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Generation of high-order Bessel beams by use of an axicon

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Abstract

We demonstrate and analyse a method for efficiently generating a high-order Bessel beam of arbitrary order by illuminating an axicon with the appropriate Laguerre–Gaussian light beam. High-order Bessel beams offer distinct advantages over other 'hollow' light beams for atom guiding. Our high-order Bessel beam generation technique offers a direct method for coupling cold atoms into this optical atom guide. © 2000 Elsevier Science B.V. All rights reserved.

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In 1987 Durnin established that the free-space Helmholtz equation has a set of solutions that are propagation-invariant [1]. These solutions have electric field amplitudes proportional to Bessel functions. The zeroth-order beam has a bright central maximum whereas the higher-order beams have a dark central core which propagates in free space without any spreading due to diffraction. Ideal Bessel beams are of infinite transverse extent and energy and thus can not be generated experimentally. However, it is possible to generate finite size approximations to Bessel beams which propagate over extended distances in a diffraction free manner [1–4].

Most of the experiments done to date consider the simplest of these beams, the zeroth-order Bessel beam. In this paper we present for the first time a highly efficient but straightforward technique for generating higher-order Bessel beams. Such higher-order Bessel beams have potential applications in alignment and indeed the notion of alignment using intensity minima produced with zone plates has been recently discussed [5]. Furthermore, we identify that high-order Bessel beams have applications for atom guiding and offer advantages over other 'hollow' light beams due to their non-diffractive nature and the fact that the radius of the central minimum can be of the order of the wavelength of light.

The first experiment by Durnin and co-workers generated a zeroth-order Bessel beam by illuminating an annular slit placed in the back focal plane of a lens with a plane wave [1]. This method is very inefficient as most of the intensity of the illuminating beam is blocked by the aperture. Much higher efficiencies of around 45% were achieved using holographic elements [2,3]. The most successful technique to generate an approximation to a zeroth-order Bessel beam is by use of a conically shaped optical element termed an axicon [4]. When illuminated by a Gaussian beam with a waist size much smaller than

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the hard aperture of the axicon, virtually the whole input intensity is converted into an approximation to a Bessel beam. However, to date axicons have only been used to generate zeroth-order Bessel beams.

Higher-order Bessel beams can be produced directly from an illuminating Gaussian beam by use of axicon-type computer generated holograms [3,6]. A more complicated holographic technique can be used to efficiently generate a superposition of odd-order Bessel beams that shows less variation of the central ring as the beam propagates [7]. In this paper we describe the generation of higher-order Bessel beams with high efficiency by illuminating an axicon with a Laguerre–Gaussian mode.

The electric field amplitude of a lth order Bessel beam is given by

$$E_l(r,\phi,z) = A\exp(\mathrm{i}k_z z)\, J_l(k_r r)\exp(\mathrm{i}l\phi)\,, \qquad (1)$$

where J_l is the lth-order Bessel function, and k_z and k_r are the longitudinal and radial components of the free-space wavevector, such that $k = 2\pi n/\lambda = \sqrt{k_z^2 + k_r^2}$. The zeroth-order beam has a central maximum, whereas all the higher-order beams have zero on-axis intensity surrounded by concentric rings of light. The central zero of the higher-order Bessel beams is due to the phase singularity of charge l associated with the azimuthal phase term $\exp(\mathrm{i}l\phi)$. As the radius of the inner ring, $r_l = \rho_l/k_r$, is determined by the position ρ_l of the first maximum of the lth-order Bessel function it increases with the order l. The transverse intensity spectrum of Bessel beams, on the other hand, does not change at all. Regardless of their order l it is always a ring of radius k_r. It is only the azimuthal phase variation $\exp(\mathrm{i}l\phi)$ across this annular spectrum that distinguishes different order Bessel beams.

This fact can be exploited for the generation of higher-order Bessel beams. All the above mentioned experimental methods used to generate zeroth-order Bessel beams convert an illuminating beam with plane wave fronts into a beam with an annular spectrum. If the illuminating plane wave is replaced by a beam with an azimuthal phase variation $\exp(\mathrm{i}l\phi)$, its azimuthal phase is conserved in the transformation, as only circular symmetric elements are involved. Thus the generated beam will have an annular transverse spectrum with an azimuthal phase variation, which gives an approximation to a Bessel beam of order l.

The illuminating beams are best described in terms of Laguerre–Gaussian (LG) modes, as they have the desired azimuthal phase variation. They form a complete orthonormal basis set for paraxial light beams and their electric field amplitude at the beam waist is given by

$$E(r,\phi,0) = A\left(2r^2/w_0^2\right)^{|l|/2} L_p^{|l|}\left(2r^2/w_0^2\right) \times\exp\left(-r^2/w_0^2\right)\exp(\mathrm{i}l\phi)\,, \qquad (2)$$

where L_p^l is a generalised Laguerre polynomial and w_0 is the waist size of the beam. The modes are characterised by two indices. The radial mode index p is related to the number of concentric rings, $p+1$, in the intensity cross-section and the azimuthal mode index l describes the charge of the phase singularity. LG modes may be generated from their Hermite–Gaussian counterparts by use of a cylindrical lens mode converter [8] or straight from the fundamental Gaussian beam by use of holographic techniques [9]. In some instances laser cavities may be made to oscillate in such modes [10].

If a single-ringed LG mode with azimuthal mode index l is used to illuminate an axicon placed at its beam waist, an approximation to a Bessel beam of order l is generated (Fig. 1). This establishes a direct link via just one optical element between LG modes and Bessel beams. For a rigorous mathematical deduction one can evaluate the Fresnel diffraction integral using the stationary phase method. The discussion is completely analogous to the one for axi-

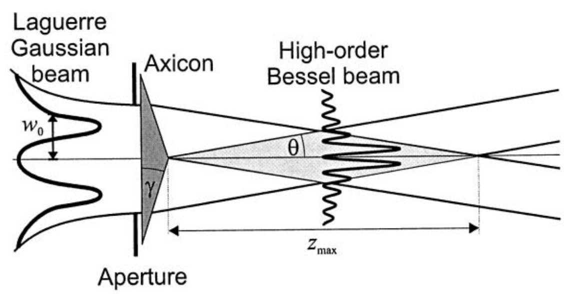

Fig. 1. Illuminating an axicon with a Laguerre–Gaussian mode generates an approximation to a higher-order Bessel beam within the shaded region. If a hole is drilled through the centre of the axicon the setup can be used for atom guiding, allowing the atoms to be funneled from the LG beam into the Bessel beam.

con-type holograms outlined in Ref. [3] and will be presented in the Appendix. Although the generated beam shows a pronounced variation in peak intensity its transverse profile close to the optic axis approximates to that of a Bessel beam of order l.

To investigate the conversion of a Laguerre–Gaussian beam into a higher-order Bessel beam experimentally we generated LG modes by illuminating a computer generated hologram [9] with a linearly polarised He–Ne laser. We used a selection of holograms to generate single-ringed beams of varying azimuthal index with an efficiency of about 30%. The LG beams had a waist size of $w_0 = 2.5$ mm and the axicon was positioned at their beam waist. A hard aperture of radius $R = 5$ mm was used to filter out the other diffraction orders of the hologram. The axicon had an internal angle of $\gamma = 1°$, generating a beam with an annular transverse spectrum of radius $k_r \approx k(n-1)\gamma$, where $n = 1.5$ is the refractive index of the axicon. The propagation distance of the generated Bessel beam can be estimated geometrically to be

$$z_{\max} = w_0 k / k_r \,. \tag{3}$$

The intensity cross-sections of the generated Bessel beams were examined by magnifying the beam using a x10 microscope objective and then recording it on a CCD camera. We generated Bessel beams with orders $l = 1$ to 4 and investigated their propagation. Their profile and propagation distance agree closely with our calculation. The radius of the inner ring of the generated first order Bessel beam is only $r_1 = 21.2$ μm and it propagates about $z_{\max} = 29$ cm without any spreading. This should be compared with a Laguerre–Gaussian beam with $l = 1$ and the same ring size at its waist, which would have a Rayleigh range of only about 4 mm.

Fig. 2 shows the profiles for different order beams at the same distance behind the axicon. The averaged radial profiles of the beams are also shown together with the theoretical intensity profile. They show a very good agreement. The alignment of the axicon gets more critical the higher the order of the Bessel beam, as higher order vortices tend to break apart into l single vortices of charge one if there is any astigmatism present in the optical system.

The Laguerre–Gaussian beam illuminating the axicon is hardly apertured and therefore its conversion into a Bessel beam is almost 100% efficient. In the experiment presented here, the overall efficiency is obviously limited by the computer generated holograms used to produce the LG beams. However, there are simple, more efficient methods to generate LG beams. By use of an open-cavity laser emitting a Hermite–Gaussian mode, which is then converted into a LG mode with a simple cylindrical lens mode converter [8], the overall efficiency may approach 100%.

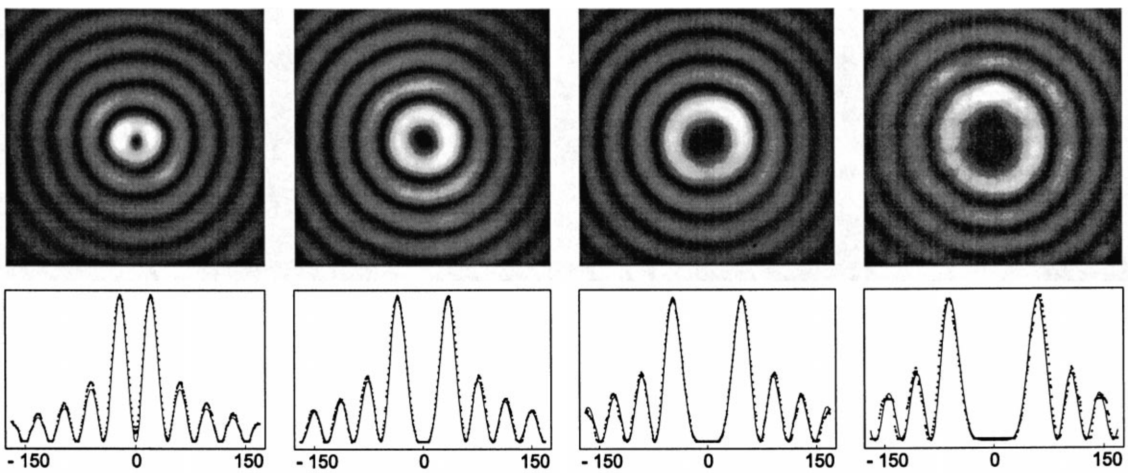

Fig. 2. Experimental beam cross-sections (355 μm × 355 μm) of Bessel beams of order $l = 1$ to 4 (from left to right) at a distance $z = 14$ cm behind the axicon. The radial profiles shown are the average of 40 azimuthal sections. They are in excellent agreement with the calculated profiles which are also shown. The radius of the inner ring increases with the order from $r_1 = 21.2$ μm to $r_4 = 61.2$ μm.

Most notably, our straightforward conversion of LG into Bessel beams could be used for atom guiding. Atom guiding along both hollow fibres and hollow light beams has generated much interest recently [11–14]. Blue-detuned LG beams have been used to guide and focus cold atoms [15]. As a result of diffraction, the cross-section of the guiding beam increases away from the focal position. By drilling a hole through the centre of the axicon, our technique for LG beam to Bessel beam conversion can be used to channel the atoms into the central minimum of the high-order Bessel beam which forms a long narrow optical guide for the atoms (Fig. 1).

A first indication for the quality of the guide can be found by looking at the optical dipole potential

$$U(\boldsymbol{r}) = \frac{\hbar\Delta}{2}\ln\left(1+\frac{I(\boldsymbol{r})/I_{\mathrm{Sat}}}{\left(1+4(\Delta/\Gamma)^2\right)}\right). \qquad (4)$$

Here, $I(\boldsymbol{r})$ is the spatially varying laser intensity, $\Delta=\omega_L-\omega_0$ is the laser frequency detuning from the atomic resonance, and Γ and I_{Sat} are the natural linewidth and the saturation intensity of the atomic transition, respectively. As an example, Fig. 3 shows the optical potential for a LG beam $(l=1)$ of waist size of 300 μm being converted into a Bessel beam. This J_1 beam has a central minimum of 15 μm radius and propagates for 5 cm without diffractive

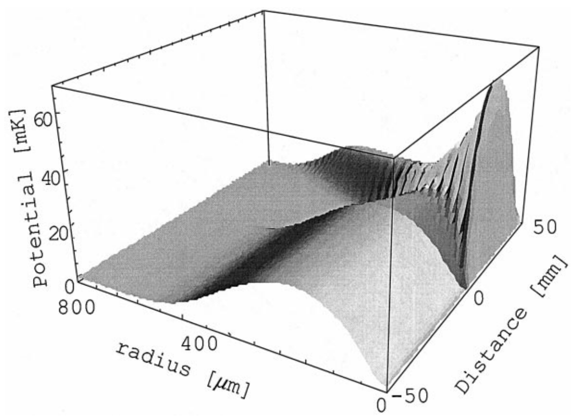

Fig. 3. Guiding potential for a $l=1$ LG beam with waist size $w_0=300$ μm converted into a Bessel beam with radius $r_1=15$ μm by an axicon sitting at position $z=0$ mm. The potential is calculated for rubidium ($\lambda=780.2$ nm, $\Gamma=2\pi\times6.1$ MHz, $I_{\mathrm{Sat}}=16\,\mathrm{W/m^2}$), where a laser power of 1 W and a frequency blue-detuning of $\Delta=3$ GHz was assumed.

spreading. The height of the optical potential for the J_1 beam increases by almost factor of 3 compared to the LG beam and the optical potential barrier is much steeper. Notably, high-order Bessel beams can have a central minimum of a size approaching the wavelength of light. In the optical region this size is comparable to the de Broglie wavelength of an ultra-cold ensemble of atoms and the atoms may propagate in modes along the Bessel beam. Thus one has the prospect of observing atom interference effects in a purely optically generated atom waveguide.

We have demonstrated a novel and efficient technique for generating a Bessel beam of arbitrary order l by illuminating an axicon with a Laguerre–Gaussian light beam of azimuthal index l. We demonstrated experimentally the generation of high-order Bessel beam from J_1 to J_4. Amongst other applications high-order Bessel beams offer distinct advantages for cold atom guiding due to their narrow central region and their non-diffracting nature.

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Appendix A

Our transformation of a Laguerre–Gaussian beam into an approximation to a Bessel beam can be analysed mathematically using the stationary phase method to evaluate the Fresnel diffraction integral as discussed in Ref. [3]. The field distribution behind the axicon illuminated by a singled-ringed Laguerre–Gaussian mode can be described by use of the Fresnel diffraction integral as

$$\begin{aligned}E(r,\phi,z) = {} & \frac{1}{\mathrm{i}\lambda z}\exp\left(\mathrm{i}k(z+r^2/2z)\right) \\ & \times\int_0^R \mathrm{d}r'\,r'\left[A\left(\sqrt{2}\,r'/w_0\right)^l \right. \\ & \left.\times\exp\left(-r'^2/w_0^2\right)\exp(-\mathrm{i}k_r r')\right] \\ & \times\exp\left(\mathrm{i}kr'^2/2z\right)\int_0^{2\pi}\mathrm{d}\phi'\,\exp(\mathrm{i}l\phi) \\ & \times\exp(-\mathrm{i}kr'r\cos(\phi-\phi')/z)\,. \end{aligned} \qquad (A.1)$$

Here the phase factor $\exp(-\mathrm{i}k_r r')$ is due to the phase retardation caused by the axicon and the field amplitude of the LG mode (Eq. (2) with $p=0$) has been factorised into its radial and azimuthal components.

The integration over the azimuthal angle ϕ' can be performed easily, giving rise to a lth-order Bessel function:

$$E(r,\phi,z) = \frac{1}{\mathrm{i}\lambda z}\exp\left(\mathrm{i}k\left(z+r^2/2z\right)\right)\exp(\mathrm{i}l\phi) \times \int_0^R f_l(r')\exp(-\mathrm{i}k\mu(r'))\mathrm{d}r', \tag{A.2}$$

where

$$f_l(r') = 2\pi(-\mathrm{i})^l \left[A\left(\sqrt{2}\,r'/w_0\right)^l \times \exp\left(-r'^2/w_0^2\right)\right] r' J_l(krr'/z) \tag{A.3}$$

and

$$\mu(r') = r'^2/2z - k_r r'/k. \tag{A.4}$$

The critical points r'_c for the principle of stationary phase in the r' integration are the zeros of the first derivative of $\mu(r')$. There is only one critical point, $r'_c = k_r z/k$, and the leading contribution of the integral then behaves as (see Chapter 7 of Ref. [16], Eqs. (3)–(8))

$$\int_0^R f_l(r')\exp(-\mathrm{i}k\mu(r'))\mathrm{d}r' \propto \frac{f_l(r'_c)\exp\left(\mathrm{i}k\mu(r'_c)\right)}{\sqrt{k\mu^{(2)}(r'_c)}}, \tag{A.5}$$

where $\mu^{(2)}(r'_c) = 1/z$ denotes the value of the second derivative of $\mu(r')$ at the critical point. This approximation is valid as long as the variation in $f_l(r')$ over the region of stationary phase is small. This is the case if the periodicity of the Bessel function is much larger than the width of the stationary phase region, requiring that $r^2 \ll z\lambda/4$ [6]. Neglecting position-independent factors the intensity in a transverse plane behind the axicon is then proportional to

$$I(r,z) \propto z^{2l+1}\exp\left(-2z^2/z_{\max}^2\right)J_l^2(k_r r), \tag{A.6}$$

where $z_{\max}$ is the 'propagation distance' as defined by Eq. (3).

Close to the optic axis, the beam generated by an axicon illuminated by a Laguerre–Gaussian beam with azimuthal index l therefore approximates to a Bessel beam of order l.

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