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An optimal and intelligent control strategy for a class of nonlinear systems: adaptive fuzzy sliding mode

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Abstract

In this paper, an optimal adaptive fuzzy sliding mode controller is presented for a class of nonlinear systems. In the proposed control, in the beginning, the boundaries of parametric uncertainties, disturbances and un-modeled dynamics are reduced using a feedback linearization approach. Next, in order to overcome the remaining uncertainties, a sliding mode controller is designed. Mathematical proof shows that the closed-loop system with the proposed control is globally asymptotically stable. Using sliding mode control causes the undesirable chattering phenomenon to occur in the control input. Next, in order to remove the undesirable chattering phenomenon, an adaptive fuzzy approximator is designed to approximate the maximum boundary of the remaining uncertainties. Another mathematical proof shows that the closed-loop system with the proposed control is globally asymptotically stable in the presence of structured and unstructured uncertainties, and external disturbances. Finally, the self-adaptive modified bat algorithm is used to determine the coefficients of the adaptive fuzzy sliding mode control and the coefficients of the membership functions of the adaptive fuzzy approximator. To investigate the performance of the proposed controller, an inverted pendulum system is used as a case study. Simulation results verify the desirable performance of the optimal adaptive fuzzy sliding mode control.

Keywords

Nonlinear control, adaptive control, self-adaptive modified bat algorithm, fuzzy logic, sliding mode control, robust control, heuristic algorithm

1. Introduction

Controlling nonlinear systems with uncertainty has always been one of the important challenges of control engineering. As a result, researchers have presented many methods for controlling these systems. Some of the conventional nonlinear control approaches, such as the feedback linearization approach and the back stepping technique, have shown a very desirable performance in controlling nonlinear systems. However, the efficiency of these approaches in controlling nonlinear systems with uncertainties has decreased significantly and in some cases they cause the closed-loop system to become unstable (Slotin and Li, 1991; Khalil, 2002). Variable structure control (VSC) is one of the powerful methods in controlling nonlinear systems with parametric uncertainties and external disturbances (Soltanpour et al., 2012a).

Sliding mode control (SMC) is one of the famous variable structure control approaches. In addition to the advantages of variable structure control, sliding mode control has other advantages such as simplicity of designing and simplicity of practical implementation. In recent years, SMC has been used for controlling nonlinear systems with uncertainties (Shafiei and Soltanpour, 2011; Khooban et al., 2012a). In these papers, interesting approaches have been presented in order to overcome the parametric uncertainties, disturbance and unmodeled dynamics.

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The designers of SMC often assume that the control could be switched from one structure to another without delay (Almutairi and Zribi, 2009; Chaouch et al., 2012; Mamani et al., 2012). However, in practice, it is impossible to achieve high-speed switching control. This is due to the switching delay computation and also because of the limitations of physical actuators, which makes them unable to handle the switching of control signal at an infinite rate. This imperfect control switching between different structures causes the system trajectory to chatter instead of sliding along the sliding surface. Basically, there are two different ways for countering the chattering phenomenon. The first way involves using higher order sliding mode (Bartolini and Pydnowski, 1993; Bartolini et al., 1998) and in the second way a boundary layer is introduced around the sliding surface and a continuous control is used within the boundary layer (Temeltas, 1998; Chiang and Hu, 1999). Normally, the boundary has a constant width and a larger width results in a smoother control signal. Although the boundary design reduces the chattering phenomenon, it does not drive the system state to the origin any longer, and steady-state error will appear. The larger boundary width results in a larger steady-state error.

In recent years, fuzzy logic has been used to increase the efficiency of SMC in controlling nonlinear systems with uncertainties. This approach is known as fuzzy sliding mode control (FSMC) by researchers (Soltanpour and Khooban, 2013). Simulation results show that FSMC is very powerful at overcoming structured and unstructured uncertainties, and external disturbances. Furthermore, the undesirable chattering phenomenon is not observed in the input control. Next, in order to increase the efficiency of the proposed control, the coefficients of the control input are optimized by using intuitive optimization algorithms (Khooban et al., 2013a; Soltanpour and Khooban, 2013). It is very simple to design and implement the proposed approaches and the control input has a low computational burden. However, the proposed controllers do not have a mathematical analysis and a proof for the stability of the closed-loop system.

Adaptive control is another approach which has been proposed by researchers to control nonlinear systems with uncertainties. In adaptive control, to overcome the uncertainties, some adaptive laws are presented which online adjust the coefficients of control input or system parameters. Of course, in adaptive control methods, system parameters converge only when the variation of parameters happens slowly (Åström and Wittenmark, 2008). Therefore, adaptive control is successful at controlling nonlinear systems that only have parametric uncertainties.

Nowadays, by combining SMC, fuzzy logic, and adaptive control concepts, the adaptive fuzzy sliding mode control (AFSMC) has been presented for controlling nonlinear systems. In Benbrahim et al. (2013), an AFSMC controller has been presented to overcome the existing uncertainties in a nonlinear system. In this method, two adaptive type-2 fuzzy systems have been used to estimate unknown functions. Simulation results show that the proposed control has a good performance in overcoming the existing uncertainties and it makes the tracking error converge to zero. Although the type-2 fuzzy logic is very flexible in overcoming the existing uncertainties in a nonlinear system, it greatly increases the computational burden of the control input. As a result, there are some problems with the practical implementation of the proposed controller.

An AFSMC controller has been designed in Han (2011). The proposed approach is only capable of overcoming parametric uncertainties, but, in addition to parametric uncertainties, nonlinear systems encounter unstructured uncertainties and external disturbances. The AFSMC controllers are also presented by Essounbouli and Hamzaoui (2006), Wang et al. (2006), Madboul et al. (2010) and Sharkawy and Salman (2011). Simulation results and mathematical proof show the desirable performance of the proposed controller. However, in the proposed approaches, many adaptive fuzzy systems have been used to estimate the unknown functions of nonlinear systems. Therefore, the control input has a high computational burden. Therefore, if a delay occurs during the calculation of the control input, it will be impossible to guarantee the stability of the closed-loop system.

In this paper, to overcome the undesirable chattering phenomenon, an optimal adaptive fuzzy sliding mode controller is presented for controlling a class of nonlinear systems. In the design of the proposed control, a combination of feedback linearization technique, fuzzy logic, adaptive control concepts, and self-adaptive modified bat algorithm has been used. The features of these methods mean that the proposed control is powerful at overcoming the uncertainties. Some considerations have been taken into account in the designing process of the proposed controller in order to make it practically implementable.

2. Problem formulation

Consider a single-input single-output (SISO) nonlinear system described by the differential equation (1) (Slotine and Li, 1991; Khalil, 2002):

$$\begin{aligned} X^{(n)}(t) &= f(X(t), t) + g(X(t), t)u(t) + d(t) \\ y(t) &= x(t) \end{aligned} \tag{1}$$

where, $X(t) = [x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)]^T$represents the n^{th} order state vector of the system, $y(t) \in R$ denotes the output of the system, $f(X(t), t)$ and $g(X(t), t)$ are known (uncertain) but bounded continuous functions, $u(t) \in R$ denotes the control input and $d(t) \in R$ represents the disturbance which is unknown but bounded, due to the external load and noise.

The control process is aimed at forcing the output $y(t)$ to follow a given bounded reference input signal $y_d(t)$. Assume that the tracking error is represented by $e(t) = y(t) - y_d(t)$ and its forward shifted values are defined as $e^{(i)} = y^{(i)}(t) - y_d^{(i)}(t)$ $(i = 1, 2, \ldots, n-1)$. Therefore, the error vector is defined as $e(t) = [e(t), \dot{e}(t), \ldots, e^{(n-1)}(t)]$.

The following assumptions are made throughout this paper.

Assumption 1 The state vector $X(t)$ is available.

Assumption 2 The desired trajectory $y_d(t)$ is once differentiable in time. Besides, $y_d(t)$ and $y_d^{(n)}(t)$ are available and have known bounds.

Assumption 3 A known continuous function of $X(t)$ upper bounds the extent of the imprecision on $f(X(t), t)$.

Assumption 4 $g(X(t), t)$ is lower and upper bounded so that $0 < \underline{g} < g(X(t), t) < \overline{g}$ where, $\underline{g}$ and $\overline{g}$ are positive constants.

Assumption 5 $d(t)$ is unknown, but it is bounded. In other words, $|d(t)| < D$ where, "D" is a known positive constant.

3. Problem formulation in state space

Equation (2) is defined in order to transform the system described by equation (1) to the state space form:

$$x(t) = x_1(t),\ \dot{x}(t) = x_2(t), \ldots, x^{(n-1)}(t) = x_n(t) \tag{2}$$

And if new variables are substituted in equation (1), the following equations will be obtained:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ &\vdots \\ \dot{x}_n(t) &= f(X(t), t) + g(X(t), t)u(t) + d(t) \\ y(t) &= x_i(t), \quad i = 1, 2, \ldots, n \end{aligned} \tag{3}$$

The following error equations will appear if we transfer the nonlinear system equations to the state space domain:

$$\begin{aligned} x_1(t) - x_{1_d}(t) = e_1(t),\ x_2(t) - x_{2_d}(t) \\ = e_2(t), \ldots, x_n(t) - x_{n_d}(t) = e_n(t) \end{aligned} \tag{4}$$

In the equations above, $x_{i_d}(t)$ denotes that the (i−1)th derivative of the desired path must be tracked by the input control. From equations (2) and (4) it can be concluded that:

$$\dot{e}_1(t) = e_2(t),\ \dot{e}_2(t) = e_3(t), \ldots, \dot{e}_{n-1}(t) = e_n(t) \tag{5}$$

4. Sliding mode control

In this section, the sliding surface is defined as

$$S(t) = c_1 e_1(t) + c_2 e_2(t) + \cdots + c_{n-1} e_{n-1}(t) + e_n(t) \tag{6}$$

where c_i, $(i = 1, \ldots, n-1)$ are constant positive factors.

The control action $u(t)$ is designed in such a way that the output is able to track a desired path. Besides, the tracking error and all its derivatives will tend to zero. Therefore, the control action is defined by the following equation:

$$u(t) = \hat{g}^{-1}(X(t), t)\left\{-\hat{f}(X(t), t) + \dot{x}_{n_d}(t) + u_f(t)\right\} \tag{7}$$

where, $\hat{g}(X(t), t)$ and $\hat{f}(X(t), t)$ represent known parts of d $f(X(t), t)$ respectively. Also, $u_f(t)$ represents the SMC input which is designed for handling structured and unstructured uncertainties and $\dot{x}_{n_d}(t)$ denotes the derivative of the desirable path $x_{n_d}(t)$ with respect to time. For the sake of brevity, from now on in this section we will use $\hat{f}$, $\hat{g}$, f and g instead of $\hat{f}(X(t), t)$, $\hat{g}(X(t), t)$, $f(X(t), t)$ and $g(X(t), t)$ respectively. Equation (7) is substituted in equation (3):

$$\dot{x}_n(t) = f + g\left[\hat{g}^{-1}\left\{-\hat{f} + \dot{x}_{n_d}(t) + u_f(t)\right\}\right] + d(t) \tag{8}$$

To equation (8), $\dot{x}_{n_d}(t)$ and $u_f(t)$ are added and subtracted.

$$\begin{aligned} \dot{x}_n(t) = f + g\left[\hat{g}^{-1}\left\{-\hat{f} + \dot{x}_{n_d}(t) + u_f(t)\right\}\right] + d(t) + \dot{x}_{n_d}(t) \\ - \dot{x}_{n_d}(t) + u_f(t) - u_f(t) \end{aligned} \tag{9}$$

Equation (9) could be reorganized as follows:

$$\dot{x}_n(t) - \dot{x}_{n_d}(t) = f - g\hat{g}^{-1}\hat{f} + (g\hat{g}^{-1} - 1)\dot{x}_{n_d}(t) + (g\hat{g}^{-1} + 1)u_f(t) + d(t) - u_f(t) \quad (10)$$

The following equations are used in order to simplify equation (10):

$$\dot{e}_n(t) = \dot{x}_n(t) - \dot{x}_{n_d}(t)$$
$$\eta = f - g\hat{g}^{-1}\hat{f} + (g\hat{g}^{-1} - 1)\dot{x}_{n_d}(t) + (g\hat{g}^{-1} + 1)u_f(t) + d(t) \quad (11)$$

Remark 1 According to equation (11), η includes all existing uncertainties. Substituting equation (11) into equation (10) results in

$$\dot{e}_n(t) = \eta - u_f(t) \quad (12)$$

In SMC design $u_f(t)$ is composed of two parts; $u_{eq}(t)$ equivalent control and $u_s(t)$ switching control (Temeltas, 1998; Chiang and Hu, 1999):

$$u_f(t) = u_{eq}(t) + u_s(t) \quad (13)$$

In the sliding phase, where $S(t)$ and $\dot{S}(t)$ are equal to zero, the equivalent term $u_{eq}(t)$ is responsible for keeping the system on the sliding surface. In the approaching phase where $S(t) \neq 0$, the switching term $u_s(t)$ is designed to meet the reaching condition $S(t)\dot{S}(t) < 0$. To design the part $u_{eq}(t)$, it is assumed that the derivative of equation (6) is equal to zero:

$$\dot{S}(t) = c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) + \dot{e}_n(t) = 0 \quad (14)$$

Substituting equation (12) in equation (14) yields

$$c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) + \eta - u_f(t) = 0 \quad (15)$$

In the design of $u_{eq}(t)$, the sliding surface is assumed to be equal to zero. Therefore, $u_{eq}(t)$ is responsible for preventing the sliding surface from changes. Based on this assumption, in this part of the design $u_s(t)$ might be considered as zero. Considering the aforementioned points and substituting equation (13) in (15):

$$c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) + \eta - u_{eq}(t) = 0 \quad (16)$$

Eventually $u_{eq}(t)$ is derived from the equation (16):

$$u_{eq}(t) = c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) + \eta \quad (17)$$

Since η is not completely known, $u_{eq}(t)$ is chosen as follows:

$$u_{eq}(t) = c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) \quad (18)$$

Now, $u_s(t)$ is designed in such a way that the sliding surface tends to zero. Thus, the following Lyapunov candidate function is introduced:

$$V(S(t)) = \frac{1}{2}S^2(t) \quad (19)$$

Calculating the derivative of the Lyapunov candidate function with respect to time yields

$$\dot{V}(S(t)) = \dot{S}(t)S(t) \quad (20)$$

From equations (6) and (20), it can be concluded that

$$\dot{V}(S(t)) = (c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) + \dot{e}_n(t))S(t) \quad (21)$$

And the following equation could be concluded from equations (12), (13) and (21):

$$\dot{V}(S(t)) = (c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) + \eta - (u_{eq}(t) + u_s(t)))S(t) \quad (22)$$

Substituting equation (18) in equation (22) results in

$$\dot{V}(S(t)) = {}'c_1\dot{e}_1(t)S(t) + c_2\dot{e}_2(t)S(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t)S(t) + \eta S(t) - c_1\dot{e}_1(t)S(t) - c_2\dot{e}_2(t)S(t) - \cdots - c_{n-1}\dot{e}_{n-1}(t)S(t) - u_s(t)S(t) \quad (23)$$

From equation (23) it can be concluded that the following condition has to be met in order to have the inequality $\dot{V}(S(t)) < 0$ satisfied:

$$\begin{cases} u_s(t) = \rho & \text{if} \quad S(t) > 0 \\ u_s(t) = -\rho & \text{if} \quad S(t) < 0 \end{cases} \quad (24)$$

where, ρ is a constant positive factor that it is $\geq \eta$. Considering equations (13), (18) and (24) we have

$$\begin{cases} u_f(t) = u^+(t) = u_{eq}(t) + \rho & \text{if} \quad S(t) > 0 \\ u_f(t) = u^-(t) = u_{eq}(t) - \rho & \text{if} \quad S(t) < 0 \end{cases} \quad (25)$$

The above equation can also be shown in the following form:

$$\begin{cases} u_f(t) = u_{eq}(t) + \rho\,\text{sign}(S(t)) \\ u_{eq}(t) = c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) \end{cases} \quad (26)$$

where, sign($*$) represents the Sign function.

5. Adaptive fuzzy sliding mode control

From equations (25) and (26), it can be concluded that the reason for the occurrence of chattering phenomenon in classic sliding mode control lies in the existence of the constant coefficient ρ and the Sign function.

Now, suppose that the control gain $\rho sign(S(t))$ is replaced by a fuzzy gain ρ. The block diagram of a typical fuzzy system is shown in Figure 1. A fuzzy system normally has one or more inputs and a single output. A system with multiple outputs could be considered as a combination of several single-output systems (Wang et al., 1997).

A fuzzy system consists of four basic parts. The fuzzification and defuzzification play the role of an interface between the fuzzy systems and the crisp systems. The rule base is composed of a set of "if...then..." rules which are derived from human experience. Each of these rules describes a relationship between the input space and the output space. For each rule, based on the relationship defined by the rule, the input fuzzy sets are mapped to an output fuzzy set by the inference engine. Then, it combines the fuzzy sets from all the rules that exist in the rule base into the output fuzzy set. This output fuzzy set is translated to a crisp value output by the defuzzification.

All these parts could be mathematically formulated. The singleton fuzzifications, center average defuzzification, Mamdani implication and product inference engine are used in this paper. Therefore, the output of the fuzzy system could be described by the following equation:

$$y = \frac{\sum_{m=1}^{M} \theta^m \prod_{i=1}^{n} \mu_{A_i^m}(x_i^*)}{\sum_{m=1}^{M} \prod_{i=1}^{n} \mu_{A_i^m}(x_i^*)} = \theta^T \Psi(x) \tag{27}$$

Where, $\theta = [\theta^1, \ldots, \theta^m, \ldots, \theta^M]^T$ represents the vector of the centers of the membership functions of y, $\Psi(x) = [\Psi(x)^1, \ldots, \Psi(x)^m, \ldots, \Psi(x)^M]^T$ denotes the vector of the height of the membership functions of y in which

$$\Psi(x)^m = \prod_{i=1}^{n} \mu_{A_i^m}(x_i^*) \Big/ \sum_{m=1}^{M} \prod_{i=1}^{n} \mu_{A_i^m}(x_i^*),$$

and M is the amount of the rules.

Then, the new control input could be written as

$$u_f(t) = u_{eq}(t) + \delta + \rho \tag{28}$$

In equation (28), $u_{eq}(t)$ is equal to equation (18) and δ is a positive constant. To design the adaptive fuzzy controller, the following candidate Lyapunov function is introduced:

$$V(S(t)) = \frac{1}{2} S^2(t) \tag{29}$$

In equation (29), V is considered as an indicator of the energy of S. The stability of the system is guaranteed by choosing a control law such that $\dot{V} \leq 0$ and $\dot{V} = 0$ only when $S = 0$. In order to compensate the system uncertainty and reduce the energy of S, a fuzzy ρ is applied in the adaptive fuzzy sliding mode control.

Equation (29) is differentiated with respect to the time and the following equation is obtained by substituting equation (18) into it:

$$\begin{aligned} \dot{V}(S(t)) &= c_1 \dot{e}_1(t) S(t) + c_2 \dot{e}_2(t) S(t) + \cdots \\ &\quad + c_{n-1} \dot{e}_{n-1}(t) S(t) + \eta S(t) \\ &\quad - c_1 \dot{e}_1(t) S(t) - c_2 \dot{e}_2(t) S(t) - \cdots \\ &\quad - c_{n-1} \dot{e}_{n-1}(t) S(t) - \delta S(t) - \rho S(t) \end{aligned} \tag{30}$$

From equation (30), it can be concluded that $\dot{V} \leq 0$ only if ρ and $S(t)$ have the same sign. On the other hand, if $\|S(t)\|$ is too large then a larger value of ρ can further guarantee the stability because ρ becomes more negative. In other words, the energy of $S(t)$ decreases more rapidly. If $S(t)$ is too small, then

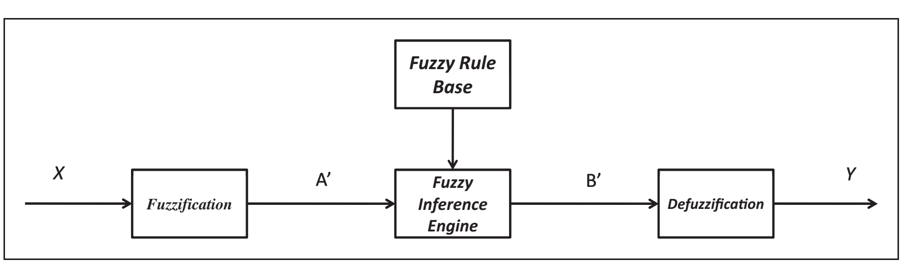

Figure 1. Diagram of a typical fuzzy system

$\rho S(t)$, $\delta S(t)$, and $\eta S(t)$ have an insignificant effect on $\dot{V}$. Under such conditions, a small value of ρ could prevent the occurrence of chattering. Finally, if $S(t) = 0$, then the value of ρ could be chosen to be equal to zero.

This is similar to the idea of applying the function at (∗). The difference is that the control gain varies along with the sliding surface all the time. Moreover, in order to guarantee that ρ is capable of compensating the system uncertainty, an adaptive law is designed. It is revealed by these analyses that the value of ρ could be decided by the value of the sliding surface $S(t)$. Therefore, the fuzzy system for ρ ought to be a SISO system, with $S(t)$ as the input and ρ as the output. Figure 1 shows the structure of the fuzzy systems. The rules in the rule base are in the following format:

$$\text{if } S(t) \text{ is } A_i^m \text{ then } \rho \text{ is } B_i^m$$

where A_i^m and B_i^m are fuzzy sets. In this paper, the same kind of membership functions, i.e. NB, NM, NS, ZE, PS, PM, PB are chosen for both $S(t)$ and ρ where, N stands for negative, P positive, B big, M medium, S small and ZE zero. These are all Gaussian membership functions defined by the following equation:

$$\mu_A(x_i) = exp\left[-\left(\frac{x_i - \alpha}{\sigma}\right)^2\right]$$

where, "A" denotes one of the fuzzy sets NB, ..., PB and x_i represents $S(t)$ or ρ. α is the center of "A" and σ is the width of "A". Despite the fact that the membership functions for $S(t)$ and ρ have the same titles, correspondingly, the values of the center and the width of the membership function with a same title for $S(t)$ and ρ are different, respectively. The parameters of the membership functions of ρ are updated online whereas, those of $S(t)$ have predefined values. Thus, the controller is an adaptive controller.

Based on the definitions of the input and output membership functions and according to the above discussion, the following rules could be decided as the rule base:

$$\begin{aligned}
&\text{if } S(t) \text{ is NB then } \rho \text{ is NB}\\
&\text{if } S(t) \text{ is NM then } \rho \text{ is NM}\\
&\text{if } S(t) \text{ is NS then } \rho \text{ is NS}\\
&\text{if } S(t) \text{ is ZE then } \rho \text{ is ZE}\\
&\text{if } S(t) \text{ is PS then } \rho \text{ is PS}\\
&\text{if } S(t) \text{ is PM then } \rho \text{ is PM}\\
&\text{if } S(t) \text{ is PB then } \rho \text{ is PB}
\end{aligned}$$

and based on our knowledge of fuzzy systems, ρ can be written as follows:

$$\rho = \frac{\sum_{m=1}^{M} \theta^m \mu_{A^m}(S(t))}{\sum_{m=1}^{M} \mu_{A^m}(S(t))} = \theta^T \Psi(S(t)) \tag{31}$$

where $\theta = \left[\theta^1, \ldots, \theta^{\mathrm{m}}, \ldots, \theta^{\mathrm{M}}\right]^{\mathrm{T}}$, $\Psi(S(t)) = [\Psi(S(t))^1, \ldots, \Psi(S(t))^m, \ldots, \Psi(S(t))^M]^T$ and

$$\Psi(S(t))^m = \prod_{i=1}^{n} \mu_{A_i^m}(S(t)) \Big/ \sum_{m=1}^{M} \prod_{i=1}^{n} \mu_{A_i^m}(S(t)).$$

θ is selected as the parameter to be updated and it is therefore called the parameter vector. $\Psi(S(t))$ is known as the function basis vector and can be considered as the weight of the parameter vector.

Define θ^* so $\rho = \theta^{*^{\mathrm{T}}} \Psi(S(t))$ is the optimal compensation for η. Based on Wang's theorem (Wang et al., 1997), there exists $\omega > 0$ which satisfies the following inequality:

$$\eta - \rho = \eta - \theta^* T \Psi(S(t)) \leq \omega \tag{32}$$

In the above inequality, ω is approximation error and it can be as small as possible. Then, define

$$\tilde{\theta} = \theta - \theta^* \tag{33}$$

From equations (31) and (33) it is concluded that

$$\rho = \tilde{\theta}^T \Psi(S(t)) + \theta^* T \Psi(S(t)) \tag{34}$$

When the details of designing adaptive fuzzy control are explained, in order to design it, the candidate Lyapunov function (29) is changed as follows:

$$\mathrm{V}(S(t)) = \frac{1}{2} S(t)^2 + \frac{1}{2\gamma} \tilde{\theta}^{\mathrm{T}} \tilde{\theta} \tag{35}$$

where, γ is a constant that is greater than 0. Equation (35) is differentiated with respect to the time and the following equation is obtained by substituting equation (18) into it:

$$\begin{aligned}
\dot{V}(S(t)) = {} & c_1 \dot{e}_1(t) S(t) + c_2 \dot{e}_2(t) S(t) + \cdots + c_{n-1} \dot{e}_{n-1}(t) S(t)\\
& + \eta S(t) - c_1 \dot{e}_1(t) S(t) - c_2 \dot{e}_2(t) S(t) - \cdots\\
& - c_{n-1} \dot{e}_{n-1}(t) S(t) - \delta S(t) - \rho S(t)\\
& + \frac{1}{2\gamma} (\dot{\tilde{\theta}}^T \tilde{\theta} + \tilde{\theta}^T \dot{\tilde{\theta}})
\end{aligned} \tag{36}$$

Simplifying equation (36) results in

$$\dot{V}(S(t)) = -\delta S(t) + (\eta - \rho)S(t) + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} \quad (37)$$

Equation (34) is substituted in equation (36) and the result is reorganized as follows:

$$\dot{V}(S(t)) = -\delta S(t) + (\eta - \theta^* T\Psi(S(t)))S(t) + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}}\tilde{\theta}^T\Psi(S(t))S(t) \quad (38)$$

Equation (38) is reorganized as follows:

$$\dot{V}(S(t)) = -\delta S(t) + (\eta - \theta^* T\Psi(S(t)))S(t) + \tilde{\theta}^T\left(\frac{1}{\gamma}\dot{\tilde{\theta}}\Psi(S(t))S(t)\right) \quad (39)$$

Considering equation (39), the adaptive rule could be chosen as follows:

$$\dot{\tilde{\theta}} = \gamma\Psi(S(t))S(t) \quad (40)$$

By choosing the above adaptive rule, equation (39) is simplified as follows:

$$\dot{V}(S(t)) = -\delta S(t) + (\eta - \theta^* T\Psi(S(t)))S(t) \quad (41)$$

From equations (32) and (41) it is concluded that:

$$\dot{V}(S(t)) \leq -\delta S(t) + \omega S(t) \quad (42)$$

Equation (42) shows that by properly choosing the coefficient δ, $\dot{V}(S(t)) \leq 0$ is satisfied and $\dot{V}(S(t)) = 0$ if $\dot{V}(S(t)) = 0$. Therefore, the closed-loop system with adaptive fuzzy sliding mode control is globally asymptotically stable in presence of all structured and unstructured uncertainties. To summarize our discussion, the proposed control input is as follows:

$$\begin{cases} u_f(t) = u_{eq}(t) + \delta + \rho \\ u_{eq}(t) = c_1\dot{e}_1(t) + c_2\dot{e}_2(t) + \cdots + c_{n-1}\dot{e}_{n-1}(t) \\ \rho = \theta^T\Psi(S(t)) \\ \dot{\theta} = \gamma\Psi(S(t))S(t) \end{cases} \quad (43)$$

6. Self-adaptive modified bat algorithm (SAMBA)

A new optimization algorithm which is based on bat algorithm (BA) is proposed in this section.

6.1. Original bat algorithm

BA is a meta-heuristic population-based algorithm which is inspired by the behavior of bats searching for food. Four simple and basic ideas constructing the main concept behind the BA are as follows (Niknam et al., 2011):

1. Each bat with the position Xi has the velocity of Vi and produces a special pulse with the frequency and loudness of fi and Ai, respectively.
2. In order to distinguish between the food and prey, the echolocation phenomenon is used.
3. Loudness Ai changes from a large value to a small value.
4. The frequency fi and rate ri of each pulse are regulated automatically during the optimization process.

Similar to the other evolutionary optimization algorithms, BA also uses a random population to start its search. The position of bats is updated using the following equation:

$$\begin{aligned} \mathbf{V}_i^{new} &= \mathbf{V}_i^{old} + f_i(Gbest - X_i);\ i = 1, \ldots, N_{Bat} \\ X_i^{new} &= X_i^{old} + \mathbf{V}_i^{new}; \quad i = 1, \ldots, N_{Bat} \\ f_i &= f_i^{\min} + \varphi_1(f_i^{\max} - f_i^{\min}); \quad i = 1, \ldots, N_{Bat} \end{aligned} \quad (44)$$

where, G_{best} is the best bat; N_{Bat} denotes the size of the population; $f_i^{\max}/f_i^{\min}$ are the maximum/minimum frequency of the ith bat and $\varphi 1$ is a random value in the range [0,1].

Another random movement is also simulated in this algorithm. In this regard, a random number β is generated randomly. If this random number is greater than ri, a new solution around the bat Xi is produced:

$$X_i^{new} = X_i^{old} + \varepsilon A_{mean}^{old};\ i = 1, \ldots, N_{Bat} \quad (45)$$

where, A_{mean}^{old} is the mean value of the bats' frequency loudness and ε is a random value in the range of [−1,1]. On the other hand, if the random value β is smaller than ri, a new solution X_i^{new} is generated randomly. The new solution X_i^{new} can be accepted if the following two conditions are met:

$$\begin{aligned} \beta &< A_i \\ f(X_i) &< f(Gbest) \end{aligned} \quad (46)$$

Meanwhile, the rate parameter and the loudness are updated as follows:

$$\begin{aligned} A_i^{new} &= \alpha A_i^{old} \\ r_i^{Iter+1} &= r_i^0[1 - exp(-\gamma \times Iter)] \end{aligned} \quad (47)$$

In equation (47), α and γ are constant values and $Iter$ represents the number of the iterations.

6.2. Self-adaptive modification method

In this section, a new self-adaptive modification technique is proposed to efficiently improve the total search capability of the BA. This modification mechanism is aimed at making use of an adaptive structure that makes the bats capable of selecting between two different modifications. Indeed, the proposed modification approach is composed of two modification methods that are described below.

6.3. Sub-modification method 1

The first modification is aimed at increasing the diversity of the bat population by using the crossover and mutation operators. The significant role of this modification in improving the performance of the optimization algorithms has been explained in various references (Niknam et al., 2011, 2014; Khooban et al., 2013b, 2014). In this regard, for each bat Xi three bats $Xm1$, $Xm2$ and $Xm3$ are chosen randomly so that $m1 \neq m2 \neq m3 \neq i$. Now, a test solution is generated using the mutation operator:

$$\begin{aligned} X_{Test} &= X_{m1} + \varphi_1(X_{m2} - X_{m3}) \\ X_{Test} &= [x_{Test},1, x_{Test},2, \ldots, x_{Test},n] \end{aligned} \tag{48}$$

where n denotes the length of the control vector. Then, the crossover operator is used for producing two new promising optimal solutions as follows:

$$\begin{aligned} X_{Test1} &= \begin{cases} x_{i,j}; & \varphi_2 < \varphi_3 \\ gbest_j; & \varphi_3 \geq \varphi_2 \end{cases} \\ X_{Test2} &= \begin{cases} x_{Test,j}; & \varphi_3 < \varphi_4 \\ gbest_j; & \varphi_4 \geq \varphi_3 \end{cases} \\ X_i &= [x_{i,1}, x_{i,2}, \ldots, x_{i,n}] \\ Gbest &= [gbest_1, gbest_2, \ldots, gbest_n] \end{aligned} \tag{49}$$

where $\varphi1$, $\varphi2$, $\varphi3$, and $\varphi4$ are random values in the range [0,1].

6.4. Sub-modification method 2

The goal of this modification method is to update the parameter α in equation (47) during the optimization adaptively.

$$\alpha^{new} = (1/2Iter)^{1/Iter} \alpha^{old} \tag{50}$$

This formula is obtained empirically by running the algorithm for several times.

In the beginning, a probability parameter is defined for the sub-modification methods (known as $Pr\theta$ for θth sub-modification method). Initially, the probability parameters of both modification methods are assumed to be equal; i.e. $Pr\theta = 0.5$ & $\theta = 1, 2$. As previously mentioned, the key idea behind this adaptive modification is to give the bats the choice of preference. Nonetheless, their probability could be increased or decreased by the successful performance of each sub-modification. It is clear that a bigger $Pr\theta$ shows more chance for θth modification to be chosen as the proper sub-modification method by the bats.

The population of bats is sorted in a descending order in each iteration. Now, a better bat solution will take a higher weighting factor:

$$WT_j = \frac{Log(N - j + 1)}{\sum_{i=1}^{n} Log(i)}; \quad j = 1, \ldots, N \tag{51}$$

where N is the number of bats in the population. Now, using the following equation, the probability success of each sub-modification method is updated:

$$Pr_\theta = Pr_\theta + \frac{WT_l}{n_{Mod_\theta}} l = 1, \ldots, n_{Mod_\theta}; \quad \theta = 1, 2 \tag{52}$$

Where n_{Mod_θ} denotes the number of bats that have chosen the θth sub-modification method. Then, using the following equation, the probability success parameters are updated at the end of each iteration:

$$Pr_\omega = \frac{Pr_\omega}{\sum_{\omega=1}^{2} Pr_\omega} \tag{53}$$

The roulette wheel mechanism is used for selecting the proper modification method by each bat, in order to keep the random characteristics of the algorithm. This process is shown in Figure 2.

```
Begin:
  For i=1:N_Bat where N_Bat is size of bat population
    If _1 ≤ Prb_1^(Iter+1)
    Select modification method 1 for the bat solution i
    ElseIf Prb_1^(Iter+1) < rand_i ≤ Prb_1^(Iter+1) + Prb_2^(Iter+1)
    Select modification method 2 for the bat solution i
    End If
  End For i
End
```

Figure 2. Pseudo code for choosing θth modification method by randomized weighted majority (RWM).

Generally speaking, a heuristic algorithm like SAMBA only requires the cost function to be checked to guide its search and no longer requires information about the system. Therefore, the mean of root of squared errors (MRSE) is considered in this paper:

$$MRSE = E(K) = 1\Big/N\sum_{i=1}^{N}|e(i)| \quad (54)$$

where N is the number of samples, i is the iteration number, $e(i)$ represents the trajectory error of the ith sample for the object, and $u(i)$ denotes the control signal.

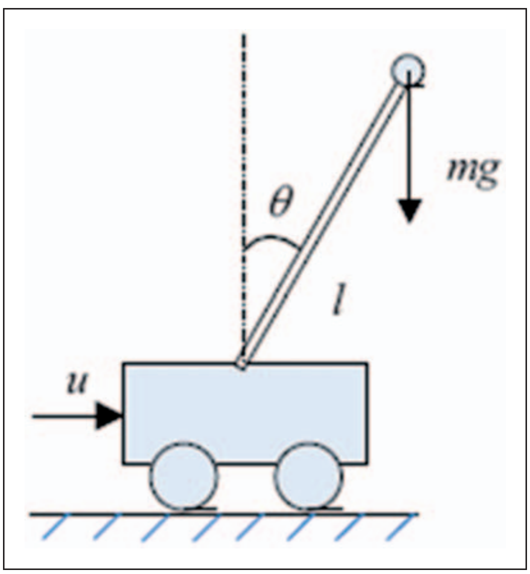

Figure 3. The inverted pendulum system..

7. Advantages of proposed control

In the design of the proposed control some important points have been considered that play a key role in the practical implementation of this approach. These key points are as follows:

1. In the design of the proposed control, a combination of feedback linearization technique, fuzzy logic, adaptive control concepts, and SAMBA optimization algorithm has been used in order to overcome parametric uncertainties, external disturbances and unmodeled dynamics. Therefore, the features of these techniques could be used in the proposed control. In other words, if fairly accurate information about the dynamics of a nonlinear system is available, the contribution of the feedback linearization part could be increased by removing the known dynamics and finally overcoming the remaining dynamics which are insignificant could be done by the adaptive part. If accurate information about the dynamics of the nonlinear system is not available, the contribution of the feedback linearization part decreases and consequently, contribution of the adaptive part of this controller increases because of an increase in the boundaries of uncertainties.
2. Paying attention to the computational burden of the proposed control is one of the most important points in designing a controller for industrial systems, because if the computational burden of the controller is high, owing to delay in online control, the stability of the closed- loop system cannot be guaranteed (Soltanpour and Fateh, 2009; Fateh, 2012a). In the proposed approach, using feedback linearization, the boundary of uncertainties decreases, and only seven rules in the rule base of

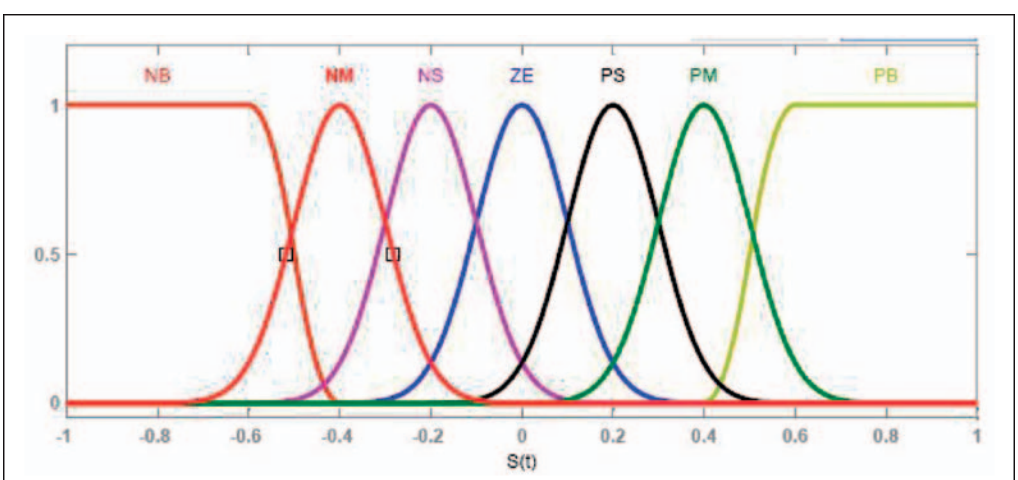

Figure 4. The membership functions of the rule base of the fuzzy inference engine.

the fuzzy system have been used. Therefore, the computational burden is very low and no delay occurs in online control of the nonlinear system.

3. In the classic sliding mode control design approach, for precaution's sake, a value much greater than the boundary of existing uncertainties is chosen as the value of the coefficient ρ (Shafiei and Soltanpour, 2009). When the amplitude of the control input increases, in order to prevent the saturation of actuators, high power actuators must be used. While, in the proposed approach, due to the existence of an adaptive law for estimating the coefficient ρ, the magnitude of the control input does not increase. Thus, using high power actuators is out of the question.
4. In most of the presented controllers, some coefficients are considered for decreasing the tracking error. Designers can decrease the tracking error by increasing these parameters but this increase causes the amplitude of the control input to increase and consequently it causes the actuators to get saturated (Fateh et al., 2012b; Soltanpour et al., 2012b). Coefficient γ is used in the adaptive law of the proposed approach. Increasing this coefficient decreases the tracking error, while it has no effect on the increase of the amplitude of the control input. This advantage is presented in the simulation section.
5. In recent years, in order to decrease the undesirable chattering phenomenon, fuzzy logic has been used in classic SMC (Soltanpour et al., 2013; Khooban et al., 2013c). Although the proposed approaches have desirable performance, they suffer from the absence of proof for the stability of the closed loop. In this paper, however, it has been proved that the closed-loop system with the proposed control is globally asymptotically stable in the presence of all uncertainties.
6. In designing the adaptive fuzzy approximator, instead of using seven rules, nine or at most 11 rules could be used. By so doing, the accuracy of tracking the desirable path greatly increases, without increasing the computational burden or the amplitude of the control input.
7. The SAMBA optimization algorithm has been used for determining the coefficients of the proposed control and determining the membership functions of the assumed part of the fuzzy rules (Khooban et al., 2012b; Niknam and Khooban, 2013; Shamsi-Nejad et al., 2013). Therefore, the amplitude of the control input will be optimal and concerns about the saturation of the actuators of the system will be out of the question.

8. Stages of designing the proposed control

To design the proposed control, the following steps must be followed:

1. Determine the tracking errors, $e_i(t)$.
2. Determine the sliding surface $S(t)$ using equation (6), through properly selecting the coefficients, c_i.
3. Determine $u_{eq}(t)$, according to equation (43).
4. Determine η by using equation (11) and existing information from known dynamics of the nonlinear system.
5. Build the rule base of fuzzy inference engine.

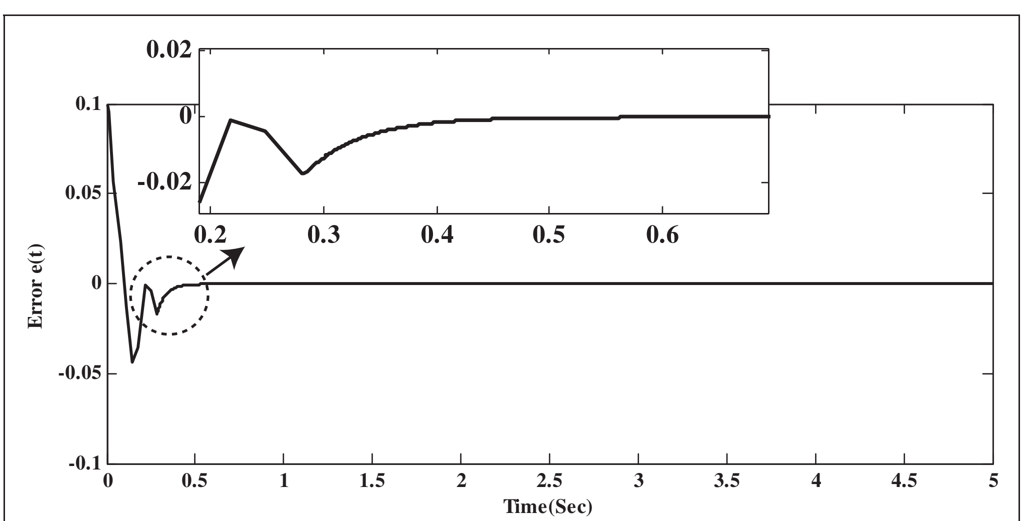

Figure 5. The error caused by applying sliding mode control.

6. Determine the adaptive law according to equation (43).
7. Determine the coefficient ρ, according to equation (43).
8. Implementation.
9. Determine the coefficients δ, γ and the coefficients c_i of sliding surface $S(t)$ by using the SAMBA optimization algorithm.

10. Change the number of rules in the rule base of the fuzzy inference engine to decrease the tracking error.

9. Simulation results

In order to demonstrate the effectiveness and robustness of the proposed approach, illustrative numerical simulation examples are provided in this section. The problem to be considered is a pole-balancing of an inverted pendulum. This system is shown in Figure 3 and is described by the following equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{mlx_2^2 \sin(x_1)\cos(x_1) - (M+m)g\sin(x_1)}{ml\cos^2(x_1) - \left(\dfrac{4}{3}\right)l(M+m)} \\ \qquad + \dfrac{-\cos(x_1)}{ml\cos^2(x_1) - \left(\dfrac{4}{3}\right)l(M+m)}u(t) + d(t) \end{cases} \tag{55}$$

where x_1 angle θ (in radians) of the pendulum from the vertical, M mass of cart, m mass of the pole, $u(t)$ force applied to the cart and $d(t)$ denotes an external disturbance. The following values are used for parameters in this simulation:

$$M = 1\,kg,\ \mathrm{m} = 0.3\,\mathrm{kg},\ \mathrm{l} = 0.5\,\mathrm{m} \text{ and } \mathrm{g} = 9.8\,\mathrm{m/s^2}.$$

In this simulation, the known parts of $f(X(t))$ and $g(X(t))$ are listed as follows:

$$\begin{cases} \hat{f}(X(t)) = \dfrac{\hat{m}\hat{l}x_2^2 - (\hat{M}+\hat{m})\hat{g}}{\hat{m}\hat{l} - \dfrac{4}{3}\hat{l}(\hat{M}+\hat{m})} \\ \hat{g}(X(t)) = \dfrac{-1}{\hat{m}\hat{l} - \dfrac{4}{3}\hat{l}(\hat{M}+\hat{m})} \end{cases} \tag{56}$$

The values considered for parameters $\hat{m}$, $\hat{l}$, $\hat{M}$ and $\hat{g}$ are equal to 90% of their real values. $e_1(t) = x_1(t) - x_{1_d}(t)$, $e_2(t) = x_2(t) - x_{2_d}(t)$ and $S(t) = c_1 e_1(t) + e_2(t)$ define the error and the sliding surface equation, respectively. The equivalent control input is $u_{eq}(t) = c_1\dot{e}_1(t)$. $|\eta|$ would be calculated as follows:

$$\|\eta\| = 0.01\,x_2^2 + 0.8 + 0.01\ \|\dot{x}_{2_d}(t)\| + 0.01\ \|u_f(t)\| \tag{57}$$

Using the SAMBA optimization algorithm the coefficients, $\gamma, \delta,$ and c_1 were determined to be equal to 2235, 5.16, and 24.23 respectively. The membership functions of the rule base of the fuzzy inference engine are shown in Figure 4.

Simulation 1 In this part of simulation $x_{1_d}(t) = 0$ and external disturbance is equal to zero. In other words, in

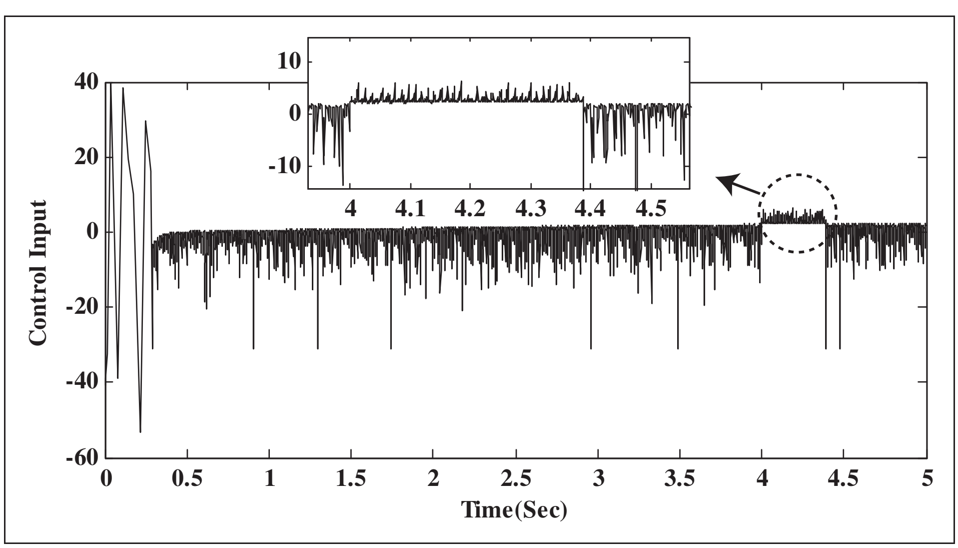

Figure 6. The control input caused by applying sliding mode control.

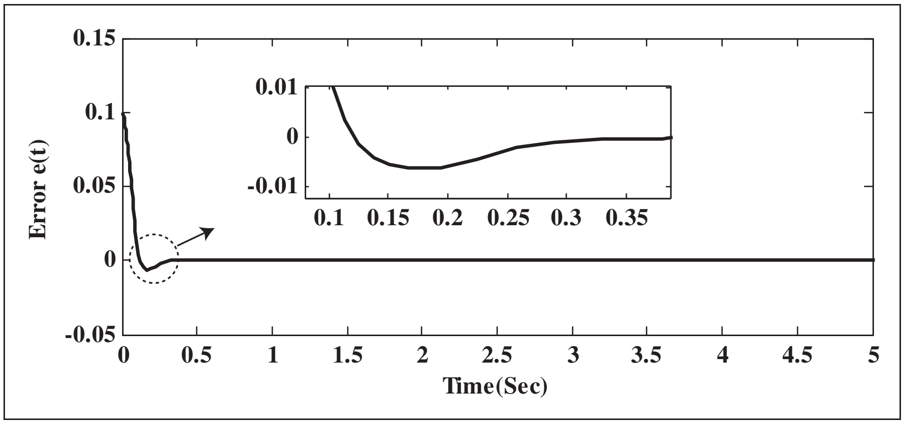

Figure 7. The error caused by applying optimal adaptive fuzzy sliding mode controller.

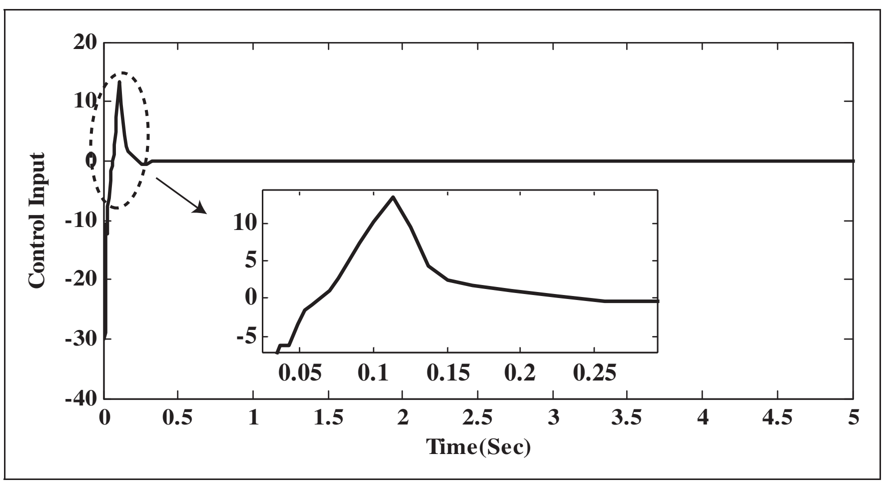

Figure 8. The control input caused by applying optimal adaptive fuzzy sliding mode controller.

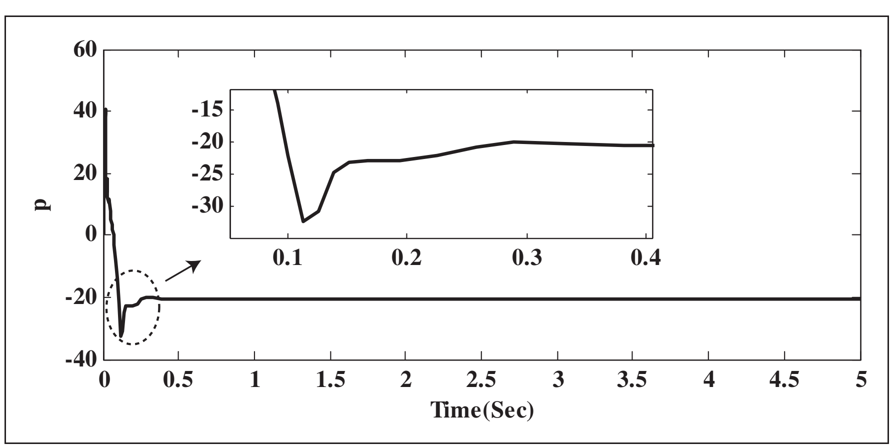

Figure 9. Approximating the ρ caused by applying optimal adaptive fuzzy sliding mode controller.

this part, the system has only the parametric uncertainties. In this stage of simulation, the performance of the proposed control is compared to the classic sliding mode controller. Once the classic sliding mode control is applied on the inverted pendulum system, according to Figure 5 it is concluded that the classic sliding mode controller shows a good performance and makes the error converge to zero in less than 0.6 s. However, Figure 6 shows that the control input of classic sliding mode has too much chattering. The occurrence of the undesirable chattering phenomenon in the mechanical system excites high-order nonlinear dynamics and consequently makes the closed-loop system unstable. After applying the proposed control, according to Figure 7, it can be seen that controller shows good performance and makes the error converge to zero in 0.3 s. In Figure 8 it can be seen that the control input does not have chattering and is within the allowed range. From Figure 9 it is concluded that, to make the error equal to zero, the adaptive law makes ρ converge to -20 after 0.3 s. The desirable performance of the proposed control can also be seen in Figure 10. According to this figure, it can be concluded that the sliding surface converges to zero in less than 0.5 seconds and it remains zero without chattering until the end of the simulation.

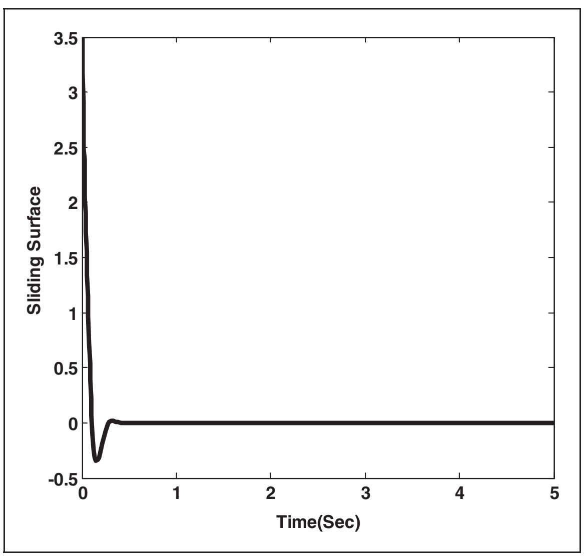

Figure 10. The sliding surface caused by applying optimal adaptive fuzzy sliding mode controller.

Simulation 2 In this part of the simulation, in order to create a more difficult challenge for the proposed control, the disturbance was considered to be $d(t) = -4\sin(3t)$ and the desired trajectory was considered to be time-variant, i.e. $x_{1_d}(t) = 0.2\sin(t)$. In this stage of the simulation, to evaluate the effect of coefficient γ, this coefficient was first considered to be equal to 500 but the next time it was considered to be equal to 2235. After simulation was carried out, it can be concluded from Figure 11 that the controller shows a good performance and makes the error converge to zero, but from Figure 12 it can be concluded that the tracking error has increased with the decrease of coefficient γ . In this figure it can be seen that with the increase of coefficient γ, the maximum tracking error decreased from 0.038 to 0.003. Therefore, we conclude that the tracking error could be significantly improved by increasing the coefficient γ in the proposed control. In most of the control approaches if the coefficient of the controller increases with a factor of 4, the amplitude of control input increases to an extent that it forces the system actuators to move towards saturation. However, from Figure 13 it is concluded that although the coefficient γ has increased with a factor of 4.47, the amplitude of the control input has not increased

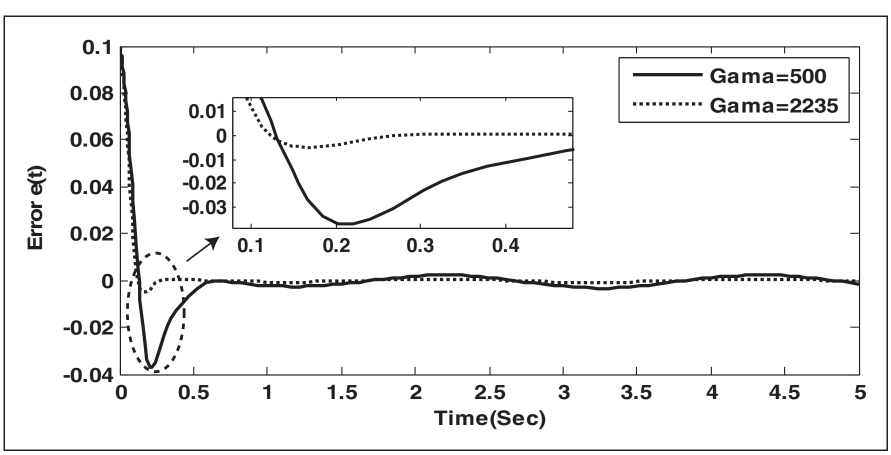

Figure 11. The tracking error when $\gamma = 500$.

significantly. Therefore, in the proposed approach, the concerns about the saturation are automatically eliminated.

In Figure 14 we can see that when$\gamma = 2235$, the sliding surface converges to zero without chattering in a very short time. Next, the control presented in Soltanpour and Khooban (2013) is implemented on the inverted pendulum system. To make a fair comparison, the conditions governing this stage of simulation are considered for the controller presented in Soltanpour and Khooban (2013). The coefficient of the controller was set to $\rho = 30$. After simulation, it can be concluded from Figure 15 that the controller presented in Khooban and Soltanpour (2013) functions properly and makes the tracking error converge to zero. Based on a comparison made between Figures 11 and 15, it can be concluded that the amplitude of the fluctuations of tracking error is higher in Figure 15. In Figure 16 it can be seen that the amplitude of control input is within a correct range. However, a comparison between Figure 13 and Figure 16 shows that the amplitude of control input in Soltanpour and Khooban (2013) is higher and it encounters the problem of chattering. It can be seen in Figure 17 that the sliding surface caused by applying the controller in Soltanpour and Khooban (2013) has high fluctuations. Therefore, these fluctuations cause the tracking error to converge to zero in a longer period of time. By investigating simulation results of this stage and the contents of Section 7 of this paper, it can be concluded that

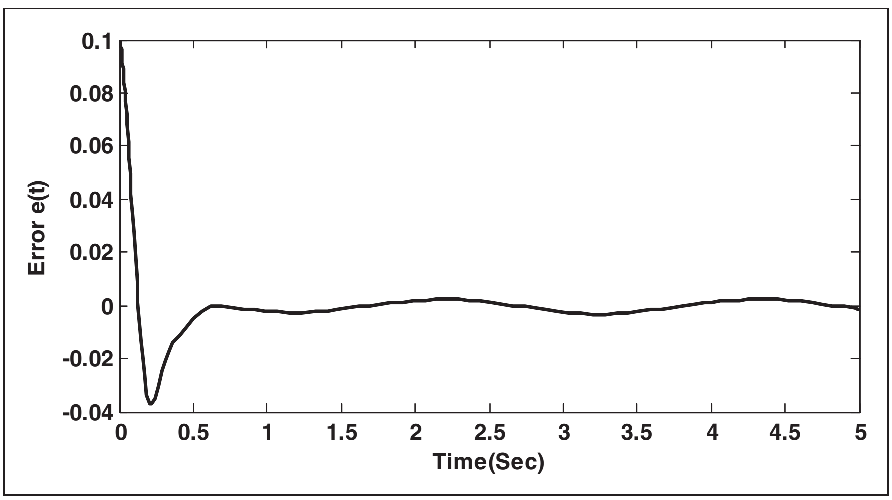

Figure 12. The decrease of tracking error with the increase of coefficient γ.

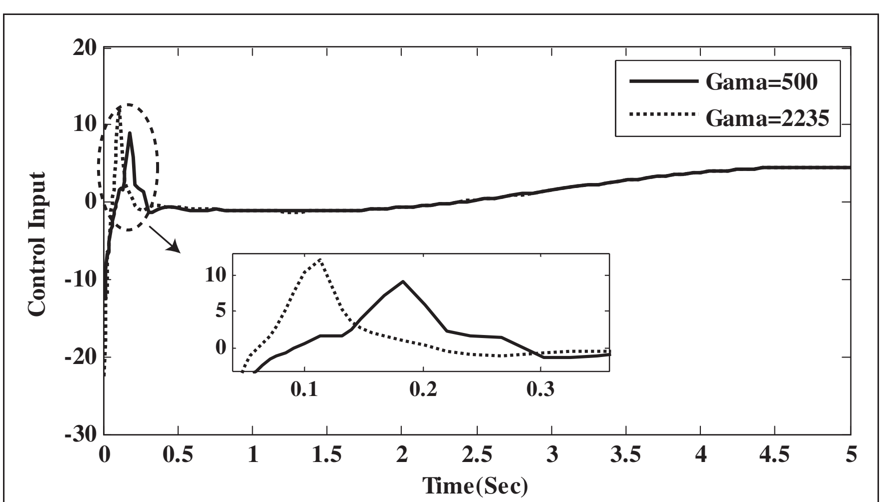

Figure 13. The control inputs with the increase of coefficient γ.

compared to the controller presented in Soltanpour and Khooban (2013), the proposed control has a better and more acceptable performance in overcoming the structured and unstructured uncertainties existing in the inverted pendulum system.

10. Conclusion

In this paper, an optimal adaptive fuzzy sliding mode controller was presented. In the proposed approach, to prevent the increase of the amplitude of control input, the boundary of uncertainties is decreased using the feedback linearization approach. Next, in order to overcome the remaining uncertainties, an adaptive fuzzy approximator is designed to determine the maximum boundary of uncertainties. The designed approximator is an appropriate substitute for the Sign function in classic sliding mode control because it operates in a way that not only provides the required conditions for overcoming uncertainties, but also totally removes the chattering effect in the control input. In the presented adaptive law, by using the coefficient the tracking error can be improved without affecting

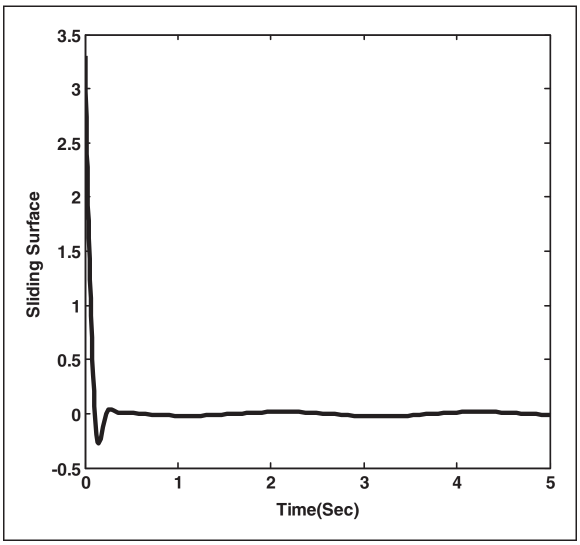

Figure 14. The sliding surface when $\gamma = 2235$.

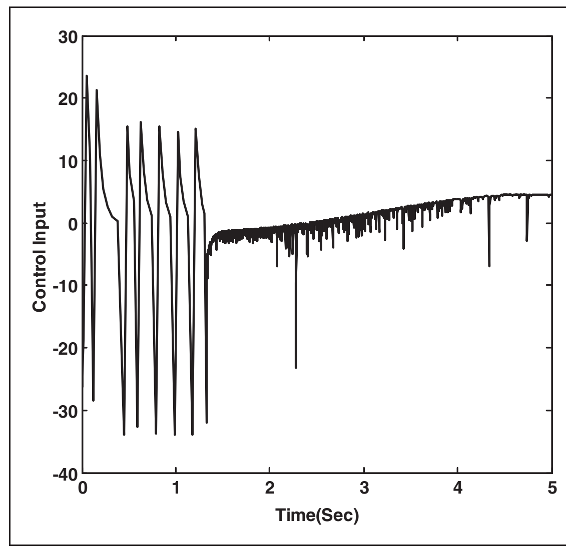

Figure 16. The control input caused by applying proposed control.

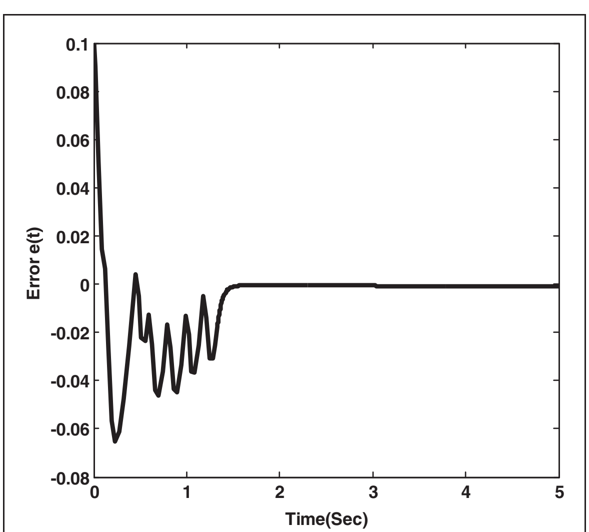

Figure 15. The tracking error caused by applying proposed control.

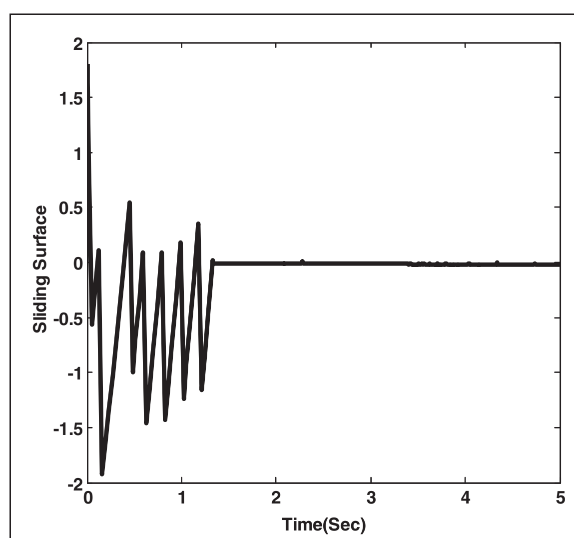

Figure 17. The sliding surface caused by applying proposed control.

the amplitude of the control input. A mathematical proof shows that the closed-loop system with the proposed controller is globally asymptotically stable in the presence of all uncertainties. Finally, to optimize the proposed controller, the coefficients of the control input were determined using the SAMBA optimization algorithm. Simulations were carried out on an inverted pendulum system in two stages. The results of these simulations show a desirable performance of the optimal adaptive fuzzy sliding mode controller.

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