A Multi-Winner Cognitive Spectrum Auction Framework with Collusion-Resistant Mechanisms

Yongle Wu, Beibei Wang, K. J. Ray Liu, and T. Charles Clancy†
Department of Electrical and Computer Engineering and Institute for Systems Research,
University of Maryland, College Park, MD 20742, USA.
{wuyl, bebewang, kjrliu}@umd.edu
†Laboratory for Telecommunications Sciences,
US Department of Defense, College Park, MD 20740, USA.
clancy@LTSnet.net

***Abstract*—Dynamic spectrum access, enabled by cognitive radio technologies, has become a promising approach to improve efficiency in spectrum utilization, and the spectrum auction is one approach in which unlicensed wireless users lease some unused bands from spectrum license holders. However, spectrum auctions are different from traditional auctions studied by economists, because spectrum resources are interference-limited rather than quantity-limited, and it is possible to award one band to multiple secondary users with negligible mutual interference. Due to its special feature, the multi-winner auction is a new concept posing new challenges in the existing auction mechanisms such as the Vickery-Clarke-Groves (VCG) mechanism. Although widely employed in other auctions, the VCG mechanism does have serious drawbacks when applied to the multi-winner auction, such as unsatisfactory revenue and vulnerability to collusive attacks. Therefore, in this paper, we propose a multi-winner spectrum auction framework, and develop suitable mechanisms for this kind of auction. In specific, the mechanism awards the bands in such a way that the spectrum efficiency is maximized, and determines prices based on the Nash bargaining solution to improve revenue and prevent collusion. We further analyze that secondary users do not have incentives to manipulate information about mutual interference which is essential to the auction. Finally, simulation results are presented to evaluate our proposed auction mechanisms.**

I. INTRODUCTION

As the demand for wireless spectrum has been growing rapidly with the deployment of new wireless applications and devices in the last decade, the regulatory bodies such as the Federal Communications Commission (FCC) have begun to consider more flexible and comprehensive uses of available spectrum [1]. With the development of cognitive radio technologies [2], dynamic spectrum access becomes a promising approach, which allows the unlicensed users (secondary users) to dynamically access the licensed bands from legacy spectrum holders (primary users) in either a non-cooperative fashion [3]–[5] or a cooperative fashion [7]–[12].

In the non-cooperative approach, primary users do not need to consider the existence of secondary users, and it is secondary users' responsibility to guarantee that their usage of the spectrum will not cause any problems to primary users. To that end, secondary users have to continuously sense the radio environment to detect the presence of the primary users. Whenever finding a spectrum opportunity, that is, when the primary users are absent from the band, secondary users are allowed to occupy the spectrum; but they must immediately vacate the band when a primary user appears. A lot of research works has been done towards this direction. For instance, in [3], the authors devised rules for secondary users to utilize available spectrum while avoiding interference with their neighbors based on a graph-theoretical model. The work in [4] examined the secondary users' access patterns to propose a feasible spectrum sharing scheme. In [5], the authors proposed a primary prioritized Markovian dynamic spectrum access scheme to optimally coordinate secondary users' spectrum access and achieve a good statistical tradeoff between efficiency and fairness.

However, as accurate detection is crucial to find such spectrum opportunities, delicate spectrum detectors have to be equipped to each secondary user. Moreover, because no realistic detector is perfect, inaccurate detection results are inevitable. Specifically, missed detection declares a spectrum opportunity in spite of primary users' presence and may impact primary users' quality of service, while false alarm fails to catch all the true opportunities, and hence the spectrum cannot be utilized in full efficiency. Recently, research also indicates that there are fundamental bounds on detection performance in low signal-to-noise ratio (SNR) in the presence of noise uncertainty [6]. To circumvent the difficulties, spectrum opportunities can be announced by primary users rather than detected by secondary users, provided collaboration between primary users and secondary users is established.

The cooperative approach can be implemented on a pricing basis, which provides both parties with incentives. Primary users would like to trade their temporarily unused bands for monetary gains. From secondary users' point of view, they also want to lease some channels to transmit their information as long as the communication gains exceed the cost. There are several previous efforts to study dynamic spectrum access via pricing and auction mechanisms. In [7], the price of anarchy was analyzed for spectrum sharing in WiFi networks. In [8], a demand responsive pricing framework was proposed to maximize the profits of legacy spectrum operators while considering the buyers' response model. An auction-based mechanism was proposed in [9] to efficiently share spectrum among secondary users in interference-limited systems. In [10], the authors considered a multi-unit sealed-bid auction for efficient spectrum allocation. In [11], a real-time spectrum auction framework with interference constraints was proposed to get a conflict-free allocation. In [12], a belief-assisted distributive pricing algorithm was proposed to achieve efficient dynamic spectrum allocation based on double auction mechanisms.

Although existing schemes have enhanced spectrum allocation efficiency through market mechanisms, some critical challenges still remain unanswered. First, in most of the current auctions, one licensed band (or a package of multiple bands) is awarded to a unique buyer who will then be called the winner, just like most other auctions studied by economists [13]. However, the spectrum resource is quite different from other goods in that it is interference-limited rather than quantity-limited, because it is reusable by wireless users geographically far apart. In some application scenarios where secondary users only need to communicate within a short range, such as a wireless personal area network (WPAN) centered around an individual person's workspace, the transmit power is quite low, and hence even users with moderate distance can simultaneously access the same band. In this case, allowing multiple winners to lease the band is an option embraced by everyone: primary users get higher revenue, secondary users get more chances to access the spectrum, and spectrum usage efficiency gets boosted as well from the system perspective. To highlight the distinction from traditional one-item-one-winner or multi-item-one-winner auctions, we coin the name "multi-winner auction" for this new one-item-multi-winner spectrum auction, in which auction outcomes (e.g., the number of winners) highly depend on the geographical locations of the wireless users.

Second, although a few papers (e.g., [9][11]) discuss spectrum auctions under interference constraints, all of them are based on the assumption that secondary users will bid their true valuations. However, with the emerging applications of mobile ad hoc networks envisioned in civilian usage, the secondary users may be selfish and only aim to maximize their own interests. Driven by self-interests, they could misrepresent their valuations in order for more profits, either individually or collusively. As auction rules significantly impact bidding strategies, designing proper auction mechanisms will help provide incentives to reveal true valuations. The Vickery-Clarke-Groves (VCG) mechanism [14] ensures that truth-telling is a dominant bidding strategy for individual buyers, but is quite vulnerable to collusion attacks, and often results in unsatisfactory seller revenues when applied to this multi-winner auction scenario. Thus, it is necessary to develop other auction mechanisms to overcome the problems.

Third, multiple-winner auction increases flexibility of the scheme, but meanwhile poses new problems such as emerging kinds of collusion. For a traditional one-winner auction with the VCG mechanism, the most effective collusion is the bidding ring collusion, where colluders greatly decrease their bids. By doing so, colluders may win the item with a dramatically low price and hence hurt the seller's interest. An auction mechanism has been proposed in [15] to protect against the bidding ring collusion by setting up an optimal reserve price. Nevertheless, the multi-winner auction makes possible new forms of collusion that need to be taken care of; otherwise, colluders will take great advantage of the flaw, with their profits increased but system efficiency decreased.

Therefore, in this paper, we propose a framework for the multi-winner cognitive spectrum auction, and develop appropriate mechanisms for this kind of auction. An auction mechanism consists of winner determination and price determination. In our proposed mechanism, the set of winners is determined by a binary linear programming problem which guarantees full spectrum efficiency, and the pricing strategy is modeled as a convex optimization problem with constraints precluding user collusion. It is shown that the proposed strategies not only improve the primary user's revenue, but also resist the possible user collusion.

The rest of this paper is organized as follows. In Section II, the model for an multi-winner cognitive spectrum auction is described. In Section III, we introduce the VCG mechanism and discuss its limitations through specific examples. In Section IV, we develop novel collusion-resistant pricing strategies to overcome the problem of the VCG mechanism. Simulation results are presented in Section V, and Section VI concludes the paper.

II. System Model

We consider a cognitive radio network where N secondary users coexist with M primary users, and primary users tend to lease their unused bands to secondary users for monetary gains. We model it as an auction where the sellers are the primary users, the buyers are the secondary users, and the auctioneer is a spectrum broker who helps coordinate the auction. Assume there is a common channel to exchange necessary information and a central bank to circulate money in the community. For simplicity, we assume each primary user owns one band exclusively, and each secondary user needs only one band. In this paper, we first consider the auction with a single band ($M = 1$), and later extend to the multi-band auction.

The system designer determines a fixed leasing period T, and the auction is done in a sealed-bid way as follows. At the beginning of each leasing period, the primary users will notify the spectrum broker whether to sell the spectrum rights for the next duration of T. Meanwhile, the potential buyers submit their bids $\boldsymbol{b} = [b_1, b_2, \ldots, b_N]$ to the spectrum broker simultaneously. According to the bids and channel availability, the broker decides both the allocation $\boldsymbol{x} = [x_1, x_2, \ldots, x_N]$ and the prices $\boldsymbol{p} = [p_1, p_2, \ldots, p_N]$, where $x_i = 1$ means secondary user i wins some band, $x_i = 0$ otherwise, and p_i is the price for secondary user i. Alternatively, we can define the set of winners as $W \subseteq \{1, 2, \ldots, N\}$, where $i \in W$ if and only if $x_i = 1$. Assume user i gains v_i from transmitting information in the leased band, his/her reward is

$$r_i = v_i x_i - p_i,\ i = 1, 2, \ldots, N. \tag{1}$$

Given all users' valuations $\boldsymbol{v} = [v_1, v_2, \ldots, v_N]$, the system utility, or the *social welfare*, can be represented by

$$U_{\boldsymbol{v}}(\boldsymbol{x}) = \sum_{i=1}^{N} v_i x_i = \sum_{i \in W} v_i, \tag{2}$$

which measures the total amount of utility realized from the multi-winner auction. An auction is *efficient* if its outcome maximizes the social welfare.

Since several secondary users with negligible mutual interference can be awarded the same band in the proposed multi-winner auction, interference relationships play an important role in the auction. For example, in Fig. 1 (a), there are four secondary cognitive base stations competing for the spectrum lease, among whom station 1 interferes with all the others due to their geographical locations. This can be characterized by a graph as in Fig. 1 (b), with edges indicating there is mutual interference between the corresponding nodes. To further simplify the representation of the interference constraints among secondary users, we adopt an $N \times N$ adjacency matrix $\boldsymbol{C}$, with $C_{ij} = 1$ if user i and user j cannot be assigned the same band, and $C_{ij} = 0$ if they can share one band with negligible interference. For instance, the matrix in Fig. 1 (c) contains all the necessary information for the auction. By collecting reports from secondary users about their locations or their neighbors, the spectrum broker can keep the matrix $\boldsymbol{C}$ updated, even if the interference constraints may change from time to time because of the slow movement of secondary users. We further assume that if two users with mutual interference attempt to access the band simultaneously, neither of them can get any gains due to strong interference.

Since the secondary users, who compete for the spectrum resources released by the primary users, want to successfully lease the band with the lowest possible price, it is reasonable to assume that all the secondary users are selfish, that is, their objectives are to maximize their own profits. Hence, a secondary user may cheat by misrepresenting his/her valuation, or a clique of the secondary users may plot collusion before participating in the auction. If several secondary users belong to the same service provider, they may even have a more facilitated way to exchange information for collusion. In a multi-winner auction, other forms of collusion beyond the bidding ring collusion are made possible by the auction rule that several buyers can be awarded the band. There are mainly two emerging kinds of collusion. The first kind is called *loser collusion*, where several losers, by raising their bid collusively, may overtake the winners and get the spectrum lease instead. The other collusion, termed *sublease collusion*, happens when several winners sublease the spectrum to others and effortlessly take away some profits that are supposed to be credited to the primary user.

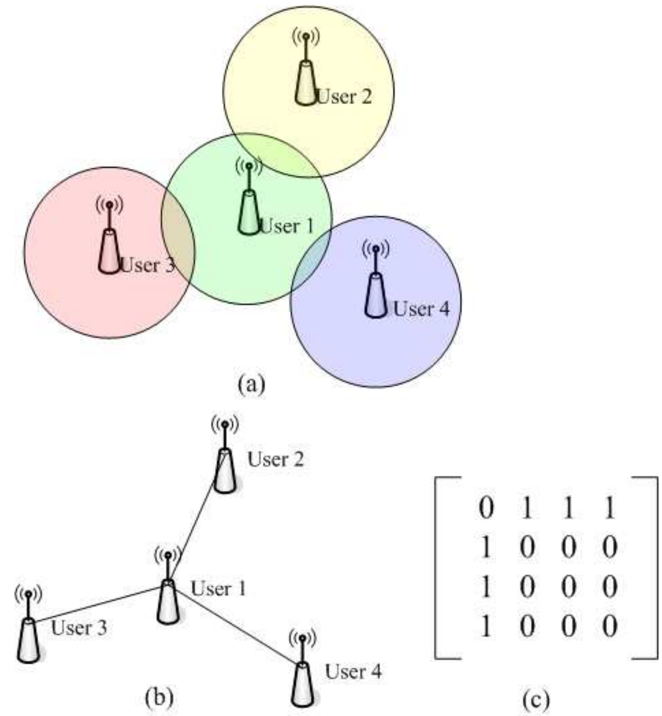


Fig. 1. Illustration of the interference structure in a cognitive spectrum auction. (a) physical model; (b) graph representation; (c) matrix representation.

III. The VCG Mechanism

The VCG mechanism has been widely employed in combinatorial auctions [14], because it ensures full system efficiency, and in most auction scenarios, it is the only mechanism that enforces truth-telling, that is, no single individual has the incentive to misrepresent his/her valuation under this mechanism. In other words, when there is no collusion, $b_i = v_i$ $(i = 1, 2, \ldots, N)$ will be adopted by all secondary users as the best bidding strategy. The VCG mechanism can also be applied to the multi-winner auction; however, several drawbacks make it less attractive. In this section, after giving a brief introduction on the VCG mechanism, we will analyze its drawbacks through specific examples.

A. Auctions with the VCG Price

In general, an auction mechanism defines rules for two consecutive steps: the rule to determine winners, and the rule to determine prices. Under the VCG mechanism, the winners are decided in such a way that the social welfare is maximized, and the price charged to each winner equals his/her "social opportunity cost" to the whole system. To make it more clear, we apply it to the multi-winner spectrum auction, and illustrate it by a simple example.

For a cognitive spectrum auction, not all combination of secondary users can access the band at the same time. Instead, only those without mutual interference can be awarded the band simultaneously, and we call it a *compatible* combination. Throughout all compatible combinations, the one with the highest social welfare will become the set of winners, i.e., the allocation vector $\boldsymbol{x}$ is determined by the following optimization problem,

$$\begin{aligned} \max_{\boldsymbol{x}} \ U_{\boldsymbol{v}}(\boldsymbol{x}) &= \sum_{i=1}^{N} v_i x_i, \\ \text{s.t. } x_i + x_j &\le 1, \ \forall i, j \text{ if } C_{ij} = 1, \\ x_i &= 0 \text{ or } 1, \ i = 1, 2, \ldots, N, \end{aligned} \tag{3}$$

where interference constraints require that secondary users with mutual interference should not be assigned the band at the same time. This is a binary integer programming (BIP) problem with N variables. For simplicity, we denote the maximizer to this problem as $\boldsymbol{x}^*$, and correspondingly, the maximum system utility attained is denoted by $U_{\boldsymbol{v}}^* = U_{\boldsymbol{v}}(\boldsymbol{x}^*)$.

Denote $\boldsymbol{v}_{-i} = [v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_N]$ which is similar to $\boldsymbol{v}$ except that the i-th entry is excluded. In order to calculate the prices, we have to show what the social opportunity cost is. Assume user i is one of the winners. If user i were absent and everyone else remained in the system, the social welfare would be $U_{\boldsymbol{v}_{-i}}^*$, which can be computed from solving (3) again with $\boldsymbol{v}$ replaced by $\boldsymbol{v}_{-i}$. On the other hand, in the real situation with user i existing, the system utility is $U_{\boldsymbol{v}}^*$, and hence the total utility of all users except i is $U_{\boldsymbol{v}}^* - v_i$. The difference, $U_{\boldsymbol{v}_{-i}}^* - (U_{\boldsymbol{v}}^* - v_i)$, is the "damage" that user i causes to the whole society, and VCG pricing requires the winners to make compensation by paying

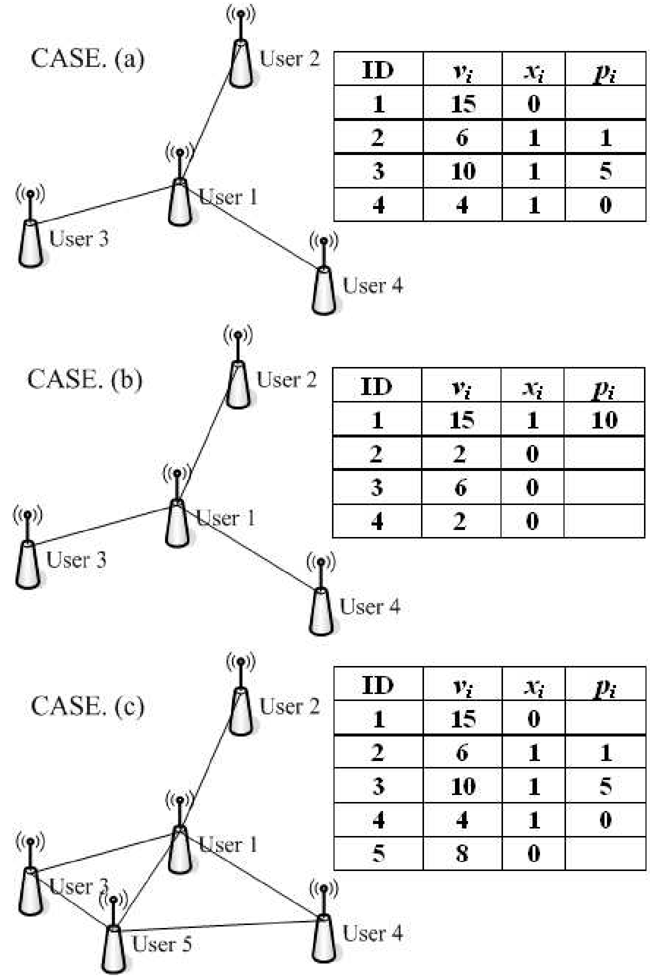
$$p_i = v_i + U_{\boldsymbol{v}_{-i}}^* - U_{\boldsymbol{v}}^*. \tag{4}$$

Take Case (a) in Fig. 2 as an example, where there are four secondary users, whose valuations are $v_1 = 15$, $v_2 = 6$, $v_3 = 10$, and $v_4 = 4$, respectively, as shown in the figure. First, we solve the efficient allocation (3) with $\boldsymbol{v} = [15, 6, 10, 4]$, and find that $\boldsymbol{x}^* = [0, 1, 1, 1]$ with $U_{\boldsymbol{v}}^* = v_2 + v_3 + v_4 = 20$, that is, leasing the band to user 2, 3, and 4 maximizes the spectrum utilization. Then, (4) is employed to calculate the price for each winner. If user 2 were absent and the other three users made their bids, the maximal social welfare would be achieved when awarding the band to user 1, i.e., $U_{\boldsymbol{v}_{-2}}^* = v_1 = 15$, which can be solved from (3) simply by taking $\boldsymbol{v}_{-2} = [15, 10, 4]$ as input. As a result, user 2 has to pay $p_2 = v_2 + U_{\boldsymbol{v}_{-2}}^* - U_{\boldsymbol{v}}^* = 1$. Similarly, user 3 needs to pay 5, and user 4 pays nothing, which are listed in Fig. 2. By the same approach, the VCG outcomes for Case (b) and (c) are easy to compute, too.

B. Drawbacks of the VCG Mechanism

However, the VCG mechanism has several shortcomings, as illustrated by cognitive spectrum auction examples in the following.

First, the seller's revenue may be low. As in Case (a) with the VCG prices, the total payment collected by the primary user is $p_2 + p_3 + p_4 = 6$, which is quite low compared to the system utility. Furthermore, there is no guarantee that the primary user's revenue is bounded away from zero. In some unfavorable cases, for example, $v_1 = v_2 = v_3 = v_4 = 10$, the primary user sells the spectrum for nothing according to the VCG price.



ID	v_i	x_i	p_i
1	15	0	
2	6	1	1
3	10	1	5
4	4	1	0

ID	v_i	x_i	p_i
1	15	1	10
2	2	0	
3	6	0	
4	2	0	

ID	v_i	x_i	p_i
1	15	0	
2	6	1	1
3	10	1	5
4	4	1	0
5	8	0	

Fig. 2. Different auctions with the VCG mechanism employed. v_i is the valuation of user i, x_i and p_i is the allocation and price to user i, respectively, determined by the VCG mechanism.

Second, the losers may take advantage of the VCG pricing by colluding. For example, in Case (b), secondary user 1 gets the spectrum lease, and user 2, 3, 4 are the losers in the VCG auction. However, if colluding and misrepresenting their valuations, they may become winners instead. For instance, they may collude to mimic Case (a) by claiming the same valuations as in Case (a), whose outcome is favorable since all colluders gain positive rewards, i.e., $r_2 = v_2 - p_2 = 1$, $r_3 = 1$, and $r_4 = 2$, respectively. The system efficiency is degraded because the spectrum resources are not assigned to the users who value them most.

Third, colluders may extract some profits from the seller by sublease collusion. Consider Case (c) where another secondary user shows up without changing the VCG outcome from Case (a). In this case, user 3 and user 4 may now collude with user 5 by subleasing the band at price $p_5 = 7$, and the income is split between them as 6 and 1. Then, both user 3 and 4 make extra profit by subleasing the band at higher prices than their leasing prices, and user 5 also benefits from subleasing since the reward is $v_5 - p_5 = 1$. Such collusion impairs the spectrum efficiency as well as the primary user's revenue.

Low revenue (even zero in some cases) and vulnerability to collusion attacks make the VCG mechanism unsuitable for the multi-winner auction. The most important reason accounting for the drawbacks of the VCG mechanism is that the VCG prices are sometimes too low to prevent collusion. Because the prices are low, even the losers who have less capacity of paying can afford the prices when they collude to win the spectrum band, and the winners have sufficient margins to make extra profits by subleasing the band to others. By increasing the prices, we can not only alleviate user collusion, but also provide the seller with a higher revenue. Since VCG pricing is the only choice to ensure both efficiency and truth-telling, we want to see whether there is such a mechanism that sacrifices either one a little bit, but overcomes the drawbacks of VCG pricing.

C. The Second-Price Auction

In traditional one-item-one-winner auction, the second-price mechanism is well-known and widely deployed. The auction rule is quite simple: the bidder with the highest bid wins the item, and pays the amount of money equal to the second highest bid. For example, still consider Case (a), if only one buyer is awarded the band, user 1 will get the spectrum lease by paying $p_1 = 10$, which equals the second highest bid made by user 3. In addition, it is well-known that submitting bids equal to their true valuations is the dominant strategy [13]. By submitting a higher bid than true valuation, the buyer may end up with paying more money than what it actually worths; and by submitting a lower bid, the buyer may lose the opportunity to win the item. Hence, bidding the true valuation is self-enforced. Moreover, the primary user's revenue in the second-price auction will not be too low, since the price equals the second highest bid.

The second-price auction is a special case of the VCG mechanism with an additional constraint that only one secondary user can get the spectrum lease, i.e., $\sum_{i=1}^{N} x_i = 1$. Then, the VCG auction rules (3) (4) will reduce to the second-price rules, respectively. As we have argued that multi-winner auction is far more efficient than the single-winner auction, the second-price auction is not a wise choice; but we will develop new mechanisms using the similar ideas to the second-price auction.

IV. Proposed Mechanism for Multi-Winner Auction

In this section, we develop proper mechanisms for the multi-winner cognitive spectrum auction to overcome the drawbacks of the VCG mechanism, and then extend the auction to a more general multi-band auction.

A. Basic Auction Mechanism

Because the goal of dynamic spectrum access is to improve the spectrum efficiency, the auction mechanisms should be designed to maximize the efficiency as well. To that end, we keep the same winner determination method as the VCG mechanism, which awards the spectrum resources to the secondary users who value them most. We solve the BIP problem (3) to determine the set of winners W. After the efficient allocation is decided, we employ new pricing strategies other than the VCG pricing. This sacrifices the enforcement of truth-telling to some extent, but yields a mechanism with higher revenue and more robustness against colluding attacks. Since the proposed pricing strategy is quite similar to the second-price mechanism, we can expect the bids will not deviate too much from the true valuations, and thus we neglect the difference between b_i and v_i in the following analysis to focus on revenue and robustness aspects of the new mechanisms.

In order to apply the second-price idea that can only be used in a single-winner auction, we have to remodel the multi-winner spectrum auction into a single-winner auction by introducing the concept of *virtual bidders*. We group secondary users with negligible interference together as virtual bidders, whose valuation equal the sum of the individual valuations. Every compatible combination of secondary users corresponds to a virtual bidder; for instance, in Case (a), the spectrum broker finds eight virtual bidders in all, whose valuations are $v(\{1\}) = 15$, $v(\{2\}) = 6$, $v(\{3\}) = 10$, $v(\{4\}) = 4$, $v(\{2,3\}) = 16$, $v(\{2,4\}) = 10$, $v(\{3,4\}) = 14$, and $v(\{2,3,4\}) = 20$, respectively. Similar to the second-price strategy, the virtual bidder with the highest bid will be awarded the band, and the total payment equals the highest bid made by the virtual bidder after the winners are removed from the system.

This can be done by solving two BIP problems in succession. First, we solve (3) to determine the set of winners W, or the virtual bidder with the highest bid. Then, we remove all the winners W from the system, solve the optimization problem again to calculate the maximum utility, denoted by $U^*_{\boldsymbol{v}_{-W}}$. The winners have to pay $U^*_{\boldsymbol{v}_{-W}}$ in total.

Now, the only unsolved problem is splitting the payment among the secondary users within the winning virtual bidder. This is quite similar to a Nash bargaining game [16] where each selfish player proposes his/her own payment during a bargaining process such that the total payment equals $U^*_{\boldsymbol{v}_{-W}}$, and it is well-known that the Nash bargaining solution (NBS), which maximizes the product of the individual payoffs, is an equilibrium [16]. In our proposed auction, no individual bargaining is necessary; instead, the spectrum broker directly sets the equilibrium prices for each winner, which is the solution to the following optimization problem,

$$\begin{aligned} \max_{\{p_i, i \in W\}} & \prod_{i \in W} (v_i - p_i), \\ \text{s.t.} & \sum_{i \in W} p_i = U^*_{\boldsymbol{v}_{-W}}, \\ & 0 \le p_i \le v_i. \end{aligned} \tag{5}$$

By using the fact that $\sum_{i \in W} v_i = U^*_{\boldsymbol{v}}$ and applying Karush-Kuhn-Tucker (KTT) conditions [17], we can write the solution to (5) as

$$p_i = \max\{v_i - \rho, 0\}, \text{ for } i \in W, \quad (6)$$

where ρ is chosen such that $\sum_{i\in W} p_i = U^*_{\boldsymbol{v}_{-W}}$. The proof is left to the appendix. In particular, denote $|W|$ as the cardinality of set W, if $\hat{p}_i \triangleq v_i - \frac{U^*_{\boldsymbol{v}} - U^*_{\boldsymbol{v}_{-W}}}{|W|} \geq 0$ for any i, $p_i = \hat{p}_i$ will be the solution. It can be seen that the payment is split in a fair way such that the profits are shared among the winners as equally as possible.

When such a pricing strategy is used, the seller's revenue is $U^*_{\boldsymbol{v}_{-W}}$, which is often relatively high. Moreover, if some losers collude to beat the winners by raising their bids, they will have to pay more than $U^*_{\boldsymbol{v}_{-W}}$; however, the payment is already beyond what the band is actually worth to them, and as a result, loser collusion is completely eliminated. The sublease collusion can be also alleviated to some extent. For example, in Case (c), the prices for the winners according to (6) are $p_2 = 13/3$, $p_3 = 25/3$, and $p_4 = 7/3$. Then, sublease collusion among user 3, 4, and 5 will fail, since user 3 and 4 will not sublease at any price lower than $p_3 + p_4 = 32/3$, and user 5 will not pay more than $v_5 = 8$ for the band. However, if user 5 has a higher valuation, say $v_5 = 11$, sublease collusion may still take place; nevertheless, in such a situation, the maximal collusion profit $v_5 - (p_3 + p_4)$ significantly drops from 3 with the VCG pricing to $1/3$ with the proposed pricing strategy.

B. Collusion-Resistant Auction Mechanism

In order to completely inhibit sublease collusion, a more complicated algorithm needs to be developed by adding more constraints. Notice that sublease collusion happens in this way: a subset of the winners $W_C \subseteq W$ sublease the band to a subset of the losers $L_C \subseteq L$, where $L = \{1, 2, \ldots, N\} - W$ denotes the set of losers. The necessary condition for the sublease collusion to happen is $\sum_{i\in W_C} p_i < \sum_{i\in L_C} v_i$, so that they can find a sublease price in between acceptable to both parties. They also have to take mutual interference into consideration: the losers in L_C have to be interference-free with each other, and they will not sublease the band if it turns out to be unusable due to interference with the users in $W - W_C$.

When W is determined by the efficient allocation strategy (3), given any colluding-winner subset $W_C \subseteq W$, the possible colluding losers must come from a subset of the losers whose members are interference-free with those users in $W - W_C$, denoted by $L(W - W_C)$. If the prices are set such that $\sum_{i\in W_C} p_i \geq \max_{L_C \in L(W-W_C)} \sum_{i\in L_C} v_i$, there will be no sublease collusion. Note that $\max_{L_C\in L(W-W_C)} \sum_{i\in L_C} v_i$ is the maximum system utility $U^*_{\boldsymbol{v}_{L(W-W_C)}}$ which can be obtained by solving the BIP problem, then the optimum collusion-resistant pricing strategy is the solution to the following problem,

$$\begin{aligned} \max_{\{p_i, i\in W\}} & \prod_{i\in W} (v_i - p_i), \\ \text{s.t.} & \sum_{i\in W_C} p_i \geq U^*_{\boldsymbol{v}_{L(W-W_C)}}, \forall W_C \subseteq W, \\ & 0 \leq p_i \leq v_i. \end{aligned} \quad (7)$$

When $W_C = W$, the constraint reduces to $\sum_{i\in W} p_i \geq U^*_{\boldsymbol{v}_{-W}}$, which incorporates the constraint in (5) as a special case.

It can be shown that (7) is a convex optimization problem with linear inequality constraints, and hence it can be efficiently solved by numerical methods [17]. The major complexity comes from solving $2^{|W|} - 1$ BIP problems in order to get the values $U^*_{\boldsymbol{v}_{L(W-W_C)}}$ for any $W_C \subseteq W$ except $W_C = \Phi$. However, in most cases, the size of $L(W - W_C)$ can be greatly reduced due to the interference constraints, and therefore, the complexity of solving the BIP problems will also be reduced.

In sum, the proposed auction mechanism first determines an efficient allocation according to (3), and then assigns a price to each winner using (7) (or (6) if computational capability is limited), which can completely (or partially) eliminate user collusion.

C. Interference Matrix

So far, our auction mechanism is based on the assumption that the underlying interference matrix $\boldsymbol{C}$ reflects the true mutual interference relationships between secondary users. However, since $\boldsymbol{C}$ comes from secondary users' own reports, it is quite possible that the selfish users manipulate this information just as what they may do with their bids. If cheating could help a loser become a winner, or help a winner pay less, the selfish users would have incentives to do so, which would compromise the efficiency of the spectrum auction. Also, the cheating behavior may happen individually or in a collusive way. Therefore, we have to carefully consider whether they have such an incentive to deviate, and if so, how to fix the potential problem.

In order to keep the matrix $\boldsymbol{C}$, the spectrum broker has to collect information from secondary users. Secondary users may report their locations in terms of coordinates, and the spectrum broker takes the responsibility to calculate the matrix according to their distances. In this way, secondary users do not have much freedom to fake an interference map in favor of themselves. Alternatively, secondary users may directly inform the spectrum broker about who are their neighbors, and hence they are able to manipulate the matrix. They can either conceal an existing interference relationship, or fabricate an interference relationship that actually does not exist.

When secondary users have little information about others, they will misrepresent the interference relationships only if they do not get punished even in the worst case. Assume user j lies about C_{jk}. When user j and k do not have interference, i.e., $C_{jk} = 0$, but user j claims $\hat{C}_{jk} = 1$, he/she may lose an opportunity of being a winner since an extra interference constraint is added. On the other hand, if $C_{jk} = 1$ but user j claims $\hat{C}_{jk} = 0$, user j may end up with winning the band together with user k, and the band cannot be used at all due to strong interference. In short, the worst-case analysis suggests secondary users have no incentive to cheat whenever information is limited.

When secondary users somehow have more information about others, they may distort the information in a more intelligent way, that is, they can choose when to cheat and how to cheat. Nevertheless, we will show that no individual would have the incentive to lie unilaterally by discussing all the possible situations in what follows. Assume user j lies about C_{jk}, and we will check whether user j gets better off by doing so.

1) Under the condition that user j is supposed to be a loser.
1a. Claim $\hat{C}_{jk} = 1$ against the truth $\hat{C}_{jk} = 0$. By doing this, user j actually introduces an additional interference constraint to himself/herself, and nothing will change, since user j is already a loser.
1b. Claim $\hat{C}_{jk} = 0$ against the truth $\hat{C}_{jk} = 1$. Removing a constraint possibly helps user j to become a winner, but in the case, user k is also one of the winners. Then, user j has to pay a band that turns out to be unusable due to strong mutual interference with user k. This is unacceptable to user j.
2) Under the condition that user j is supposed to be a winner.
2a. Claim $\hat{C}_{jk} = 0$ against the truth $\hat{C}_{jk} = 1$. If user j is the only one among the winners who has interference with user k, it will take user k into the winner set, which will in turn make user j suffer from the mutual interference.
2b. Claim $\hat{C}_{jk} = 1$ against the truth $\hat{C}_{jk} = 0$. If user k is not a winner, doing this will change nothing. If user k is indeed a winner, user j takes the risk of throwing himself/herself out of the winner set. Even if user j has enough information to secure he/she can still be a winner, kicking out user k does not necessarily make user j pay less.

In sum, no individual has the incentive to cheat even if there is enough information to make the intelligent cheating possible.

Now consider the situation when a group of secondary users are able to distort the information collusively. From the analysis above, collusion makes no difference except the case (2b). By kicking out some winners, the colluding winners may welcome their allies to join in the family of winners. For instance, assume user sets $\{1, 2, 3\}$ and $\{1, 2, 4\}$ are compatible combinations, and $\{1, 2, 3\}$ is the winner set. If user 1 and 4 belong to the same group of interest, user 1 will claim $\hat{C}_{13} = 1$ to kick user 3 out and make $\{1, 2, 4\}$ the winner set instead.

Since fabricating an interference relationship to an innocent user as in case (2b) is the only way that colluders get benefit, a conservative rule could make their efforts in vain, that is, the spectrum broker only sets C_{jk} to 1 when both user j and k declare they have mutual interference. Although colluding user j wants to claim an false interference relationship with an innocent user k, $C_{jk} = 0$ always holds no matter what user j claims, because user k always truthfully reveals there is no interference with user j. Therefore, colluding users will lose their incentives to cheat because they cannot make any difference.

In sum, we examine secondary users' incentives to lie about the underlying interference relationships, and realize no single user or group of users would have incentive to cheat individually or collusively, when the spectrum broker employ the conservative rule to determine the interference matrix $\boldsymbol{C}$ from secondary users' reports.

D. Multi-Band Extension

The proposed auction mechanism can be easily extended to a more general case when M primary users want to lease their unused bands or a single primary user divides the band into M subbands for lease. In other words, there are M bands $(M > 1)$ available for the secondary users. We assume all the secondary users are interested in only one band, and they are indifferent to different primary bands.

In the multi-band spectrum auction, the allocation strategy can be similarly determined by the following BIP problem,

$$\begin{aligned}
&\max_{\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^M} U_{\boldsymbol{v}}(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^M) = \sum_{m=1}^{M} \sum_{i=1}^{N} v_i x_i^m, \\
&\text{s.t. } x_i^m + x_j^m \leq 1, \ \forall i, j \text{ if } C_{ij} = 1, \forall m, \\
&\quad \sum_{m=1}^{M} x_i^m \leq 1, \ \forall i, \\
&\quad x_i^m = 0 \text{ or } 1, i = 1, 2, \ldots, N; m = 1, 2, \ldots, M,
\end{aligned} \tag{8}$$

where $x_i^m = 1$ implies secondary user i leases a band from primary user m, and $x_i^m = 0$ otherwise. Similarly, we can denote the sets of winners as $W_1, W_2, \ldots, W_M$. Actually, this is a natural extension of (3) except for an additional constraint requiring that each secondary user can lease at most one band.

However, the new BIP problem has MN variables, which may be very computation-consuming when M is large. Therefore, we propose a greedy algorithm to reach approximate efficiency by solving the following N-variable BIP problems M times, i.e., for $m = 1, 2, \ldots, M$,

$$\begin{aligned}
&\max_{\boldsymbol{x}} U_{\boldsymbol{v}}(\boldsymbol{x}) = \sum_{i=1}^{N} v_i x_i, \\
&\text{s.t. } x_i + x_j \leq 1, \ \forall i, j \text{ if } C_{ij} = 1, \\
&\quad x_i = 0 \text{ or } 1, \ \text{if } i \in L^{(m)}, \\
&\quad x_i = 0, \ \text{if } i \notin L^{(m)}.
\end{aligned} \tag{9}$$

In (9), we initialize $L^{(1)} = \{1, 2, \ldots, N\}$. After assigning the primary band m to the set $W^{(m)}$ in the m-th iteration, we update set $L^{(m+1)}$ as $L^{(m+1)} = L^{(m)} - W^{(m)}$.

Similar to the second-price strategy in the single-unit auction, an $(M + 1)$-st price strategy can be applied in an auction with M items for sale. The price equals the highest rejected bid. By replacing $U^*_{\boldsymbol{v}_{-W}}$ in (5) with $U^*_{\boldsymbol{v}_{-\left(\cup_{j=1}^{M} W_j\right)}}$, we can derive an analogous pricing strategy for the multi-band scenario. Furthermore, analogy to the pricing strategy (7) is also available for multi-band auction by adding more constraints to make sublease collusion unprofitable.

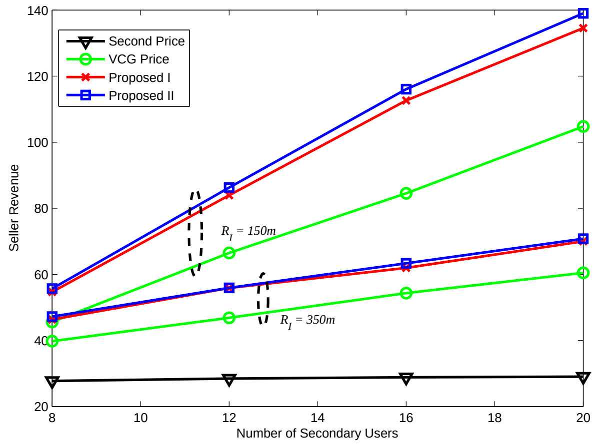


Fig. 3. Seller's revenue when different auction mechanisms are employed.

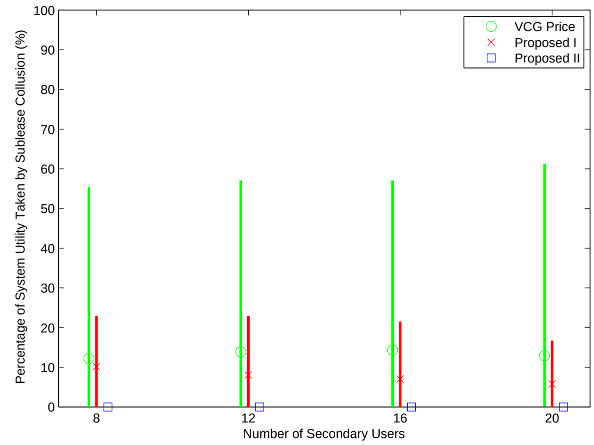


Fig. 4. Normalized collusion gains when different auction mechanisms are employed.

V. Simulation Results

In this section, we evaluate the performance of the proposed collusion-resistant multi-winner spectrum auction mechanisms by computer experiments. Consider a 1000×1000 m^2 area, in which N secondary users are uniformly distributed. Assume each secondary user is a cognitive base station with R_I-meter coverage radius, that is, two users at least $2R_I$ meters away can share the same band without mutual interference. The valuations of different users $\{v_1, v_2, \ldots, v_N\}$ are assumed to be i. i. d. random variables uniformly distributed in $[20, 30]$.

First, we consider the one-band auction, i.e., $M = 1$. Fig. 3 shows the seller's revenue versus the number of secondary users when different auction mechanisms are employed, with $R_I = 150$ or 350. The result is averaged over 100 independent runs, in which the locations and valuations of the N secondary users are generated randomly. When the second-price auction which assigns the band to the user with the highest valuation is used, the spectrum resources do not get fully utilized, and hence the revenue is low. On the other hand, the other three mechanisms guarantee the efficiency of spectrum utilization, but the primary user's revenue differs when various pricing strategies are used. Here, we refer to pricing strategies (4), (5), (7) as "VCG price", "Proposed I", and "Proposed II", respectively. As shown in the figure, the proposed methods can significantly improve the primary user's revenue, e.g., nearly 15% increase compared to the VCG outcome when $R_I = 350$, and 30% increase when $R_I = 150$. This means the proposed algorithms have better performance when more secondary users are admitted to lease the band simultaneously.

Moreover, the proposed auction mechanisms can effectively combat user collusion. We use the percentage of the system utility taken away by colluders to represent the vulnerability to sublease colluding attacks. Fig. 4 demonstrates the results from 100 independent runs, when the number of the secondary users are 8, 12, 16, and 20, respectively. We use a line segment to represent the range of the results, and a marker to represent their mean. With the VCG pricing strategy, colluders could steal away more than 10% of the social welfare on average, while in the worst case, they may even grasp up to half of the system utility. If using pricing strategy (5) instead, the system will be more robust against colluder attacks, as colluding gains drop considerably. Furthermore, the proposed strategy (7) can completely prevent user collusion, as shown in the figure.

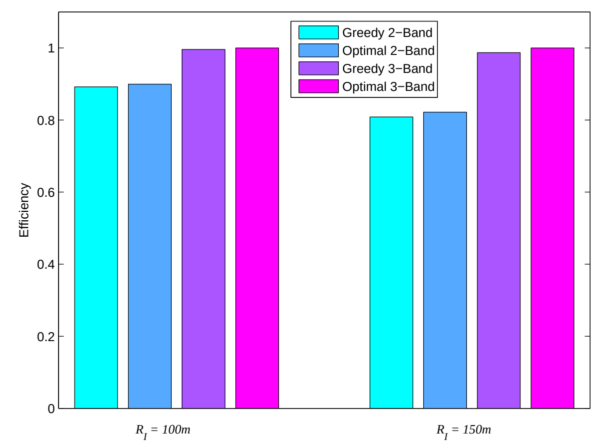


Fig. 5. Approximate efficiency of the greedy algorithm compared to the optimal solution.

Finally, we show that for the multi-band auction ($M > 1$) the proposed greedy algorithm can approximately achieve the efficient allocation. As illustrated in Fig. 5, the normalized system utilities are evaluated for both the greedy and optimal algorithms. We can see that for both two-band and three-band auction cases, the proposed greedy algorithm (9) can achieve a comparable outcome to the optimal solution to (8).

VI. Conclusions

In this paper, we propose a multi-winner auction framework for the spectrum auction in cognitive radio networks, in which secondary users can lease some unused bands from primary users. After showing that the VCG mechanism has several drawbacks by several examples, we propose new auction mechanisms which guarantee full efficiency of the spectrum utilization, yield higher revenue to primary users, and help eliminate user collusion as well. Since the auction outcome largely depends on the interference constraints reported by secondary users themselves, we investigate whether secondary users can take advantages by distorting this information, and conclude they will not. We further extend the one-band auction mechanism to the multi-band case, and propose a greedy algorithm that can achieve almost the same efficiency as the optimal solution with reduced complexity.

APPENDIX

Proof of eqn. (6):

Proof: Let $q_i = v_i - p_i$ for $i \in W$ and use the fact $\sum_{i\in W} v_i = U^*_{\boldsymbol{v}}$, the optimization problem (5) is equivalent to the following convex optimization problem

$$\begin{aligned} \min_{\{q_i, i\in W\}} & -\sum_{i\in W} \log q_i, \\ \text{s.t. } & \sum_{i\in W} q_i = U^*_{\boldsymbol{v}} - U^*_{\boldsymbol{v}_{-W}} \triangleq \Delta U, \\ & q_i - v_i \le 0, \\ & q_i > 0. \end{aligned} \tag{10}$$

Introducing Lagrange multiplier λ and $\mu_i \ge 0, i \in W$, the Lagrangian function is

$$L(\boldsymbol{q}, \lambda, \boldsymbol{\mu}) = -\sum_{i\in W} \log q_i + \lambda(\sum_{i\in W} q_i - \Delta U) + \sum_{i\in W} \mu_i (q_i - v_i), \tag{11}$$

from which KKT conditions can be obtained as follows,

$$\begin{aligned} & q_i = \frac{1}{\lambda + \mu_i}, \mu_i \ge 0, \mu_i(q_i - v_i) = 0, \; i \in W, \\ & \sum_{i\in W} q_i = \Delta U. \end{aligned} \tag{12}$$

After some manipulations, the solution is

$$q_i = \begin{cases} v_i & v_i \le \rho \\ \rho & v_i > \rho \end{cases}, \tag{13}$$

with $\rho \triangleq \frac{1}{\lambda}$. Finally, $p_i = v_i - q_i$ yields (6). ■

REFERENCES

[1] Federal Communications Commission, "Spectrum policy task force report," *FCC Document ET Docket No. 02-135*, Nov. 2002.

[2] J. Mitola III, "Cognitive radio: an integrated agent architecture for software defined radio," Ph.D. Thesis, KTH Royal Institute of Technology, Stockholm, Sweden, 2000.

[3] H. Zheng, and C. Peng, "Collaboration and fairness in opportunistic spectrum access," in *Proc. of IEEE International Conference on Communications (ICC'05)*, pp. 3132–3136, Seoul, May 2005.

[4] S. Keshavamurthy, and K. Chandra, "Multiplexing analysis for spectrum sharing," *Proc. of IEEE MILCOMM'06*, pp. 1–7, Washington D.C., Oct. 2006.

[5] B. Wang, Z. Ji, and K. J. R. Liu, "Primary-prioritized Markov approach for dynamic spectrum access," *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'07)*, pp. 507–515, Dublin, Apr. 2007.

[6] R. Tandra, and A. Sahai, "SNR walls for feature detectors," *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'07)*, pp. 559–570, Dublin, Apr. 2007.

[7] M. M. Halldorson, J. Y. Halpern, L. Li, and V. S Mirrokni, "On spectrum sharing games," *Proc. ACM on Principles of distributed computing*, pp. 107–114, 2004.

[8] O. Ileri, D. Samardzija, and N. B. Mandayam, "Demand responsive pricing and competitive spectrum allocation via a spectrum server," *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'05)*, pp. 194–202, Baltimore, Nov. 2005.

[9] J. Huang, R. Berry, and M. L. Honig, "Auction-based spectrum sharing," *ACM/Springer Mobile Networks and Apps.,*, vol. 11, no. 3, pp. 405–418, June 2006.

[10] C. Kloeck, H. Jaekel, and F. K. Jondral, "Dynamic and local combined pricing, allocation and billing system with coginitive radios," *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'05)*, pp. 73–81, Baltimore, Nov. 2005.

[11] S. Gandhi, C. Buragohain, L. Cao, H. Zheng, and S. Suri, "A general framework for wireless spectrum auctions," *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN'07)*, pp. 22–33, Dublin, Apr. 2007.

[12] Z. Ji, and K. J. R. Liu, "Belief-assisted pricing for dynamic spectrum allocation in wireless networks with selfish users," in *Proc. IEEE Int'l Conference on Sensor, Mesh, and Ad Hoc Communications and Networks (SECON)*, pp. 119–127, Reston, Sep. 2006.

[13] V. Krishna, *Auction theory*, Academic Press, 2002.

[14] P. Cramton, Y. Shoham, and R. Steinberg, *Combinatorial auctions*, MIT Press, 2006.

[15] Z. Ji, and K. J. R. Liu, "Multi-stage pricing game for collusion-resistant dynamic spectrum allocation," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 182–191, Jan. 2008.

[16] M. J. Osborne, *An introduction to game theory*, Oxford University Press, 2004.

[17] S. P. Boyd, and L. Vandenberghe, *Convex optimization*, Cambridge University Press, 2004.