On the Performance of Cooperative Relaying Protocols in Wireless Networks

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***Abstract*— Cooperative relaying recently emerged as a viable option for future wireless networks. By simultaneously exploiting path loss savings known from relaying scenarios and the diversity inherent to any scheme involving spatially separated transmitters, this technique is able to leverage gains from both relaying and spatial diversity techniques. In this paper, we study different cooperative relaying protocols and compare their performance with that of direct transmission and conventional relaying. We investigate under which conditions the developed techniques provide gains over other approaches. Our results confirm that cooperative relaying is an effective means of enhancing the performance of wireless systems whenever temporal and frequency diversity is scarce.**

I. Introduction

The rising demand for high data rate services in current and future wireless networks calls for advanced strategies at various layers. A frequently considered concept is to allow intermediate nodes to assist in the transmission of information from a source to a destination node. A good overview of the state-of-the-art of such *relaying* schemes is found in [13].

As an extension of this approach, *cooperative relaying*[1] exploits the inherent spatial diversity of the relay channel by allowing mobile terminals to co-operate. More generally, taking the original signal copy sent by the source node into account, these systems exploit useful side information that conventional relaying systems unnecessarily regard as interference. In this paper, we generally focus on the "two-hop" case – the transmission from source to destination is assisted by a single relay. More complex schemes can of course be envisaged and have been studied for example in [2], [3], [12]. However, this simplest case of (cooperative) relaying already yields substantial insights into the fundamental challenges and trade-offs faced by such techniques.

The remainder of this document is structured as follows: Section II reviews the concept of cooperative relaying and discusses the main benefits and challenges. We continue by describing the investigated protocols in Section III before assessing their performance in terms of outage probabilities and SNR gain over direct transmission in Section IV, for narrow-band quasi-static environments where space is the only source of diversity. After discussing implementation issues in Section V, we draw conclusions in Section VI.

[1]also known as user cooperation diversity [14], cooperative diversity [11], virtual antenna arrays [4], coded cooperation [8] or distributed turbo codes [15]

II. Fundamentals

A. Introduction to Cooperative Relaying

We consider scenarios as depicted in Figure 1, which include a single relay and where all nodes feature only a single antenna. As is true for all relaying protocols, cooperative relaying schemes suffer from the "orthogonality constraint"[2], calling for the assignment of orthogonal resources for reception and transmission at the relay. Without loss of generality, we focus on the time division case and divide the available channel into two orthogonal subchannels in the time domain. Note that in order to achieve the same end-to-end spectral efficiency, we then need to double the spectral efficiency on each of the individual links in this case.

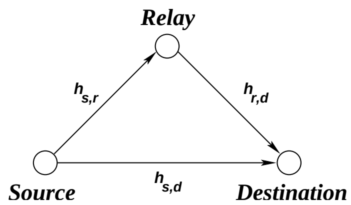

Fig. 1. Illustration of a cooperative relaying scenario. The source sends a broadcast message to destination and relay. The relay then forwards additional information about the source message to the destination, which appropriately combines the received data.

Cooperative relaying protocols can be classified according to their *forwarding strategy* as:

1) *Amplify-and-Forward*: The relay acts as an analog repeater, resulting in noise enhancement in the relay path.
2) *Decode-and-Forward*: The relay fully decodes, again encodes and retransmits the received message, possibly propagating decoding errors that may lead to a wrong decision at the destination.
3) *Decode-and-Reencode*: The relay fully decodes the received message, but constructs a codeword differing from the source codeword – thus enabling parallel channel coding. This approach can be seen as an distributed incremental redundancy technique. The main problem is again error propagation.

Protocols that require the relay to decode the received message are clearly favorable for implementation purposes since amplify-and-forward style protocols require either the storage of large amounts of analog data (time division) or

[2]denoting the inability of current RF implementations to simultaneously receive and transmit at the same frequency

complicated and expensive transceiver structures (frequency division). The reader is referred to [16] for a detailed discussion of advantages and disadvantages.

We may further categorize the transmission schemes based on the protocol nature:

1) *Fixed Protocols* where the relay *always* forwards (a processed version of) its received message,
2) *Adaptive Protocols* where the relay uses a threshold rule to decide autonomously whether to forward or not, and
3) *Feedback Protocols* where the relay only assists the transmission when explicitly required by the destination.

A combination of these options yields 9 different cooperative relaying protocols. Static Amplify-and-Forward networks have been extensively discussed in [6], a number of Amplify- and Decode-and-Forward protocols have been thoroughly investigated in [11] and Decode-and-Reencode protocols have been the subject of evaluation in [9], [15].

In this paper, we will focus our attention on decoding relays and non-fixed protocols, as these constitute the most promising option for future wireless networks.

B. Benefits

For statistically independent channels between all nodes in a cooperative relaying system as depicted in figure 1, it has been shown in [11] that full 2^{nd} order diversity ($\Delta = 2$) can be achieved asymptotically. The term diversity order relates to the slope of the outage probability curve when plotted against the SNR, that is:

$$p^{out} \propto \frac{1}{\mathrm{SNR}^{\Delta}}, \qquad \mathrm{SNR} \gg 1. \tag{1}$$

While this result implies that for sufficiently high SNR (and hence, low enough outage probability), cooperative relaying will always outperform direct transmission, the applicability of such systems depends on the question whether it achieves gains over direct transmission in the *practically relevant* regime of interest – where desired outage probabilities (block error rates) assuming the existence of (Hybrid) ARQ or other retransmission techniques usually range from 0.1% to 10%. It has to be stressed that the spatial diversity gain constitutes the main advantage of *cooperative* over *conventional* relaying protocols.

Further gains can be achieved in *asymmetric*[3] network constellations. Throughout this paper, we let the relay scale its transmit power in such a way that it achieves the same *average* received SNR at the destination as the source, in order to maximize diversity gains[4]. As a result, it needs lower transmit power than the source whenever its distance to the destination is inferior to the source-destination distance[5]. We can thus minimize the energy needed for retransmission, i.e. the “cost” for conveying additional information about the source message to the destination. The relay evidently needs some sort of power control to be able to perform the appropriate transmit power scaling. However, since only long term statistics (the average path loss) need to be available, this results only in a negligible increase in network complexity.

Due to the broadcast nature of the wireless medium, the relay will likewise experience better receive conditions than the destination whenever its average path loss to the source is lower than that of the destination. Note that these two gains may be exploited simultaneously by choosing a relay node with appropriate location. While not affecting the diversity order of the system, the SNR gains obtained in such asymmetric scenarios are essential to make cooperative relaying attractive in the low SNR regime (i.e., for the desired outage probabilities).

C. Challenges

In order to ensure fair performance comparison of all investigated transmission schemes, we need to normalize the required resources, i.e. energy, time, and bandwidth. Note that time and bandwidth are inherently related by the spectral efficiency of the system. That is, by achieving the same spectral efficiency, we ensure that the same time and bandwidth constraints can be met, regardless of the specific partitioning of these resources in the considered system model. In the following, we assume our system to be allocated a certain time slot of duration T, system bandwidth B and transmit power constraint P. Under our time division system model, all investigated protocols occupy the whole system bandwidth B.

As stated in the introduction, the main drawback of any relaying scenario is the orthogonality constraint. Note that in contrast to conventional relaying, this constraint has in fact a beneficial side effect for cooperative schemes: for the studied two-hop schemes, it enables the orthogonal reception of the information transmitted from source and relay at the destination. Otherwise, the destination receiver would be required to employ sophisticated successive interference cancellation or even multiuser detection techniques in order to separate the signal streams.

However, this constraint imposes the requirement to increase the spectral efficiency on the individual links by a factor of two. It is found that for the considered target outage probabilities, this drawback will outweigh any gain from diversity and/or path loss savings when we aim for high spectral efficiencies and restrict ourselves to repetition-coding bases protocols. Remember that this drawback is inherent to any relaying scenario, i.e., cooperative still outperforms conventional relaying due to its exploiting the spatial diversity of the channel.

D. Channel Model

We model all channels as Rayleigh flat fading with additive white Gaussian noise. The channel remains constant during the time required to transmit one block of data from node i to

[3]Throughout this paper, we will refer to a network where the links between source, relay(s) and destination experience equal *average* path loss as a “symmetric network”. The term “asymmetric network” is used for all other possible network constellations.

[4]One may also use the large SNR approximation for the outage probability (cf. Section IV) to derive an optimum power allocation between source and relay, as done in [7].

[5]We assume a log-distance path loss model, hence the expressions average path loss and distance are dual of each other and we will use them exchangeably.

node j (quasi-static or block fading). Channel coefficients $h_{i,j}$ (cf. Figure 1) are modelled as zero-mean, circularly symmetric complex Gaussian random variables, such that $|h_{i,j}|^2$ are exponentially distributed with mean $\sigma^2_{i,j}$. Phases $\angle h_{i,j}$ are uniformly distributed over $[0, 2\pi)$. Noises (after sampling at the receiver) are modelled as zero-mean mutually independent, circularly-symmetric, complex Gaussian random sequences with variance N_0.

For the path loss, we assume a log-distance model, i.e., the received power decreases linearly with distance, on a logarithmic scale. An interesting approach taken in [11] suggests to model the effects of path loss into the variance of the fading variables by observing that the SNR at a specific node j obtained by transmission from node i can be written as:

$$\begin{aligned} \mathrm{SNR}_{i,j} &= \left[\mathrm{SNR}\left(\frac{d_0}{d_{i,j}}\right)^{\alpha}\right]|h_0|^2 \\ &= \mathrm{SNR}\left[\left(\frac{d_0}{d_{i,j}}\right)^{\alpha}|h_0|^2\right] \\ &= \mathrm{SNR}|h_{i,j}|^2 \qquad (2) \end{aligned}$$

where $|h_0|$ is a fading coefficient with unit variance $\sigma_0^2 = 1$, $d_{i,j}$ is the distance between transmitter and receiver, α is the path loss exponent, and SNR is the signal-to-noise ratio attained by the transmitter at a receiver at reference distance d_0. Without loss of generality, we define $d_0 = d_{s,d}$ and therefore $\sigma^2_{s,d} = 1$. Throughout this paper, we can now model the effects of path loss in the following way:

$$\sigma^2_{i,j} = \left(\frac{d_{s,d}}{d_{i,j}}\right)^{\alpha} \equiv r_{i,j}^{-\alpha}. \qquad (3)$$

where we have introduced the variable $r_{i,j} \equiv d_{i,j}/d_{s,d}$ as the normalized distance between two nodes. This approach allows for a convenient study of the effects of geometry on the performance of the proposed protocol.

III. The Protocols

A. Reference cases

We use direct transmission (D) and transmit diversity (T) as reference cases for our performance evaluation. Both schemes can use the full time slot T for transmission. For transmit diversity, the source distributes its transmit power equally over the two antennas. This can be seen as an application of the well known Alamouti scheme [1].

B. Conventional Relaying (L3DF)

Transmission takes place in two phases of duration $T/2$: during the first time slot the source transmits its information to the relay, which fully decodes and again encodes the message prior to retransmission in the second time slot. This is a classical store-and-forward operation, also known as layer 3 decode-and-forward (L3DF) which can only extend range or save transmit power but achieves no diversity gains.

C. Adaptive Decode-and-Forward Schemes

The aim of adaptive cooperative relaying protocols is to prevent error propagation by letting the relay forward the received message only when it has been correctly decoded, i.e., when source and relay have the same information available. From an information theoretic point of view, this is the case when the source-relay link supports the desired rate, that is, no outage occurs on the source-relay link (cf. the outage analysis in the following section). In practice, however, the source message must be encoded in such a way that the relay is able to verify the integrity of the decoded message[6].

1) Complex Adaptive Decode-and-Forward (CAdDF): This protocol has been introduced in [11]. Again, we use two time slots of duration $T/2$. In time slot 1 the source transmits its message in a broadcast manner to relay and destination. In case the relay was able to correctly recover the source message, it uses the second time slot to transmit an additional signal copy to the destination. Otherwise, the source retransmits its signal. Note that the source must hence be aware of the decoding status of the relay. This can for example be achieved via a simple feedback mechanism.

2) Simple Adaptive Decode-and-Forward (SAdDF): Identifying the need for feedback in the CAdDF protocol and dwelling on the notion that repetition coding is not very effective over a static channel, it was proposed in [5] to refrain from letting the source repeat its message if the relay was not able to decode. This Simple AdDF (SAdDF) protocol can be seen as an easily implementable extension to conventional relaying. Whenever the relay decodes, it forwards an additional copy of the source message over an essentially uncorrelated channel, otherwise it remains silent and the destination has to rely solely on the source message for decoding. The destination may detect the latter case by the absence of sufficient signal strength.

3) Detached Adaptive Decode-and-Forward (DAdDF): A more complex protocol, aiming to avoid any silence phase, has been introduced in [17]. The idea is to achieve a slightly higher end-to-end throughput and save transmit power by omitting the second phase whenever the relay was not able to recover the source's message. This protocol requires the relay to send a broadcast feedback message to inform source and destination of its decoding failure. For further detail, the reader is referred to [17].

D. Adaptive Decode-and-Reencode Schemes

Adaptive decode-and-forward protocol versions offer a good trade-off between implementation simplicity and performance. However, their performance remains limited due to their repetition coding nature. In fact, it has been shown in [11] that repetition coding is the most significant drawback of cooperative relaying protocols – the orthogonality constraint affects performance to a lower extent, which is especially relevant for the high spectral efficiency regime.

[6]This can be achieved either via a cyclic redundancy check (CRC) being part of the source message, or by standard error detection (and correction) codes such as block codes. LDPC, which for large enough block lengths make only few undetected errors, might be an interesting option for practical wireless systems.

This result is intuitively clear considering the inefficiency of repetition coding, as the resulting benefits come only in an accumulation of signal-to-noise ratios. The more desirable case, however, would be the accumulation of mutual information. This can be done by using for example rate compatible punctured codes (RCPC). Under our (somewhat restricted) assumptions of two transmission phases of equal length, one solution would be to let the source encode its message with a rate R_c code and then use appropriate puncturing to obtain an effective code rate of $2R_c$ that is used for the broadcast transmission to relay and destination. The relay will decode the received message and check its integrity via an appropriate error detection technique. In case it has correctly decoded, it re-encodes the message again with code rate R_c and now punctures exactly those bits that formed the message it received. It obtains a codeword that is completely different from the codeword transmitted during the first phase. The destination can then assemble the two codewords and has received a message with overall code rate R_c. A more flexible approach towards such "cooperative coding" protocols has been proposed in [9], [15].

For purposes of exhibition, we limit ourselves to extending the simple and complex adaptive decode-and-forward protocols to use this decode-and-reencode approach. We denote the resulting protocols Complex Adaptive Decode-and-Reencode (CAdDR) and Simple Adaptive Decode-and-Reencode (SAdDR) protocols, respectively.

E. Distributed Hybrid ARQ (DHARQ)

Using incremental instead of repetition coding, we have already overcome one major drawback of cooperative relaying. Further gain can be expected if we try to completely avoid the necessity for two phase transmission. This can be done by allowing for feedback from the destination: additional information is provided by the relay *only upon explicit request*.

A protocol that minimizes the number of retransmissions can be defined as follows: the source transmits half of its message during the first time slot of duration $T/2$ in a broadcast manner to relay and destination. Both stations try to decode this message and send feedback on their decoding status in a broadcast manner to the other nodes. Consequently, at the end of the first time slot, all nodes have the necessary information to act appropriately during the (optional) second time slot. In case the destination was not able to correctly decode the source message *and* the relay was able to decode the source message, the latter will send additional redundancy in the second time slot. The destination then assembles the complete codeword and retries decoding. In case of decoding failure, a block error is declared. The same occurs when destination and relay simultaneously fail to decode.

IV. Performance in the Slow Fading Regime

We investigate the slow fading case where diversity is available only in the spatial domain. Note that this is the most optimistic case for cooperative relaying, since the full benefits of spatial diversity can be leveraged and the relative merits of distributed spatial diversity decrease as other forms of diversity become available on the individual links (e.g. frequency diversity). A commonly used information theoretic performance measure is the outage probability versus SNR. We define an outage as the event that the maximum average mutual information I between source and destination is inferior to the spectral efficiency R desired for transmission. The outage probability is simply the probability that an outage event occurs: $p^{out} \equiv \Pr[I < R]$.

A. Reference cases

The mutual information for direct transmission is obviously:

$$I_D = \mathrm{ld}\big(1 + |h_{s,d}|^2 \mathrm{SNR}\big) \tag{4}$$

from which the outage probability is easily derived knowing that $|h_{i,j}|^2$ are exponentially distributed with parameter $\sigma_{i,j}^{-2}$:

$$p_D^{out} = 1 - \exp\left(-\frac{2^R - 1}{\sigma_{s,d}^2 \mathrm{SNR}}\right). \tag{5}$$

In the large SNR regime, p_D^{out} is well approximated by

$$\begin{aligned} p_D^{out} &\approx \underbrace{\frac{1}{\sigma_{s,d}^2}}_{g_D} \frac{2^R - 1}{\mathrm{SNR}}, \qquad \mathrm{SNR} \gg 1 \\ &\equiv g_D \cdot e_D \cdot \frac{1}{\mathrm{SNR}}, \end{aligned} \tag{6}$$

where $g_D = 1/\sigma_{s,d}^2 = 1$ is a geometry factor and e_D a spectral efficiency factor. From the definition of Δ in (1), we see that the protocol achieves only first order diversity.

Assuming that the two transmitter antennas face statistically similar channels to the destination ($\sigma_{s1,d}^2 = \sigma_{s2,d}^2 = \sigma_{s,d}^2$), the outage probability for transmit diversity is given by:

$$p_T^{out} = 1 - \left(1 + \frac{2(2^R - 1)}{\sigma_{s,d}^2 \mathrm{SNR}}\right) \exp\left(-\frac{2(2^R - 1)}{\sigma_{s,d}^2 \mathrm{SNR}}\right) \tag{7}$$

which in the limit for large SNR transforms to:

$$\begin{aligned} p_T^{out} &\approx \underbrace{\frac{2}{\sigma_{s,d}^4}}_{g_T} \left(\frac{2^R - 1}{\mathrm{SNR}}\right)^2, \qquad \mathrm{SNR} \gg 1 \\ &\equiv g_T \cdot e_T \cdot \frac{1}{\mathrm{SNR}^2} \end{aligned} \tag{8}$$

B. Conventional Relaying (L3DF)

The mutual information between source and destination in such a scenario is limited by the weaker of the two involved single links:

$$I_{L3DF} = \frac{1}{2} \min\{\mathrm{ld}(1 + |h_{s,r}|^2 \mathrm{SNR}_S), \mathrm{ld}(1 + |h_{r,d}|^2 \mathrm{SNR}_R)\} \tag{9}$$

where $\mathrm{SNR}_S = 2p\mathrm{SNR}$ is the SNR resulting from the power adaptation at the source and $\mathrm{SNR}_R = 2(1-p)\mathrm{SNR}$ is the SNR resulting from power scaling at the relay. The factor p, $0 < p < 1$ allows for shifting power between the first and the second transmission phase.

The outage probability can then be shown to be:

$$\begin{aligned} p_{L3DF}^{out} &= \Pr\big((I_{s,r} < R) \vee (I_{r,d} < R)\big) \\ &= 1 - \prod_{i=1,2} \big(1 - \Pr(I_i < R)\big) \\ &= 1 - \exp\left(- \left(\frac{2^{2R}-1}{\sigma_{s,r}^2 SNR_S} + \frac{2^{2R}-1}{\sigma_{r,d}^2 SNR_R} \right) \right) \\ &\approx \underbrace{\left(\frac{1}{2p\sigma_{s,r}^2} + \frac{1}{2(1-p)\sigma_{r,d}^2} \right)}_{g_{L3DF}} \frac{2^{2R}-1}{\text{SNR}} \\ &\equiv g_{L3DF} \cdot e_{L3DF} \cdot \frac{1}{\text{SNR}} \end{aligned} \tag{10}$$

As expected, the protocol only achieves first order diversity. Deriving the second term in (10) for p allows for optimizing the power fraction p to achieve optimum performance. After some manipulations, we obtain for the optimum power allocation:

$$p = \frac{1}{1 + \frac{\sigma_{s,r}}{\sigma_{r,d}}}. \tag{11}$$

C. Adaptive Decode-and-Forward Schemes

Since they follow a similar approach, we will first derive a common framework for *all* considered AdDF protocols and then parameterize this to obtain the desired results.

In order to perform correct energy normalization, we need to determine the *average* energy used by the different protocols for transmission of a single source message:

$$E_{AdDF} = \frac{T}{2} \big(\Pr(D)(P_S + P_R) + \Pr(\bar{D}) k P_S \big), \tag{12}$$

where P_S and P_R are the transmission powers of source and relay, respectively. The event that the relay successfully decodes the source message is denoted by D. Similarly we define $\bar{D}$ as the event that the relay fails to decode. The factor k allows for modelling whether the source may repeat its message or not, i.e.,

$$k = \begin{cases} 1, & \text{for SAdDF and DAdDF} \\ 2, & \text{for CAdDF} \end{cases} \tag{13}$$

Since it must be the aim of our protocol to achieve diversity gains and hence minimize the occurrence of the latter event, we assume that $\Pr(D) \approx 1$. Our results confirm that this assumption is usually fulfilled in the regime of interest, at outage probabilities around 10^{-2}. We can now write:

$$E_{AdDF} \approx \frac{T}{2}\big(P_S + P_R\big) = \frac{T}{2} P_S \left(1 + \left(\frac{d_{r,d}}{d_{s,d}} \right)^{\alpha} \right). \tag{14}$$

where the transmission power of the relay is scaled such that it attains the same SNR at the destination receiver as the source, in order to maximize diversity gains.

The energy used for direct transmission is apparently $E_D = P_S T$. To allow for fair performance comparison, we need to scale the original SNR for direct transmission by E_D/E_{AdDF} to obtain the equivalent $SNR^{'}$ used for cooperative transmission:

$$\begin{aligned} \frac{E_{AdDF}}{E_D} &= \frac{\frac{T}{2} P_S (1 + r_{r,d}^{\alpha})}{P_S T} = \frac{1}{2}(1 + r_{r,d}^{\alpha}) \qquad (15) \\ \text{SNR}^{'} &= \frac{2\ \text{SNR}}{1 + r_{r,d}^{\alpha}}. \qquad (16) \end{aligned}$$

The maximum average mutual information of the adaptive decode-and-forward protocols can now be written as:

$$I_{AdDF} = \begin{cases} \frac{1}{2}\text{ld}\big(1 + \text{SNR}^{'}|h_{s,d}|^2 + \text{SNR}^{'} r_{r,d}^{\alpha} |h_{r,d}|^2\big); & D \\ \frac{1}{2}\text{ld}(1 + k\text{SNR}^{'}|h_{s,d}|^2); & \bar{D} \end{cases} \tag{17}$$

where k and D are defined as above. Note that the mutual information for Simple and Detached AdDF are equal, as the DAdDF protocol has the possibility to skip the second phase. However, the protocols differ in the realized end-to-end throughput.

Consider now the term $r_{r,d}^{\alpha}|h_{r,d}|^2$ in (17), which is the concatenation of the fading coefficient between relay and destination and the power scaling by the relay. As we have already outlined, our structure allows for modelling the power scaling (as well as the path loss) into the fading coefficient:

$$r_{r,d}^{\alpha}|h_{r,d}|^2 = \left(\frac{d_{r,d}}{d_{s,d}} \right)^{\alpha} |h_{r,d}|^2 = \frac{|h_{r,d}|^2}{\sigma_{r,d}^2} = |h_{r,d}^{'}|^2$$

where $|h_{r,d}^{'}|^2$ is the *effective* fading coefficient between relay and destination, *including* the power adjustment of the relay and hence having a variance ${\sigma'}_{r,d}^2 = \sigma_{r,d}^2/\sigma_{r,d}^2 = 1 = \sigma_{s,d}^2$. Note that power scaling and path loss are concatenated in such a way that the resulting effective fading coefficient is statistically independent, but identically distributed to the source-destination channel. This result very nicely illustrates the fact that the relay's power scaling ensures that it attains the same *average* SNR at the destination as the source.

We can now use this knowledge to reformulate (17) appropriately:

$$I_{AdDF} = \begin{cases} \frac{1}{2}\text{ld}\big(1 + \text{SNR}^{'}(|h_{s,d}|^2 + |h'_{r,d}|^2)\big); & D \\ \frac{1}{2}\text{ld}(1 + k\text{SNR}^{'}|h_{s,d}|^2); & \bar{D} \end{cases} \tag{18}$$

The decoding event D at the relay is obviously defined by $\frac{1}{2}\text{ld}(1 + \text{SNR}^{'}|h_{s,r}|^2) > R$, which translates into $|h_{s,r}|^2 > t(\text{SNR}^{'})$ with t defined by:

$$t = \frac{2^{2R}-1}{\text{SNR}^{'}}. \tag{19}$$

Now the outage probability can be formulated easily:

$$\begin{aligned} p_{AdDF}^{out} &= \Pr(|h_{s,r}|^2 \geq t)\ \Pr(|h_{s,d}|^2 + |h'_{r,d}|^2 < t) \\ &\quad + \Pr(|h_{s,r}|^2 < t)\ \Pr(k|h_{s,d}|^2 < t) \end{aligned}$$

Using the approach from [11], we evaluate p_{DADF}^{out}:

$$\begin{aligned}\frac{p_{AdDF}^{out}}{t^2(\mathrm{SNR}^{'})} &= \Pr(|h_{s,r}|^2 \geq t) \\ &\times \frac{\Pr(|h_{s,d}|^2 + |h_{r,d}^{'}|^2 < t)}{t^2(\mathrm{SNR}^{'})} \\ &+ \frac{\Pr(|h_{s,r}|^2 < t)}{t(\mathrm{SNR}^{'})} \frac{\Pr(k|h_{s,d}|^2 < t)}{t(\mathrm{SNR}^{'})}.\end{aligned}$$

$$\begin{aligned}\lim_{\mathrm{SNR}^{'}\to\infty} \frac{p_{AdDF}^{out}}{t^2(\mathrm{SNR}^{'})} &= 1 \cdot \frac{1}{2\sigma_{s,d}^2\sigma_{s,d}^2} + \frac{1}{\sigma_{s,r}^2}\frac{1}{k\sigma_{s,d}^2} \\ \lim_{\mathrm{SNR}^{'}\to\infty} p_{AdDF}^{out} &= \frac{2\sigma_{s,d}^2 + k\sigma_{s,r}^2}{2k\sigma_{s,d}^4\sigma_{s,r}^2}\left(\frac{2^{2R}-1}{\mathrm{SNR}^{'}}\right)^2\end{aligned}$$

Full second order diversity is achieved ($p^{out} \propto \mathrm{SNR}^{-2}$) for all adaptive decode-and-forward protocols. The outage probability in the high SNR regime can hence be approximated by:

$$p_{AdDF}^{out} = \frac{2\sigma_{s,d}^2 + k\sigma_{s,r}^2}{2k\sigma_{s,d}^4\sigma_{s,r}^2}\left(\frac{2^{2R}-1}{\mathrm{SNR}^{'}}\right)^2, \quad \mathrm{SNR}^{'} \gg 1 \tag{20}$$

which, using the definitions of $\sigma_{s,d}^2$, $\sigma_{s,r}^2$ and $\mathrm{SNR}^{'}$ leads to:

$$\begin{aligned}p_{AdDF}^{out} &= \underbrace{\frac{2r_{s,r}^{\alpha} + k}{2k}\left(\frac{r_{r,d}^{\alpha}+1}{2}\right)^2}_{g_{AdDF}}\left(\frac{2^{2R}-1}{\mathrm{SNR}}\right)^2 \\ &\equiv g_{AdDF} \cdot e_{AdDF} \cdot \frac{1}{\mathrm{SNR}^2}.\end{aligned} \tag{21}$$

Clearly, an optimum for g_{AdDF} can be found independently of R and SNR. Unfortunately, deriving a closed form expression for the optimal position of the relay becomes too involved. However, the relay should obviously be located on the line between source and destination. By numerical evaluation, it can be found that for example $r_{s,r} = 1 - r_{r,d} = 0.5$ is a good approximation for $\alpha = 3$ for the Simple AdDF protocol (Figure 3 confirms this notion).

D. Adaptive Decode-and-Reencode Schemes

Having formulated the energy normalization and outage probabilities for adaptive decode-and-forward protocols, the extension to their reencoding counterparts is quite easy. Instead of accumulating SNRs, we now accumulate mutual information by performing parallel channel coding:

$$I_{AdDR} = \begin{cases} \frac{1}{2}\sum \mathrm{ld}(1 + \mathrm{SNR}^{'}|h_i|^2); & D \\ \frac{k}{2}\mathrm{ld}(1 + \mathrm{SNR}^{'}|h_{s,d}|^2); & \bar{D} \end{cases} \tag{22}$$

where k and D are defined as before. The sum of the mutual information in the upper term is over the mutual information of the source-destination and relay-destination link, respectively. Using the framework developed in [10] now allows us to derive an asymptotic expression for the outage probability:

$$\begin{aligned}p^{out}\mathrm{SNR}'^2 &= \Pr\left(\mathrm{ld}(1 + \mathrm{SNR}^{'}|h_{s,r}|^2) \geq R\right) \\ &\times \mathrm{SNR}'^2 \Pr\left(\sum \mathrm{ld}(1 + \mathrm{SNR}^{'}|h_i|^2) < R\right) \\ &+ \mathrm{SNR}^{'} \Pr\left(\mathrm{ld}(1 + \mathrm{SNR}^{'}|h_{s,r}|^2) < R\right) \\ &\times \mathrm{SNR}^{'} \Pr\left(\frac{k}{2}\mathrm{ld}(1 + \mathrm{SNR}^{'}|h_{s,d}|^2) < R\right)\end{aligned}$$

$$\begin{aligned}\lim_{\mathrm{SNR}^{'}\to\infty} p_{AdDR}^{out}\mathrm{SNR}'^2 &= 1 \cdot \frac{2^{2R}(\ln(2)2R - 1) + 1}{\sigma_{s,d}^2\sigma_{s,d}^2} \\ &+ \frac{2^2R - 1}{\sigma_{s,r}^2}\frac{2^{2R/k} - 1}{\sigma_{s,d}^2}.\end{aligned} \tag{23}$$

For $\mathrm{SNR}^{'} \gg 1$ the outage probability is hence:

$$p_{AdDR}^{out} \approx \frac{2^{2R}(\ln(2)2R - 1) + 1}{\sigma_{s,d}^4\mathrm{SNR}'^2} + \frac{(2^{2R} - 1)(2^{2R/k} - 1)}{\sigma_{s,r}^2\sigma_{s,d}^2\mathrm{SNR}'^2}. \tag{24}$$

Again, using the definitions of $\sigma_{i,j}$ and SNR' we can see that

$$\begin{aligned}p_{AdDR}^{out} &\approx \left(\frac{r_{r,d}^{\alpha}+1}{2}\right)^2\Bigg(2^{2R}(\ln(2)2R - 1) + 1 \\ &\quad +(2^{2R} - 1)(2^{2R/k} - 1)r_{s,r}^{\alpha}\Bigg)\frac{1}{\mathrm{SNR}^2} \\ &\equiv g_{AdDR} \cdot e_{AdDR}(r_{s,r}) \cdot \frac{1}{\mathrm{SNR}^2}.\end{aligned} \tag{25}$$

Contrary to the previously studied protocols, the effects of geometry and spectral efficiency are no longer separable.

E. Distributed Hybrid ARQ (DHARQ)

It is easily seen that the Distributed HARQ outage event is equivalent to the Simple AdDR ($k = 1$) outage event: an outage occurs whenever relay and destination simultaneously fail to decode or when parallel channel coding from source and relay does not provide sufficient mutual information to achieve the desired rate R. The difference lies in the achieved spectral efficiency: while the AdDR protocols need to double their link rate in order to achieve a rate R in the high SNR regime, the DHARQ protocol can operate at rate R since the destination will decode successfully in a high number of cases, in the regime of interest. The outage probability can hence be approximated by:

$$\begin{aligned}p_{DHARQ}^{out} &\approx \frac{2^R(\ln(2)R - 1) + 1}{\sigma_{s,d}^4\mathrm{SNR}^2} + \frac{(2^R - 1)^2}{\sigma_{s,r}^2\sigma_{s,d}^2\mathrm{SNR}^2} \\ &\equiv g_{DHARQ} \cdot e_{DHARQ}(r_{s,r}) \cdot \frac{1}{\mathrm{SNR}^2}.\end{aligned} \tag{26}$$

Since in the regime of interest, the relay does not frequently transmit, we let the source transmit with power P over the full time slot T. To now incorporate the effects of slightly decreased end-to-end throughput and transmission power required by the relay, we define A as the event that the destination fails to decode and the relay is able to decode, which corresponds to the relay sending additional redundancy to the destination. The probability of this event is obviously:

$$\Pr(A) = \Pr\left(|h_{s,d}|^2 < \frac{2^R - 1}{\text{SNR}}\right) \Pr\left(|h_{s,r}|^2 > \frac{2^R - 1}{\text{SNR}}\right). \quad (27)$$

The achieved spectral efficiency R' is hence:

$$R' \approx \frac{R}{1 + \Pr(A)}, \qquad \text{SNR} \gg 1, \quad (28)$$

and the SNR required to achieve the original rate R is then approximated by:

$$\text{SNR}'' \approx \text{SNR} \frac{2^R + 1}{2^{R'} + 1} \frac{1 + \Pr(A)}{1 + \Pr(A) r_{r,d}^{\alpha}}, \qquad \text{SNR} \gg 1, \quad (29)$$

where the first factor accounts for normalization of the rate and the second for normalization of transmit powers.

F. SNR Gain over Direct Transmission

What is even more important than the absolute performance of a protocol is its performance with respect to direct transmission. In order to obtain a meaningful measure for this expression, we will define the *SNR gain over direct transmission* as the quotient of the SNRs required by direct and cooperative transmission in order to attain the same outage probability.

Having formulated all asymptotic outage probabilities for a certain protocol type θ in the form

$$p_\theta^{out} \approx g_\theta e_\theta \frac{1}{\text{SNR}_\theta^{\Delta(\theta)}}, \qquad \text{SNR}_P \gg 1 \quad (30)$$

now allows for very convenient study of the SNR gain by solving the above equation for the SNR required to achieve a certain outage probability p^{out}:

$$\text{SNR}_\theta = \left(\frac{g_\theta e_\theta}{p^{out}}\right)^{\frac{1}{\Delta(\theta)}}. \quad (31)$$

The SNR gain over direct transmission readily follows:

$$\begin{aligned} \text{G} &\equiv \frac{\text{SNR}_D}{\text{SNR}_\theta} = \frac{g_D e_D}{p^{out}} \left(\frac{p^{out}}{g_\theta e_\theta}\right)^{\frac{1}{\Delta(\theta)}} \\ &= \frac{2^R - 1}{\left(g_\theta e_\theta\right)^{\frac{1}{\Delta(\theta)}}} \left(p^{out}\right)^{\frac{1}{\Delta(\theta)} - 1} \end{aligned} \quad (32)$$

Evaluating the above expression for the adaptive decode-and-forward protocols yields:

$$\text{G}_{AdDF} \equiv \sqrt{\frac{1}{p^{out} \cdot g_{AdDF}} \frac{2^R - 1}{2^{2R} - 1}}. \quad (33)$$

Similar expressions can be easily obtained for all other investigated protocols. To obtain a more expressive result for the Simple AdDF protocol, (33) can be expressed in a logarithmic manner:

$$\begin{aligned} \text{G}_{SAdDF}[\text{dB}] &\approx 5\Big(-\log_{10} p^{out}\Big) + \Big(4.5 - 2^{3-\alpha}\Big) \\ &\quad -\Big(3R + 2^{2-R}\Big) \end{aligned} \quad (34)$$

using appropriate approximations as well as the definition of g_{AdDF} and $r_{s,r} \approx 0.5$. The maximum gain due to geometry is obviously 4.5 dB while we loose 3 dB whenever we increase the spectral efficiency by 1 bit/channel use, for high values of R. On the other hand, the SNR gain increases linearly with the required magnitude of the outage probability ($-\log_{10} p^{out}$). This is intuitively clear since direct transmission achieves only first order diversity while cooperative transmission achieves second order diversity (cf. Figure 2). Using equation (34), we can easily calculate the *maximum* SNR gain of cooperative over direct transmission. If we wish to achieve an outage probability of 10^{-2} at spectral efficiency 2 bit/channel use in an suburban environment with $\alpha = 3$, the maximum SNR gain, achieved when the relay is located halfway between source and destination, will be roughly 7 dB. Figures 2 and 3 confirm this result.

G. Results

Figure 2 shows the outage probability of the different investigated protocols for a spectral efficiency of $R = 2$ bit/channel use for a path loss exponent $\alpha = 3$. The relay is located halfway between source and destination ($r_{s,r} = 0.5$). Plots are created from closed form expressions, where possible. Monte-Carlo simulations obtained comparable results (not shown). For the protocols using incremental coding we obtained the curves by means of Monte-Carlo simulation and plotted (closed form) asymptotic performance as dashed gray lines for comparison. Direct transmission as well as a transmit diversity system with two transmit and one receive antenna using the well known Alamouti space time code [1] for transmission are depicted for performance comparison. Observe that in the limit for large SNR, all cooperative protocols achieve full second order diversity. Conventional relaying is limited to first order diversity and performs even worse than direct transmission, even in this rather low rate regime.

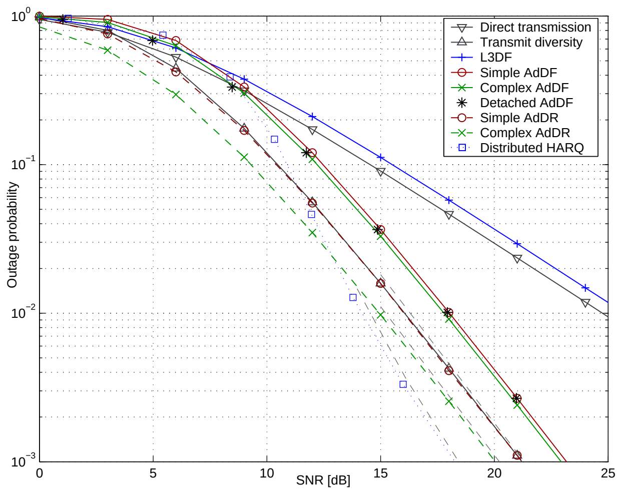

Fig. 2. Performance of the different conventional and cooperative relaying protocols in terms of outage probability versus SNR for spectral efficiency R=2 bit/channel use. Full second order diversity is achieved by all cooperative protocols.

By placing the relay halfway between source and destination, cooperative relaying protocols are able to fully exploit the nonlinear properties of path loss. The "cost of repetition" is greatly reduced and the SNR loss with respect to transmit diversity reduces to approximately 2 dB for the adaptive decode-and-forward protocols. Note that the Detached AdDF protocol draws benefit from omitting the second phase only in the low SNR regime, where the source-relay channel is often in outage. Its performance finally converges to that of the Simple AdDF protocol. The Complex AdDF protocol can leverage some additional gain from the repetition coding by the source. It is clearly visible, however, that this gain is not substantial.

Larger gains from cooperative transmission can be achieved by using incremental redundancy features instead of repetition coding approaches. Simple and Complex AdDR achieve additional gains over their AdDF counterparts of 2-3dB. However, the influence of the quality of the source-relay channel becomes more significant. This is also confirmed by the results in figure 3. The highest gains are obtained by the Distributed Hybrid ARQ protocol. Observe that the protocol only performs for low outage probabilities since otherwise the high number of outages on the source-relay and direct path dominate performance.

In the following plots, results were obtained from closed form expression for the large SNR regime, using (32) with the corresponding protocol, while markers indicate results obtained from Monte Carlo simulations. We see that analytical results and simulations agree very well.

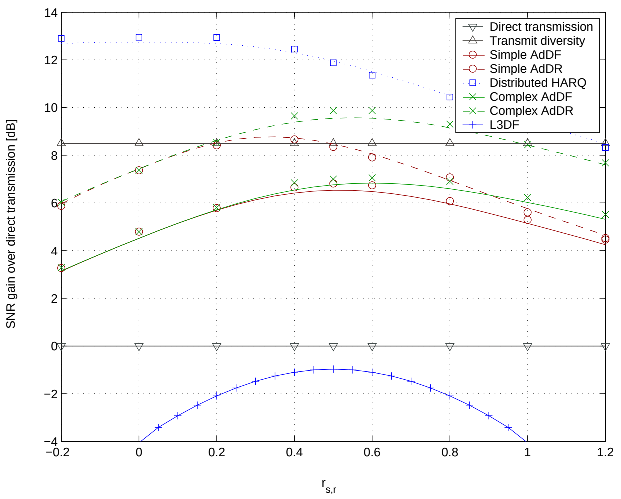

Fig. 3. SNR gain of different protocols over direct transmission. $r_{s,r}$ denotes the position of the relay between source and destination, normalized to the source-destination distance. The maximum SNR gain for most cooperative protocols is obtained for $r_{s,r} \approx 0.5$. For the distributed Hybrid ARQ protocol however, it is favorable to place the relay as close to the source as possible – to ensure an almost perfect source-relay channel.

The influence of the relay position on the SNR gain is depicted in Figure 3, again for a spectral efficiency of 2 bit/channel use. The relay is located on the line between source and destination, i.e., $r_{s,r} = 1 - r_{r,d}$. The maximum SNR gain for conventional relaying is obtained if the relay is located halfway between source and destination – this is an expected result.

However, the performance differences between Simple and Complex protocol versions and Decode-and-Forward vs. Decode-and-Reencode yield valuable insight into the related challenges and benefits. For the Simple AdDF, a relay position halfway between source and destination is optimal since it yields a good trade-off between good receive conditions for the relay and transmit power savings. The Complex AdDF shows a more robust performance and the relay can be placed closer towards the destination. This illustrates the benefit from repetition coding by the source, making the performance of the protocol less susceptible to imperfections in the source-relay link.

This effect is also visible for the adaptive decode-and-reencode version. Yielding higher gains over direct transmission and hence operating with lower transmit power at the source, their performance is largely dependent on a good source-relay channel. The optimal relay position is therefore shifted by roughly 0.2 towards the source, compared to AdDF protocols. The Simple AdDR protocol clearly suffers from this fact – when placing the relay close to the destination, its performance falls back to that of Simple AdDF, over which it yields a gain of $\approx$ 3dB under optimal conditions. When we allow for feedback from the destination, this effect is even more pronounced: for the Distributed Hybrid ARQ it is preferable to place the relay as close to the source as possible – path loss reduction does not bring any benefits since the relay repeats only rarely. The most substantial gains are due to the spatial diversity and the variable rate that adapts to the channel conditions.

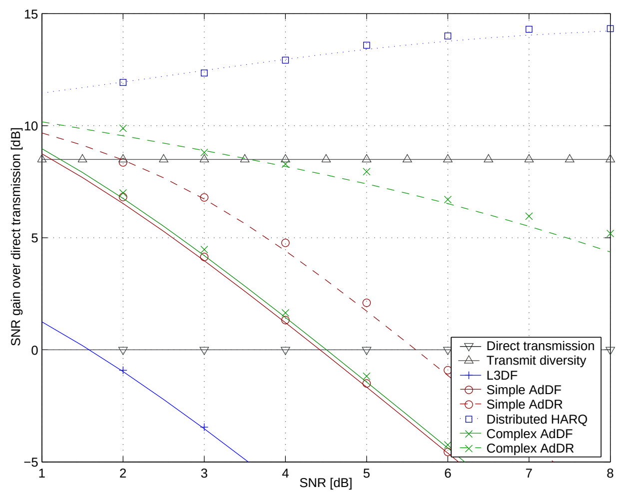

Fig. 4. SNR gain of different protocols over direct transmission as a function of the spectral efficiency R. Theis located halfway between source and destination, i.e., $r_{s,r} = 0.5$. Decode-and-Forward type protocols all lose 3dB per increase in spectral efficiency of 1 bit/channel use. Decode-and-Reencode protocols however, show more stable performance even as the transmission rate increases.

Figure 4 shows the SNR gains different relaying protocols achieve over direct transmission, as a function of the desired end-to-end spectral efficiency. We see that the gain for conventional relaying as well as AdDF type cooperative relaying protocols decreases by $\approx$ 3dB each time we increase the spectral efficiency by 1 bit/channel use – making these protocols unsuitable for high spectral efficiencies. The same holds for the Simple Adaptive Decode-and-Reencode protocol which for high spectral efficiencies is clearly limited by the inefficiency of the source-relay channel. The Complex Adaptive Decode-and-Reencode however *always* achieves gains over direct transmission, even for a spectral efficiency of 8 bit/channel use. This can be explained by the gain from parallel channel coding which can be optimally exploited by this protocol. Whenever the source-relay channel is not able to support the desired rate, the protocol simply falls back to the performance of direct transmission. The highest performance is again achieved by the Distributed Hybrid ARQ protocol, whose gains even increase as spectral efficiency increases. This is motivated by the fact that coding gains increase with the target spectral efficiency. This has been elaborately discussed in [10].

V. Implementation Considerations

While carrying out the performance analysis in this paper under a strict normalization of overall transmission power as well as end-to-end spectral efficiency for all protocols, we have so far assumed rather idealistic conditions regarding the implementation of the proposed protocols. Most notably, all investigated protocols require the relay to decode and again encode the received message prior to retransmission. The resulting processing delay would have to be taken into account as well, for a fair performance comparison. Similar arguments hold for the feedback channel required for some of the investigated protocols.

Further issues that need to be considered in a careful analysis of any relaying technique are the resulting signaling overhead, route setup delay, and security considerations when allowing an intermediate terminals to fully decode a message intended for other users. Last but not least the question of how to incentivize customers to make their terminals available for relaying purposes, i.e., the question of appropriate billing, needs to be answered.

VI. Conclusions

In this work, we gave an overview of cooperative relaying protocols and compared their performance with that of direct transmission, transmit diversity, and conventional relaying. We investigated how this performance is affected by the position of the relay as well as the target spectral efficiency. Our results indicate that repetition coding is the main drawback of cooperative relaying protocols and that performance can be drastically improved by using incremental redundancy techniques instead of simple decode-and-forward operation.

Using decode-and-reencode" approaches, cooperative relaying techniques are able to provide gains of around 5dB over direct transmission, even in the high spectral efficiency regime. By allowing for feedback from the destination, these gains are even higher and Distributed Hybrid ARQ protocol outperforms direct transmission by as much as 12dB, for all investigated target transmission rates. These results confirm that cooperative relaying is a very powerful means of improving link performance in wireless networks.

It should be remembered that our focus has so far been only on the slow fading regime, i.e., space is the only source of diversity. Future research should address the more general case, where diversity is available also in other dimensions (time and frequency) and the relative merits of exploiting spatial diversity through cooperative relaying can be expected to be significantly lower. A first step in this direction has been taken in [18].

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