MEMORY OBSERVER-BASED OUTPUT FEEDBACK CONTROL FOR LINEAR TIME-DELAY SYSTEMS

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Abstract: This paper considers synthesis problems of stabilizing dynamic output feedback controllers for linear time-delay systems via infinite-dimensional Linear Matrix Inequality (LMI) approach. We derive an existence condition and an explicit formula of dynamic output feedback controllers for linear time-delay systems, which guarantee the internal stability of the closed loop systems. The derived dynamic output feedback controllers can be interpreted as controllers which consist of memory state feedback controllers and memory observers. Next, we introduce a technique to reduce conditions for synthesis in the form of infinite-dimensional LMIs to a finite number of LMIs, and present a feasible algorithm for synthesis of controllers based on the finite-dimensional LMIs. Finally we demonstrate the efficacy of the proposed dynamic output feedback controllers by a numerical case study.

Keywords: linear time-delay systems, output feedback control, stabilization, linear matrix inequality

1. INTRODUCTION

The fact that the state space of linear time-delay systems is infinite-dimensional leads generally to infinite-dimensional characterizations for analysis and synthesis in linear time-delay systems. For example it is well known that the optimal LQ control for linear time-delay systems is given memory, i.e. infinite-dimensional, state feedback form whose feedback gains are characterized by the infinite-dimensional Riccati equations; as for state feedback control synthesis, we could say that memory state feedback controllers achieve better performance than memoryless state feedback controllers (T. Azuma and Uchida, 2002; J. He and Lee, 1998; K. Ikeda and Uchida, 2001; Loiseau and Brethe, 1996; Louisell, 1991). Of course, the infinite-dimensional characterizations give us contrary hard problems in computations and implementations. Our concern is to find a feasible approach to such infinite-dimensional tasks in synthesis for linear time-delay systems.

Recently the Linear Matrix Inequality (LMI) approach (S. Boyd and Balakrishnan, 1994; Dullerud and Paganini, 2000) has been developed in analysis and synthesis problems for linear time-delay systems and its advantages in numerical computations are presented (Choi and Chung, 1995; Choi and Chung, 1997; de Souza and Li, 1999; J. He and Lee, 1998); however, the approach is mostly developed under some finite-dimensional assumptions assured by a special form of Lyapunov functional in analysis and/or a memoryless controller form in synthesis. One exception which does not require such finite-dimensional assumptions is a series of the works by Gu (Gu, 1997*a*; Gu, 1997*b*); he proposes a discretization technique, which can characterize a general Lyapunov functional with a finite number of LMIs. As more recent references on LMI for linear time-delay systems(de Souza, 2000) (and references inside) and (A. Fattouh and Dion, 2000) should be mentioned; a synthesis problem of state feedback with delay is discussed in (de Souza, 2000) and a memoryless state feedback is designed for a system with distributed time-delays in (A. Fattouh and Dion, 2000). Those methods are developed under the assumption that the full state is directly available. However, in most practical situations, the actual state is not available directly. Thus it is important to consider output feedback control synthesis problems.

In this paper, we derive an existence condition of stabilizing dynamic output feedback controllers for linear time-delay systems in the form of infinite-dimensinal LMIs, which is an extension developed in our work (T. Azuma and Uchida, 2002). The derived dynamic output feedback controllers are based on the Lyapunov functional that is a natural extension of the functional to solve the state feedback control problem of linear time-delay systems, where the basic structure of the Lyapunov function to solve H_∞ output feedback control problem of linear systems without delays proposed in (Uchida and Fujita, 1989; Uchida and Fujita, 1992) is adopted as the key idea to construct the functional. Thus the derived dynamic output feedback controllers can be interpreted as controllers which consist of memory state feedback controllers and observers. Because these observers have a special structure such as the memory type, we call these observers as "memory observers". Next we reduce the infinite-dimensional LMIs to a finite-dimensional LMIs by applying the technique proposed in results (T. Azuma and Uchida, 1997; T. Azuma and Fujita, 2000). Finally, we demonstrate the efficacy of the derived output feedback controllers by a numerical case study.

2. SYSTEM DESCRIPTION AND PROBLEM FORMURATION

Consider the following linear time-delay system defined on the time interval $[0,\infty)$ and described by

$$\begin{aligned}\dot{x}(t) &= A_0x(t) + A_1x(t-h) + Bu(t),\\ y(t) &= Cx(t),\\ x(\beta) &= \phi(\beta),\ -h \le \beta \le 0,\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the internal variable, $u(t) \in \mathbb{R}^r$ is the control input, $y(t) \in \mathbb{R}^m$ is the measurement output, $\phi(\beta) \in L_2([-h,0];\mathbb{R}^n)$ is a continuous initial function. The parameter h denotes the time delay and $h > 0$.

The purpose of this paper is to design *dynamic output feedback controllers* which stabilize the linear time-delay system (1). In this paper, we use the following functional for stabilization of the linear time-delay system (1).

$$\begin{aligned}V(x_s) = {} & x'(t)Mx(t) + e'(t)\gamma^2T^{-1}e(t)\\ & + \int_{-h}^{0} x'(t+\beta)Qx(t+\beta)d\beta\\ & - \int_{-h}^{0} x'(t+\beta)Qe(t+\beta)d\beta\\ & - \int_{-h}^{0} e(t+\beta)'Qx(t+\beta)d\beta\\ & + \int_{-h}^{0} e'(t+\beta)(\gamma^2H+Q)e(t+\beta)d\beta\\ & + x'(t)\int_{-h}^{0} Mx(t+\beta)d\beta\\ & + e'(t)\int_{-h}^{0} \gamma^2T^{-1}e(t+\beta)d\beta\\ & + \int_{-h}^{0} x'(t+\alpha)Md\alpha x(t)\\ & + \int_{-h}^{0} e'(t+\alpha)\gamma^2T^{-1}d\alpha e(t)\\ & + \int_{-h}^{0}\int_{-h}^{0} x'(t+\alpha)(S(\alpha,\beta)+M)x(t+\beta)d\alpha d\beta\\ & - \int_{-h}^{0}\int_{-h}^{0} x'(t+\alpha)S(\alpha,\beta)e(t+\beta)d\alpha d\beta\\ & - \int_{-h}^{0}\int_{-h}^{0} e'(t+\alpha)S(\alpha,\beta)x(t+\beta)d\alpha d\beta\\ & + \int_{-h}^{0}\int_{-h}^{0} e'(t+\alpha)(\gamma^2J(\alpha,\beta)\\ & + S(\alpha,\beta) + \gamma^2T^{-1})e(t+\beta)d\alpha d\beta,\end{aligned} \tag{2}$$

where

$$\begin{aligned}& x_s = (x(t), x_t, e(t), e_t),\\ & x_t = \{x(t+\beta) | -h \le \beta \le 0\},\\ & e_t = \{e(t+\beta) | -h \le \beta \le 0\},\\ & M,\ Q,\ T,\ H \in \mathbb{R}^{n\times n},\\ & S(\alpha,\beta),\ J(\alpha,\beta) \in L_2([-h,0]\times[-h,0];\mathbb{R}^{n\times n}),\end{aligned}$$

and $e(t)$ denotes the error $e(t) = x(t) - x_u(t)$ in which $x_u(t)$ denotes the state of dynamic output feedback controllers. The parameter γ is a free parameter in case of synthesis problems for stabilization of the linear time-delay system (1) via dynamic output feedback controllers. The parameter γ influences feasibility of dynamic output feedback controllers. If H_∞ contol problems are considered, γ is the L_2 gain of the closed loop systems.

Remark 1. We have a result concerning about stabilization problems of the system (1) by using this functional. This is a natural extension of the functional to solve the state feedback control problem of linear time-delay systems, whose basic structure is based on the Lyapunov function to solve H_∞ output feedback control problem of linear systems without delays proposed in (Uchida and Fujita, 1989; Uchida and Fujita, 1992).

In this paper, we use a notation,

$$\Theta(\alpha,\beta)=\begin{bmatrix}\Theta_0 & \Theta_1(\beta)\\ \Theta_1'(\alpha) & \Theta_2(\alpha,\beta)\end{bmatrix}>0,$$
$$^\forall\alpha,\ \beta\in[-h,0],$$

which means that Θ_0 and $\Theta_2(\alpha,\beta)$ are symmetric, that is $\Theta_0=\Theta_0'$ and $\Theta_2(\alpha,\beta)=\Theta_2'(\alpha,\beta)=\Theta_2(\beta,\alpha)$, and symmetric matrix,

$$\frac{1}{2}\left(\Theta(\alpha,\beta)+\Theta'(\alpha,\beta)\right)=$$
$$\begin{bmatrix}\Theta_0 & \frac{1}{2}\left(\Theta_1(\alpha)+\Theta_1(\beta)\right)\\ \frac{1}{2}\left(\Theta_1'(\alpha)+\Theta_1'(\beta)\right) & \frac{1}{2}\left(\Theta_2(\alpha,\beta)+\Theta_2(\beta,\alpha)\right)\end{bmatrix},$$

is positive definite for each $(\alpha,\beta)\in[-h,0]\times[-h,0]$, where " $'$ " denotes transposition of vector and matrix. Note that, if a matrix function $\Theta(\alpha,\beta)>0$ is continuous in (α,β), there exists a positive number λ such that $\Theta(\alpha,\beta)\geq\lambda I$ for all $(\alpha,\beta)\in[-h,0]\times[-h,0]$, where I denotes identity matrix.

3. OUTPUT FEEDBACK CONTROL SYNTHESIS

We have the following main theorem for stabilization problems of the system (1) using dynamic output feedback controllers based on the Lyapunov functional (2).

Theorem 2. Given $\gamma\in\mathbb{R}$, if there exist constant matrices $N,\ L,\ R,\ H,\ Z_0,\ Y_0$ and continuously differentiable matrix functions $Z_{01}(\beta),\ Y_{01}(\beta),\ X(\alpha,\beta),\ J(\alpha,\beta)$ which satisfy the following inequalities for $^\forall\alpha\in[-h,0]$ and $^\forall\beta\in[-h,0]$,

$$\Theta^{st}(\alpha,\beta):=\begin{bmatrix}\Theta_{11}^{st}(\alpha,\beta) & \gamma^{-1}\Theta_{12}^{st}\\ \gamma^{-1}(\Theta_{12}^{st})' & \Theta_{22}^{st}(\alpha,\beta)\end{bmatrix}<0,$$

$$\begin{bmatrix}N & \gamma^{-1}I\\ \gamma^{-1}I & R\end{bmatrix}>0, \tag{3}$$

$$L>0,\ H>0,\ X(\alpha,\beta)>0,\ J(\alpha,\beta)>0,$$

where

$$\Theta_{11}^{st}(\alpha,\beta)=\left[\begin{array}{c|c|c}\begin{matrix}A_0N+NA_0'\\ -B_2Z_0-Z_0'B_2'\\ +L+2N\end{matrix} & A_1N-N & \begin{matrix}NA_0'+X(0,\beta)+N\\ -Z_0'B_2'-B_2Z_{01}(\beta)\end{matrix}\\ \hline NA_1'-N & -L & NA_1'-X(-h,\beta)-N\\ \hline \begin{matrix}A_0N+X(\alpha,0)+N\\ -B_2Z_0-Z_{01}'(\alpha)B_2'\end{matrix} & \begin{matrix}A_1N-N\\ -X(\alpha,-h)\end{matrix} & \begin{matrix}-B_2Z_{01}(\beta)-Z_{01}'(\alpha)B_2'\\ -\left(\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial\beta}\right)X(\alpha,\beta)\end{matrix}\end{array}\right],$$

$$\Theta_{12}^{st}=\begin{bmatrix}2I & A_1-I & -A_0+I\\ -I & 0 & 0\\ A_0+I & A_1-I & 0\end{bmatrix},$$

$$\Theta_{22}^{st}(\alpha,\beta)=\left[\begin{array}{c|c|c}\begin{matrix}RA_0+A_0'R\\ -Y_0C_2-C_2'Y_0'\\ +H+2R\end{matrix} & RA_1-R & \begin{matrix}A_0'R+J(0,\beta)+R\\ -C_2'Y_0'-Y_{01}(\beta)C_2\end{matrix}\\ \hline A_1'R-R & -H & A_1'R-J(-h,\beta)-R\\ \hline \begin{matrix}RA_0+J(\alpha,0)+R\\ -Y_0C_2-C_2'Y_{01}'(\alpha)\end{matrix} & \begin{matrix}RA_1-R\\ -J(\alpha,-h)\end{matrix} & \begin{matrix}-Y_{01}(\beta)C_2-C_2'Y_{01}'(\alpha)\\ -\left(\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial\beta}\right)J(\alpha,\beta)\end{matrix}\end{array}\right],$$

then the closed loop system with the output feedback controller

$$\begin{aligned}\dot{x}_u(t)&=A_{K0}x_u(t)+A_{K1}x_u(t-h)\\ &\quad+\int_{-h}^{0}A_{K2}(\beta)x_u(t+\beta)d\beta\\ &\quad+B_{K0}y(t)+\int_{-h}^{0}B_{K01}(\beta)y(t+\beta)d\beta,\\ u(t)&=C_{K0}x_u(t)+\int_{-h}^{0}C_{K01}(\beta)x_u(t+\beta)d\beta,\end{aligned} \tag{4}$$

where

$$\begin{aligned}A_{K0}&=A_0+BC_{K0}-B_{K0}C+\gamma^{-2}TE,\\ A_{K1}&=(I+\gamma^{-2}TM)A_1,\\ A_{K2}(\beta)&=(I+\gamma^{-2}TM)BC_{K01}(\beta)-B_{K01}(\beta)C,\\ B_{K0}&=TY_0,\\ B_{K01}(\beta)&=TY_{01}(\beta),\\ C_{K0}&=-Z_0M,\\ C_{K01}(\beta)&=-Z_{01}(\beta)M,\\ T&=(I-\gamma^{-2}PM)^{-1}P,\\ E&=MA_0+A_0'M-MBZ_0M,\end{aligned}$$

is internally stable, where $M:=N^{-1},\ P:=R^{-1}$.

Though the detail of the proof of this theorem is omitted, the key idea in the proof is explained.

The key idea is to use the output feedback control synthesis technique for linear systems with no time-delay proposed in papers (Uchida and Fujita, 1989; Uchida and Fujita, 1992). According to this output feedback control synthesis technique, the state of the closed loop system $x_{cl}(t)$ is defined as follows,

$$x_{cl}(t) = \begin{bmatrix} x(t) \\ x(t) - x_u(t) \end{bmatrix}.$$

For linear time-delay system (1), the closed loop system with the controller (4) is given as

$$\begin{aligned}\dot{x}_{cl}(t) &= A_{cl0}x_{cl}(t) + A_{cl1}x_{cl}(t-h) \\ &\quad + \int_{-h}^{0} A_{cl01}(\beta)x_{cl}(t+\beta)d\beta, \qquad (5)\end{aligned}$$

where

$$A_{cl0} = \begin{bmatrix} A_0 + B_2C_{K0} & -B_2C_{K0} \\ -\gamma^{-2}TE & A_0 - B_{K0}C_2 + \gamma^{-2}TE \end{bmatrix},$$
$$A_{cl1} = \begin{bmatrix} A_1 & 0 \\ -\gamma^{-2}TMA_1 & A_1 + \gamma^{-2}TMA_1 \end{bmatrix},$$
$$A_{cl01}(\beta) =$$
$$\left[\begin{array}{c|c} B_2C_{K01}(\beta) & -B_2C_{K01}(\beta) \\ \hline -\gamma^{-2}TMB_2C_{K01}(\beta) & \begin{array}{c}\gamma^{-2}TMB_2C_{K01}(\beta) \\ -B_{K01}(\beta)C_2\end{array} \end{array}\right].$$

Using the condition (3) in Theorem 2, we can prove that the functional (2) is a Lyapunov functional for the closed loop system (5).

Now considering the element of the state of the closed loop system $x_{cl}(t)$, a condition $x(t) = x_u(t)$ is satisfied in the steady state if the internal stability of the closed loop system is assured. Thus *the dymanics of $x_u(t)$ can be interpreted as an observer of the state $x(t)$ for the linear time-delay system* (1). The dynamics $x_u(t)$ in (4) can be rewritten as

$$\begin{aligned}\dot{x}_u(t) &= (A_0 + \gamma^{-2}TE)x_u(t) \\ &\quad + (A_1 + \gamma^{-2}TMA_1)x_u(t-h) \\ &\quad + \int_{-h}^{0} \gamma^{-2}TMBC_{K01}(\beta)x_u(t+\beta)d\beta \\ &\quad + Bu(t) + B_{K0}(y(t) - Cx_u(t)) \\ &\quad + \int_{-h}^{0} B_{K01}(\beta)(y(t+\beta) - Cx_u(t+\beta))d\beta.\end{aligned}$$

Considering that this dynamics is an observer for the linear time-delay system (1), the last term of this dynamics denotes the integral of the observer error $y - Cx_u$. So we call this observer as the "*memory observer*". This memory observer is a new one with the special structure for the linear time-delay system (1). *Thus the derived dynamic output feedback controller* (4) *can be understood as the controller which consists of the memory state feedback contoller and the memory observer.* The fact that the dynamics $x_u(t)$ in (4) properly acts as an observer for linear time-delay system (1) is shown in the numerical example.

Remark 3. Theorem 2 is proposed for the dynamic output feedback control synthesis of the linear time-delay system (1). This theorem is an extension of the result of the memory state feedback control synthesis for linear time-delay systems (T. Azuma and Uchida, 2002).

4. REDUCTION TO A FINITE NUMBER OF LMI CONDITIONS

Inequalities in Theorem 2 depend on parameters α and β. It seems difficult to solve these infinite-dimensional (parameter-dependent) inequalities directly. In our approach, we reduce these infinite-dimensional inequalities to a finite number of LMIs by using the technique in (T. Azuma and Uchida, 1997; T. Azuma and Fujita, 2000), and obtain the solution of the infinite-dimensional inequalities by computing the finite number of LMIs.

Here we restrict solution in their theorems to the following forms,

$$\begin{aligned}X(\alpha,\beta) &= X_0 + g_1(\alpha,\beta)X_1 + \cdots + g_{l_X}(\alpha,\beta)X_{l_X}, \\ J(\alpha,\beta) &= J_0 + h_1(\alpha,\beta)J_1 + \cdots + h_{l_J}(\alpha,\beta)J_{l_J}, \\ Z_{01}(\beta) &= Z_0^{01} + p_1(\beta)Z_1^{01} + \cdots + p_{l_Z}(\beta)Z_{l_Z}^{01}, \\ Y_{01}(\beta) &= Y_0^{01} + q_1(\beta)Y_1^{01} + \cdots + q_{l_Y}(\beta)Y_{l_Y}^{01},\end{aligned} \quad (6)$$

where $g_i,\ h_i : \mathbb{R}^2 \to \mathbb{R}$ is continuous differentiable function of α and β such that

$$g_i(\alpha,\beta) = g_i(\beta,\alpha),\ h_i(\alpha,\beta) = h_i(\beta,\alpha),$$

and $p_i,\ q_i : \mathbb{R} \to \mathbb{R}$ is a continuous differentiable function of β and the unknown matrices satisfy

$$\begin{aligned}&X_i \in \mathbb{R}^{n\times n},\ X_i' = X_i\ (i = 0,\ ,1,\cdots,\ l_X), \\ &J_i \in \mathbb{R}^{n\times n},\ J_i' = J_i\ (i = 0,\ ,1,\cdots,\ l_J), \\ &Z_i^{01} \in \mathbb{R}^{l\times n}\ (i = 0,\ 1,\cdots,\ l_Z), \\ &Y_i^{01} \in \mathbb{R}^{l\times n}\ (i = 0,\ 1,\cdots,\ l_Y).\end{aligned}$$

Note that equations (6) satisfy matrix inequalities (3). Then inequalities in Theorem 2 can be written in the form of the following parameter dependent LMI condition,

$$F_0(M) + f_1(\xi)F_1(M) + \cdots + f_r(\xi)F_r(M) < 0, \quad (7)$$

where $\xi \in \Xi = \{[\alpha\ \beta]' \mid \alpha \in [-h,0],\ \beta \in [-h,0]\}$, $f_i : \mathbb{R}^2 \to \mathbb{R}$ is a continuous function of α and β, and a symmetric matrix function F_i depends affinely on the unknown matrix $M = [X_0,\cdots,\ X_{l_X},\ J_0,\cdots,\ J_{l_J}, Z_0^{01},\cdots,\ Z_{l_Z}^{01}\ Y_0^{01},\cdots,\ Y_{l_Y}^{01}]$. The parameter dependent LMI condition (7) can be reduced to a finite number of LMI conditions as follows.

Theorem 4. (T. Azuma and Uchida, 1997) Let $\{p_1,\ p_2,\ \cdots,\ p_q\}$ be vertices of a convex polyhedron which includes the curved surface T,

$$T = \{[f_1(\xi)\ f_2(\xi)\ \cdots\ f_r(\xi)]' \mid \xi \in \Xi\}.$$

Assume that there exists M which satisfies the following LMI condition for all $p_i (i = 1,2,\cdots,q)$,

$$F_0(M) + p_{i1}F_1(M) + \cdots + p_{ir}F_r(M) < 0, \quad (8)$$

where p_{ij} is the jth element of p_i. Then M satisfies (7) for all $\xi \in \Xi$.

A general technique to construct a convex polyhedron which includes the curved surface T is proposed in (T. Azuma and Uchida, 1997). In the special case that $r = 2s$

$$f_i(\alpha, \beta) = \begin{cases} f_i(\alpha), \ i = 1, \ 2, \cdots, \ s, \\ f_i(\beta), \ i = s+1, \ s+2, \cdots, \ 2s, \end{cases},$$

$f_i(\alpha)$ and $f_i(\beta)$ are polynomial functions of α and β respectively, we can use a simple technique to construct such a convex polyhedron, which is given by the paper (K. Ikeda and Uchida, 2001). If the parameter is scalar and f_i is given as a general polynomial function, a technique to construct a convex polyhedron which includes the curved surface T is proposed in the paper (T. Azuma and Fujita, 2000) and less conservative results can be obtained by using this technique.

5. NUMERICAL EXAMPLE

In this section, we illustrate the efficacy of the memory output feedback controller proposed in Theorem 2. Consider the following time-delay system,

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1 x(t-h) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

where the system parameter is given as follows,

$$A_0 = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, \ A_1 = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \ h = 1.0,$$

and the initial state is given as

$$x(\beta) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ -h \le \beta < 0, \quad x(0) = \begin{bmatrix} 10 \\ 5 \end{bmatrix}.$$

Here note that the open loop system ($u(t) = 0$) is unstable(See Fig. 1).

To use the technique of the previous section 4, we restrict solutions of Theorem 2 as follows,

$$\begin{aligned} X(\alpha, \beta) &= X_0 + (\alpha + \beta)X_1 + (\alpha^2 + \beta^2)X_2 \\ J(\alpha, \beta) &= J_0 + (\alpha + \beta)J_1 + (\alpha^2 + \beta^2)J_2 \\ Z_{01}(\beta) &= Z_0^{01} + \beta Z_1^{01} + \beta^2 Z_2^{01} \\ Y_{01}(\beta) &= Y_0^{01} + \beta Y_1^{01} + \beta^2 Y_2^{01}. \end{aligned} \quad (9)$$

The value of γ is chosen as 100 by considering feasibility of controllers in Theorem 2. Finally the finite number of LMIs is 30 and the computation time is 7 [sec] by using MATLAB on the computer with Athron-1GHz and 512MB-memory. System parameters of the obtained dynamic output feedback controller (4) are given as

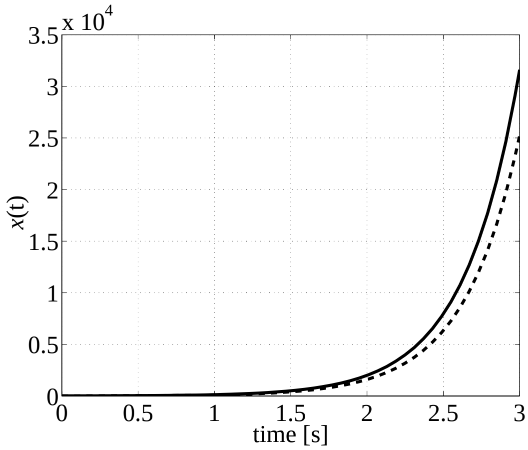

Fig. 1. The initial value response of the open loop system

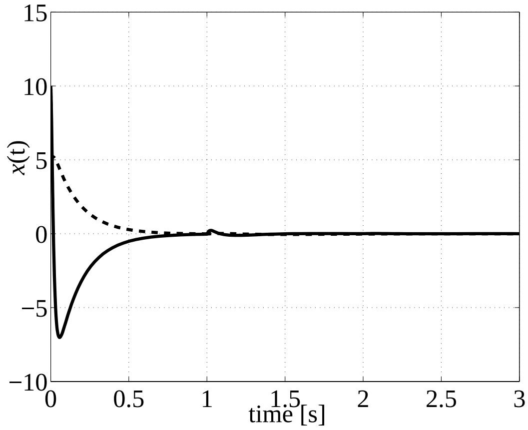

Fig. 2. The initial value response of the closed loop system

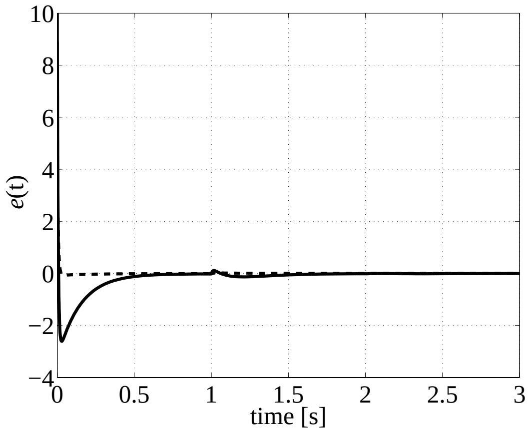

Fig. 3. The response of the estimated error $x(t) - x_u(t)$

$$A_{K0} = \begin{bmatrix} -71.996 & -511.41 \\ 2.9336 & -172.79 \end{bmatrix},$$

$$A_{K1} = \begin{bmatrix} 1.0026 & 2.0052 \\ 0.0010161 & 0.0020322 \end{bmatrix},$$

$$A_{K2}(\beta) = \begin{bmatrix} -32.084 & -410.22 \\ -0.032516 & -146.57 \end{bmatrix} + \beta \begin{bmatrix} -20.779 & -18.5 \\ -0.021058 & 0.061289 \end{bmatrix}$$
$$+\beta^2 \begin{bmatrix} 6.8301 & -30.575 \\ 0.0069221 & -14.62 \end{bmatrix},$$

$$B_{K0} = \begin{bmatrix} 436.52 \\ 171.72 \end{bmatrix},$$
$$B_{K01}(\beta) = \begin{bmatrix} 375.25 \\ 146.53 \end{bmatrix} + \beta \begin{bmatrix} -2.396 \\ -0.0825 \end{bmatrix} + \beta^2 \begin{bmatrix} 38.4 \\ 14.628 \end{bmatrix},$$
$$C_{K0} = \begin{bmatrix} -72.826 & -76.707 \end{bmatrix},$$
$$C_{K01}(\beta) = \begin{bmatrix} -32.0 & -34.883 \end{bmatrix} + \beta \begin{bmatrix} -20.725 & -20.842 \end{bmatrix} + \beta^2 \begin{bmatrix} 6.8124 & 7.8042 \end{bmatrix}.$$

Using this controller, we obtain the simulation result depicted in Fig. 2 and Fig. 3. Fig. 2 shows the initial value response of the state $x(t) = [x_1(t)\ x_2(t)]'$ of the closed loop system, where the solid line denotes $x_1(t)$ and the dotted line denotes $x_2(t)$. The state $x(t)$ of the closed loop system converges zero. Fig. 3 shows the response of the observer error $e(t) = x(t) - x_u(t) = [e_1(t)\ e_2(t)]'$, where the solid line denotes $e_1(t)$ and the dotted line denotes $e_2(t)$. The error $e(t)$ also converges zero.

6. CONCLUSION

In this paper, dynamic output feedback controller synthesis problems for linear time-delay systems via infinite-dimensional LMI approach were considered. An existence condition for synthesis problems of dynamic output feedback controllers was derived in the form of infinite-dimensional LMIs. The derived dynamics of output feedback controllers $x_u(t)$ can be interpreted as observers for linear time-delay systems. Because the observers have a special structure such as the memory type, we call the observers as "memory observers". Thus derived dynamic output feedback controllers consisted of memory state feedback controllers and memory observers. A technique to reduce the infinite-dimensional LMIs to a finite-dimensional LMIs, which provide feasible formulas, was also shown . Finally we demonstrated the efficacy of the derived dynamic output feedback controller and our proposed approach by a numerical example.

REFERENCES

A. Fattouh, O. Senme and J. M. Dion (2000). h_∞ controller and observer design for linear systems with point and distributed time-delay. *Proc. of IFAC Workshop on Linear Time Delay Systems* pp. 225–230.

Choi, H. H. and M. J. Chung (1995). Memoryless h_∞ controller design for linear systems with delayed state and control. *Automatica* **31**, 917–919.

Choi, H. H. and M. J. Chung (1997). An lmi approach h_∞ controller design for linear time-delay systems. *Automatica* **33**, 737–739.

de Souza, C. E. (2000). Stability and stabilizability of linear state-delay systems with multiplicative noise. *Proc. of IFAC Workshop on Linear Time Delay Systems* pp. 21–26.

de Souza, C. E. and X. Li (1999). Delay-dependent robust h_∞ control of uncertain linear state-delayed systems. *Automatica* **35**, 1313–1321.

Dullerud, G. and F. Paganini (2000). *A Course in Robust Control Theory.* Springer.

Gu, K. (1997*a*). Constrained lmi set in the stability problem of linear uncertain time-delay systems. *Proc. of the 1997 American Control Conference* pp. 3657–3661.

Gu, K. (1997*b*). Discretization of lyapunov functional for uncertain time-delay systems. *Proc. of the 1997 American Control Conference* pp. 505–509.

J. He, Q. Wang and T. Lee (1998). h^∞ disturbance attenuation for state delayed systems. *Systems & Control Letters* **33**, 105–114.

K. Ikeda, T. Azuma and K. Uchida (2001). Infinite-dimensional lmi approach to analysis and synthesis for linear time-delay systems. *KYBERNETIKA* **37**, 505–520.

Loiseau, J. J. and D. Brethe (1996). An effective algorithm for finite spectrum assignment of single input systems with delay. *Proc. of Symposium "Modeling, Analysis and Simulation", IEEE-IMACS Conference Computation Engineering in Systems Applications.*

Louisell, J. (1991). *A Stability Analysis for a Class of Differential-Delay Equations having Time-Varying delay.* Vol. Lecture Notes in Mathematics No 1745. Springer-Verlag.

S. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan (1994). *Linear Matrix Inequalities in System and Control Theory.* Vol. 15. SIAM studied in applied mathematics.

T. Azuma, K. Ikeda, T. Kondo and K. Uchida (2002). Memory state feedback control synthesis for linear systems with time delay via a finite number of linear matrix inequalities. *Computers and Electrical Engineering* **28**, 217–228.

T. Azuma, R. Watanabe and K. Uchida (1997). An approach to solving parameter-dependent lmi conditions based on finite number of lmi conditions. *Proc. of the 1997 American Control Conference* pp. 510–514.

T. Azuma, R. Watanabe, K. Uchida and M. Fujita (2000). A new lmi approach to analysis of linear systems depending on scheduling parameter in polynomial forms. *Automatisierungstechnik* **48**, 199–204, http://www–fujita.ee.t.kanazawa–u.ac.jp/~azuma/pub.html.

Uchida, K. and M. Fujita (1989). On the central controller: Characterizations via differential games and leqg control problems. *Systems & Control Letters* **13**, 9–13.

Uchida, K. and M. Fujita (1992). Finite horizon h_∞ control problems with terminal penalty. *IEEE Trans. Automat. Contr.* **37**, 1761–1767.