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Security adoption and influence of cyber-insurance markets in heterogeneous networks☆

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ABSTRACT

Hosts (or nodes) in the Internet often face epidemic risks such as virus and worm attack. Despite the awareness of these risks and the importance of network/system security, investment in security protection is still scare, and hence epidemic risk is still prevalent. Deciding whether to invest in security protection is an *interdependent process*: security investment decision made by one node can affect the security risk of others, and therefore affect their decisions also. The first contribution of this paper is to provide a fundamental understanding on how "network externality" with "node heterogeneity" may affect security adoption. Nodes make decisions on security investment by evaluating the epidemic risk and the expected loss. We characterize it as a *Bayesian network game* in which nodes only have the local information, e.g., the number of neighbors, and minimum common information, e.g., degree distribution of the network. Our second goal is to study a new form of risk management, called *cyber-insurance*. We investigate how the presence of a competitive insurance market can affect the security adoption and show that if the insurance provider can observe the protection level of nodes, the insurance market is a positive incentive for security adoption if the protection quality is not very high. We also find that cyber-insurance is more likely to be a good incentive for nodes with higher degree. Conversely, if the insurance provider cannot observe the protection level of nodes, we verify that partial insurance can be a non-negative incentive, improving node's utility though not being an incentive.

1. Introduction

Network security is a major problem in communication networks. One of its most common manifestations is in form of virus, worms and botnet spreading, which we call the *epidemic risk*. In these epidemic risks, hosts (or nodes) which are infected become the sources of new infections, and adversaries can use these compromised nodes to generate new attacks. Epidemic risk is highly damaging, e.g., the Code Red worm [1] has infected thousands of computers and induced huge financial loss. To counter this risk, there have been great efforts in both the research and industrial fronts to come up with techniques and tools (i.e., anti-virus software, intrusion detection systems, firewalls, etc.) to detect virus/worms. Despite the sophistication of these tools, only a small percentage of hosts adopt some form of security protection, making epidemic risk still prevalent. In this paper, instead of discussing the technology side of security, we discuss the security adoption in economic language. We argue that it may better explain the low adoption level of security products.

Note that a node's decision of whether to adopt some security measures is not a simple individual and independent process, but rather, *depends* on the decisions of many other nodes in the network. Nodes which decide not to invest

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in security protection, also put other nodes at security risk. This *network externality effect* caused by the spreading of epidemic influences the degree of adoption of security measure. *Our first contribution in this paper is to provide a theoretical understanding on how network externality effect with node heterogeneity may influence security adoption in a network of interconnected nodes (i.e., the Internet).* The externality effect with heterogeneity has significant implication for a policy maker aiming to boost the security level in that by subsidizing early adopters, later adopters will naturally follow.

Modeling such decision and security problem requires the combination of epidemic theory and game theory. While extensive studies in the traditional literature have been dedicated to epidemic theory [2,3], few works have addressed the problems of strategic behavior of security investment. In a realistic situation, nodes which make decision in security investment usually do not have complete information about the network topology or knowledge of other nodes. As a result, it is difficult for them to accurately evaluate the epidemic risk and other nodes' influence on itself. In this paper, we model the security investment as a *Bayesian network game* where nodes only have the local information of their degree and the minimum common information of network's degree distribution. In contrast to graphical game [4], in which complete topology is given and analysis is complicated, we show that using the Bayesian network game, one can elegantly tradeoff using partial topology information while making analysis tractable.

By using the Bayesian network game, *we show how heterogeneous nodes, characterized by their degree, can estimate their epidemic risk and make decisions on security investment with incomplete information*. We show that nodes with higher degree are more likely to be infected by epidemic, making the secure measure less effective for nodes with higher degree in terms of the reduction in infection probability. Moreover, nodes with higher degrees are more sensitive to externality, i.e., they are more likely to be affected by others' decision. The final adoption fraction of nodes with different degrees depends on their relative loss from epidemic.

While protection measures may limit the spread of virus/worms, another way to manage the epidemic risk is to transfer the risk to a third-party, which is called *cyber-insurance* [5]: nodes pay certain premium to insurance companies in return for compensation in the virus outbreaks. The two main challenges in cyber-insurance are: *adverse selection* and *moral hazard* [5,6]. The problem of adverse selection arises when the insurance provider cannot distinguish between high and low risk nodes. The combination of self-protection and insurance raises the problem of moral hazard, in which nodes covered by insurance may take fewer secure measures, or even falsify their loss. Moral hazard happens when the insurance provider cannot observe the protection level of nodes. In this paper, we address the moral hazard problem which is especially serious in cyber-insurance. We investigate the effect of cyber-insurance on security adoption under a competitive insurance market. *Our second contribution is to show the conditions under which cyber-insurance is an incentive, with and without moral hazard*. We find that cyber-insurance without moral hazard is an incentive for security adoption if the initial secure condition is bad and the quality of secure measure is not very high. Moreover, cyber-insurance is more likely to be an incentive for nodes with high degree. We verify that partial insurance coverage can be a non-negative incentive for secure adoption with moral hazard.

This paper is outlined as follows. In Section 2, we present the epidemic and security investment models. In Section 3, we show how heterogeneous nodes can determine their infection probability and decide on proper security investment. In Section 4, we investigate the effect of the insurance market, both with and without moral hazard, on security adoption. Validations and performance evaluations are presented in Section 5. Section 6 gives related work. Finally, in Section 7, we briefly summarize and discuss several ways in which our model could be improved.

2. Mathematical models

Let us first present the mathematical models on how nodes make decision on security investment. The model mainly derives from that of [7,8] with some modification. Our models include: (a) *epidemic model*: to characterize the spread of virus or malware in a network, (b) *investment model*: to characterize node's decision in security investment, and (c) *Bayesian network game*: given the epidemic and investment models, how nodes make decision under the incomplete information setting. We summarize some of the notations in Table 1 for reference.

Epidemic model: the interaction relation of N nodes is denoted by the undirected graph $G = (V, E)$ with the vertex set V, $|V| = N$ and the edge set E. For $i, j \in V$, if $(i, j) \in E$, then nodes i and j are neighbors and we use $i \sim j$ to denote this relationship. Let $S = \{healthy, infected\}$ represent the set of states each node can be in. If node i is infected (healthy), then $S_i = 1$ ($S_i = 0$). Each infected node can contaminate its neighbors independently with probability q. Note that this is similar to the *bond percolation process* [3] in which every edge is occupied with probability q. Each node has an *initial state* of being infected or not. This can represent whether the node has been attacked by the adversary. Let us denote it by s_i where $s_i = 1$ if node i is initially infected and $s_i = 0$ otherwise. Hence, at the steady state, a node is infected either because it is initially infected, or it contracts virus from its infected neighboring nodes. The final state of node i can be expressed in the following recursive equation:

$$1 - S_i = (1 - s_i) \prod_{\forall j: j \sim i} (1 - \theta_{ji} S_j) \quad \forall i \in V, \tag{1}$$

where θ_{ji} is a random variable indicating whether the edge (i, j) is occupied or not. According to previous discussion, θ_{ji} is a Bernoulli random variable with $\Pr(\theta_{ji} = 1) = q$. Since an infected node will incur some financial loss, a node needs to

Table 1
Notations.

Summary of notations		
q	:	Infection probability of an edge
x_i	:	Investment in security of node i
l_i	:	Loss of node i if get attacked
$p(x_i)$	:	Initial infection probability with investment x_i
p_i,	:	Final infection probability of node i
p^+	:	Initial infection probability without security investment
p^-	:	Initial infection probability with fixed amount of investment
$\{p_k\}_{\underline{K}}^{\overline{K}}$	:	Degree distribution of graph G
$F_k(l)$	:	Loss distribution of nodes with degree k
$\{\tilde{p}_k\}_{\underline{K}}^{\overline{K}}$	:	Neighbor degree distribution
$\{q_k\}_{\underline{K}'}^{\overline{K}'}$	:	Excess degree distribution
ϕ_k	:	Infection probability of nodes with degree k
λ_k	:	Fraction of adopters for nodes with degree k
$u_i(w_i)$	:	Utility of nodes i with wealth w_i
$U(\pi, X)$	:	Utility of buying insurance amount X at price π
$U_{\mathcal{N}}(\pi, X)$	:	Utility of buying insurance amount X at price π when taking action $\mathcal{N}$
$U_{\mathcal{S}}(\pi, X)$	:	Utility of buying insurance amount X at price π when taking action $\mathcal{S}$

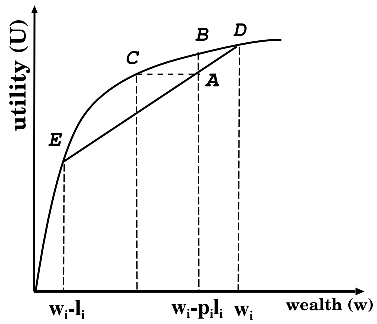


Fig. 1. Risk-averse utility function.

decide whether to invest in self-protection to reduce the potential financial loss. Let us present the model to help a node in making such a decision.

Investment model: the investment model is mainly formulated using game theoretical terms; readers not familiar with game theory can refer to [9]. Node i has an initial wealth $w_i \in \mathbb{R}_+$. A node's utility $u_i(w)$ is a function of wealth $w \in \mathbb{R}_+$. We consider that nodes are *risk averse*, i.e., the utility function is strictly increasing and concave in w, i.e., $u_i'(w) > 0$ and $u_i''(w) < 0$. Fig. 1 depicts a risk averse utility function. In this paper, we consider the *constant relative risk averse* utility function commonly used in the economic literature [10]:

$$u(w) = \frac{w^{1-\sigma}}{1-\sigma}, \quad 0 < \sigma < 1, \tag{2}$$

where σ is a parameter for the degree of risk aversion. The condition $0 < \sigma < 1$ is added to ignore the case of $\sigma = 1$ and also for tractability of analysis later on. For node i, the utility function is given by the above utility function with parameter σ_i. If node i is infected, then it will incur a financial loss of $l_i \in \mathbb{R}_+$. For node i, the expected utility is as shown in Fig. 1. D is the initial utility point; E is the utility point after getting infected. C is the expected utility. To reduce the potential financial loss, a node can consider some self-protection measures or purchasing insurance. In the first part of this paper, we consider the case of self-protection. In the second part of this paper, we consider both cases and study the influence of insurance market on security protection.

A node's investment in self-protection can reduce the probability of being infected initially. For the amount of investment x, the probability of being infected initially is $p(x)$, which is a continuous differentiable decreasing function of x. In particular, we assume that the effort of security investment is separable with the wealth. Similar assumptions have been made in [11,12]. If node i invests x_i in secure protection, the expected utility is

$$p_i u_i(w_i - l_i) + (1 - p_i) u_i(w_i) - x_i, \tag{3}$$

where p_i is the *final* probability that node i will be infected. p_i contains two parts: the probability of being infected initially, given by $p(x_i)$, and the probability of getting infected from neighbor nodes. For simplicity of analysis, we assume that the choice of node i regarding security self-protection is a binary decision: either the node invests unit amount with a cost of c_i, or it does not invest at all. We use the action set $A = \{\mathcal{S}, \mathcal{N}\}$ to denote the behavior, where $\mathcal{S}$ denotes taking secure measure and $\mathcal{N}$ otherwise. If it decides to invest, the node can still be infected with probability p^-. Otherwise, it will be infected with probability p^+. Obviously we have $0 < p^- < p^+ < 1$. Let $a = (a_1, \ldots, a_i, \ldots, a_N) = (a_i, a_{-i})$ be an *action profile*. Given the action profile a_{-i} of other nodes, node i makes the decision by maximizing its expected utility. If node i takes action $\mathcal{N}$, the expected utility is:

$$p_i(\mathcal{N}, a_{-i})u_i(w_i - l_i) + (1 - p_i(\mathcal{N}, a_{-i}))u_i(w_i) \tag{4}$$

where $p_i(\mathcal{N}, a_{-i})$ is the final probability of node i being infected when it initially did not adopt security protection. On the other hand, the expected utility of a node which initially subscribed to security protection (or action $\mathcal{S}$) is:

$$p_i(\mathcal{S}, a_{-i})u_i(w_i - l_i) + (1 - p_i(\mathcal{S}, a_{-i}))u_i(w_i) - c_i \tag{5}$$

where $p_i(\mathcal{S}, a_{-i})$ is the final probability of a node being infected when it is initially subscribed to some self-protection measures with cost of c_i. Note that $p_i(\mathcal{S}, a_{-i})$ and $p_i(\mathcal{N}, a_{-i})$ are functions of p^- and p^+, the contagion probability q and the graph G since it controls the infection process.

Each node needs to consider whether it should subscribe to some self-protection measures. The decision is based on the cost of investing in security measure, as well as the risk loss of being infected. The decision is non-trivial because one has to consider the *network externality effect*. In particular, node i will choose to invest in security protection if and only if

$$c_i < (p_i(\mathcal{N}, a_{-i}) - p_i(\mathcal{S}, a_{-i}))(u_i(w_i) - u_i(w_i - l_i)). \tag{6}$$

Bayesian network game: according to Inequality (6), each node needs to have the complete information of the network topology G so as to make the proper decision. However, it is almost impossible in practice for each node to have the complete information of G. Instead, each node can only have some *local information* on G, i.e., a node may only know its neighbors, and in some cases, only know the number of neighbors it is to interact with. Second, it is impossible to know the exact loss of other nodes in a large network.

Here, we assume that nodes only have the *minimum common information*, that is, the knowledge of the degree distribution of G, as well as the distribution of financial loss of nodes caused by virus. Assume that the degree distribution of the graph is $\{p_k\}_{\underline{K}}^{\overline{K}}$, where $\overline{K}$ is the maximum degree and $\underline{K}$ is the minimum degree. In this paper, we consider the *asymptotic case* that N, the number of nodes, tends to infinity and the degree distribution converges to the fixed probability distribution $\{p_k\}_{\underline{K}}^{\overline{K}}$. For nodes with degree k, the loss distribution is given by the CDF $F_k(l)$. We assume that the cost of secure measure is the same for all nodes which have the same degree and we denote this as c_k. Furthermore, these nodes have the same utility function u_k and the same initial wealth w_k. Nodes make decision on security investment based on the information of degree and loss. According to the discussion in the investment model, a node should know the probability of getting infected before deciding on security investment. Since nodes do not have the complete information, they should estimate these probabilities based on the limited common information. Next, we derive this infection probability with the partial topology information.

3. Analysis for strategic security adoption

Let us show that how nodes make decisions on security investment and how to determine the final security protection level.

3.1. General case

Determining the final infection probability for a node is a difficult problem because of the complex network structure. In this work, we assume that a node only knows the degree distribution and consider the network topology as a *random graph* [3] with a given degree distribution $\{p_k\}_{\underline{K}}^{\overline{K}}$. Thus, nodes do not need to know the full network topology G to determine the final infection probability. Although real networks are not random graphs [3] and they have some characteristics, e.g., high clustering coefficient, community structure, etc., that are not possed by a random graph, recent study [13] has shown that random graph approximation is very often accurate for a real network. Thus, it is reasonable to assume that the network topology is a random graph; especially here we consider an incomplete information case. With the assumption, each node can compute its final infection probability using the following methodology.

Estimating the probability: a node can calculate its final infection probability by constructing a *local mean field tree* [14]. Fig. 2 illustrates the local tree structure of node i which has degree k. For ease of illustration, let us say that none of these nodes take secure measure, i.e., the initial infection probability is p^+ for all nodes in this subsection. We will show how to relax this in the later section.

The children of node i in the local mean field tree are denoted as v_c, $c \in [1, k]$. The triangle under each child node v_c denotes another tree structure. Based on the results in [3,14], for any node i, the local topology of a large random graph G can be modeled as a tree rooted at node i with high probability. In other words, we transform G to a tree rooted at node i (or local mean field of node i). Node i can be independently influenced by each subtree rooted at v_c. For every subtree rooted at v_c, it consists of its subtrees. Using this *recursive* structure, we derive the total infection probability that other nodes in G can impose on node i.

First we divide nodes into levels. The root node i is at the zero level. The neighbors of node i is at the first level and so on. Let Y_j be the final state of node j, $j \neq i$, *conditioned on its parent in the tree structure is not infected*, and y_j be the initial state

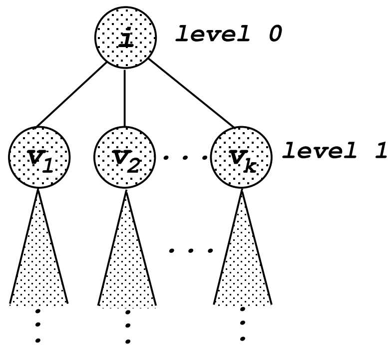


Fig. 2. Local mean field tree for node i with degree k.

of node j. For the root node i, we use S_i to denote its final state and s_i to denote its initial state, and then we have

$$1 - S_i = (1 - s_i) \prod_{\forall j: j \sim i} (1 - \theta_{ji} Y_j). \tag{7}$$

The above equation indicates that the root node i is either initially infected, or it can be infected by its neighbors. The state of its neighbors conditioned on that the root node i is not infected is also determined by the state of the children of the neighbors in the tree structure, or one can express it recursively as:

$$1 - Y_j = (1 - y_j) \prod_{\forall l: l \to j} (1 - \theta_{lj} Y_l) \quad \forall j \neq i, \tag{8}$$

where $l \to j$ denotes that l is a child of j in the tree structure. To solve Eq. (8), we need to know the degree distribution of a child node, i.e., the neighbor degree distribution. This degree distribution can be expressed as:

$$\tilde{p}_k = \frac{k p_k}{\sum_{k=\underline{K}}^{\overline{K}} k p_k} = \frac{k p_k}{\bar{d}},$$

where $\bar{d}$ is the average degree of nodes in G. The number of edges of a child excluding the edge connecting to its parents is called the *excess degree* [3]. Let $\underline{K}' = \max\{0, \underline{K} - 1\}$ and $\overline{K}' = \max\{0, \overline{K} - 1\}$. The excess degree distribution of a child is

$$q_k = \tilde{p}_{k+1} = \frac{(k+1) p_{k+1}}{\bar{d}}, \quad k \in [\underline{K}', \overline{K}']. \tag{9}$$

As in [14], if nodes are at the same level of the tree structure, then their states are independent of each other. Let ρ_n, $n \geq 1$ be the probability that a node at the nth level is infected conditioned on its parent is not infected. By Eq. (8), we have

$$1 - \rho_n = (1 - p^+) \sum_{k=\underline{K}'}^{\overline{K}'} q_k (1 - q\rho_{n+1})^k.$$

ρ_1 is the average probability that a child node of the root node i will be infected conditioned on the root node is not infected. When we scale up the network (or let $n \to \infty$), define $\rho \triangleq \lim_{n \to \infty} \rho_1$, and then ρ is determined by the solution of the fixed point equation

$$1 - \rho = (1 - p^+) \sum_{k=\underline{K}'}^{\overline{K}'} q_k (1 - q\rho)^k.$$

By Eq. (7), for a node with degree k, the infection probability is

$$\phi_k = 1 - (1 - p^+)(1 - q\rho)^k. \tag{10}$$

Security adoption: in the previous subsection, we show how a node can compute the infection probability with incomplete information. The calculation is based on the assumption that none of the nodes take secure adoption, so that the initially infection probability is p^+. In here, we show how to use this infection probability for strategy selection. Let λ_k be the fraction of nodes with degree k which take action $\mathcal{S}$. Then by applying the method shown above, we have the following proposition.

Proposition 1. *If λ_k fraction of the nodes with degree k takes secure measure, ρ is given by the unique solution of the fixed point equation in* $[0, 1]$*:*

$$\rho = 1 - \sum_{k=\underline{K}'}^{\overline{K}'} q_k(1 - p^+ + \lambda_{k+1}(p^+ - p^-))(1 - q\rho)^k. \tag{11}$$

For a node with degree k, if it decides to take secure measure, then by Eq. (10), the infection probability is

$$\phi_k(\mathcal{S}, \lambda_{\underline{K}}, \ldots, \lambda_{\overline{K}}) = 1 - (1 - p^-)(1 - q\rho)^k. \tag{12}$$

If it does not invest in protection measure, the probability for this node to get infected is

$$\phi_k(\mathcal{N}, \lambda_{\underline{K}}, \ldots, \lambda_{\overline{K}}) = 1 - (1 - p^+)(1 - q\rho)^k. \tag{13}$$

The infection probability reduction for a node with degree k is

$$\phi_k(\mathcal{N}) - \phi_k(\mathcal{S}) = (p^+ - p^-)(1 - q\rho)^k. \tag{14}$$

Note that this infection probability reduction decreases as degree increases. This implies that higher degree nodes have *less incentive* to invest in protection measure.

Corollary 1. *ρ, given by the solution of fixed point Eq. (11), has a unique solution in* $[0, 1]$*, and $\rho(\lambda_{\underline{K}}, \ldots, \lambda_{\overline{K}})$ is a decreasing function of $\lambda_k, \forall k \in [\underline{K}, \overline{K}]$.*

Proof. Let

$$g(\rho, \lambda_{\underline{K}}, \ldots, \lambda_{\overline{K}}) = 1 - \sum_{k=\underline{K}'}^{\overline{K}'} q_k(1 - p^+ + \lambda_{k+1}(p^+ - p^-))(1 - q\rho)^k.$$

Obviously, $g(\rho, \lambda_{\underline{K}}, \ldots, \lambda_{\overline{K}})$ is an increasing function of ρ.

$$g(0, \lambda_{\underline{K}}, \ldots, \lambda_{\overline{K}}) = 1 - \sum_{k=\underline{K}'}^{\overline{K}'} q_k(1 - p^+ + \lambda_{k+1}(p^+ - p^-)) > 0,$$

$$g(1, \lambda_{\underline{K}}, \ldots, \lambda_{\overline{K}}) = 1 - \sum_{k=\underline{K}'}^{\overline{K}'} q_k(1 - p^+ + \lambda_{k+1}(p^+ - p^-))(1 - q)^k < 1.$$

We can see that the fixed point equation $\rho = g(\rho)$ has at least one solution. Taking the second order derivative with respect to ρ, we have

$$g_{\rho\rho} = -\sum_{k=\underline{K}'}^{\overline{K}'} q_k(1 - p^+ + \lambda_{k+1}(p^+ - p^-))k(k-1)(1 - q\rho)^{k-2}q^2 < 0.$$

$g(\rho)$ is a concave function. Let ρ^* be one of the solutions, i.e., $\rho^* = g(\rho^*)$. Then by concavity of $g(\rho)$, $g_\rho(\rho^*) < 1$. Otherwise, $g(\rho^*) = g(0) + \int_0^{\rho^*} g_\rho(\rho)d\rho > \rho^*$. Then for $0 < \rho < \rho^*$, $g(\rho) > \rho$, for $\rho^* < \rho < 1$, $g(\rho) < \rho$. As a result, the fixed point equation $\rho = g(\rho, \lambda_{\underline{K}}, \ldots, \lambda_{\overline{K}})$ has a unique solution in [0, 1].

Let $\lambda_k^1 < \lambda_k^2$, and $\rho_1 = g(\rho_1, \lambda_k^1)$ and $\rho_2 = g(\rho_2, \lambda_k^2)$. Since $g(\rho, \lambda_k)$ is a decreasing function of λ_k for all $k \in [\underline{K}, \overline{K}]$ and $\lambda_k^1 < \lambda_k^2$, we have $g(\rho_2, \lambda_k^1) > g(\rho_2, \lambda_k^2) = \rho_2$. By the same argument in proving uniqueness, we can get $\rho_1 > \rho_2$. As a result, the solution of $\rho = g(\rho, \lambda_k)$ is a decreasing function of $\lambda_k, \forall k \in [\underline{K}, \overline{K}]$. □

Remark. Combining Corollary 1 with Eq. (14), we see that the reduction in infection probability by taking security measure increases as other nodes adopt security measure. This shows the *network externality effect*, i.e., the value of security measure increases as more nodes invest in self-protection.

This externality effect is first modeled in [15,16] and later verified in [7,8]. We complement their results by studying the externality effect with heterogeneity characterized by node degree.

Sensitivity analysis: nodes with different degrees have different sensitivities to the externality effect. Define $\widetilde{\phi}_k = \phi_k(\mathcal{N}) - \phi_k(\mathcal{S}) = (p^+ - p^-)(1 - q\rho)^k$. Assume that ρ decreases by a small amount $\Delta\rho$, and then $\Delta\widetilde{\phi}_k = (p^+ - p^-)(1 - q\rho)^{k-1}kq\Delta\rho$, and the relative change is given by $\frac{\Delta\widetilde{\phi}_k}{\widetilde{\phi}_k} = \frac{kq\Delta\rho}{(1-q\rho)}$, which indicates that sensitivity to the network externality effect is proportional to the degree.

A node with degree k will invest if and only if the utility with secure measure is higher than that without secure measure, or

$$\begin{aligned} c_k &< (\phi_k(\mathcal{N}) - \phi_k(\mathcal{S}))(u_k(w_k) - u_k(w_k - l)) \\ &= (p^+ - p^-)(1 - q\rho)^k (u_k(w_k) - u_k(w_k - l)). \end{aligned}$$

Note that the loss distribution of nodes with degree k is $F_k(l)$. Since the infection probability varies with the fraction of security adopters, we consider the *self-fulfilling expectations equilibrium* [17] in analyzing the final adoption extent. Nodes form a shared expectation that the fraction of the nodes has adopted security measure and if each of them makes decision based on this expectation, then the final fraction is indeed the initial expectation.

Let l_k^* be the minimum value that satisfies the above inequality in the equilibrium, and then λ_k^*, the fraction of node of degree k taking the secure measure, is given by the equation $\lambda_k^* = 1 - F_k(l_k^*)$. Summarizing the previous analysis, we have the following proposition.

Proposition 2. *Nodes with degree k take the secure measure if their loss is greater than l_k^*. The final fraction of nodes with degree k that invest in self-protection is λ_k^*. l_k^* and λ_k^* are solutions of the following fixed point equations:*

$$\lambda_k^* = 1 - F_k(l_k^*), \tag{15}$$

$$c_k = (p^+ - p^-)(1 - q\rho^*)^k (u_k(w_k) - u_k(w_k - l_k^*)), \tag{16}$$

where ρ^* is given by the solution of the following equation

$$\rho^* = 1 - \sum_{k=\underline{K}'}^{\overline{K}'} q_k (1 - p^+ + \lambda_{k+1}^* (p^+ - p^-))(1 - q\rho^*)^k. \tag{17}$$

We first propose an algorithm to compute the equilibrium point in Proposition 2, and then we prove the existence of equilibrium points by showing that the algorithm will lead to a feasible answer.

Algorithm 1 Compute the equilibrium

Input: The error tolerance ϵ.
Output: The equilibrium point $\{\lambda_k^*\}_{\underline{K}}^{\overline{K}}$.
for all $k \in [\underline{K}, \overline{K}]$ **do**
  $\lambda_k^0 \leftarrow 0$
  $\lambda_k^1 \leftarrow 1$
end for
Compute ρ^0 by substituting λ_k^0 in Eq. (17).
$t \leftarrow 0$
repeat
  $t \leftarrow t + 1$
  Compute l_k^t by substituting ρ^t in Eq. (16).
  Compute λ_k^t by substituting l_k^t in Eq. (15).
  Compute ρ^{t+1} by substituting λ_k^t in Eq. (17).
until $\max_k |\lambda_k^{t-1} - \lambda_k^t| < \epsilon$
return $\lambda_k^* = \lambda_k^t$.

Corollary 2. *Fixed point equation (15)–(17) have at least one solution.*

Proof. In Algorithm 1, we start from $\lambda_k^0 = 0$ and do the iteration. Obviously $\lambda_k^1 \geq \lambda_k^0 = 0$. If there does not exist k such that $\lambda_k^1 > \lambda_k^0$, then $\lambda_k^* = 0$ is an equilibrium point. Otherwise, with Corollary 1, it is easy to prove that $\rho^{t+1} \leq \rho^t$ and $\lambda_k^{t+1} \geq \lambda_k^t$ by induction. For the iteration process, λ_k^t is non-decreasing with t and is bounded with $\lambda_k^t \leq 1$. So λ_k^t will converge to the minimum equilibrium point, which is one of the solutions of the fixed point equations in Proposition 2. □

The above proof also shows the dynamics of the adoption process. Initially, $\lambda_k^0 = 0$, based on this belief nodes make decisions. Then they update the belief and continue to update their decision. The above proof shows the convergence of this dynamic process.

Corollary 3. *The equilibrium points given by fixed point Eqs. (15)–(17) are monotone, i.e., if $\mathbf{\Lambda}^{*1} = (\lambda_{\underline{K}}^{*1}, \ldots, \lambda_k^{*1}, \ldots, \lambda_{\overline{K}}^{*1})$ and $\mathbf{\Lambda}^{*2} = (\lambda_{\underline{K}}^{*2}, \ldots, \lambda_k^{*2}, \ldots, \lambda_{\overline{K}}^{*2})$ are two equilibrium points, then we have either $\mathbf{\Lambda}^{*1} \geq \mathbf{\Lambda}^{*2}$ or $\mathbf{\Lambda}^{*1} \leq \mathbf{\Lambda}^{*2}$ and there exists at least one $k \in [\underline{K}, \overline{K}]$ such that $\lambda_k^{*1} \neq \lambda_k^{*2}$.*

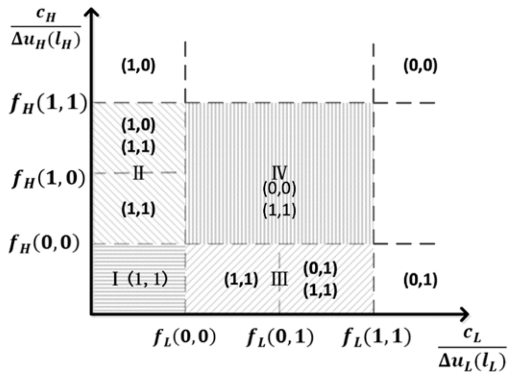


Fig. 3. Equilibrium in two types case.

Proof. We prove the above corollary by contradiction. Assume there exist k_1 and k_2 such that $\lambda^{*1}_{k_1} < \lambda^{*2}_{k_1}$ and $\lambda^{*1}_{k_2} > \lambda^{*2}_{k_2}$. Since $\lambda^{*1}_{k_1} < \lambda^{*2}_{k_1}$, by Eqs. (15) and (16), we have $\rho^{*1} > \rho^{*2}$. Since $\rho^{*1} > \rho^{*2}$, by similar analysis in Corollary 2, we can conclude $\lambda^{*1}_{k_2} \le \lambda^{*2}_{k_2}$, which contradicts $\lambda^{*1}_{k_2} > \lambda^{*2}_{k_2}$. □

The above corollaries prove the existence and monotonicity of equilibrium points. In the following, we study the *multiplicity* and *monotonicity* of the equilibrium points by considering a special case.

3.2. Analysis of node heterogeneity: two types case

To provide more insight on how different nodes can influence each other, let us consider a special case where there are two types of nodes: nodes with low degree k_L and nodes with high degree k_H, $k_H > k_L$. We assume that the cost of self-protection for low degree nodes is c_L and the loss due to being infected is l_L. On the other hand, if $k = k_H$, the cost of self-protection is c_H and the loss is l_H. Note that in Proposition 2, we did not explicitly impose any restriction on the CDF $F_k(l)$. So Proposition 2 still applies to the case when the loss is the same for all nodes with given degree.

Nodes will invest in self-protection if their utility with investment is greater than that without investment; hence

$$\lambda_L = \Pr((\phi_L(\mathcal{N}) - \phi_L(\mathcal{S}))(u_L(w_L) - u_L(w_L - l_L)) \ge c_L),$$
$$\lambda_H = \Pr((\phi_H(\mathcal{N}) - \phi_H(\mathcal{S}))(u_H(w_H) - u_H(w_H - l_H)) \ge c_H).$$

Note that the probabilities $\phi_L(\mathcal{S})$, $\phi_L(\mathcal{N})$ and $\phi_H(\mathcal{S})$, $\phi_H(\mathcal{N})$ are functions of λ_L and λ_H. We can compare the utilities to determine the fraction of users that will invest in self-protection. Define $\Delta u_L(l_L) \triangleq u_L(w_L) - u_L(w_L - l_L)$ and $f_L(\lambda_L, \lambda_H) \triangleq (\phi_L(\mathcal{N}) - \phi_L(\mathcal{S})) = (p^+ - p^-)(1 - q\rho)^{k_L}$. For $k = k_L$, the utility gap is

$$f_L(\lambda_L, \lambda_H)\Delta u_L(l_L) - c_L,$$

where $f_L(\lambda_L, \lambda_H)$ is the reduction in probability for nodes being finally infected if they invest in self-protection. Similarly, for $k = k_H$, define $\Delta u_H(l_H) \triangleq u_H(w_H) - u_H(w_H - l_H)$ and $f_H(\lambda_L, \lambda_H) \triangleq (\phi_H(\mathcal{N}) - \phi_H(\mathcal{S})) = (p^+ - p^-)(1 - q\rho)^{k_H}$, the utility gap is:

$$f_H(\lambda_L, \lambda_H)\Delta u_H(l_H) - c_H.$$

By Corollary 1, $f_L(\lambda_L, \lambda_H)$ and $f_H(\lambda_L, \lambda_H)$ are increasing functions in λ_L and λ_H, which indicates that λ_L and λ_H degenerate to indicator functions. In other words, either no nodes invest in self-protection, or all of them invest in self-protection.

Nodes can decide whether to make investment or not by comparing the expected profit of investment $f_L(\lambda_L, \lambda_H)\Delta u_L(l_L)$ ($f_H(\lambda_L, \lambda_H)\Delta u_H(l_H)$) with the cost c_L (c_H) for nodes with low (high) degree. It is equivalent to compare $f_L(\lambda_L, \lambda_H)$ ($f_H(\lambda_L, \lambda_H)$) with $\frac{c_L}{\Delta u_L(l_L)}$ ($\frac{c_H}{\Delta u_H(l_H)}$). The possible equilibrium points are shown in Fig. 3.

We divide them into four cases:

Case I: If $c_L/\Delta u_L(l_L) < f_L(0, 0)$, $c_H/\Delta u_H(l_H) < f_H(0, 0)$, then there is a unique equilibrium point $(\lambda^*_L, \lambda^*_H) = (1, 1)$ where all nodes invest in self-protection. Even if initially none of the nodes invest in self-protection, the profit of investment exceeds the cost regardless of the degree of nodes and eventually, all nodes will purchase self-protection tools.

Case II: If $c_L/\Delta u_L(l_L) < f_L(0, 0)$, $c_H/\Delta u_H(l_H) > f_H(0, 0)$, then all nodes with degree $k = k_L$ will invest in self-protection because the profit of investment for low degree nodes exceeds the cost, while the profit is smaller than the cost for high degree nodes.

- If $c_H/\Delta u_H(l_H) < f_H(1, 0)$, then all nodes with degree k_H will invest in self-protection. The profit of investment for nodes with high degree increases since nodes with low degree invest in security protection. Hence, the investment in security by nodes with degree k_L will incentivize nodes with degree k_H to invest also. There is a unique equilibrium point $(\lambda^*_L, \lambda^*_H) = (1, 1)$.

- If $f_H(1, 0) < c_H/\Delta u_H(l_H) < f_H(1, 1)$, there exists a *tipping point* λ_H^T, such that $f_H(1, \lambda_H^T) = \frac{c_H}{\Delta u_H(l_H)}$. This implies that if we can offer self-protection to λ_H^T fraction of nodes with degree k_H for free, then this will incentivize all nodes with high degree to invest. There are two equilibrium points $(\lambda_L^*, \lambda_H^*) = (1, 0)$ and $(\lambda_L^*, \lambda_H^*) = (1, 1)$.
- If $c_H/\Delta u_H(l_H) > f_H(1, 1)$, all nodes with degree k_H will not perform self-protection. There is only one equilibrium point $(\lambda_L^*, \lambda_H^*) = (1, 0)$.

Case III: If $c_L/\Delta u_L(l_L) > f_L(0, 0)$, $c_H/\Delta u_H(l_H) < f_H(0, 0)$, then all nodes with degree k_H will take self-protection measure.

- If $c_L/\Delta u_L(l_L) < f_L(0, 1)$, then all nodes with degree k_L will invest in self-protection. In this case, the investment in security by nodes with degree k_H will incentivize nodes with degree k_L to invest in self-protection. There is only one equilibrium point $(\lambda_L^*, \lambda_H^*) = (1, 1)$.
- If $f_L(0, 1) < c_L/\Delta u_L(l_L) < f_L(1, 1)$, there exists a *tipping point* λ_L^T, such that $f_L(\lambda_L^T, 1) = c_L/\Delta u_L(l_L)$. There are two equilibrium points $(\lambda_L^*, \lambda_H^*) = (0, 1)$ and $(\lambda_L^*, \lambda_H^*) = (1, 1)$.
- If $c_L/\Delta u_L(l_L) > f_L(1, 1)$, all nodes with degree k_L will not invest in self-protection. There is only one equilibrium point $(\lambda_L^*, \lambda_H^*) = (0, 1)$.

Case IV: If $f_L(0, 0) < c_L/\Delta u_L(l_L) < f_L(1, 1)$, $f_H(0, 0) < c_H/\Delta u_H(l_H) < f_H(1, 1)$, then there exists a *tipping point* λ_L^T and λ_H^T. Two possible equilibrium points are $(\lambda_L^*, \lambda_H^*) = (0, 0)$ and $(\lambda_L^*, \lambda_H^*) = (1, 1)$. However, there are other possible equilibrium points in this region. We omit the analysis to avoid getting too involved.

The tipping point induced by externality effect has significant implication for security provider and also for policy maker aiming to promote the security adoption. For security providers, setting an initially low price will promote the security adopters, when the fraction exceeds the tipping point, a large fraction of hosts will purchase the product. Policy makers can increase the adoption fraction by subsiding the initial security adopter so as to boost the initial fraction above the tipping point.

3.2.1. *Impact of topology on the externality effect*

Because the externality effect is caused by the virus transmission on the network, it is interesting to investigate the effect of topology on the externality effect. We keep k_L and p_{k_L}, p_{k_H} fixed and increase k_H to see how it impacts the adoption fraction. To write it our explicitly, ρ is determined by the fixed point equation

$$\begin{aligned}\rho = g(\lambda_L, \lambda_H, \rho) &\triangleq 1 - [q_{(k_L-1)}(1 - p^+ + \lambda_L(p^+ - p^-))(1 - q\rho)^{k_L} \\ &\quad + q_{(k_H-1)}(1 - p^+ + \lambda_H(p^+ - p^-))(1 - q\rho)^{k_H}],\end{aligned}$$

where $q_{(k_L-1)} = \frac{q_{k_L}k_L}{q_{k_L}k_L + q_{k_H}k_H}$ and $q_{(k_H-1)} = \frac{q_{k_H}k_H}{q_{k_L}k_L + q_{k_H}k_H}$. Since $q_{(k_L-1)} + q_{(k_H-1)} = 1$, $g(\lambda_L, \lambda_H, \rho)$ increases as k_H increases $\forall \lambda_L, \lambda_H, \rho$. As a result, ρ determined by the fixed point equation increases as k_H becomes greater. Note that $f_L(\lambda_L, \lambda_H) = (p^+ - p^-)(1 - q\rho)^{k_L}$, $f_H(\lambda_L, \lambda_H) = (p^+ - p^-)(1 - q\rho)^{k_H}$ are decreasing functions of ρ. Thus $f_L(\lambda_L, \lambda_H)$ and $f_H(\lambda_L, \lambda_H)$ both decreases as k_H increases, making both low degree and high degree nodes less likely to adopt the secure measure.

3.2.2. *Contagion probability dependent on secure decisions*

Previously we consider the contagion probability q is independent of nodes' decision, which is a limitation of the model. It will make the analysis much harder. Moreover, equilibrium may not exist in this situation. Assume that a node takes measure $\mathcal{S}$, the contagion probability becomes q^-; otherwise if it takes measure $\mathcal{N}$, the contagion probability is q^+. Then we have

$$\begin{aligned}\phi_k(\mathcal{S}) &= 1 - (1 - p^-)(1 - q^-\rho)^k, \\ \phi_k(\mathcal{N}) &= 1 - (1 - p^+)(1 - q^+\rho)^k,\end{aligned}$$

and

$$\phi_k(\mathcal{N}) - \phi_k(\mathcal{S}) = (1 - p^-)(1 - q^-\rho)^k - (1 - p^+)(1 - q^+\rho)^k,$$

which is no longer a monotone function of ρ. It depends on q^+ and q^- and this makes the equilibrium analysis complicated. Similar to that of [7], we analyze a special case when $q^- = 0$, which is called "strong protection". Then

$$\phi_k(\mathcal{N}) - \phi_k(\mathcal{S}) = 1 - p^- - (1 - p^+)(1 - q^+\rho)^k,$$

is a decreasing function of ρ, which indicates that a node's incentive to adopt secure measure decreases as other nodes adopt secure measures. This is exactly a free-rider problem and it coincides with the result of [7]. Next, we show that the equilibrium may not exist. Now $f_L(\lambda_L, \lambda_H)$ and $f_H(\lambda_L, \lambda_H)$ are decreasing functions of λ_L and λ_H. Consider $c_L/\Delta u_L(l_L) > f_L(0, 0)$ and $f_H(0, 1) < c_H/\Delta u_H(l_H) < f_H(0, 0)$; then equilibrium do not exist. We see that nodes with low degree will not adopt secure measures since $c_L/\Delta u_L(l_L) > f_L(0, 0)$; thus the only possible equilibrium are $(0, 1)$ and $(0, 0)$. We can exclude $(0, 0)$ since $c_H/\Delta u_H(l_H) < f_H(0, 0)$. However, if nodes with high degree decide to adopt secure measures, the benefit of adopting secure measures declines to such an extent that they have no incentive to adopt secure measure again. This can be easily justified by the condition $f_H(0, 1) < c_H/\Delta u_H(l_H)$. Hence nodes will vacillate between the two points and no equilibrium exist.

4. Analysis for the cyber-insurance market

In here, we consider *cyber-insurance* and analyze its impact on security adoption.

4.1. Supply of insurance

The presentation of insurance model in this subsection tailors that in the economic literature [6] to adapt to the model in our paper.

Let us say the insurance provider offers insurance at the price of $\pi < 1$. Nodes which buy insurance at the premium of πX from the insurance provider will be compensated X for the loss incurred if they are infected. Given the price π, node will choose to buy the amount of insurance that maximizes its utility. Define $\phi_k(\mathcal{S})(\phi_k(\mathcal{N}))$ as the probability that a node with degree k will be infected if it subscribes (does not subscribe) to a secure measure. In this paper, we consider cyber-insurance without adverse selection, in which the insurance provider can observe the degree of a node, hence the risk type of a node (high degree indicates the high risk level). Thus, in the following, we drop the subscript k where the meaning is clear for general presentation. A node will choose the amount of insurance that maximizes

$$U(\pi, X) = \phi u(w - l + (1 - \pi)X) + (1 - \phi)u(w - \pi X) - x, \tag{18}$$

where x is the wealth spent on security protection. When a node chooses $\mathcal{N}$, ϕ becomes $\phi(\mathcal{N})$, $x = 0$. When a node chooses $\mathcal{S}$, ϕ becomes $\phi(\mathcal{S})$, $x = c$. Assume the insurance provider is risk neutral, so they only care about the expected wealth. If a node buys X amount of insurance, then the profit of the insurance is $(\pi - \phi)X$. In here, we consider a competitive market so the insurance provider has to offer the insurance at the price $\pi = \phi$, or the *actuarially fair price* [18].

Lemma 1. *When the insurance is offered at the actuarially fair price, the optimal insurance coverage is a full insurance coverage, i.e., a node will buy insurance amount l, which is equal to the loss. The maximal expected utility is $\max_X U(\phi, X) = u(w - \phi l) - x$, i.e., when a node chooses $\mathcal{N}$, the maximal expected utility is $u(w - \phi(\mathcal{N})l)$, when a node chooses $\mathcal{S}$, the maximal expected utility is $u(w - \phi(\mathcal{S})l) - c$.*

Proof. A node will optimize

$$U(\phi, X) = \phi u(w - l + (1 - \phi)X) + (1 - \phi)u(w - \phi X) - x. \tag{19}$$

Taking the derivative of $U(\phi, X)$ with respect to X, we have

$$U'(\phi, X) = \phi(1 - \phi)\left[u'(w - l + (1 - \phi)X) - u'(w - \phi X)\right].$$

Since $u(w)$ is an increasing and concave function, $u'(w)$ is a decreasing and positive function. When $X < l$, $U'(\phi, X) > 0$; when $X > l$, $U'(\phi, X) < 0$. The expected utility is maximized at $X = l$, and the optimal expected utility is $u(w - \phi l) - x$. □

In Fig. 1, the expected utility without the insurance market is point C, i.e., nodes feel that they lose more than the expected wealth loss because of the risk aversion. With the insurance market, the expected utility improves from point C to point B.

Lemma 2. *When the insurance is offered at price $\pi > \phi$, the optimal insurance coverage is partial insurance coverage, i.e., a node will buy insurance coverage less than l. The maximal expected utility is $u(w - \phi l - \delta(\phi, \pi)) - x$., where $\delta(\phi, \pi) > 0$.*

Proof. Similar to the proof of Lemma 1, a node optimizes

$$U(\pi, X) = \phi u(w - l + (1 - \pi)X) + (1 - \phi)u(w - \pi X) - x.$$

The first order differentiation of $U(\pi, X)$ is

$$U'(\pi, X) = \phi(1 - \pi)u'(w - l + (1 - \pi)X) - (1 - \phi)\pi u'(w - \pi X).$$

It is easy to verify that $U'(\pi, l) < 0$ since $\pi > \phi$. The second order derivative is

$$U''(\pi, X) = \phi(1 - \pi)^2 u''(w - l + (1 - \pi)X) + (1 - \phi)\pi^2 u''(w - \pi X).$$

Since $u(w)$ is concave, $u''(w) < 0$, it follows that $U''(\pi, X) < 0$. $U(\pi, X)$ is a concave function of X. Also, $U'(\pi, l) < 0$, so the optimal solution is smaller than l. As a result, the optimal insurance converge is partial coverage. Let the optimal expected utility be $u(w - \phi l - \delta(\phi, \pi)) - x$. Since $U(\phi, X) > U(\pi, X)$, $u(w - \phi l) - x = \max_X U(\phi, X) > \max_X U(\pi, X) = u(w - \phi l - \delta(\phi, \pi)) - x$, we can get $\delta(\phi, \pi) > 0$. □

Remark. Lemma 1 shows that the expected utility with the insurance market is $u(w - \phi l) - x > \phi u(w - l) + (1 - \phi)u(w) - x$. The utility of a node is *improved* by the insurance market with the fair price. But if the contract is at an unfair price, the utility improvement is smaller according to Lemma 2.

One problem with the combination of insurance and self-protection is *moral hazard*, which happens when the insurance provider cannot observe the protection level of a node. Insurance coverage may discourage the node to take self-protection measure to prevent the losses from happening, or even to encourage nodes to cause the loss and make insurance claims. Here, we examine the effect of the insurance market on the self-protection level. We consider the two cases, one is without moral hazard, where the insurance provider can observe the protection level of a node, and the other is with moral hazard, where the insurance provider does not have any information about the protection level of a node. Without the moral hazard, the insurance provider can discriminate against the nodes with protection measure and those without protection measure. We investigate whether the insurance market will help to incentivize nodes to take secure measure. For the case with moral hazard, we investigate whether the insurance provider can design contracts so that the insurance market is not a negative incentive.

4.2. Cyber-insurance without moral hazard

Security adoption with the cyber-insurance market: because the insurance provider can observe the protection level of a node, the insurance provider will offer insurance price of $\phi(\mathcal{S})$ (or $\phi(\mathcal{N})$) for those nodes with (or without) security protection. According to Lemma 1, nodes will buy the full insurance regardless of its protection level. As a result, the expected utility for nodes without protection is $u(w-\phi(\mathcal{N})l)$ and the expected utility for nodes with protection is $u(w-\phi(\mathcal{S})l)-c$. Thus, with the insurance market, a node will invest in security protection if and only if

$$c < g(l,\rho) \triangleq u(w-\phi(\mathcal{S})l) - u(w-\phi(\mathcal{N})l).$$

Note that $g(l,\rho)$ is a function of ρ because $\phi(\mathcal{S})$ and $\phi(\mathcal{N})$ can be expressed in ρ.

Lemma 3. *The function $g(l,\rho) \triangleq u(w-\phi(\mathcal{S})l) - u(w-\phi(\mathcal{N})l)$ increases with respect to the loss l.*

Proof. Substituting $u(w)$ with $\frac{w^{1-\sigma}}{1-\sigma}$, we can get the first order derivative of $g(l,\rho)$:

$$g_l = -\frac{\phi(\mathcal{S})}{(w-\phi(\mathcal{S})l)^{\sigma}} + \frac{\phi(\mathcal{N})}{(w-\phi(\mathcal{N})l)^{\sigma}}. \tag{20}$$

It is easy to verify that $g_l > 0$ since $\phi(\mathcal{N}) > \phi(\mathcal{S})$. □

Lemma 4. *The function $g(l,\rho) \triangleq u(w-\phi(\mathcal{S})l) - u(w-\phi(\mathcal{N})l) = u(w-(1-(1-p^-)(1-q\rho)^k)l) - u(w-(1-(1-p^+)(1-q\rho)^k)l)$ is decreasing with respect to ρ.*

Proof. Similarly, taking the first order derivative we can get:

$$g_\rho = lkq(1-q\rho)^{k-1}[(1-p^+)(w-l+(1-p^+)(1-q\rho)^k l)^{-\sigma} - (1-p^-)(w-l+(1-p^-)(1-q\rho)^k l)^{-\sigma}].$$

It is easy to verify that function $h(p) \triangleq (1-p)(w-l+(1-p)(1-q\rho)^k l)^{-\sigma}$ decreases with p. Thus, $h(p^+) < h(p^-)$ and $g_\rho < 0$. □

Lemma 3 indicates that nodes with higher loss are more likely to invest in security. From Lemma 4 we know that positive network externality still exists even in the presence of the insurance market. Similar to the analysis in Section 3, we can arrive in the following proposition regarding the adoption fraction with the insurance market:

Proposition 3. *With the insurance market, nodes with degree k will take the secure measure if their loss is greater than $l_k^{*\mathrm{I}}$. The final fraction of nodes with degree k that will invest in self-protection is $\lambda_k^{*\mathrm{I}}$. $l_k^{*\mathrm{I}}$ and $\lambda_k^{*\mathrm{I}}$ are solutions of the following fixed point equations:*

$$\lambda_k^{*\mathrm{I}} = 1 - F_k(l_k^{*\mathrm{I}}), \tag{21}$$

$$c_k = u_k(w_k - \phi(\mathcal{S})l_k^{*\mathrm{I}}) - u_k(w_k - \phi(\mathcal{N})l_k^{*\mathrm{I}}), \tag{22}$$

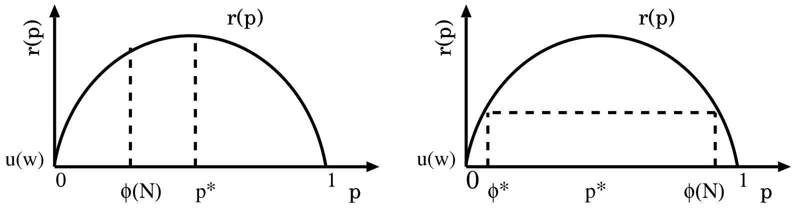
where $\rho^{*\mathrm{I}}$ is given by the solution of the following equation

$$\rho^{*\mathrm{I}} = 1 - \sum_{k=\underline{K}'}^{\overline{K}'} q_k(1-p^+ + \lambda_{k+1}^{*\mathrm{I}}(p^+ - p^-))(1-q\rho^{*\mathrm{I}})^k. \tag{23}$$

Previous corollaries following Proposition 2 on the existence and monotonicity of equilibrium points also hold here. Comparing Proposition 3 with Proposition 2, we can recognize the only difference lies in Eqs. (22) and (16). Buying insurance improves node's utility, and hence changes their decision on security protection as well. In the following, we examine the effect of the insurance market on security adoption. An overall and detailed analysis needs calculating out all the equilibrium points and comparing the equilibrium points specified by the two propositions, which is quite complicated. Instead, we examine the effect from the local point of view, but still provide enough insight.

Incentive analysis: according to the previous analysis, a node will take secure measure if

$$c < c_{NI} \triangleq (\phi(\mathcal{N}) - \phi(\mathcal{S}))(u(w) - u(w-l)), \tag{24}$$



(a) Insurance is not an incentive. (b) Insurance is an incentive.

Fig. 4. Thresholds of $\phi(S)$.

where c_{NI} is the threshold without the insurance market. With the insurance market, nodes will take secure measure if and only if

$$c < c_I \triangleq u(w - \phi(\mathcal{S})l) - u(w - \phi(\mathcal{N})l), \tag{25}$$

where c_I denotes the threshold with the insurance market.

In order for the insurance market to be a good incentive for self-protection, we should have $c_{NI} < c_I$, i.e.,

$$c_I - c_{NI} = u(w - \phi(\mathcal{S})l) + \phi(\mathcal{S})(u(w) - u(w - l)) - [u(w - \phi(\mathcal{N})l) + \phi(\mathcal{N})(u(w) - u(w - l))] > 0.$$

Define $r(p) \triangleq u(w - pl) + p(u(w) - u(w - l))$, and then the above condition becomes $r(\phi(\mathcal{S})) > r(\phi(\mathcal{N}))$. Next we investigate under what condition the above inequality will hold. Consider the function $r(p)$, we have the following lemma.

Lemma 5. *$r(p)$ is a concave function of p, there exists a unique p^* that maximizes $r(p)$.*

Proof. Substituting $u(w)$ with $\frac{w^{1-\sigma}}{1-\sigma}$, we can derive the second order derivative of $r(p)$:

$$r''(p) = -\sigma l^2 (w - pl)^{-\sigma-1}.$$

Since $r''(p) < 0$, $r(p)$ is a concave function with respect to p. Because $r(0) = r(1) = u(w)$, there exists a unique optimal point $p^* \in (0, 1)$ that maximizes $r(p)$. □

Proposition 4. *If the initial infection probability $\phi(\mathcal{N})$ is greater than p^* and the quality of self-protection is not too high, i.e., $\phi(\mathcal{N}) - \phi(\mathcal{S})$ is low enough, insurance will be a good incentive for self-protection.*

Proof. We want $r(\phi(\mathcal{S})) > r(\phi(\mathcal{N}))$ conditioned on $\phi(\mathcal{N}) > \phi(\mathcal{S})$. Let ϕ^* be the minimum value such that $r(\phi^*) = r(\phi(\mathcal{N}))$. If $\phi(\mathcal{N})$ is smaller than the optimal value p^*, as shown in Fig. 4(a), then $\phi^* = \phi(\mathcal{N})$. In this case, it is impossible for insurance to be an incentive for self-protection. Otherwise if $\phi(\mathcal{N})$ is bigger than the optimal value p^*, then $\phi^* < \phi(\mathcal{N})$. In this case, if $\phi^* < \phi(\mathcal{S}) < \phi(\mathcal{N})$, then the insurance market will be a good incentive for self-protection. The feasible region of $\phi(\mathcal{S})$ is shown in Fig. 4(b). □

Fig. 4(a) shows the case where $\phi(\mathcal{N})$ is smaller than the optimal value p^* that maximizes $r(p)$. In this case, it is impossible for insurance to be an incentive. In Fig. 4(b), $\phi(\mathcal{N})$ is greater than p^*. If $\phi(\mathcal{S})$ is within the region $[\phi^*, \phi(\mathcal{N})]$, then insurance is a good incentive for security adoption. From Fig. 4(b), we can see that insurance will be more likely to be an incentive with large $\phi(\mathcal{N})$ and small $\phi(\mathcal{N}) - \phi(\mathcal{S})$. Hence, if the initial secure situation is bad and the protection quality of secure measure is not too high, then the insurance market is a positive incentive for self-protection; otherwise, the insurance market is a negative incentive, i.e., if a node adopts secure measure without insurance, it may decide not to adopt secure measure with the insurance market.

We can study the effect of cyber-insurance on nodes with *different degrees* based on above analysis. For $k_1 < k_2$, we have $\phi_{k_1}(\mathcal{S}) < \phi_{k_2}(\mathcal{S})$, $\phi_{k_1}(\mathcal{N}) < \phi_{k_2}(\mathcal{N})$ and $\phi_{k_1}(\mathcal{N}) - \phi_{k_1}(\mathcal{S}) < \phi_{k_2}(\mathcal{N}) - \phi_{k_2}(\mathcal{S})$. In other words, nodes with higher degree have higher initial infection probability and the protection measure will be less effective to nodes with higher degree. As a result, the insurance market will be more likely to be an incentive for nodes with higher degree. (A quantitative conclusion needs to examine the influence of wealth and loss difference for nodes with different degrees.)

Whether insurance will be an incentive greatly depends on the parameters. Generally speaking, cyber-insurance can be a positive incentive for all nodes, a negative incentive for all nodes and a negative incentive for low degree nodes, but a positive incentive for high degree nodes. We provide extensive numerical results in Section 5 to demonstrate the above cases.

4.3. Cyber-insurance with moral hazard

With moral hazard, the insurance provider cannot observe the protection level of the nodes. As a result, the insurance contract cannot be differentiated for nodes with a different protection level. Instead, with the insurance contracts given, nodes will choose the behavior that maximizes their expected utility. It is possible that nodes will choose not to invest in self-protection if the insurance can cover part of the loss. In this case, insurance is a negative incentive for self-protection. Here, we investigate whether it is possible to design a contract that is *not* a negative incentive for self-protection.

In a competitive insurance market, the only possible equilibrium is that the insurance provider offers the contracts at the price $\phi(\mathcal{S})$ ($\phi(\mathcal{N})$) and the nodes choose (not) to invest in self-protection. If the price is at $\phi(\mathcal{S})$, but nodes choose not to invest in self-protection, then the expected profit of the provider $(\phi(\mathcal{S}) - \phi(\mathcal{N}))X < 0$. The provider will not offer such insurance since it will lead to a loss. On the other hand, if the insurance provider sells the contracts at the price of $\phi(\mathcal{N})$, but nodes choose to invest in self-protection, then the expected profit is $(\phi(\mathcal{N}) - \phi(\mathcal{S}))X > 0$. Since the market is competitive, the positive profit will lead to competition and the insurance provider who offers contracts at price $\phi(\mathcal{S})$ will survive.

We first consider the case when the insurance provider can offer the full insurance coverage. Nodes can choose the optimal amount of insurance coverage. We show that in this case *$\mathcal{S}$-equilibrium*, contracts are sold at the price of $\phi(\mathcal{S})$ and nodes choose $\mathcal{S}$, is impossible. If nodes choose $\mathcal{N}$, the optimal expected utility is $\max_X U_{\mathcal{N}}(\phi(\mathcal{S}), X) \geq U_{\mathcal{N}}(\phi(\mathcal{S}), l) = u(w-\phi(\mathcal{S})l)$, where we use $U_{\mathcal{N}}(\phi(\mathcal{S}), X)$ to denote the utility of node choosing $\mathcal{S}$ and X amount of insurance. If nodes choose $\mathcal{S}$, by Lemma 1, the optimal expected utility is $u(w - \phi(\mathcal{S})l) - c < u(w - \phi(\mathcal{S})l)$. So nodes will choose $\mathcal{N}$ if full insurance coverage can be offered. In other words, $\mathcal{S}$-equilibrium does not exist under full insurance coverage. Full coverage insurance is never an incentive for security adoption with moral hazard. The reason why full coverage insurance is not an incentive is that if nodes get infected, loss will be covered fully regardless whether they take secure measure or not by paying the same premium. As a result, the investment on security protection is not necessary.

One solution to the moral hazard problem is partial coverage against loss [19]. Partial insurance can incentivize nodes to invest in self-protection by exposing them to certain risk loss. Consider the $\mathcal{S}$-equilibrium, the insurance provider offers the contract at price $\phi(\mathcal{S})$ and the maximal insurance coverage is W. We already showed $W < l$. In a partial insurance contract, a node cannot decide the amount of coverage by maximizing its utility. If a node chooses $\mathcal{N}$, its maximal expected utility is

$$U_{\mathcal{N}}(\phi(\mathcal{S}), W) = \phi(\mathcal{N})u(w - l + (1 - \phi(\mathcal{S}))W) + (1 - \phi(\mathcal{N}))u(w - \phi(\mathcal{S})W). \tag{26}$$

If a node chooses $\mathcal{S}$, its maximal expected utility is

$$U_{\mathcal{S}}(\phi(\mathcal{S}), W) = \phi(\mathcal{S})u(w - l + (1 - \phi(\mathcal{S}))W) + (1 - \phi(\mathcal{S}))u(w - \phi(\mathcal{S})W) - c. \tag{27}$$

The S-equilibrium exists if and only if

$$\begin{aligned}\Delta(W) &= U_{\mathcal{S}}(\phi(\mathcal{S}), W) - U_{\mathcal{N}}(\phi(\mathcal{S}), W) \\ &= (\phi(\mathcal{N}) - \phi(\mathcal{S}))(u(w - l + (1 - \phi(\mathcal{S}))W) - u(w - \phi(\mathcal{S})W)) - c \geq 0.\end{aligned} \tag{28}$$

It is easy to find out that $\Delta(W)$ is a strictly decreasing function of W. We want to find out whether there exist W such that $W \in [0, l]$ and $\Delta(W) \geq 0$. From previous analysis, we know $\Delta(l) < 0$, i.e., when full insurance is offered, nodes will choose $\mathcal{N}$. If $W = 0$, it indicates that no insurance is provided. $\Delta(0)$ is the expected utility gap when no insurance is provided. If $\Delta(0) < 0$, i.e., nodes will not invest in self-protection without insurance market, it is impossible to find out W such that $\Delta(W) > 0$ due to the monotonicity of $\Delta(W)$. Thus, cyber-insurance can never be a positive incentive for self-protection. However, if $\Delta(0) > 0$, i.e., nodes will invest in self-protection without insurance market, we can always find such W such that $\Delta(W) = 0$ since $\Delta(W)$ is a continuous function of W. As a result, the maximal insurance coverage which can be offered by the insurance provider so that $\mathcal{S}$-equilibrium is possible is:

$$W_{\max} = \arg\{\Delta(W) = 0\}. \tag{29}$$

Here we show that cyber-insurance with moral hazard can never be positive incentive for security adoption. Though cyber-insurance cannot be positive incentive, we demonstrate it can be non-negative, which still has practical meaning. The cyber-insurance with moral hazard can improve hosts' welfare while not impeding them from investing in security.

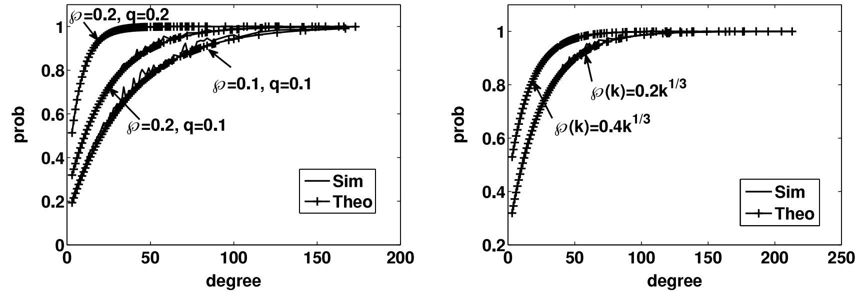
In the competitive insurance market without moral hazard, the expected utility of nodes who choose $\mathcal{S}$ with insurance market is $u(w-\phi(\mathcal{S})l)-c$. With moral hazard, the maximal insurance coverage is $W_{\max}$. Then the maximal expected utility for nodes choosing $\mathcal{S}$ is $U_{\mathcal{S}}(\phi(\mathcal{S}), W_{\max})$. Since $W_{\max} < l$, we have $U_{\mathcal{S}}(\phi(\mathcal{S}), W_{\max}) < u(w - \phi(\mathcal{S})l) - c$. In other words, nodes' welfare is hurt by the moral hazard. If the insurance provider offers full insurance, nodes will, on the contrary, choose $\mathcal{N}$. Partial insurance with the maximal contract $W_{\max}$ will make it worthwhile for nodes to invest in self-protection.

5. Simulation and numerical results

We present simulation and numerical results to investigate the influence of various parameters in this section.

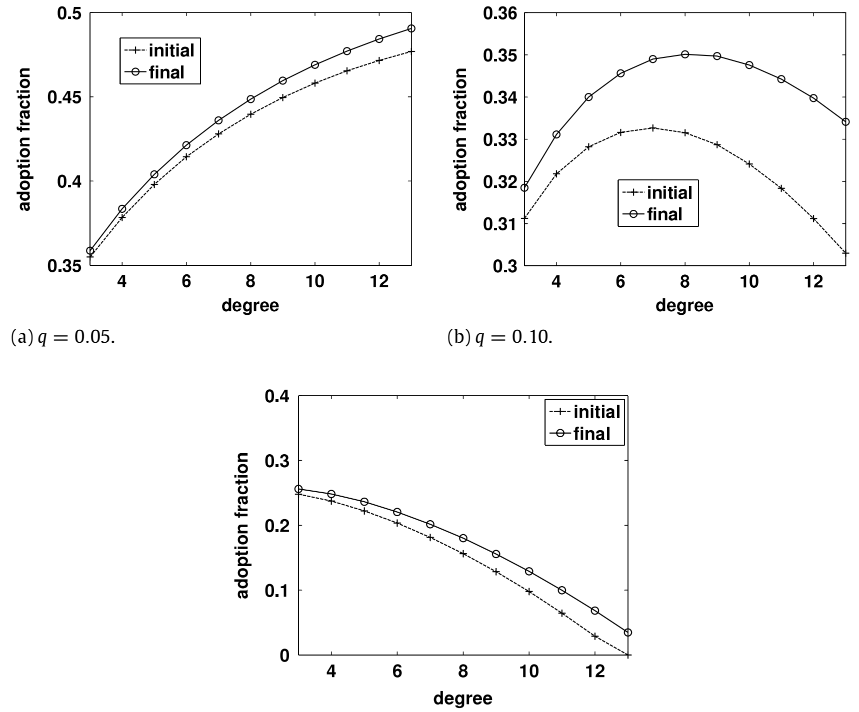
Validating final infection probability: we consider a large graph with power-law degree distribution [20]. We want to verify the accuracy of using the mean field on these power law graphs. We use the popular *Generalized Linear Preference (GLP)* method to generate power law graphs [21]. Parameters were selected so that the power law exponent $\gamma = -3$. We generate graphs with 10,000 nodes and approximately 30,000 edges. The minimum degree is 3 and the maximum degree is approximately 200. First, we verify the case when all the nodes have the same probability of being infected initially. The result is shown in Fig. 5(a). Initially, every node is infected with the same probability $\wp$ and every edge is occupied with probability q. We calculate the probability that nodes with certain degree is infected. Fig. 5(a) shows that the simulation verifies the theoretical results. One can also observe that the infection probability is an increasing function of node's degree. When $\wp$ and q increases, the infection probability also increases.

Next, Fig. 5(b) shows the infection probability of nodes with different degrees under different initial infection probability. For both curves, q is set to be 0.1. For the curve above, we set the initial probability for nodes with degree k to be



(a) Homogeneous infection prob. (b) Heterogeneous infection prob.

Fig. 5. Determine final infection prob. via local mean field.



(a) $q = 0.05$. (b) $q = 0.10$.

(c) $q = 0.15$.

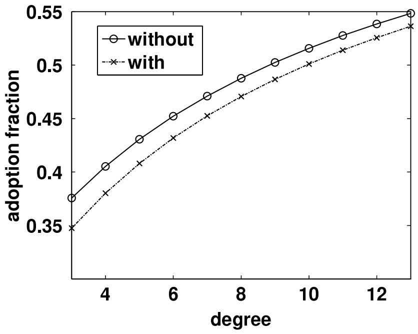
Fig. 6. Externality effect on nodes with different degrees.

$\wp(k) = 0.4/k^{\frac{1}{3}}$. The probability decreases with degree. For the curve below, we set the initial infection probability to be $\wp(k) = 0.2k^{\frac{1}{3}}$. The probability increases with degree. From the figure, we see that the local mean field technique is very accurate and the theoretical results accurately match with simulation results.

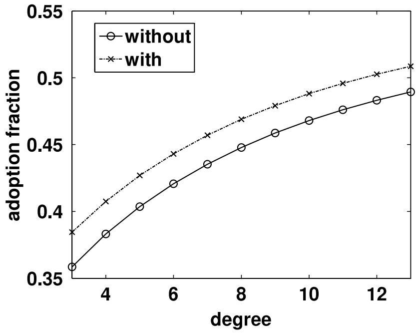
Security adoption: let us investigate how parameters can influence the fraction of nodes with different degrees in adopting secure measures. We consider a graph G with power law distribution with $\gamma = -3$; minimum and maximum degrees are 3 and 13, respectively. Here the maximum degree is set small for the convenience of selecting other parameters. With very large maximum degree, even a small q will make the infection probability $\phi_k(\mathcal{N})$ or $\phi_k(\mathcal{S})$ very big because of the power relationship. However, our results still apply when the maximum degree is large.

We set the degree of risk of aversion of the utility function $\sigma = 0.5$, the same for all node. The initial wealth of nodes with degree k is $w_k = 10 * k + 50$. The loss follows uniform distribution from 0 to half of the initial wealth. The cost of secure measure of all nodes is $c = 0.3$. Initially, all nodes without (with) secure measure are infected initial with probability $p^+ = 0.3$ ($p^- = 0.2$). Having fixed the above parameters, we choose to change q to calculate the fraction of adopters with different degrees because nodes with different degrees are mainly differentiated via the term $(1 - q\rho)^k$, in which q plays an important role. We want to examine the effect of heterogeneity by setting different q.

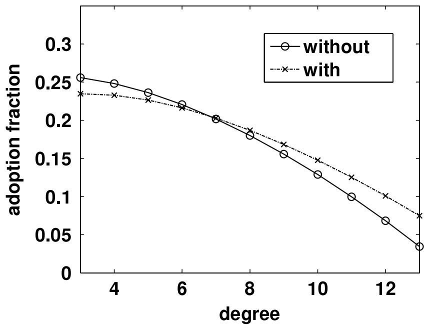
We show the initial fraction and final fraction of adoption in Fig. 6. Here the initial fraction means that every node assumes that other nodes will not adopt secure measure and makes its decision based on this assumption. The final fraction means the



(a) Negative incentive.



(b) Positive incentive.



(c) Negative and positive incentive.

Fig. 7. Effect of cyber-insurance on security adoption.

fraction given by the minimum equilibrium point in Proposition 2. Due to the positive externality effect, the final fraction is greater than the initial fraction. We plot them to examine the externality effect. From Fig. 6(a) to (c), we set q to be 0.05, 0.10 and 0.15 respectively. The figures show that the adoption fraction of nodes with every degree decreases as q increases. This indicates that the spreading effectiveness can inhibit adoption of secure measure. In Fig. 6(a), the adoption fraction increases with degree, and in Fig. 6(b), the adoption fraction initially increases with degree, and then decreases with degree, while in Fig. 6(c), the adoption fraction decreases with degree. Comparing these three figures, we see that there is no general rule regarding the fraction of adopters as a function of the degree. It greatly depends on the parameters. However, we can see in all figures that the gap between the final adoption fraction and the initial adopt fraction increases with degree, indicating nodes with higher degree will be incentivized better than nodes with lower degree. This agrees with our previous result that higher degree nodes are more sensitive to the externality effect.

Influence of cyber-insurance: we claim in the previous section that insurance can be a negative incentive for all nodes, a positive incentive for all nodes and a negative incentive for low degree nodes but a positive incentive for high degree nodes. We demonstrate these cases through numerical results. In Fig. 7(a), we set the parameters $p^+ = 0.3$, $p^- = 0.2$ and $q = 0.02$. We see that the fraction of nodes which adopt the secure measure without the insurance market is greater than that with insurance market. This is because the infection probability without secure measure is low. In Fig. 7(b), we set the parameters $p^+ = 0.8, p^- = 0.7$ and $q = 0.02$. As the figure shows, the insurance market is a positive incentive. In this case, the infection probability without secure measure is high and the protection quality is low. In Fig. 7(c), we set the parameters $p^+ = 0.8, p^- = 0.7$ and $q = 0.15$. In contrast to Fig. 7(a), q is greater, making the infection probability for low degree nodes small while for high degree nodes big. Thus insurance is a negative incentive for low degree nodes, but a positive incentive for high degree nodes.

6. Related work

Recently there has been growing research in the economics of information security [22,23]. Several models are proposed to study the strategic behavior of security investment. [15,16] are the earliest work to consider strategic security investment and to find externality effect. [24] assumes that security investment is continuous and considers the cases when the security of one agent depends on others by the summation, weakest, best of the investment effort of all agents. They find the overall security investment is highly relevant with how the security condition depends on each other. All these papers do not incorporate the effect of network topology. Others assume that the graph topology is given. The authors in [25] combine the N-intertwined epidemic model with game theory and model nodes' strategic behavior. The model is based on the complete information of topology. In [26,27], the level of security is determined by weights assigned to a topology and no infection process is modeled. [28] generalized the work of [24] to consider topology and also assume that the network topology information is incomplete for agents. Our model contrasts [28] is that the security investment is discrete and security dependence is caused by epidemic spreading. [29–31] are extension of [24] by considering some parameters of [24] such as loss or attack probability is incomplete information and model it as a Bayesian security game. [7,8] are the closely related to our work. The network topology is modeled as a homogeneous random graph while real networks are with the power law degree distribution. Also, they do not consider the interaction among those nodes. In contrast, we consider the interaction of nodes by studying a Bayesian network game. Our modeling result provides significant insight on the influence of node heterogeneity on the adoption extent, sensitivity to network externality and cyber-insurance as an incentive. [32] also models the externality effect by assuming that the loss is different among the nodes and finds multiple equilibrium. There is a coordination problem in reaching better equilibrium. Our work can be seen as a step further in including heterogeneous factors of nodes. We consider that nodes know their degree in estimating the infection probability, which makes the model more practical and reasonable. [33] is our previous extended abstract in considering network heterogeneity, which is defined by setting degree thresholds to divide the nodes into classes. This work generalizes the previous work and also considers the effect of cyber-insurance.

Insurance was studied in the economic literature long time ago [18,19]. But these literatures lack to consider many characteristics specific to a computer network, such as the interdependence of security, heterogeneity considered in this work. Cyber-insurance was proposed to manage the security risk [34] but is only modeled recently [5,35,11]. A key concern is whether cyber-insurance is an incentive for security adoption. [35] concludes that competitive and monopoly insurance markets are not incentive with moral hazard and competitive insurance market is an incentive without moral hazard. In contrast, we find competitive the insurance market is an incentive conditioned that the protection quality is not high. The authors do not consider the heterogeneity in modeling cyber-insurance. We consider heterogeneity and show that cyber-insurance is more likely to be an incentive for node with higher degree. [11] assumes the effort on security protection is continuous and find that the competitive cyber-insurance market, if exist, cannot be incentive for security investment with moral hazard. In [36], the authors consider there are two types of negatively correlated failures, security related and non-security related. They propose a new type of cyber-insurance in which loss is partially covered. When cyber-insurance is mandatory, this type of new insurance will be preferred than traditional cyber-insurance in which all loss is covered. These papers on cyber-insurance, including ours, all show that moral hazard is the obstacle for cyber-insurance to be incentive for security investment.

7. Discussion

Modeling strategic behavior in security adoption helps us to understand what are the factors that could result in under investment. In this paper, we show, via a Bayesian network game formulation, how "network externality" with "node heterogeneity" can affect security adoption in a large communication network. We also investigate the effect of cyber-insurance on the protection level. We establish the conditions under which cyber-insurance is a positive incentive without moral hazard. Under the situation of moral hazard, we verify that partial insurance can be a non-negative incentive.

There are several ways to extend the result of our work. The first is to follow the paper to continue to analyze cyber-insurance with both moral hazard and adverse selection (the insurance provider cannot distinguish between high and low risk(degree) nodes). The second is to consider that the effort on security investment is continuous, which is more practical. The third direction is to incorporate the strategic behavior of adversaries, which can overcome the weakness of our paper by assuming that all the nodes have the same probability of being attacked. It is interesting to see how the behavior of adversaries may impact adoption of security measures. We also hope to get the real data on the parameters defined in our paper to verify the model, which we artificially set in the simulation.

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