also note that the direction of the edges as shown in the figure is same as that of the actual energy flow. (A source node **S** and a sink node **T** have been added for convenience, and does not have any physical significance.)

B. VCG mechanism for PEV charging with elastic supply

The socially optimal goal of PEV charging is that of maximizing the "economic surplus", defined as the total valuation of the energy allocated minus the total cost of supply, subject to the the charging constraints. This is expressed as,

$$\max \quad S(\mathbf{q}) = \sum_{k \in \mathcal{K}} v_k\left(\sum_{t \in T_k} q_k^t\right) - \sum_{t \in \mathcal{T}} C_t\left(D_t + \sum_{k \in \mathcal{K}} q_k^t\right), \quad (1)$$

$$s.t. \quad \sum_{t \in T_k} q_k^t \leq \alpha_k, \ k \in \mathcal{K}, \quad (2)$$

$$q_k^t = 0, t \notin T_k; \quad q_k^t \geq 0, t \in T_k; \ k \in \mathcal{K}. \quad (3)$$

Let $\mathcal{D}_k = \{q_k^t, t = 1, \ldots, T, \text{subject to (2) and (3)}\}$ be the feasibility constraint set for the charging of PEV k. Then all feasible schedule vectors (for all PEVs) that satisfy (2) and (3) must be contained in $\mathcal{D} = \mathcal{D}_1 \times \mathcal{D}_2 \times \ldots \times \mathcal{D}_K$. Therefore, (2) and (3) can be replaced by a single constraint $\mathbf{q} \in \mathcal{D}$. Specifically, the VCG mechanism requires the *user* (PEV agent) k to make a payment based on the *social opportunity cost*, expressed as, $\pi_k = \sum_{j \in \mathcal{K} \setminus \{k\}} \left(v_j(Q_{j,-k}^*) - v_j(Q_j^*) \right) +$

$\sum_{t \in \mathcal{T}} \left(C_t(Q^{*,t}) - C_t(Q_{-k}^{*,t}) \right)$. Here Q_j^* ($Q^{*,t}$) represents the energy allocation to user j (total load at time t, resp.) under socially optimal energy allocation (one that solves (1)-(3)) when all users (including user k) are present. Also $Q_{j,-k}^*$ ($Q_{-k}^{*,t}$) represents the energy allocation to user j (total load at time t, resp.) under socially optimal energy allocation when user k is absent from the auction. The VCG payment policy ensures that rational users (acting in self-interest) do not have any incentive to declare their valuation functions untruthfully.

There are practical difficulties however in implementing a VCG mechanism as is, since it requires users to declare their entire valuation functions. Firstly, users (PEV agents in our case) themselves may not be fully aware of their entire valuation functions. Furthermore, communicating the entire valuation function (which is a continuous function) within a close approximation degree requires very high message complexity. These factors motivate the need to look at VCG-like mechanisms that require users to submit their bids in some simple form that is both convenient to users and requires low message complexity. When the bid space is restricted, however, the challenges are (i) How can the auction mechanism be designed so that rational users do not have any incentive to declare the bids untruthfully? ii) How can the socially optimal allocation be computed based on the submitted bids?

C. PSP auctions and Related work

When the bid message complexity is restricted, dominant strategy implementation of socially optimal allocation is not possible any more in general settings [16], [19], [18], [17]. The *Progressive Second Price (PSP)* auction mechanism proposed in [15], [16], which requires each user to submit a 2-dimensional (quantity and price) bid however retains incentive compatibility in a limited yet meaningful sense (i.e., in the price dimension, for the chosen quantity), while attaining social optimality at Nash equilibrium. The simplicity of the PSP mechanism is also appealing, which has led to several extensions and applications of this approach in multiple contexts [22], [21], [20], [17], [14], [23]. In this paper, we analyze the PSP mechanism in the context of PEV charging, taking into account the elasticity of the energy supply; we naturally term it as *elastic-supply Progressive Second Price (es-PSP)* auction. The consideration of heterogeneous charging time constraints across users makes the analysis of the PSP mechanism substantially more complex than that for a single-resource model [15], [16], [22], [23].

While our bipartite network model is still a special case of the more general network model in [17] (which however considers fixed quantities of resources being auctioned off), we provide a broader and stronger set of results than [17] for our model, while exploiting the elasticity of the resource and strict convexity of the associated cost curve. In particular, while Proposition 1 of our paper reflects a similar result shown in [17], results similar to Proposition 2 and 3 have not been claimed in [17]. In fact, Proposition 2 does not seem to hold for the fixed-quantity resource (inelastic supply) model (see Example 1 in [17]), or likely requires additional restrictions on the equilibrium such as all users bid *truthfully* (see Proposition 3 in [16]).

PSP auctions for scheduling of PEV (more generally, elastic) loads has been recently considered in [23], [24], which also consider flexible resource supplies associated with convex costs functions. Compared to our PEV charging model, [23] considers a single time slot (single resource); [24] considers multiple time slots (resources) but all PEVs having homogeneous charging time constraints. We not only consider a more general and realistic model where PEVs can have different charging (time) constraints (reflecting the true multi-resource aspect of the model where each agent can access/use only a certain set of resources), but also prove stronger results. In particular, the main result in [24] parallels Proposition 1 of our paper, but for homogeneous charging time constraints; in addition, we prove two important results (Propositions 2 and 3), parallels of which have not been shown before for this model and context.

Finally, while our consideration of the PEV charging problem implies that our network model is a (possibly incomplete) bipartite graph (Figure 1), most of our analysis does not explicitly use the specific network model, but relies more on general convexity assumptions/conditions. Therefore, we believe our theoretical results (as outlined in the next section) would extend to a broader set of network models (topologies), although an exploration of that is deferred to future work.

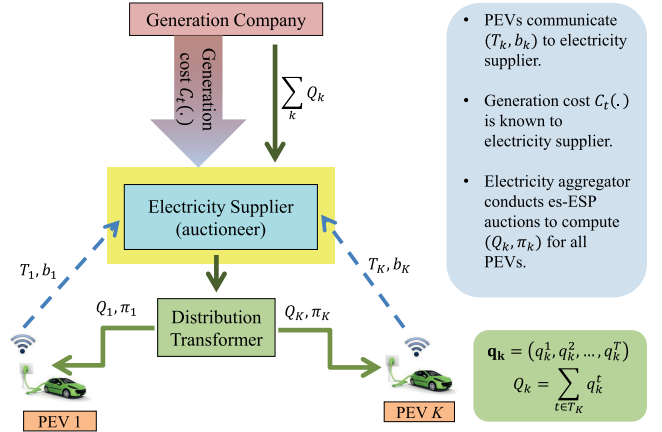### III. ELASTIC-SUPPLY PROGRESSIVE SECOND PRICE (*es*-PSP) AUCTION MECHANISM

Fig. 2. Schematic diagram of *es*-PSP auction mechanism.

A. Auction mechanism and Dispatch rules

We first describe the auction mechanism, based on the PSP auction as proposed in [15], [16], but extending it to our context by considering heterogeneous charging time constraints of the PEVs and elastic supply cost curves. A schematic diagram describing our elastic-supply PSP (*es*-PSP) auction mechanism is provided in Figure 2. In this mechanism, the bid of any user (PEV agent) $k \in \mathcal{K}$ is given by $b_k = (a_k, p_k)$ where a_k is the amount of energy demanded by user k, and p_k is the price per unit of electricity it is willing to pay. Let $\mathcal{B}_k \subseteq \mathbb{R}_+^2$ to be the set of all possible bids by user k. Let $\mathcal{B}_k^T \subset \mathcal{B}_k$ denote the set of all *truthful* bids where the bid price reflects the *marginal valuation*

of the bid quantity, i.e. the 2-d bid is of the form $(a_k, v'_k(a_k))$. A general bid vector (truthful or untruthful) $\mathbf{b}$ is then defined as $\mathbf{b} = (b_1, b_2, \ldots, b_K)$. Also let $\mathbf{b}_{-k}$ denote the set of bids of all users other than k, i.e. $\mathbf{b}_{-k} = (b_1, b_2, \ldots b_{k-1}, b_{k+1}, \ldots b_K)$. In our *es*-PSP mechanism, once the bid b_k and time constraints T_k are reported to the auctioneer (PEV load serving entity in our case), it solves the following optimization problem for optimal allocation of the energy for charging the PEVs,

$$\max \; \bar{S}(\mathbf{q}) = \sum_{k \in \mathcal{K}} \underbrace{\Big(\sum_{t \in T_k} q_k^t \Big) p_k}_{l_k} - \sum_{t \in \mathcal{T}} \underbrace{C_t(D_t + \sum_{k \in \mathcal{K}} q_k^t)}_{g_t}, \quad (4)$$

$$s.t. \quad \sum_{t \in T_k} q_k^t \le a_k, \; k \in \mathcal{K}, \quad (5)$$

$$\mathbf{q} \in \mathcal{D}. \quad (6)$$

Note that the constraint (6) just captures the feasibility constraints (2)-(3).[1] The part $\sum_{k \in \mathcal{K}} l_k$ represents the revenue obtained by the auctioneer (supplier, representing the EV load serving entity) through sale of the resource (electric energy) to the users (PEV agents). The part $\sum_{t \in \mathcal{T}} g_t$ represents the cost incurred by the auctioneer (PEV load serving entity) in purchasing the same resource from the energy generation companies (market). In effect, the objective function $\sum_{k \in \mathcal{K}} l_k - \sum_{t \in \mathcal{T}} g_t$ can be viewed as the profit of the auctioneer based on the bids declared by the users. Similar in principle to the VCG mechanism, payments that need to be made by any user k reflects its "social opportunity cost", expressed as,

$$\pi_k = \sum_{j \in \mathcal{K} \setminus \{k\}} \Big(\tilde{Q}_{j,-k} - \tilde{Q}_j \Big) p_j + \sum_{t \in \mathcal{T}} \Big(C_t(\tilde{Q}^t) - C_t(\tilde{Q}_{-k}^t) \Big).$$

Here $\tilde{Q}_j$ ($\tilde{Q}_{j,-k}$) represents the energy allocation to user j when all users including (except) user k are present in the auction. The time-dependent load terms $\tilde{Q}^t$ and $\tilde{Q}_{-k}^t$ are similarly defined.

For a given schedule (and corresponding payments) as computed by the *es*-PSP auctioneer, the utility of any user k is a function of its own bid as well as the bids of others, and is expressed as

$$u_k(b_k, \mathbf{b}_{-k}) = v_k(b_k, \mathbf{b}_{-k}) - \pi_k(b_k, \mathbf{b}_{-k}) \quad (7)$$

$$= v_k(\tilde{Q}_k) - \sum_{j \in \mathcal{K} \setminus \{k\}} \Big(\tilde{Q}_{j,-k} - \tilde{Q}_j \Big) p_j$$

$$- \sum_{t \in \mathcal{T}} \Big(C_t(\tilde{Q}^t) - C_t(\tilde{Q}_{-k}^t) \Big) \quad (8)$$

$$= \overbrace{v_k(\tilde{Q}_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \tilde{Q}_j p_j - \sum_{t \in \mathcal{T}} C_t(\tilde{Q}^t)}^{U_k(b_k, \mathbf{b}_{-k})}$$

$$- \underbrace{\sum_{j \in \mathcal{K} \setminus \{k\}} \tilde{Q}_{j,-k} p_j + \sum_{t \in \mathcal{T}} C_t(\tilde{Q}_{-k}^t)}_{h_k(\mathbf{b}_{-k})}. \quad (9)$$

Note that in (9), the term $h_k(\mathbf{b}_{-k})$ depends only on the bids of the other users. Therefore, given the bids of others $\mathbf{b}_{-k}$, a rational user k would look towards choosing its bid b_k so as to maximize the term $U_k(b_k, \mathbf{b}_{-k})$ subject to (5) and (6), and the given tie-breaking rule (as discussed later). Since this term depends on the allocation of the *es*-PSP mechanism (when all users including k is present), with slight abuse of notation we refer to this term later in this paper simply as a function of the corresponding schedule ($\tilde{\mathbf{q}}$) or allocation vector ($\tilde{\mathbf{Q}}$), as $U_k(\tilde{\mathbf{q}})$ or $U_k(\tilde{\mathbf{Q}})$. In this paper, we look at the properties of the game in which each user k, who is assumed to be aware of the *es*-PSP payment and allocation policy, and the bids and constraints of the other users, attempts to choose its 2-d bid b_k so as to maximize $U_k(b_k, \mathbf{b}_{-k})$.

B. Preliminaries

Definition 1: For a two-dimensional bid $b_k = (a_k, p_k) \in \mathcal{B}_k$, an equivalent *ramp function* $\hat{w}_{k,b_k} : \mathbb{R}_+ \to \mathbb{R}_+$ is defined as $\hat{w}_{k,b_k}(x) = p_k \min(x, a_k)$.

Note: For any "truthful" bid $b_k \in \mathcal{B}_k^T$ with bid quantity a_k, the ramp function can be represented as $\hat{v}_{k,a_k}$ which is same as earlier, but with $p_k = v'_k(a_k)$, i.e. $\hat{v}_{k,a_k}(x) = v'_k(a_k) \min(x, a_k)$. This is illustrated in Figure 3.
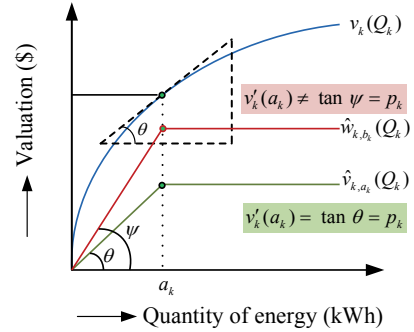

Fig. 3. Ramp function representation of 2-d bid b_k: here $\hat{w}_{k,b_k}(Q_k)$ represents a (possibly *untruthful*) bid, and $\hat{v}_{k,a_k}(Q_k)$ represents a *truthful* bid, corresponding to bid quantity a_k.

Note that replacing the terms l_k in the *es*-PSP allocation objective $\bar{S}(\mathbf{q})$ in (4) by their corresponding ramp functions, allows us to ignore (5) and optimize only with respect to (6), i.e. the feasibility constraints (2)-(3). It is easy to see that any schedule vector that optimizes $\bar{S}$ (with l_k's replaced by their ramp functions) under $\mathbf{q} \in \mathcal{D}$, will not realize an allocation vector $\tilde{\mathbf{Q}}$ such that $\tilde{Q}_k > a_k$ for any k. Since the supply cost functions are (strictly) increasing in the load, if $\tilde{Q}_k > a_k$ for any k, then the objective $\bar{S}(\mathbf{q})$ could be increased by adjusting the corresponding schedule vector $\tilde{\mathbf{q}}$ so as to reduce $\tilde{Q}_k$ to a_k. Due to the nature of the ramp functions, such adjustment would not change $\sum_{k \in \mathcal{K}} l_k$ but reduces $\sum_{t \in \mathcal{T}} g_t$ in (4) thus improving $\bar{S}$. Hence $\tilde{\mathbf{q}}$ cannot be optimal. Therefore, the energy allocation problem in the *es*-PSP mechanism can be equivalently expressed as

$$\max \; \bar{S}(\mathbf{q}) = \sum_{k \in \mathcal{K}} \hat{w}_{k,b_k}(\sum_{t \in T_k} q_k^t) - \sum_{t \in \mathcal{T}} C_t(D_t + \sum_{k \in \mathcal{K}} q_k^t), \quad (10)$$

$$s.t. \quad \mathbf{q} \in \mathcal{D}. \quad (11)$$

Note that constraints (5) are implied by the definition of the ramp functions, and therefore only constraint (6), i.e. $\mathbf{q} \in \mathcal{D}$ need to be accounted for in the optimization.

Recall from the analysis in (7)-(9) that each rational user seeks to maximize $U_k(b_k, \mathbf{b}_{-k})$ subject to (5) and (6) and the tie-breaking rule (which we will describe shortly). Expressed in terms of ramp functions, this becomes equivalent to maximizing $U_k(\tilde{\mathbf{q}})$ (where $\tilde{\mathbf{q}}$ is a schedule vector resulting from the *es*-PSP auction for the bid vector $\mathbf{b}_k$), given by

$$U_k(\tilde{\mathbf{q}}) = v_k(\tilde{Q}_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,b_j}(\tilde{Q}_j) - \sum_{t \in \mathcal{T}} C_t(\tilde{Q}^t), \quad (12)$$

where $\mathbf{q}$ satisfies $\mathbf{q} \in \mathcal{D}$.

Finally, note that the allocation resulting from the *es*-PSP mechanism may not be unique, when two users bid the same price. To resolve this, we assume that the auctioneer utilizes a *tie-breaking* rule which is known to every user participating in the auction. This tie-breaking rule allows the auctioneer to determine a unique allocation for the users even if some of the price bids are equal. Any fixed tie-breaking rule works for our purpose; for definiteness, we assume that the users submitting the the same price bids are prioritized in increasing order of their indices: a higher PEV index gets the user (PEV agent) a higher priority in allocation.

Lemma 1: Assuming a fixed tie-breaking rule, the allocation vector $\mathbf{Q}$ in any solution of the *es*-PSP mechanism is unique.

Proof: Consider a given bid vector $\mathbf{b}$, for which $\hat{\mathbf{q}}$ is an optimal schedule vector (possibly non-unique) resulting from (4)-(6) (or equivalently, (10)-(11)) and the fixed tie-breaking rule. We want to show that the allocation vector $\hat{\mathbf{Q}} = \mathbf{M}\hat{\mathbf{q}}$ is unique, even though the optimal schedule vector $\hat{\mathbf{q}}$ may be non-unique.

It is easy to see from the strict convexity of $C_t(Q^t)$ in Q^t, that $\hat{Q}^t \ \forall t \in \mathcal{T}$ is unique for all optimal schedule vectors $\hat{\mathbf{q}}$. Therefore, it follows that the total flow (of energy) given by $f(\hat{\mathbf{q}}) = \sum_{t \in \mathcal{T}} \hat{Q}^t$ is a constant under any optimal schedule.

Now, for the sake of contradiction let us assume that $\hat{\mathbf{Q}}$ is not unique i.e. there exists some $\check{\mathbf{Q}}$ which is realized by a scheduling vector $\check{\mathbf{q}}$, such that $\hat{\mathbf{Q}} \neq \check{\mathbf{Q}}$. Let us order the users (from top to bottom in the bipartite graph representation) in increasing order of their price bids; users with the same price bids are ordered in the increasing order of their indices. Let us renumber the indices of the users (PEVs) now according to this new order. Let m be the smallest index user (in this new order just defined) in which the two allocations differ. Without loss of generality, let us assume $\hat{Q}_m > \check{Q}_m$. Since $f(\hat{\mathbf{q}}) = f(\check{\mathbf{q}})$ (as argued before, the total energy allocation is the same in any optimal schedule), there must exist an index $r > m$, with $\check{Q}_r > \hat{Q}_r$, such that we can direct some positive flow $\delta > 0$ flow from m to r in the solution $\hat{\mathbf{q}}$ (see Figure 4). (Note that the flow $\check{\mathbf{q}} - \hat{\mathbf{q}}$ can be resolved into a set of *path flows*, each of which start and end at a PEV node, such as the one shown in Figure 4.) Note that r is such that either (i) $p_r > p_m$, or (ii) $p_r = p_m$ and user r has a higher priority than m according to the tie-breaking rule. In case (i) the flow redirection from m to r improves the objective $\bar{S}(\hat{\mathbf{q}})$; in case (ii) the solution

$\hat{\mathbf{q}}$ could not have satisfied the tie-breaking rule. In either case, we arrive at a contradiction to our assumption that $\hat{\mathbf{q}}$ optimizes $\bar{S}$ subject to the tie-breaking rule, thereby proving the result. ∎
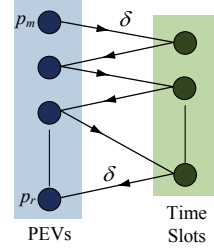

Fig. 4. Flow redirection that i) either increases the objective value $\bar{S}$, or ii) directs flow from a low priority user to a higher priority user as defined by the tie-breaking rule.

C. Relation between Nash equilibrium and Social optimality

In this section we provide the main results stating the relationship between the Nash equilibrium and Social optimality of the *es*-PSP mechanism. Let $\mathbf{q}^* = (\mathbf{q}_1^*, \mathbf{q}_2^*, ..., \mathbf{q}_K^*)$ be the socially optimal schedule vector that realizes a socially optimal allocation vector $\mathbf{Q}^* = (Q_1^*, Q_2^*, \ldots, Q_K^*)$ Note that the optimal schedule $\mathbf{q}^*$ can be non-unique, but due to the strict concavity of $v_k(Q_k)$ in Q_k, it follows that the optimal allocation vector $\mathbf{Q}^*$ is unique.

Proposition 1: Truthful bidding at the socially optimal allocations is a Nash equilibrium of the *es*-PSP mechanism, i.e., the bids $b_k^* = (Q_k^*, v_k'(Q_k^*))$ for $k \in \mathcal{K}$, constitute a Nash equilibrium of the *es*-PSP mechanism.

Proof: Consider the bid vector $\mathbf{b}^* = (b_1^*, b_2^*, \ldots, b_K^*)$ where $b_k^* = (Q_k^*, v_k'(Q_k^*)) \ \forall k \in \mathcal{K}$. The allocation problem solved by the auctioneer in this case is

$$\max \ \hat{S}(\mathbf{q}) = \sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(Q_k) - \sum_{t \in \mathcal{T}} C_t(Q^t), \quad (13)$$

subject to $\mathbf{q} \in \mathcal{D}$ (and the tie-breaking rule), where $\hat{v}_{k,Q_k^*}(Q_k) = v_k'(Q_k^*) \min(Q_k, Q_k^*)$ is the truthful ramp function corresponding to the socially optimal allocation for user k. Since $\mathbf{q}^*$ optimizes $S(\mathbf{q})$ in (1) subject to $\mathbf{q} \in \mathcal{D}$, from [25] we can say,

$$\partial S(\mathbf{q}^*) \cap \Gamma^+(\mathbf{q}^*) \neq \phi. \quad (14)$$

Here, $\partial S(\mathbf{q}^*)$ is the set of sub-gradients of the function $S(\mathbf{q})$ at $\mathbf{q} = \mathbf{q}^*$; in our case, since $S(\mathbf{q})$ is differentiable for all $\mathbf{q}$, $\partial S(\mathbf{q}^*)$ will just consist of the gradient of $S(\mathbf{q})$ at $\mathbf{q} = \mathbf{q}^*$. Also, $\Gamma^+(\mathbf{q}^*)$ represents the conjugate to the cone of feasible directions in $\mathcal{D}$, at the point $\mathbf{q} = \mathbf{q}^*$. We can see that the only difference in (1) and (13) is that $v_k(Q_k)$ has been replaced by $\hat{v}_{k,Q_k^*}(Q_k)$. Also, note that $\hat{v}_{k,Q_k^*}(Q_k)$ is non-differentiable with respect to Q_k at Q_k^*. Further, the component corresponding to q_k^t (for any $t \in T_k$) in any sub-gradient of $\hat{S}(\mathbf{q})$ at $\mathbf{q} = \mathbf{q}^*$ is given by $\lambda v_k'(Q_k^*)$ for $0 \leq \lambda \leq 1$, which contains $v_k'(Q_k^*)$ ($\lambda = 1$) case. Thus, we can argue that,

$$\partial S(\mathbf{q}^*) \subset \partial \hat{S}(\mathbf{q}^*). \quad (15)$$

From (14) and (15), we can write,

$$\partial \hat{S}(\mathbf{q}^*) \cap \Gamma^+(\mathbf{q}^*) \neq \phi, \quad (16)$$

which implies that $\mathbf{q}^*$ also optimizes $\hat{S}(\mathbf{q})$ subject to $\mathbf{q} \in \mathcal{D}$.

Now for any $\mathbf{q} = \bar{\mathbf{q}}$ that optimizes $\hat{S}(\mathbf{q})$ subject to $\mathbf{q} \in \mathcal{D}$ and the tie-breaking rule, the corresponding allocation vector $\bar{\mathbf{Q}}$ is unique (from Lemma 1). We claim that $\bar{\mathbf{Q}} = \mathbf{Q}^*$. To see this, let us assume for the sake of contradiction, $\bar{\mathbf{Q}} \neq \mathbf{Q}^*$. Note that any optimal allocation $\bar{\mathbf{Q}}$ must satisfy $\bar{Q}_k \leq Q_k^*$, $\forall k \in \mathcal{K}$. Also note that $\hat{v}_{k,Q_k^*}(\bar{Q}_k) = v_k'(Q_k^*)\bar{Q}_k$ for all such allocations. Since $\hat{S}(\mathbf{q}^*) = \hat{S}(\bar{\mathbf{q}})$, we have

$$\sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(Q_k^*) - \sum_{t \in \mathcal{T}} C_t(Q^{*,t}) = \sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(\bar{Q}_k) - \sum_{t \in \mathcal{T}} C_t(\bar{Q}^t).$$
(17)

From the strict convexity of $C_t(\cdot)$, it follows that Q^t, $\forall t \in \mathcal{T}$ must be unique in any optimal solution. Therefore, $Q^{*,t} = \bar{Q}^t$, $\forall t \in \mathcal{T}$. Thus, $\sum_{t \in \mathcal{T}} C_t(Q^{*,t}) = \sum_{t \in \mathcal{T}} C_t(\bar{Q}^t)$. Thus,

$$\sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(Q_k^*) = \sum_{k \in \mathcal{K}} \hat{v}_{k,Q_k^*}(\bar{Q}_k).$$

$$\Rightarrow \sum_{k \in \mathcal{K}} v_k'(Q_k^*)(Q_k^* - \bar{Q}_k) = 0.$$
(18)

Since $v_k'(\cdot) > 0$ at all points (we have assumed the valuation functions to be (strictly) increasing), it follows that $Q_k^* = \bar{Q}_k$, $\forall k \in \mathcal{K}$. Thus, $\mathbf{Q}^* = \bar{\mathbf{Q}}$. This shows that given the bid vector $\mathbf{b}_{-k}^*$, i.e. when every other agent $j \in \mathcal{K} \setminus \{k\}$ bids the ramp function $\hat{v}_{j,Q_j^*}(\cdot)$, the allocation is $\mathbf{Q}^*$ provided agent k bids $\hat{v}_{k,Q_k^*}(Q_k)$.

Now from (12), given $\mathbf{b}_{-k}^*$, a rational (selfish) user k's objective is to maximize $U_k^*(\mathbf{q})$, given by

$$U_k^*(\mathbf{q}) = v_k(Q_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{v}_{j,Q_j^*}(Q_j) - \sum_{t \in \mathcal{T}} C_t(Q^t). \quad (19)$$

Comparing $U_k^*(\mathbf{q})$ and $S(\mathbf{q})$ (defined in (1)), we see that the only differences are the replacement of $v_j(Q_j)$ by $\hat{v}_{j,Q_j^*}(Q_j)$, $\forall j \in \mathcal{K} \setminus \{k\}$. Using similar arguments as provided earlier in this proof (when comparing the sub-gradients of $S(\mathbf{q})$ and $\bar{S}(\mathbf{q})$ at $\mathbf{q} = \mathbf{q}^*$), it follows that

$$\partial S(\mathbf{q}^*) \subset \partial U_k^*(\mathbf{q}^*). \quad (20)$$

From (14) and (20), we can write,

$$\partial U_k^*(\mathbf{q}^*) \cap \Gamma^+(\mathbf{q}^*) \neq \phi. \quad (21)$$

This shows that $U_k^*(\mathbf{q})$ is maximized at $\mathbf{q} = \mathbf{q}^*$ subject to (11) and the tie-breaking rule, provided $\mathbf{b}_{-k} = \mathbf{b}_{-k}^*$. Now suppose that given the bids of other users remains fixed at $\mathbf{b}_{-k}^*$, user k deviates by bidding b_k', which realizes in a schedule vector $\mathbf{q}'$ (and corresponding allocation vector $\mathbf{Q}'$), as a result of the *es*-PSP auction. Since $\mathbf{q}^*$ optimizes $U_k^*(\mathbf{q})$ subject to (11) and the tie-breaking rule, $U_k^*(\mathbf{q}') \leq U_k^*(\mathbf{q}^*)$. Thus we see that given $\mathbf{b}_{-k} = \mathbf{b}_{-k}^*$, agent k has no incentive to change its bid from $\hat{v}_{k,Q_k^*}$ or equivalently $b_k^* = (Q_k^*, v_k'(Q_k^*))$. Therefore the bid vector $\mathbf{b}^* = (b_k^*, \mathbf{b}_{-k}^*)$ is a Nash equilibrium of the *es*-PSP mechanism. ∎

Proposition 2 shows that truthful biding at socially optimal allocations is a Nash equilibrium of the *es*-PSP auction. From the proof of Proposition 2, it can also be seen that for this bidding strategy, each user k get the quantity Q_k^* that its asks for, i.e., the optimal energy allocation resulting from the

auction when the bid vector is $\mathbf{b}^*$ is $\mathbf{Q}^*$. This still keeps open the possibility of inefficient Nash equilibria. The next result however shows that the allocation at *any* Nash equilibria of the *es*-PSP mechanism is efficient.

Proposition 2: The allocation at any Nash equilibrium of the *es*-PSP mechanism is socially efficient.

Proof: Consider a bid vector $\check{\mathbf{b}}$ that is at Nash equilibrium, and let $\hat{w}_{k,\check{b}_k}(\cdot)$, $\forall k \in \mathcal{K}$ be the corresponding ramp functions. (We know from Proposition 1 that a Nash equilibrium to the *es*-PSP mechanism exists.) Let $\check{\mathbf{q}}$ to be a schedule vector resulting from the *es*-PSP mechanism for this bidding strategy, and the corresponding allocation vector be $\check{\mathbf{Q}}$.

Now from (12), given the bids of other users $\check{\mathbf{b}}_{-k}$, user k seeks to maximize $\check{U}_k(\mathbf{q})$, given by

$$\check{U}_k(\mathbf{q}) = v_k(Q_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,\check{b}_j}(Q_j) - \sum_{t \in \mathcal{T}} C_t(Q^t). \quad (22)$$

We first argue that $\check{U}_k(\mathbf{q})$ is maximized by $\check{\mathbf{q}}$, $\forall k \in \mathcal{K}$ subject to $\mathbf{q} \in \mathcal{D}$ and the tie-breaking rule. To see this, for sake of contradiction, suppose that for any $k \in \mathcal{K}$, $\check{U}_k(\mathbf{q})$ is not maximized at $\check{\mathbf{q}}$. In other words, there exists some schedule vector $\bar{\mathbf{q}} \neq \check{\mathbf{q}}$ that maximizes $\check{U}_k$ subject to $\mathbf{q} \in \mathcal{D}$ and the tie-breaking rule; let $\bar{\mathbf{Q}} (\neq \check{\mathbf{Q}})$ be the corresponding allocation vector. Let $\partial \check{U}_k(\mathbf{q})$ be the set of sub-gradients of the function $\check{U}_k(\mathbf{q})$ at $\mathbf{q}$, and $\Gamma^+(\mathbf{q})$ be the conjugate to the cone of feasible directions in $\mathcal{D}$ at $\mathbf{q}$. Then since $\mathbf{q} = \check{\mathbf{q}}$ optimizes $\check{U}_k(\mathbf{q})$, from [25] we can write (sub-gradient constrained optimality condition):

$$\partial \check{U}_k(\bar{\mathbf{q}}) \cap \Gamma^+(\bar{\mathbf{q}}) \neq \phi. \quad (23)$$

Now let user k unilaterally change its bid from $\check{b}_k$ to $\bar{b}_k = (\bar{Q}_k, v_k'(\bar{Q}_k))$, and let $\hat{v}_{k,\bar{Q}_k}(\cdot)$ be the corresponding (truthful) ramp function. We will show that user k gains for this deviation. Since the bids of the other users are kept fixed at $\check{\mathbf{b}}_{-k}$, allocation will be determined by the auctioneer by maximizing $\bar{S}(\mathbf{q})$ given by

$$\bar{S}(\mathbf{q}) = \hat{v}_{k,\bar{Q}_k}(Q_k) + \sum_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,\check{b}_j}(Q_j) - \sum_{t \in \mathcal{T}} C_t(Q^t), \quad (24)$$

subject to (11) and the tie-breaking rule. Note that the only difference in $\bar{S}(\mathbf{q})$ and $\check{U}_k(\mathbf{q})$ is the replacement of $v_k(Q_k)$ in $\check{U}_k(\mathbf{q})$ by $\hat{v}_{k,\bar{Q}_k}(Q_k)$ in $\bar{S}(\mathbf{q})$. Using arguments as before, thus we have, $\partial \check{U}_k(\bar{\mathbf{q}}) \subset \partial \bar{S}(\bar{\mathbf{q}})$. This fact and (23) gives

$$\partial \bar{S}(\bar{\mathbf{q}}) \cap \Gamma^+(\bar{\mathbf{q}}) \neq \phi. \quad (25)$$

This implies that $\bar{\mathbf{q}}$ also optimizes $\bar{S}(\mathbf{q})$.

From Lemma 1, we know that the corresponding allocation $\bar{\mathbf{Q}}$ is unique. This implies that when the other users' bids are remain fixed at $\check{\mathbf{b}}_{-k}$ and user k changes its bid to $\bar{b}_k = (\bar{Q}_k, v_k'(\bar{Q}_k)) \neq \check{b}_k$, the allocation resulting from the *es*-PSP auction must be $\bar{\mathbf{Q}}$, which improves $\check{U}_k(\mathbf{q})$ beyond its value at Nash equilibrium $\check{U}_k(\check{\mathbf{q}})$. This provides incentive for user k to change its bid from $\check{b}_k$, contradicting the fact that the bid vector $\check{\mathbf{b}}$ is at Nash equilibrium. Therefore, our supposition was wrong, implying that $\check{U}_k(\mathbf{q})$ is maximized at $\check{\mathbf{q}}$ for all $k \in \mathcal{K}$.

Define $C(\mathbf{q}) = \sum\limits_{t \in \mathcal{T}} C_t(Q^t)$. From the first order (necessary) conditions for optimality of $\check{U}_k(\mathbf{q})$ at $\mathbf{q} = \check{\mathbf{q}}$ along the direction of $\mathbf{q}_k$, and identifying the fact that $\sum\limits_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,\check{b}_j}(Q_j)$ in (22) is independent of $\mathbf{q}_k$, we can write

$$[\nabla_{\mathbf{q}_k} v_k(\check{Q}_k) - \nabla_{\mathbf{q}_k} C(\check{\mathbf{q}})]_{\mathcal{D}_k} = 0. \quad (26)$$

Note that (26) holds for all $k \in \mathcal{K}$. Now consider $S(\mathbf{q})$ in (1) for computing the social optimum subject to (11). The corresponding first order conditions for optimality (which are both necessary and sufficient in this case, due to the convexity of $S(\mathbf{q})$ in $\mathbf{q}$) of $S(\mathbf{q})$ subject to (11) are given as,

$$[\nabla_{\mathbf{q}_k} v_k(Q_k) - \nabla_{\mathbf{q}_k} C(\mathbf{q})]_{\mathcal{D}_k} = 0, \ \forall k \in \mathcal{K}. \quad (27)$$

Note that (26) when considered for all $k \in \mathcal{K}$, is the same as the conditions in (27). Therefore the schedule vector $\check{\mathbf{q}}$ (which realizes an allocation vector of $\check{\mathbf{Q}}$) also maximizes $S(\mathbf{q})$, and is there socially optimal. This also implies that the allocation vector $\check{\mathbf{Q}}$ at any Nash equilibrium of the *es*-PSP mechanism equals the unique socially optimal allocation $\mathbf{Q}^*$. ∎

Proposition 2 should not be interpreted as the uniqueness of the Nash equilibrium bids. It can be shown that there may exist multiple (many) bid vectors $\check{\mathbf{b}}$ that are at Nash equilibrium; some examples will be provided in the full version of the paper. Proposition 2 only implies that the allocation vector at all Nash equilibria is the same, and is socially optimal. Note however that this (socially optimal) allocation vector is in general realizable by multiple schedule vectors.

D. Truthful price-bid declaration

Recall that Proposition 1 shows the existence of a *truthful* bid that is a Nash equilibrium. In the proof of Proposition 2, we have also used truthful bidding in constructing potentially better bids for a user, given the bids of others. These are not accidental, as can be seen from Proposition 3, stated below. The result shows that given the bids of other users (which need not be at Nash equilibrium or result in socially optimal allocation), any user cannot be worse of by declaring its price bid truthfully for the quantity bid it declares (the quantity bid that optimizes its individual utility, given other users' bids).

Proposition 3: Given the bids of other users, $\mathbf{b}_{-k}$, there exists a *truthful best bid* for user k, $\tilde{b}_k(\mathbf{b}_{-k}) \in \mathcal{B}_k^T$.

Proof: From (12), given the bids of other users $\mathbf{b}_{-k}$, user k seeks to maximize

$$U_k(\mathbf{q}) = v_k(Q_k) + \sum\limits_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_{j,b_j}(Q_j) - \sum\limits_{t \in \mathcal{T}} C_t(Q^t), \quad (28)$$

subject to $\mathbf{q} \in \mathcal{D}$ and the tie-breaking rule. Let an optimal schedule vector (that maximizes $U_k(\mathbf{q})$) be $\mathbf{q} = \tilde{\mathbf{q}}$ which realizes an allocation vector $\tilde{\mathbf{Q}} = (\tilde{Q}_1, \tilde{Q}_2, \ldots, \tilde{Q}_K)$. While $\tilde{\mathbf{q}}$ can be non-unique, owing to strict concavity of $v_k(Q_k)$ with respect to Q_k, it follows that $\tilde{Q}_k$ is unique. From an argument similar to that in the proof of Lemma 1, we can show that $\tilde{Q}_j, \ \forall j \in \mathcal{K} \setminus \{k\}$ are unique as well, when the tie-breaking rule is taken into account. Thus, $\tilde{\mathbf{Q}}$ is unique. Note that for optimality of (28) at $\tilde{\mathbf{q}}$, it follows from [25] that,

$$\partial U_k(\tilde{\mathbf{q}}) \cap \Gamma^+(\tilde{\mathbf{q}}) \neq \phi. \quad (29)$$

Now let us define the following "truthful" bid for user k: $\tilde{b}_k(\mathbf{b}_{-k}) = (\tilde{Q}_k, v_k'(\tilde{Q}_k))$. Then $\tilde{b}_k$ can be represented as the ramp function $\hat{v}_{k,\tilde{Q}_k}(\cdot)$. Now when user k bids $\tilde{b}_k$ in the *es*-PSP mechanism, while the bids of other users remain fixed at $\mathbf{b}_{-k}$, the auctioneer computes the energy allocation by maximizing $\tilde{S}(\mathbf{q})$ given by

$$\tilde{S}(\mathbf{q}) = \hat{v}_{k,\tilde{Q}_k}(Q_k) + \sum\limits_{j \in \mathcal{K} \setminus \{k\}} \hat{w}_j(Q_j) - \sum\limits_{t \in \mathcal{T}} C_t(Q^t), \quad (30)$$

subject to (11) and the tie-breaking rule. Compare $U_k(\mathbf{q})$ and $\tilde{S}(\mathbf{q})$. The only differences are replacement of $v_k(Q_k)$ in $U_k(\mathbf{q})$ by $\hat{v}_{k,\tilde{Q}_k}(Q_k)$ in $\tilde{S}(\mathbf{q})$. Hence, arguing as before (see proofs of Propositions 1 and 2), $\partial U_k(\tilde{\mathbf{q}}) \subset \partial \tilde{S}(\tilde{\mathbf{q}})$. Combining this with (29) we get $\partial \tilde{S}(\tilde{\mathbf{q}}) \cap \Gamma^+(\tilde{\mathbf{q}}) \neq \phi$, which implies that $\tilde{\mathbf{q}}$ also optimizes $\tilde{S}(\mathbf{q})$. Since the allocation vector correspond to any optimum solution of $\tilde{S}(\mathbf{q})$ subject to the tie-breaking rule is unique (Lemma 1), it follows that if user k submits a bid of $\tilde{b}_k$ when the other bids are kept at $\mathbf{b}_{-k}$, the resulting allocation is $\tilde{\mathbf{Q}}$. Since $\tilde{\mathbf{Q}}$ maximizes $U_k(\mathbf{q})$, therefore the truthful bid $\tilde{b}_k = (\tilde{Q}_k, v_k'(\tilde{Q}_k))$, which depends on $\mathbf{b}_{-k}$, represents the user's best bid given $\mathbf{b}_{-k}$. The result follows. ∎

IV. NUMERICAL STUDY

In our simulation study, we consider a simple residential distribution network having a single distribution transformer of rating 200 kVA, and shared by 24 PEVs each having a valuation function of the form $v_k(x) = \kappa(1 - e^{-rx})$ where κ and r are constants that define the concavity of the valuation function. We assume that the PEVs are classified into four *types*: each type i being represented by a unique set of κ_i and r_i values. For our study, we chose $\kappa_1 = 10; \kappa_2 = 9; \kappa_3 = 11; \kappa_4 = 7$ and $r_1 = 0.1; r_2 = 0.11; r_3 = 0.12; r_4 = 0.09$. We assume that all PEVs are available to charge in all time slots. The supply cost functions are taken as $C_t = c(\sum\limits_{k \in \mathcal{K}} q_k^t)^2$ where c is an increasing function of D_t. In this setting, we first compute the socially optimal energy allocation; Figure 5 shows the overall load (over time) resulting from this allocation, along with the base (non-PEV) demand. We note from the figure the approximate *valley filling* nature of the load curve: the socially optimal allocation based on (1)-(3) allocates PEV charging mostly in the time slots that have less inelastic load, thereby attaining (albeit approximately) the desirable objective of flattening the overall load curve.

Next we demonstrate that for the *es*-PSP mechanism, truthful bidding at this socially optimal allocation is a Nash equilibrium. The socially optimal allocations for the PEVs of types 1,2,3 and 4 respectively are 7.07 kWh, 6.34 kWh, 8.10 kWh and 2.73 kWh, respectively. Now, we study the *es*-PSP mechanism in which we demonstrate the effect of increasing the quantity of bid (a_k) for each agent k in an entire range of interest of charging, while the bids of all other 23 agents are held constant at their socially optimal values. We assume truthful bidding, so determining the quantity a_k also determines the per-unit price p_k (which can be obtained from agent k's valuation function).

We observe from Figure 6 that, agents of type 1 gain maximum utility by truthful bidding at the socially optimal
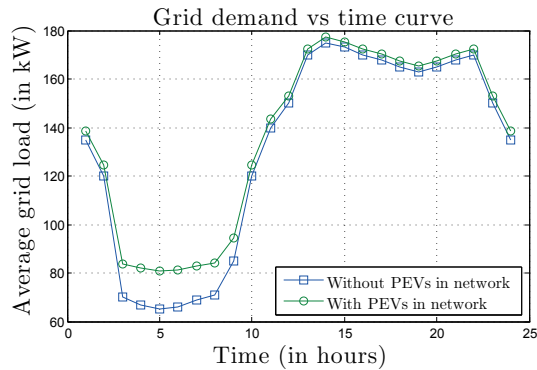
Fig. 5. Approximate load-valley minimization obtained at social optimum.
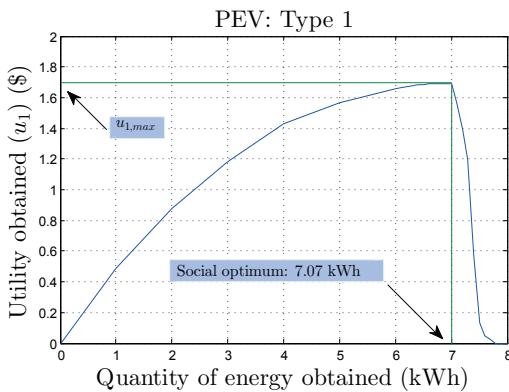

Fig. 6. Utility received by a type 1 agent by varying its quantity bid unilaterally, while other agents' bids are kept at their socially optimal values.

quantities, at which point there is no incentive to unilaterally deviate and gain a greater utility. (Results for the other three types (2, 3 and 4) are found to show similar trends as in Figure 6.) We also validated that this equilibrium point (as which the the users declare their price bids truthfully) is stable with respect unilateral changes in the bid price as well, while keeping the bid quantity the same. These observations are consistent with our theoretical results in Propositions 1 and 3, although the result is Proposition 1 is stronger in that it claims the stability of truthful bidding at the socially optimal allocation with respect to changes in both bid price and quantity dimensions at the same time. A more extensive simulation study of the properties of our auction mechanism in more general settings will be presented in a full version of this work.

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