Role of Cognitive Theory in the Study of Learning Disability in Mathematics

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Abstract

Gersten, Jordan, and Flojo (in this issue) provide the beginnings of an essential bridge between basic research on mathematical disabilities (MD) in young children and the application of this research for the early identification and remediation of these forms of learning disability. As they acknowledge, the field of MD is in the early stages of development, and thus recommendations regarding identification measures and remedial techniques must be considered preliminary. I discuss the importance of maintaining a tight link between theoretical and empirical research on children's developing numerical, arithmetical, and mathematical competencies and future research on learning disabilities in mathematics. This link will provide the foundation for transforming experimental procedures into assessment measures, understanding the cognitive strengths and weaknesses of children with these forms of learning disability, and developing remedial approaches based on the pattern of cognitive strengths and weaknesses for individual children.

Gersten, Jordan, and Flojo (in this issue) address several key issues related to the extension of basic research on learning disabilities (LD) in mathematics to the development of early identification measures and remedial strategies. These topics are at the horizon of the field and are the ultimate applied goals of research in this area, and Gersten et al. are to be commended for tackling these difficult and complicated issues. As Gersten et al. state, the interplay between theory and empirical research in the area of reading disabilities occurred over the course of more than 20 years. These efforts have yielded a very rich knowledge base on the cognitive and brain correlates of phonetics-based reading disabilities and have led to the development of valid identification measures and remedial techniques (e.g., Shaywitz et al., 2004). Gersten et al. note that the state of knowledge in the area of mathematical disabilities is in its infancy, comparatively speaking. Thus, any proposals regarding the early identification of and instructional implications for LD in mathematics should be viewed as important but necessarily preliminary, as Gersten et al. acknowledge. In this commentary, I focus on the role of theory in guiding research in LD in mathematics and in guiding the ultimate development of early identification measures.

Standardized Achievement Tests

The road to the development of assessment measures specifically for mathematical disabilities (MD) perforce runs through existing standardized achievement tests. These tests, however, should only be viewed as initial screening measures—that is, as a means to identify children who might have a cognitive disability that interferes with mathematical learning. As Gersten et al. (in this issue) note, research on MD is often based on the study of children who have achievement test scores below the 35th percentile. This cutoff is necessary because we do not yet have the measures needed to differentiate between children with an actual cognitive disability and children who have low achievement scores for other reasons. This is the result of the nature of achievement tests; specifically, the sampling of items that assess an array of numerical, arithmetical, and mathematical domains. Because many children with MD have cognitive deficits that interfere with quantitative learning in some domains but not in others, the use of these tests results in the overestimation of their competencies in some areas and their underestimation in others (Jordan, Hanich, & Kaplan, 2003). Moreover, many children who score low on achievement tests for any single assessment do not appear to have an underlying cognitive deficit and, in fact, often show average test scores on later assessments (Geary, 1990; Geary, Hamson, & Hoard, 2000).

The use of experimental and neuropsychological measures that are more sensitive than standardized achievement tests suggests that between 5% and 8% of elementary school children have some form of specific cognitive deficit (e.g., difficulties remembering number combinations) that interferes with the learning and understanding of numerical and arithmetical concepts, procedures, or facts (Badian, 1983; Kosc, 1974; Ostad, 1998; Shalev, Manor, & Gross-Tsur, 1993). At this time, it is not clear how these deficits will affect learning in other areas of mathematics, such as algebra or geometry, or whether there are different forms of deficit that disrupt learning in these specific domains and, thus, will not become apparent until later grades. In other words, low mathematics achievement scores are used as a starting point in attempts to identify children with MD for further study and should not be used in and of themselves as a diagnostic marker.

As Gersten et al. (in this issue) discuss, a core goal of research on MD thus includes the development of measures that are more sensitive than are standardized achievement tests to the specific deficits of children with these forms of LD. An attendant and crucial issue concerns the choice of candidate measures of specific potential deficits, and this is where reliance on theory becomes essential. I agree with Gersten et al. that there has been too much theory and not enough empirical study in the area of mathematics education. Nonetheless, theory in cognitive and developmental psychology has provided the foundation for much of the progress in our understanding of the early core deficits of children with MD (Geary, 1993; Russell & Ginsburg, 1984; Shalev et al., 1993) and will continue to provide an anchor for the further development of assessment measures.

The Role of Theory

The breadth and complexity of the field of mathematics results in instructional challenges for most teachers and children (Geary, 1995) and makes the systematic study of LD especially daunting. There is considerable potential to confuse difficulty in learning complex material with an actual cognitive disability—that is, a disability that impedes learning even with appropriate instruction (e.g., Fuchs, Fuchs, & Prentice, 2004). To illustrate the challenge, a mathematical disability can result from deficits in the ability to represent or process information in one or several of the many subareas of mathematics (e.g., base-10 arithmetic versus geometric theorems) or in one or a set of procedural or conceptual features within each subarea (e.g., use of base-10 arithmetic for trading versus conceptual understanding of this system). To narrow the focus of the search for LD, our approach has been to apply theory and methods used to study mathematical development in academically typical children to the study of children with low achievement in mathematics (e.g., Geary & Brown, 1991).

Unfortunately, in most mathematical domains, such as geometry and algebra, not enough is known about the cognitive systems that support the typical learning of the associated competencies to provide a systematic framework for the study of MD. Fortunately, cognitive theory and experimental methods are well developed in the areas of number, counting, and arithmetic (Briars & Siegler, 1984; Gelman & Meck, 1983; McCloskey, Aliminosa, & Macaruso, 1991; Siegler & Shrager, 1984) and also are well developed in other cognitive domains, such as working memory, that contribute to learning across these subareas of mathematics (Tronsky & Royer, 2002). These theoretical models and experimental methods have provided the foundation for the study of the cognitive deficits of children with MD and have led to some important discoveries (for reviews, see Geary, 2004; Geary & Hoard, 2002).

In the area of arithmetic, these models and measures have allowed researchers to pinpoint areas in which children with MD are developmentally delayed in comparison to their peers and areas in which there appear to be more persistent cognitive deficits. In particular, these children are delayed in the sophistication and skilled use of counting procedures to solve simple arithmetic problems, but many of these children eventually catch up to their peers. In contrast, many children with MD have a persistent deficit in the ability to store number combinations in or retrieve them from long-term memory (Geary, 1993; Jordan et al., 2003). Cognitive theory has led to several hypotheses regarding the basic source of this deficit, and, although the issue remains to be completely resolved, recent studies have suggested that there may be two forms of retrieval deficit (Barrouillet, Fayol, & Lathulière, 1997; Geary et al., 2000). One appears to involve a straightforward deficit in the ability to retrieve facts from a semantics-based long-term memory network. The second form results from a disruption of the retrieval process due to difficulties in inhibiting the retrieval of irrelevant associations. For instance, when solving the problem 4 + 8, these children might retrieve 5 or 9, or retrieve both 5 and 9, because these are the counting string associates (i.e., 1, 2, 3, 4, 5 . . .) of 4 and 8, respectively.

Equally important, the measures developed from theoretical models of typical development, such as those used to assess children's conceptual understanding of counting or numerical relationships, can be used as the basis for the development of standardized measures for the identification of patterns of cognitive strengths and weaknesses of children with low mathematical achievement and potential LD, as noted by Gersten et al. (in this issue) and Shalev et al. (1993). The majority of these measures were initially used in experimental research on children's conceptual understanding and procedural skills in the domains of number, counting, and arithmetic, and were intimately linked to theories of children's emerging competencies in these domains. It is important to maintain this theoretical and empirical link to research on the typical development of children's mathematical competencies.

Conclusion

Gersten et al. (in this issue) lay the foundation that will eventually provide the bridge between basic research on children's MD and the application of this research to the development of early identification measures and remedial techniques for these children. As Gersten et al. acknowledge, research on these forms of LD is only in the beginning stages vis-à-vis the progress that has been made on phonetic forms of reading disability. Thus, as they state, Gersten et al.'s conclusions must be considered a beginning point and not a final word on the issues of the identification and remediation of MD. At this point in the development of the field, I believe it is essential that we maintain a tight link between the vibrant and growing research base on children's developing mathematical competencies (for a review, see Geary, in press) and research on children's LD in mathematics. This link will provide a foundation for understanding the nature and potential cause of these forms of LD; a considerable number of experimental procedures and measures that might be used to develop tests for the identification of mathematical learning disabilities; and a starting point for approaching the remediation of these disabilities.

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