RANSAC-LEL: AN OPTIMIZED VERSION WITH LEAST ENTROPY LIKE ESTIMATORS

Cosimo Distante

Istituto Nazionale di Ottica - CNR
Arnesano (Le), Italy
(cosimo.distante@ino.it)

Giovanni Indiveri

Dipartimento Ingegneria Innovazione
Università del Salento via Monteroni, 73100 Lecce, Italy
(giovanni.indiveri@unisalento.it)

ABSTRACT

The paper proposes a robust estimation method which implements, in cascade, two algorithms: (i) a Random Sample and Consensus (RANSAC) algorithm and (ii) a novel nonlinear prediction error estimator minimizing a cost function inspired by the mathematical definition of Gibbs entropy. The minimization of the nonlinear cost function allows to refine the Consensus Set found with standard RANSAC in order to reach optimal estimates of geometric transformation parameters under image stitching context. The method has been experimentally tested and compared with a standard RANSAC-MSAC algorithm where noticeable improvements are recorded in terms of computational complexity and quality of the stitching process, namely of the mean squared symmetric re-projection error.

***Index Terms*—** Image matching, Homography Estimation, RANSAC-LEL

1. INTRODUCTION

Robust estimation in computer vision has been applied to many problems that range from line and plane estimation [1] to foundamental matrix estimation [2] etc. Any robust estimator technique must find the parameter vector being at the same time prone to the presence of outliers that are in disagreement with the assumed model. Several robust estimation methods have been proposed in the last years: maximum likelihood estimators, least median of squares (LMeds) [3] and later the most popular Random Sample and Consensus (RANSAC) [4] which has become a standard. RANSAC has gained more interests to several robust estimation problems due to its ability to detect and clean datasets with more than 50% of outliers. Repeatedly, subsets of the input data (for example, a set of tentative correspondences) are randomly selected (with replacement), and model parameters are computed by fitting these subsets. In the second step, the quality of the parameters is evaluated on the input data.

However, RANSAC requires to select some tuning coefficients such as the error tolerance which is unknown in advance in many real world problems. The performance of standard RANSAC can be improved by modifying its cost function using an M-Estimator Sample and Consensus (MSAC) [5] approach, but this also requires a user-specified error tolerance. The process is terminated when the probability of finding a better model becomes lower than a user-specified probability. Optimized version of RANSAC have been proposed in [6] to speed-up the estimation process, and in [7] for image matching process using regional affine filters to classify inliers against outliers. In this paper the RANSAC method to estimate homography is introduced and compared with the proposed RANSAC-LEL technique inspired by the results in [8]. Experimental results suggest the superiority of the proposed technique in terms of precision of the solution (MSE) and computational complexity.

2. AN ENTROPY-LIKE ESTIMATOR

Let us consider a sample set $\mathbf{X} = \{[\mathbf{x}', \mathbf{x}]_i\}_{i=1}^N$ of point correspondeces $\mathbf{x}' \leftrightarrow \mathbf{x}$ between two images. We consider $\mathcal{A}_\theta$ a model representing a geometric transformation to be estimated with respect to noisy computed correspondeces $\mathbf{X}$. To this aim, given $\theta([\mathbf{x}', \mathbf{x}]_1, \ldots, [\mathbf{x}', \mathbf{x}]_h)$ the parameter vector estimated with $h << N$ observations and representing a minimal sample set (for *equi-form*, *affine* and *projective* $h = 2, 3, 4$ respectively). The optimal estimated model is given by $\mathbf{x}' = \mathcal{A}_\theta \mathbf{x}$ which produces an error given by the symmetric transfer function. Let us define the error associated to the solution $\mathcal{A}_\theta$ and computed over each correspondence as given by the $i-th$ residual

$$\begin{aligned} r_i^2 &= e([\mathbf{x}', \mathbf{x}]_i, \mathcal{A}_\theta) = \\ &= d(\mathbf{x}_i, \mathcal{A}_\theta^{-1}\mathbf{x}'_i)^2 + d(\mathbf{x}'_i, \mathcal{A}_\theta \mathbf{x}_i) \quad i = 1, \ldots N \end{aligned} \tag{1}$$

with d the Euclidean distance. In conventional RANSAC terminology, the residual defined in (1) defines a *Consensus Set* (CS) defined by the following:

$$S(\theta) = \{[\mathbf{x}', \mathbf{x}] \in \mathbf{X} : e([\mathbf{x}', \mathbf{x}]_i, \mathcal{A}_\theta) \le \delta\} \tag{2}$$

Given the residual r_i as in equation (1), let us define [8]:

$$D = \sum_{j=1}^N r_j^2, \tag{3}$$

namely the *Least Square* (LS) estimation cost. Then define the *relative squared residuals* q_i as

$$\text{if } D \neq 0 \implies q_i := \frac{r_i^2}{\sum_{j=1}^{N} r_j^2} \; : \; q_i \in [0,1], \; \sum_{i=1}^{N} q_i = 1, \tag{4}$$

and finally

$$H = \begin{cases} 0 & \text{if } D = 0 \\ -\frac{1}{\log N} \sum_{i=1}^{N} q_i \log q_i & \text{otherwise} \end{cases} \tag{5}$$

namely H enjoys all the mathematical properties of a normalized *entropy* function associated to the sequence of "probability" - like $q_i \; : \; i = 1, 2, \ldots, N$. In particular:

$$H \in [0,1] \tag{6}$$

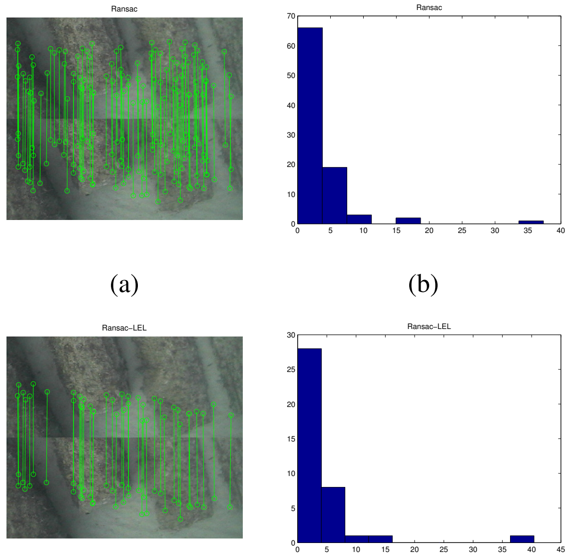
$$H = 0 \text{ iff } \begin{cases} r_i = 0 \quad \forall \quad i \in [1,N] \\ \text{or} \\ \exists\,! \; i^* : r_{i^*} \neq 0 \text{ and } r_i = 0 \quad \forall \quad i \neq i^* \end{cases} \tag{7}$$

$$H = 1 \text{ iff } r_i^2 = r_j^2 \neq 0 \quad \forall \quad i,j \in [1,N]. \tag{8}$$

Notice that the hypothesis that $D \neq 0$ in the definition (5) is needed just to prevent the singular situation occurring when the LS fit is perfect. This is not a practical limit as prior to computing H one can always check if the LS fit is perfect. In Physics the entropy of a system admitting N discrete states with probabilities $p_1, p_2, \ldots, p_N$ is computed as $-\sum_{i=1}^{N} p_i \log p_i$. It is a very well known fact that such function is a very sensitive measure of the distribution of the probabilities. Configurations with only a fraction of highly probable states have a much lower entropy of configurations where most states are approximately equally probable. Motivated by this fact, the function H is defined with the aim of computing a robust estimate of the model parameter vector θ. In particular given that the entropy-like function H as defined by equation (5) depends on $\boldsymbol{\theta}$ through the residuals r_i, the following estimator is proposed:

$$\hat{\theta}_{LEL} := \arg\min_{\boldsymbol{\theta}} H \tag{9}$$

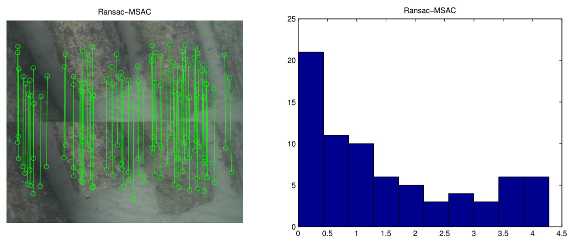
where LEL stands for Least Entropy-Like [8]. The idea behind the $\hat{\theta}_{LEL}$ estimator defined in (9) is that such estimate will correspond either to making all the residuals null, or to making the relative squared residuals as little equally distributed as possible according to the entropy-like function H, the available data and the model structure. The key to robustness with respect to outliers is related to the fact that the devised penalty function does not directly measure the (weighted) mean square error (that as known tends to level out or "low pass" residuals), but only the distribution of the relative squared errors. A more detailed analysis of the properties of this estimator, including the singular situations associated with the (practically impossible) perfect fit case $r_i = 0 \; \forall \; i$, can be found in [8]. According with the experience so far acquired with simulated [8] and real range data for plane estimation [9] [1] the computation of $\hat{\theta}_{LEL}$ can be successfully performed locally and numerically from an initialization value sufficiently close to the real value of θ.



(a) (b)

(c) (d)

Fig. 1. Comparisons of inliers between weak Ransac (a-b) and the successively optimized LEL solution (c-d). Note the reduction of the respective consensus sets dimensions. Compare results in (c) and (d) with those obtained with standard RANSAC-MSAC of figure 2.



(a) (b)

Fig. 2. Results from standard RANSAC-MSAC that produces 60 noisy correspondeces.

3. EXPERIMENTAL SETUP

The dataset is composed of a sequence of underwater images (379×172 pixels) acquired from a ROV while inspecting an interesting archeological site acquired in the Porto Cesareo's (Lecce, Italy) Marine Protected Area (images were acquired from an area that contains seven columns in cipolin marble dating to the Roman age). An affine transformation is used for stitching. SURF detector [10] and correlations are used to extract and establish keypoints correspondences. In general, LEL estimator can be applied after two different RANSAC procedures: (1) RANSAC-LEL using "weak" RANSAC procedure, i.e. assuming a very high probability of inliers (resulting in a small number of iterations); (2) using optimal RANSAC-MSAC with a robust Consensus Set. In either cases, LEL estimator outperform RANSAC-MSAC alone. In the first configuration the speed-up is very noticeable. We will describe the first procedure and compare to classical RANSAC-MSAC [5]. Algorithm 1 shows the RANSAC-LEL pseudo-code. Both the algorithms are able to detect the affine transformation, and the two solutions provide different Consensus Sets as well as different symmetric transfer errors. To make RANSAC-MSAC robust, we have set $P_{Inliers} = 0.6$ and no limit to maximum iterations. Instead, RANSAC-LEL uses $P_{Inliers} = 0.999$ and few classical RANSAC [4] interations in order to have an initial estimate (in a least square sense) of the solution θ_{RANSAC} to successively optimize the least entropy-like cost function. This initial stage brings to a wanted large and noisy Consensus set CS_{RANSAC} due to a high P_{Inlier}. Then the minimization of the non-linear eq. (5) is performed with Levenberg Marquardt method (similarly to [9]), that allows to smoothly switch from the GaussNewton method to the steepest descent. The estimation vector that in our experiments regards and affine transformation, $\hat{\theta}_k \in \mathbb{R}^6$ is updated as follows:

$$\hat{\theta}_{k+1} := \hat{\theta}_k - \gamma(\nabla^2_\theta \mathbf{H} + \mu \cdot diag \nabla^2_\theta \mathbf{H})^{-1} \nabla^2_\theta \mathbf{H}^\top \qquad (10)$$

where it moves toward the solution of the optimization problem along the direction where the Gradient $\nabla_\theta \mathbf{H} \in \mathbb{R}^{1\times 6}$ is smaller. The solution found θ_{LEL} with the optimization stage is then used to compute the symmetric transfer error and to histogram the residuals (figure 1d). The distribution reveals a significant accumulation near zero, where the Consensus Set is defined. To this aim, a suitable threshold δ as in eq. (2) is found after kernel density estimation (Parzen windows with Gaussian kernels $K(\cdot)$) of residuals r_i

$$\hat{f}_{h_{\hat{\theta}}}(r; [r_{1\hat{\theta}}, \ldots, r_{N\hat{\theta}}]) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{h_{\hat{\theta}}} K\left(\frac{r - r_i}{h_{\hat{\theta}}}\right) \qquad (11)$$

and finding first local minimum on the right side of the first mode near zero as shown in figure 3. So far, this threshold defines a new consensus set CS_{LEL} which is always smaller than CS_{RANSAC}. Thus with CS_{LEL} a new solution is re-estimated using classical least squares method. Figure 1a-b shows correspondeses and residual distribution from the weak RANSAC CS_{RANSAC}, which follows in figure 1c-d the results of the entropy cost minimization by starting from the solution obtained with weak RANSAC. LEL estimator gives always smaller consensus sets but with more precise correspondences, i.e. with the a reduced mean squared symmetric transfer error. The results obtained with fewer iterations can be compared with the robust and complex RANSAC-MSAC of figure 2a-b. Figure 4 shows comparisons of the mean squared symmetric transfer error between RANSAC-MSAC and the proposed technique (weak) RANSAC-LEL. The figure also shows another experiment that allows to prove that LEL estimator always reduces residual errors even when applied after the RANSAC-MSAC method. The computational complexity has been measured in terms of elapsed time to find correspondences and estimate homography, and are reported in figure 5. It is interesting to note that RANSAC-LEL has a noticeable lower complexity, i.e. a faster computation. The mosaic built with 20 images, is shown in figure 6. Note that the images have not been rectified and no radiometric correction has been performed. In conclusion we proved that the novel least entropy-like estimator produces better estimates either used in cascade to a weak RANSAC algorithm or to the robust RANSAC-MSAC with a smaller number of $P_{Inliers}$.

Algorithm 1 RANSAC-LEL Algorithm

1: Given data set $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^N$ of noisy correspondences
2: Get Consensus Set (CS_{RANSAC}) with weak RANSAC iterations $P_{Inliers} = 0.9999$ (results in less than 10 iterations) and compute θ_{RANSAC} with least squares
3: Compute θ_{LEL} by optimizing LEL cost function of eq. (5) by using Levenberg Marquardt (LM) method with initial guess $\theta_0 = \theta_{RANSAC}$
4: Recompute residuals with new model θ_{LEL} and points in CS_{RANSAC}
5: Compute kernel density estimation of new residuals with bandwidth $h_\theta = (1/4) \cdot MAD(r_1, \ldots, r_{|CS_{RANSAC}|})$ and find new threshold after first mode as shown in figure 3
6: Get new Consensus Set (CS_{LEL}) with the found threshold
7: Recompute the new model θ with CS_{LEL} using least squares

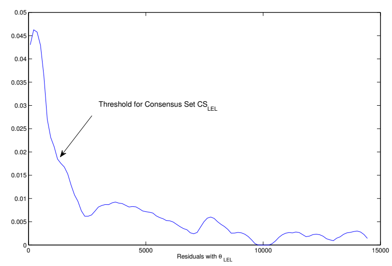


Fig. 3. Kernel density estimation of residuals with Gaussian Kernels and threshold setup.

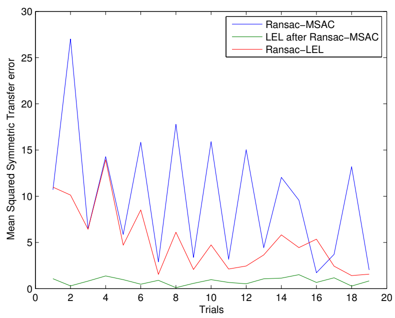


Fig. 4. Cost functions based on symmetric transfer errors for the RANSAC-LEL ($P_{Inliers} = 0.9999$, $\sigma = 0.89$), RANSAC-MSAC ($P_{Inliers} = 0.6$, $\sigma = 0.8$), and LEL applied after RANSAC-MSAC.

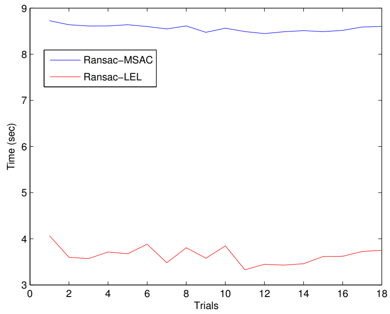


Fig. 5. Time in seconds needed to build the mosaic from 20 underwater images.

Fig. 6. Results of the image stitching process with RANSAC-LEL.

4. REFERENCES

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