FORECASTING NEW ADOPTIONS: A COMPARATIVE EVALUATION OF THREE TECHNIQUES OF PARAMETER ESTIMATION

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ABSTRACT

This research will sue multiple criteria mathematical programming to estimate the parameters of a diffusion model forecasting model for new products. The criteria that will be used for the parameter estimation are minimization of the sum of squares error and attainment of goal levels for errors (positive or negative).

Key Words: Forecasting, Nonlinear Programming, Regression, Bass model

INTRODUCTION

Forecasting new adoptions after a product introduction is an important marketing problem. The Bass model for forecasting is most appropriate for forecasting sales of an innovation (more generally, a new product) where no closely competing alternatives exist in the marketplace. Managers need such forecasts for new technologies or major product innovations before investing significant resources in them.

The Bass model offers a good starting point for forecasting the long-term sales pattern of new technologies and new durable products under two types of conditions [1] [2]:

1. The firm has recently introduced the product or technology and has observed its sales for a few time periods; or
2. The firm has not yet introduced the product or technology, but it is similar in some way to existing products or technologies whose sales history is known.

In this paper, we introduce a forecasting model developed by Frank Bass that has proven to be particularly effective in forecasting the adoption of innovative and new technologies in the market place. We then use three techniques: nonlinear programming, linear regression, and minimizing the sum of deviations to estimate the parameters of the Bass model and compare their performance using dataset of a new movie [3] [4].

The model has three parameters that must be estimated.

m = the number of people estimated to eventually adopt the new product

A company introducing a new product is obviously interested in the value of this parameter.

q = the coefficient of imitation

This parameter measures the likelihood of adoption due to a potential adopter being influenced by someone who has already adopted the product. It measures the “word-of -mouth” effect influencing purchases.

p = the coefficient of innovation

This parameter measures the likelihood of adoption, assuming no influence from someone who has already purchased (adopted) the product. It is the likelihood of someone adopting the product due to her or his own interest in the innovation.

Let,

C_{t-1} = number of people (or a multiple of that number, such as sales) who have adopted the product through time $t-1$.

Therefore, $m - C_{t-1}$ is the number of potential adopters remaining at time $t-1$. We refer to time interval between time $t-1$ and time t as time period t.

The likelihood of adoption due to imitation is:

$$q(C_{t-1}/m),$$

Where C_{t-1}/m is the fraction of the number of people estimated to adopt the product by time $t-1$.

The likelihood of adoption due to innovation is simply p, the coefficient of innovation. Thus the likelihood of adoption is:

$$p + q(C_{t-1}/m)$$

Thus, FR_t, the forecast of the number of new adopters during time period t, is:

$$FR_t = (p + q(C_{t-1}/m))(m - C_{t-1}) \quad (1)$$

The equation (1) is known as the Bass forecasting model.

Let S_t denote the actual sales in period *t* for *t* = 1,2,…N. The forecast in each period and the corresponding forecast error E_t is defined by:

$$FR_t = (p + q(C_{t-1}/m))(m - C_{t-1})$$

$$E_t = FR_t - S_t$$

NONLINEAR PROGRAMMING

Nonlinear programming is used to estimate the parameters of the Bass forecasting model [5].

Minimizing the sum of errors squared, our nonlinear programming formulation is:

$$Min \sum_{t=1}^{N} E_t^2 \quad (2)$$

Subject to:

$$FR_t = (p + q(C_{t-1}/m))(m - C_{t-1}), \quad t = 1, 2, \ldots.., N \quad (3)$$

$$E_t = FR_t - S_t, \quad t = 1, 2, \ldots.., N \quad (4)$$

LINEAR REGRESSION

The conditional likelihood that a customer will adopt the innovation exactly at time *t* since introduction, given that the customer has not adopted before that time is [1] [6]:

$$L(t) = \frac{f(t)}{1 - F(t)} \quad (5)$$

Where,

$F(t)$ = A nondecreasing function, probability that someone in the target segment will adopt the innovation by time t.
$f(t)$ = The rate at which the probability of adoption is changing at time t.

Bass (1969) proposed that $L(t)$ be defined to be equal to:

$$L(t) = p + \frac{q}{m} C(t) \quad (6)$$

Where

$C(t) =$ Number of customers (or a multiple of that number, such as sales) who have already adopted the innovation by time t
m = A parameter representing the total number of customers in the adopting target segment, all of whom will eventually adopt the product;
p = coefficient of innovation (or coefficient of external influence); and
q = coefficient of imitation (or coefficient of internal influence).

$$f(t) = \left[p + \frac{q}{m} C(t) \right] \left[1 - F(t) \right] \quad (7)$$

$$S(t) = pm + (q-p)C(t) - \frac{q}{\overline{N}} \left[C(t) \right]^2 \quad (8)$$

$$S(t) = a + bm(t-1) + cm^2(t-1) \quad (9)$$

We can calculate the Bass model parameters:

$$m = \frac{-b - \sqrt{b^2 - 4ac}}{2c};$$

$p = \frac{a}{m}$; and

$q = p + b$

We need the sales data for at least three periods to estimate the model. To be consistent with the model, $m > 0, b \geq 0,$ and $c < 0$.

MINIMIZE THE LEAST ABSOLUTE DEVIATION

$$Min \sum_{t=1}^{N} \left| FR_t - S_t \right| \quad (10)$$

Where, $E_t = \left| FR_t - S_t \right|$

Equivalent Linear Program:

$$Min \sum_{t=1}^{N} d_t$$

Subject to:

$$FR_t = (p + q(C_{t-1}/m))(m - C_{t-1}), \quad t = 1, 2, ..., N \quad (11)$$

$$E_t = FR_t - S_t, \quad t = 1, 2, ..., N \quad (12)$$

$$\begin{array}{l} E_t + d_t = 0 \\ E_t - d_t = 0 \end{array}, \quad t = 1, 2, ..., N \quad (13)$$

Where, $d_t \geq 0$ is the deviation or the forecast error

Our results indicate that all of above techniques of the Bass model parameter estimation are comparable, however, the technique: minimize the least absolute deviation has a slight edge over the other two techniques.

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