On the Approximation of Correlated Non-Gaussian Noise Pdfs using Gaussian Mixture Models

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Abstract

Gaussian mixture probability density functions (pdfs) have been popular for modeling non-Gaussian noise. The majority of non-Gaussian noise research has been restricted to independent and identically distributed observation sequences due to the difficulty in characterizing multidimensional pdf's. There has been very few studies on the ability of Gaussian mixture pdfs to model correlated non-Gaussian noise processes. In this paper, we initiate such a study and demonstrate that in practical cases, Gaussian mixture pdfs with a small number of mixing terms can give good approximations to non-Gaussian noise pdfs. Some general models for correlated non-Gaussian interference and noise are reviewed. The focus is on three approaches. The first is the Gaussian mixture model approach. The second is an approach based on spherically invariant random vectors. The final approach involves the combination of linear filters and nonlinearities, generally in an *ad-hoc* manner. The three approaches are compared and the Gaussian mixture model is shown to approximate models generated from the other approaches.

1 Introduction

Gaussian mixture models have attracted attention for many years [1, 2, 3, 4]. A simple two term mixture probability density function (pdf) for a scalar observation is given by

$$f(x) = (1-\epsilon)\eta(x) + \epsilon h(x), \tag{1}$$

where ϵ is a small positive constant, η is a Gaussian pdf, and h is some other pdf with heavier tails. When h is also a Gaussian pdf with a variance larger than that of η, (1) is called a *Gaussian mixture model*. Mixture models of the form of (1) have been used by many investigators to model heavy-tailed non-Gaussian noise pdfs. The mixture model has also been found to provide a good fit to empirical noise data in many cases.

The mixture noise pdf model of (1) has been found to be appropriate for modeling impulse noise which can be considered to be a train of randomly occurring narrow pulses in a background of Gaussian noise [5]. Suppose that the impulsive component of a noise waveform is expressed as

$$I(t) = \sum_{k=-\infty}^{\infty} A_k \, p(t-t_k). \tag{2}$$

Here the A_k, $k = -\infty,...,\infty$, are independent and identically distributed (iid) amplitudes and the t_k, $k = -\infty,...,\infty$, are assumed to be generated by a Poisson point process. The pulse shape p is determined

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by the receiver filter response. Richter and Smits [6] derived an approximation for the pdf f_I of samples of $I(t)$ as

$$f_I(x) = (1 - \upsilon T_p)\delta(x) + \upsilon T_p h_I(x), \tag{3}$$

for $\upsilon T_p \ll 1$. Here υ is the rate parameter of the Poisson point process and T_p is the width of the pulse p. h_I is a density function which depends on the pulse shape p and on the density function of the A_k. When an independent Gaussian background noise is added to $I(t)$, the first-order density function of the total noise process becomes a convolution of f_I with η, resulting in the noise density f of (1) in which ϵ is now υT_p and h is the convolution of h_I and η.

Based on a representation of the impulsive component of the noise similar to, but more general than that given by (2), Middleton [1, 2, 3, 4] derived his canonical class A model. Middleton obtained an expansion of the noise pdf f as an infinite weighted sum of Gaussian densities with decreasing weights for Gaussian densities with increasing variances. Consider the univariate probability density function of the normalized, unit-variance Middleton noise pdf model. It has a Gaussian component and an independent additive interference component arising from a Possion mechanism. The overall noise density may be approximated as [1, 2, 3, 4]

$$f(x) = \sum_{m=0}^{\infty} \frac{e^{-A}A^m}{m!} \frac{1}{\sqrt{2\pi\delta_m^2}} e^{-x^2/(2\delta_m^2)}. \tag{4}$$

Here the parameter A is called the impulsive index and a small value of A implies highly impulsive interference. δ_m^2 is determined by some physical parameters.

If we keep only the first M terms in the sum (4), and use the proper normalization, an approximation of Middleton's class A model is obtained. For several cases of practical interest a rather small value of M (for example, $M = 2$ or 3) is found to be sufficient to give excellent approximation. Such numerical studies have been reported in [7], [14] and [16].

The majority of non-Gaussian noise research has been restricted to iid observation sequences because of the difficulty in characterizing multidimensional pdf's. However, in many practical situations, the noise is temporally or spatially correlated, or both. If the processing scheme is based upon the iid assumption, then the resulting estimation and detection is only suboptimum. For example, in radar and communication systems, closely spaced sensors in an array can cause the received interference, which is a part of the noise, at each sensor to be highly correlated. Correlation may arise during propagation or may be induced by filtering of uncorrelated noise. If η and h in (1) represent multidimensional Gaussian pdfs, then a Gaussian mixture model for correlated noise results. In fact, a straightforward extension involves considering more than two terms in (1). There are very few studies, if any, on the ability of a Gaussian mixture pdf to model correlated non-Gaussian noise processes. In this paper, we initiate such a study.

2 General Classes of Models

It is perhaps surprising that there are very few general models for correlated non-Gaussian interference and noise. A search of the literature reveals that most models fall within the following three categories.

1. **Spherically invariant random process (SIRP)**. SIRPs are widely used in modeling correlated background clutter in radar signal detection [8]. A physical justification for this model is given here. Consider the received signals reflected from some scatterers. For a particular scatterer, the reflected signal might be modeled as Gaussian. However, the power of the reflected signals from different scatterers may vary. Thus the background clutter can be modeled by an SIRP process defined by (5), which is an

integral of the reflected signals over all scatterers. A spherically invariant random vector (SIRV) Y can be generated by $Y = xG$, where x is a positive random variable with pdf $f_x(x)$ and G is a $N \times 1$ Gaussian random vector with zero mean and covariance matrix M, which is independent of x. If M is given, the pdf of x determines the pdf of Y, hence it is called the characteristic pdf of the SIRP. The pdf of Y is given by

$$f_Y(y) = (2\pi)^{-N/2}|M|^{-1/2} \int_0^{\infty} x^{-N} exp\left(-\frac{y^T M^{-1} y}{2x^2}\right) f_x(x) dx. \qquad (5)$$

In [9] and [10], an introduction to SIRPs is given and a SIRP library is constructed which provides the proper choice of $f_x(x)$ to model Gaussian, Laplace, Cauchy, and Student-t distributed SIRPs. Another important class of noise models, sub-Gaussian alpha stable noise [11][12][1] also belongs to the SIRP category with slight modification. The general model for generating a sub-Gaussian alpha stable noise vector is $Y = x^{1/2}G$, where x is an alpha stable random variable and G is a Gaussian random vector.

2. **Gaussian mixture model**. A general form of a Gaussian mixture pdf is given by

$$f(x) = \sum_{n=1}^{N} \epsilon_n f_n(x) \qquad (6)$$

where $\sum_{n=1}^{N} \epsilon_n = 1$ and each $f_n(x)$ is a, possibly multivariate, Gaussian pdf. If x is an N-dimensional vector we call (6) an N-dimensional Gaussian mixture model. This model can be seen to be a generalization to a truncated version of Middleton's class A model given in (4). In order to model heavy tailed cases, typically some terms of $f(x)$ have very large variance with small *mixing ratio* ϵ_n, while other terms have small variance but large ϵ_n. Thus impulsive noise samples, those coming from the large variance terms, occur once in a while in a Gaussian noise background. In [14], the scalar Gaussian mixture model is used for signal detection in uncorrelated noise cases. This method is further developed for use in signal detection in correlated non-Gaussian noise cases in [15]. A physical justification for the scalar version of (6) was provided in the introduction and [1]-[6]. A physical justification for the multivariate version of (6) is provided in [17] for communication applications.

3. **Use of various combinations of linear filters and nonlinearities driven by Gaussian noise**. There are many topologies that are possible and particular topologies are frequently chosen in an *ad-hoc* manner. Many generalization are also possible. For example, using a Volterra series is possible [18]. This would replace the filters and nonlinearities to implement a general nonlinearity with memory. Also, the input can be non-Gaussian. Using such ideas one could generate interesting models such as those discussed in [19]. In a common implementation, the iid samples (Gaussian or non-Gaussian) are input to a filter. After filtering, correlation is introduced into the samples to produce an autoregressive (AR), moving average (MA) or ARMA process [20]. If the iid inputs are Gaussian, non-Gaussianity can be introduced by a zero memory nonlinearity (ZNL). If the input is non-Gaussian, this ZNL may be omitted. Examples are the MA linear model used by Maras [21], and AR(1) model used by Middleton [22]. A simple extension involves summing several processes. For example, one correlation model used by some authors can be expressed as

$$y(k) = x_N(k) + u(k), \quad k = 0, 1, 2, ... \qquad (7)$$

[1]For a general discussion of alpha stable models see [11] and [12]. These models include some symmetric stable models that are not sub-Gaussian (SIRV) [11, pp. 37-42]. However, sub-Gaussian appears to be more popular.

Here, $y(k)$ is the correlated noise, and $x_N(k)$ is a correlated Gaussian process. $u(k)$ is defined as

$$u(k) = a_k I(k), \tag{8}$$

where $a_k \sim N(0, \delta_I^2)$ and $I(k) = 1$ with probability ϵ, otherwise $I(k) = 0$. Thus $u(k)$ represents an impulse train with random amplitude a_k which is Gaussian with zero mean and variance δ_I^2. The occurrence of the impulses is modeled with an iid Bernoulli random process. At each sample time, an impulse occurs with probability ϵ, where $0 < \epsilon < 1$. In [23], this model is extended to introduce correlation in both the Gaussian part and the impulsive part. The model used in [23] is depicted in Fig. 1 with

$$y(k) = x_N(k) + x_I(k) \tag{9}$$

The process $y(k)$ is the sum of a correlated Gaussian noise $x_N(k)$ (the *nominal part*) and a correlated *impulsive part* $x_I(k)$. Specifically, the nominal part is given by

$$x_N(n) = \sum_{k=0}^{M_1} h_N(k) e(n-k), \tag{10}$$

where $e(k)$ is iid zero-mean Gaussian process with variance δ_e^2. Similarly, the impulsive part $x_I(k)$ is generated by

$$x_I(n) = \sum_{k=0}^{M_2} h_I(k) u(n-k). \tag{11}$$

Here, $h_N(k)$ and $h_I(k)$ are impulse responses of stable linear systems. $u(k)$ is defined in (8). $e(k)$ and $u(k)$ are assumed to be independent. It is also assumed that ϵ is small, and $\delta_I^2 \gg \delta_e^2$. The linear filter h_I creates a correlated impulsive transient that lasts over several time samples.

3 Approximating SIRPs with Mixture Models

In order to understand the relationship, we first consider the class of *elliptically symmetric pdfs* [24]. An elliptically symmetric pdf can be expressed as

$$f_e(x|\mu, \Sigma) = |\Sigma|^{-1/2} q((x-\mu)' \Sigma^{-1} (x-\mu)), \tag{12}$$

where x is a $N \times 1$ random vector, μ is a $N \times 1$ vector and Σ is a $N \times N$ positive definite matrix. In (12), q is a function on $[0, \infty)$ satisfying $\int_{R^N} q(u^T u) du = 1$, $u \in R^N$. Consider the continuous mixture

$$g_e(x|\mu, \Sigma) = \int_0^\infty a^{-N/2} |\Sigma|^{-1/2} \zeta((x-\mu)' \Sigma^{-1} (x-\mu)/a) g(a) da, \tag{13}$$

where $g(a)$ is a pdf on $(0, \infty)$. When ζ is a normal pdf, (13) is called a normal mixture. Thus the pdf of an SIRV is a normal mixture from (5) and (13).

From Lemma 1.3 of [24], we know that the necessary and sufficient condition for a pdf of form (12) to be a normal mixture is that q satisfies

$$(-1)^k [(d^k/ds^k) q(s)] \geq 0, \quad k = 1, 2, ... \tag{14}$$

If the integral in (13), where ζ is a normal pdf, is approximated with a finite sum then a finite-term Gaussian mixture model in (6) results. Consider a scalar random variable example of (12) where f_e has a *generalized Gaussian* noise distribution [5] which has the form

$$f_e(x|\mu, \Sigma) = \frac{k}{2A(k)\Gamma(1/k)} e^{-[|x|/A(k)]^k}, \tag{15}$$

where

$$A(k) = \left[\delta^2 \frac{\Gamma(1/k)}{\Gamma(3/k)}\right]^{1/2}. \tag{16}$$

Here Γ is the gamma function $\Gamma(a) = \int_0^\infty x^{a-1}e^{-x}dx$. A generalized Gaussian pdf is determined by two constants, the variance δ^2 and a rate-of-exponential-decay parameter $k > 0$. When $k = 2$ we get the Gaussian density function, and $k = 1$ gives the double-exponential (Laplace) noise distribution. For small values of k the tails of f_k decay more slowly than Gaussian tails, thus $0 < k < 2$ determines a "heavy tailed" noise distribution and $k > 2$ determines "light tailed" noise. It has been demonstrated in [25] that a generalized Gaussian with $0 < k \leq 2$ can be represented as a SIRP, but this is not true for $k > 2$, since the conditions in (14) are not satisfied. This shows that non-heavy tailed noise is not exactly modeled by a continuous Gaussian mixture model. However, in most practical situations, we are interested only in heavy-tailed noise. The results imply that heavy tailed noise can be modeled by a finite-term Gaussian mixture model as in (6) with enough terms.

Based on the previous discussion, the pdf of a SIRV should be well approximated by the Gaussian mixture model in (6). This has been shown to be true for some specific cases. In [14], a one dimensional Gaussian mixture density is used to successfully approximate the bivariate isotropic Cauchy distribution. In [16], a multidimensional Gaussian mixture density is used to approximate SIRP noise. Simulation results show that the approximation works well in all cases studied. The same method also works well in approximating sub-Gaussian alpha stable noise [16].

We have argued that a Gaussian mixture model, as in (6), that makes use of multidimensional Gaussian pdfs can model any continuous Gaussian mixture as in (13) provided N in (6) is large enough. However, such a discrete mixture would use matrices for each term of (6) that are scalar multiples of one another from (13). This is a very special case of (6). In general, the covariance matrices of each term of (6) can be completely different. Thus in some sense, the model in (6) provides more flexibility than a discrete approximation of (13).

4 Approximating Noise Generated using Filters and Nonlinearities with a Mixture Model

Here, we are particularly interested in the model in [23], since the correlation structure in this model appeared to be one of the most complicated in this category. For simplicity, we first consider the case where h_N and h_I from (10) and (11) are finite impulse response (FIR) filters with duration $M_1 + 1$ and $M_2 + 1$ respectively. Also, suppose that we are interested in the Nth order pdf of $y(k)$ in (9).

The input sequence $e(k)$ in (10) is a stationary process. Thus the output $x_N(k)$ is still a stationary process. At time k, the output $x_N(k)$ is a weighted sum of $M_1 + 1$ Gaussian random variables, thus still a Gaussian. However, correlation is introduced between any adjacent $M_1 + 1$ noise samples.

The situation in the impulsive branch is a little more complicated, because an impulse in $u(k)$ occurs with probability ϵ at each time. Thus each sample of $u(k)$ can contain a large variance Gaussian sample, which models an impulsive noise sample, or nothing. The key to determining how to model the Nth order pdf of $x_I(k)$ with an N-dimensional mixture model is to determine the number of possible distributions which can occur due to different patterns of impulses in the sequence $u(k)$. In fact, counting the number of possible distributions is equivalent to counting the number of binary patterns of length $N + M_2$. This can be seen directly from (11). Thus for an Nth order pdf model, the number of possible distributions is 2^{N+M_2}. Under the assumption that a given pattern has occurred, we are always summing Gaussian random variables in (11) and (9). Thus for any particular pattern, $x_I(k)$ in (11) is Gaussian and so is $y(k)$ in (9). Thus a Gaussian mixture model, with a number of terms equal to the number of patterns, is appropriate. In summary, we can use an N-dimensional Gaussian mixture pdf with 2^{N+M_2}

terms to exactly represent the Nth order pdf of the output process $y(k)$. Each term in the mixture pdf describes the distribution corresponding to one possible pattern of impulses in the sequence $u(k)$. It is key to note that the $x_N(k)$ process is always added in and it always has same correlation structure, thus the number of terms in the Gaussian mixture pdf is decided only by $x_I(k)$. If N or M_2 is large, then the number of terms needed in the mixture pdf will be large also. However, in some practical cases, less terms can be used to approximate the Nth order pdf of $y(k)$, since some patterns are very unlikely to occur. This is reasonable since we assumed that impulses occur with probability ϵ, $0 < \epsilon \ll 1$.

In the above analysis, we chose FIR filters. The limited length of impulse response makes the analysis easier. For systems that arise from practical applications, we almost always find correlation that drops off with time. Thus we can approximate these systems with FIR filters.

We have generated some noise samples using the model in [23]. The EM algorithm developed in [15] was employed to estimate the parameters of Gaussian mixture pdf. The resulting Gaussian mixture pdf was compared with the histogram of the sample data. In some cases, with only a small number of terms in the mixture model, the approximation is already quite good. As an example, consider the following model.

Example 1:

$$\begin{aligned} x_N(k) &= e(k) + e(k-1) \\ x_I(k) &= u(k) + 0.5\,u(k-1) \end{aligned} \tag{17}$$

Here we used $M_1 = M_2 = 1$. We wanted to approximate the $2nd$ order pdf of $y(k)$ in (9). According to our analysis, $2^{2+1} = 8$ terms will be needed in the Gaussian mixture density to completely describe the $2nd$ order pdf of $y(k)$. We generated $200,000$ noise samples and used the EM based algorithm in [15] to estimate the parameters in the mixture model. Only 2 terms were used in the Gaussian mixture pdf for approximation. Fig. 2 gives the histogram and the contour plot of sample data. Fig.3 shows the plots of the estimated pdf. In this example, although only 2 terms is used, the approximation is already reasonably good. In our second example, we selected $M_1 = M_2 = 1$ and $h_N(k) = h_I(k)$ as in the following.

Example 2:

$$\begin{aligned} x_N(k) &= e(k) + 0.5\,e(k-1) \\ x_I(k) &= u(k) + 0.5\,u(k-1) \end{aligned} \tag{18}$$

In this case, the system is equivalent to passing the nominal and impulsive noise through one FIR filter. Fig. 4 and Fig. 5 give the histogram and estimated pdf respectively. In this example, we also used 2 terms in the Gaussian mixture model for approximation yet very good results are obtained.

In a recent paper [26], a correlated non-Gaussian process is generated by passing iid generalized Gaussian white noise through an IIR filter to generate an AR(1) process, and it was shown that the pdf of the output process can be approximated by another generalized Gaussian. Recall that in Section 3, we have already shown that a generalized Gaussian can be approximated by a Gaussian mixture pdf under certain conditions. Thus the results in [26] appear to support the idea that mixture models can be used for approximating models in the third category.

5 Conclusion

In this paper, we discuss the approximation of correlated non-Gaussian processes using Gaussian mixture models. First, a review of some general models for correlated non-Gaussian interference and noise is given. The three models discussed here are the Gaussian mixture model, the spherically invariant random process model, and a model which involves the combination of linear filters and nonlinearities. The three models are then analyzed and the Gaussian mixture model is shown to be able to approximate the other two models with high fidelity when enough terms are used in the mixture pdf.

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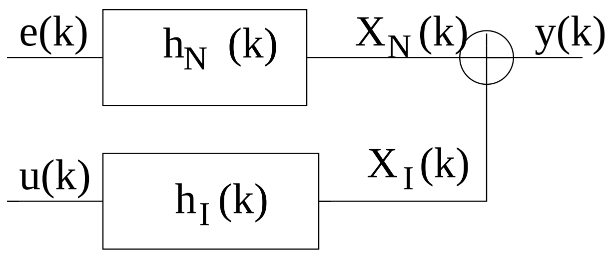
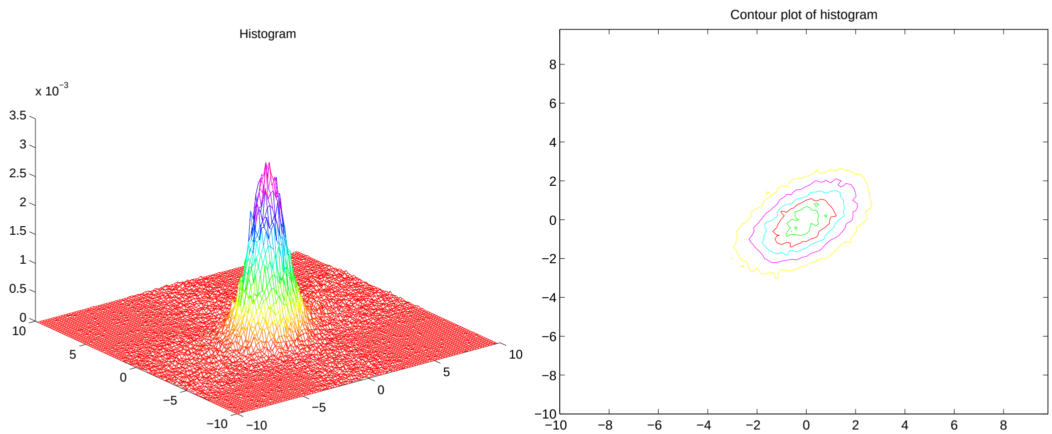
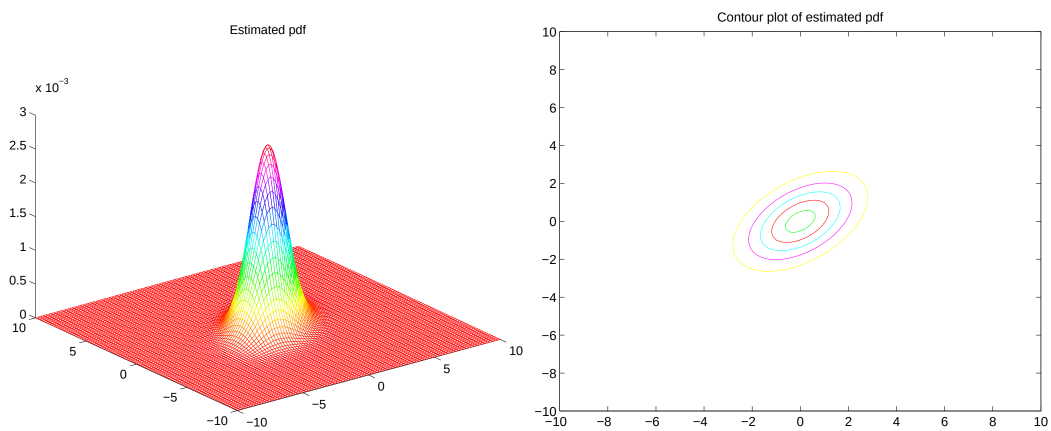


Figure 1: Correlated impulsive noise model in [23]



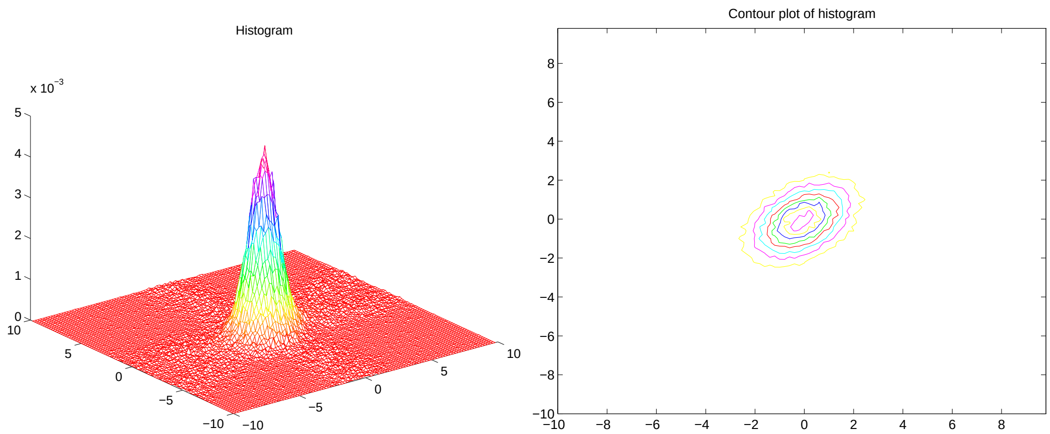
(a). Histogram of $2nd$ order pdf (b). Contour plot of histogram pdf

Figure 2: Histogram of sample data in Example 1.



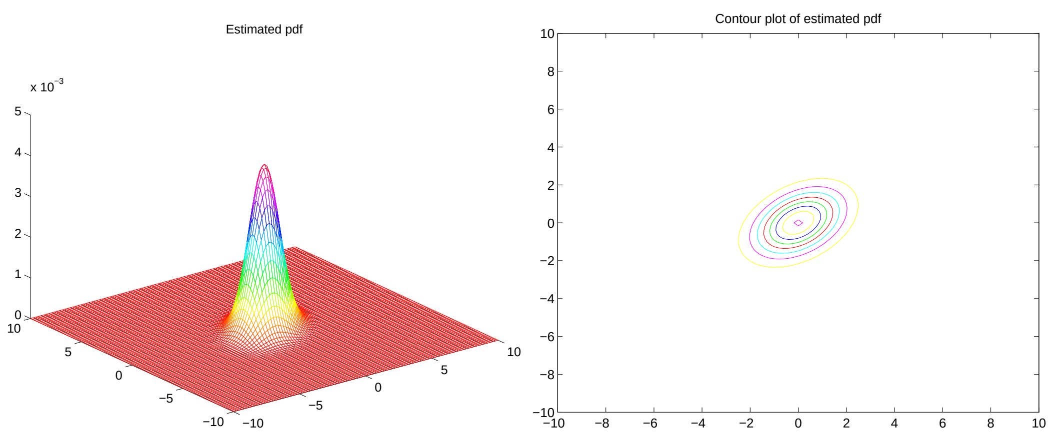
(a). Estimated $2nd$ order pdf (b). Contour plot of estimated pdf

Figure 3: Estimated pdf of sample data in Example 1.



(a). Histogram of $2nd$ order pdf (b). Contour plot of histogram pdf

Figure 4: Histogram of sample data in Example 2.



(a). Estimated $2nd$ order pdf (b). Contour plot of estimated pdf

Figure 5: Estimated pdf of sample data in Example 2.