

---

# Network Analyzer Error Models and Calibration Methods

by  
Doug Rytting



---

This paper is an overview of error models and calibration methods for vector network analyzers.

---

# Presentation Outline

---

Network Analyzer Block Diagram and Error Model

System Error Model for Error-Correction

One-Port Error Model and Calibration

Two-Port Error Models and Calibration

12-Term Method

8-Term Method

16-Term Method



---

A system error model will be derived from a generic network analyzer block diagram. This error model will then be simplified to the standard one-port and 12-term two-port models used the past 30 years.

Newer 8-term and 16-term models will then be introduced and the modern calibration approaches described.

---

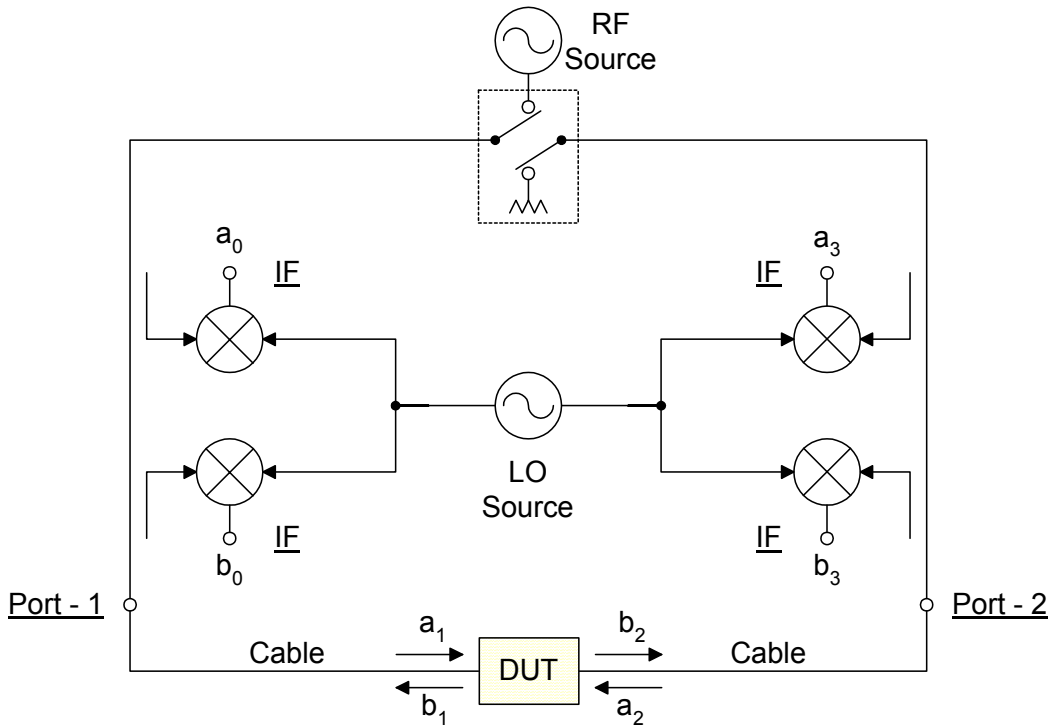
# Network Analyzer Block Diagram and Error Model



---

First the block diagram for a network analyzer is described and the hardware flow graph is defined.

# Network Analyzer Block Diagram



This is a generic block diagram of a 4 channel network analyzer. The source can be switched to excite port-1 or port-2 of the device under test (DUT). The switch also provides a  $Z_0$  termination for the output port in each direction. Directional couplers are used to separate the incident, reflected and transmitted waves in both the forward and reverse direction. Mixers are used to down convert the RF signals to a fixed low frequency IF. The LO source is tuned to the frequency of the RF + IF.

The s-parameters of the DUT can be defined as follows:

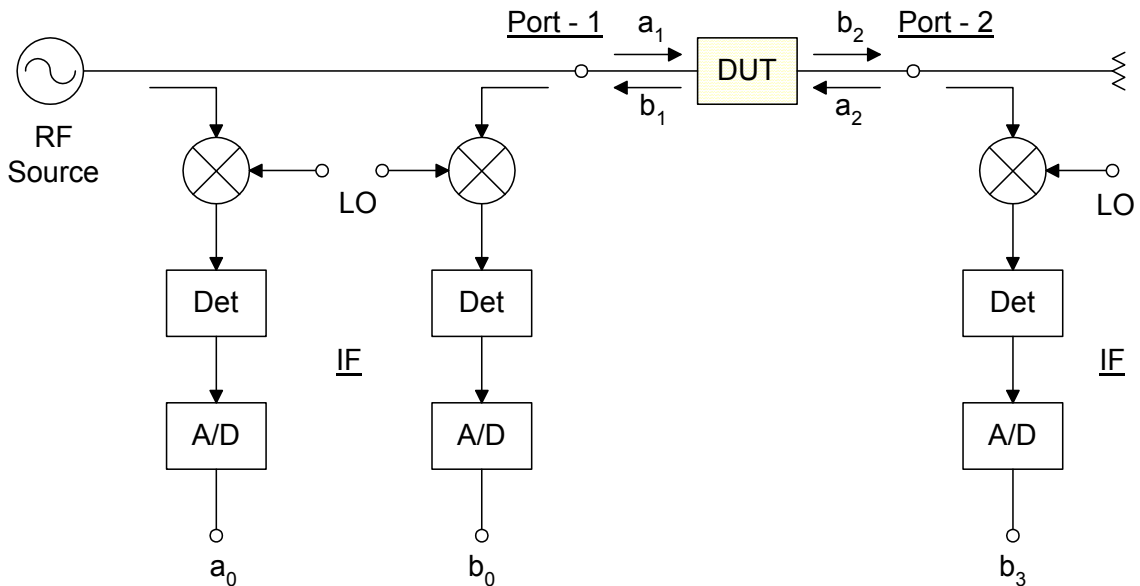
$$S_{11} = b_1/a_1, \text{ switch in forward direction}$$

$$S_{21} = b_2/a_1, \text{ switch in forward direction}$$

$$S_{12} = b_1/a_2, \text{ switch in reverse direction}$$

$$S_{22} = b_2/a_2, \text{ switch in reverse direction}$$

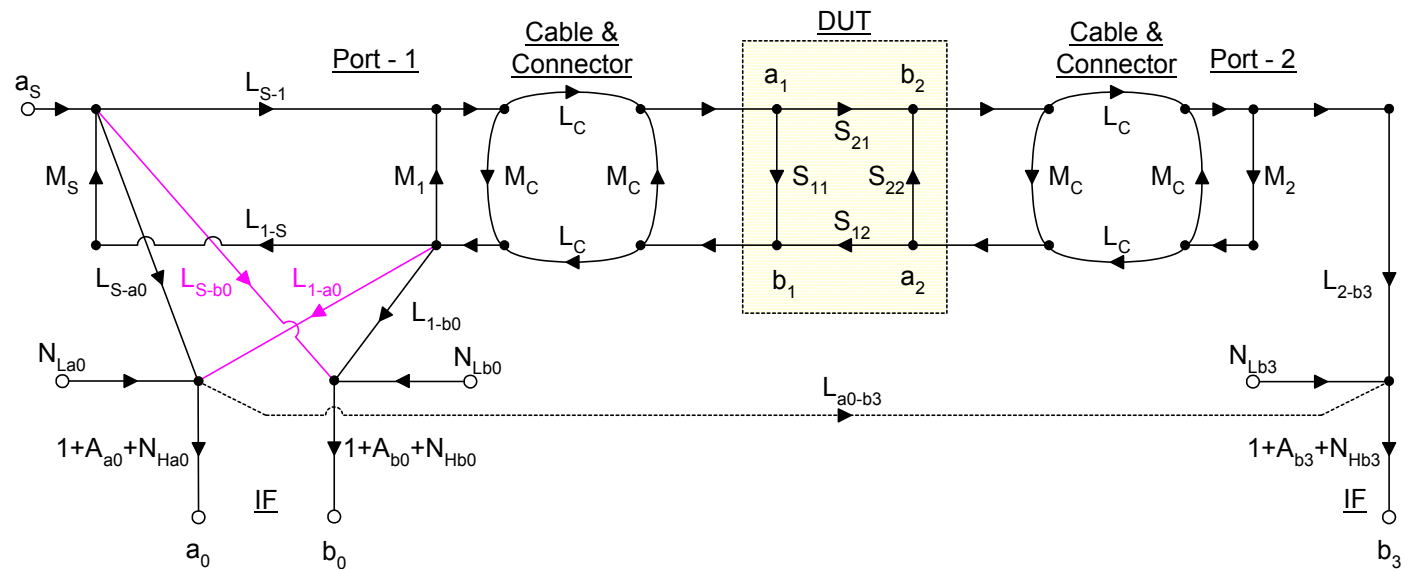
# Forward Direction Block Diagram



This block diagram shows the measurement system switched to the forward direction. Each of the IF signals are detected and digitized and the real and imaginary terms are measured. From this data the magnitude and phase can be calculated.

In most modern network analyzers the A/D digitizes directly at the IF and the detection is done in the digital domain. The resultant digitized versions of the DUT waves ( $a_0$ ,  $b_0$ , and  $b_3$ ) are a scaled version of the actual waves at the DUT ( $a_1$ ,  $b_1$ , and  $b_2$ ).

# Error Model for Forward Direction



Branches With No Label = 1

From the block diagram a flowgraph can be developed showing all the possible signal paths. These paths not only include the main desired signals but the loss, match errors, and leakage errors, of the network analyzer along with the cables, connectors, or probes that connect to to DUT.

Also included in this model are the IF, A/D and detector non linearities and the system noise.

# Error Model Definitions

$a_1$  = Incident Signal at Port-1  
 $b_1$  = Reflected Signal at Port-1  
 $a_2$  = Incident Signal at Port-2  
 $b_2$  = Transmitted signal at Port-2

$a_s$  = Source Port  
 $a_0$  = Measured Incident Port  
 $b_0$  = Measured Reflected Port  
 $b_3$  = Measured Transmitted Port

$L_{S-1}$  = Loss from Source to Port-1  
 $L_{1-S}$  = Loss from Port-1 to Source  
 $L_{S-a_0}$  = Loss from Source to  $a_0$   
 $L_{S-b_0}$  = Loss from Source to  $b_0$  (Directivity)  
 $L_{1-a_0}$  = Loss from Port-1 to  $a_0$  (Directivity)  
 $L_{1-b_0}$  = Loss from Port-1 to  $b_0$   
 $L_{2-b_3}$  = Loss from Port-2 to  $b_3$   
 $L_{a_0-b_3}$  = Loss from  $a_0$  to  $b_3$  (Leakage)  
 $L_C$  = Loss of Cables

$S_{11}$  = Refl Coef of DUT at Port-1  
 $S_{21}$  = Forward Trans Coef of DUT  
 $S_{12}$  = Reverse Trans Coef of DUT  
 $S_{22}$  = Refl Coef of DUT at Port-2

$M_1$  = Match at Port-1  
 $M_2$  = Match at Port-2  
 $M_S$  = Match of Source  
 $M_C$  = Match of Cables

$N_{La0}$  = Low Level Noise at  $a_0$   
 $N_{Lb0}$  = Low Level Noise at  $b_0$   
 $N_{Lb3}$  = Low Level Noise at  $b_3$   
 $N_{Ha0}$  = High Level Noise at  $a_0$   
 $N_{Hb0}$  = High Level Noise at  $b_0$   
 $N_{Hb3}$  = High Level Noise at  $b_3$

$A_{a_0}$  = Dynamic Accuracy at  $a_0$  (Linearity)  
 $A_{b_0}$  = Dynamic Accuracy at  $b_0$  (Linearity)  
 $A_{b_3}$  = Dynamic Accuracy at  $b_3$  (Linearity)



The above table gives the description of each of the branches and the key nodes for the flow graph. This provides a very complete model for the network analyzer. However it is possible to reduce the flow graph without any loss in accuracy. This reduced flow graph is much easier to analyze and will be discussed next.

---

# System Error Model for Error-Correction

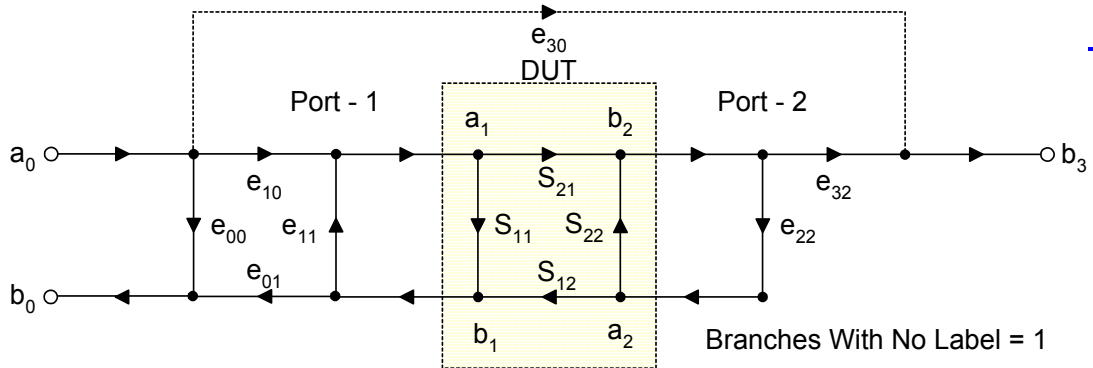


---

The simplified system error model is described. This system model will be used to develop the error correction procedure.



# System Error Model for Forward Direction



**Directivity**  $e_{00} \cong \frac{L_{S-b0}}{L_{S-a0}}$

**Leakage**  $e_{30} \cong L_{a0-b3}$

**Reflection Tracking**  $e_{10}e_{01} \cong \frac{L_{S-1}L_{1-b0}}{L_{S-a0}} - \frac{L_{S-1}L_{1-a0}L_{S-b0}}{(L_{S-a0})^2}$

**Transmission Tracking**  $e_{10}e_{32} \cong \frac{L_{S1}L_{2-b3}}{L_{S-a0}}$

**Port-1 Match**  $e_{11} \cong M_2 - \frac{L_{S-1}L_{1-a0}}{L_{S-a0}}$

**Port-2 Match**  $e_{22} \cong M_2$

There are also Errors caused by the Converter, IF, Cables and Connectors

There are also Six Terms in the Reverse Direction



The resultant system error model is the forward portion of the well known 12-term error mode. Each of the branches have an accurate relationship to the original hardware oriented flow graph presented earlier. The 6 forward terms described above show a simplified set of equations relating the two flow graphs.

The directivity error is caused primarily by the coupler leakage or ‘coupler directivity.’ This error is also increased by cable and connector match errors between the measurement coupler and the DUT. The reflection and transmission tracking is caused by reflectometer and mixer tracking as well as cable length imbalance between the measured ports. The match error is the mathematical ratioed port match error that is not necessarily the ‘raw’ port match. The leakage error is through the LO path of the mixers. It is not the leakage of the switch and this model assumes the switch leakage is negligible.

# Improvements with Error-Correction

## ERRORS REMOVED

$L_{S-1}$  = Loss from Source to Port-1  
 $L_{1-S}$  = Loss from Port-1 to Source  
 $L_{S-a_0}$  = Loss from Source to  $a_0$   
 $L_{S-b_0}$  = Loss from Source to  $b_0$  (Directivity)  
 $L_{1-a_0}$  = Loss from Port-1 to  $a_0$  (Directivity)  
 $L_{1-b_0}$  = Loss from Port-1 to  $b_0$   
 $L_{2-b_3}$  = Loss from Port-2 to  $b_3$   
 $L_{a_0-b_3}$  = Loss from  $a_0$  to  $b_3$  (Leakage)  
 $L_C$  = Loss of Cables

$M_1$  = Match at Port-1  
 $M_2$  = Match at Port-2  
 $M_S$  = Match of Source  
 $M_C$  = Match of Cables

## ERRORS REMAINING

$N_{La_0}$  &  $N_{Ha_0}$  = Noise at  $a_0$   
 $N_{Lb_0}$  &  $N_{Hb_0}$  = Noise at  $b_0$   
 $N_{Lb_3}$  &  $N_{Hb_3}$  = Noise at  $b_3$

$A_{a_0}$  = Dynamic Accuracy at  $a_0$  (Linearity)  
 $A_{b_0}$  = Dynamic Accuracy at  $b_0$  (Linearity)  
 $A_{b_3}$  = Dynamic Accuracy at  $b_3$  (Linearity)

Drift after Error-Correction

Stability after Error-Correction

Repeatability

Errors of Calibration Standards



A linear calibration procedure is then applied to remove as many of the errors as possible. The loss and match errors can be greatly reduced depending on the accuracy of the calibration standards used.

However, the noise and linearity errors can not be reduced using a simple linear calibration procedure. In fact the noise and linearity errors increase a small amount.

Once the network analyzer is calibrated the drift, stability, and repeatability errors will degrade the system performance. This usually means that the system will need to be recalibrated at some interval depending on the system usage, environment and required accuracy.

---

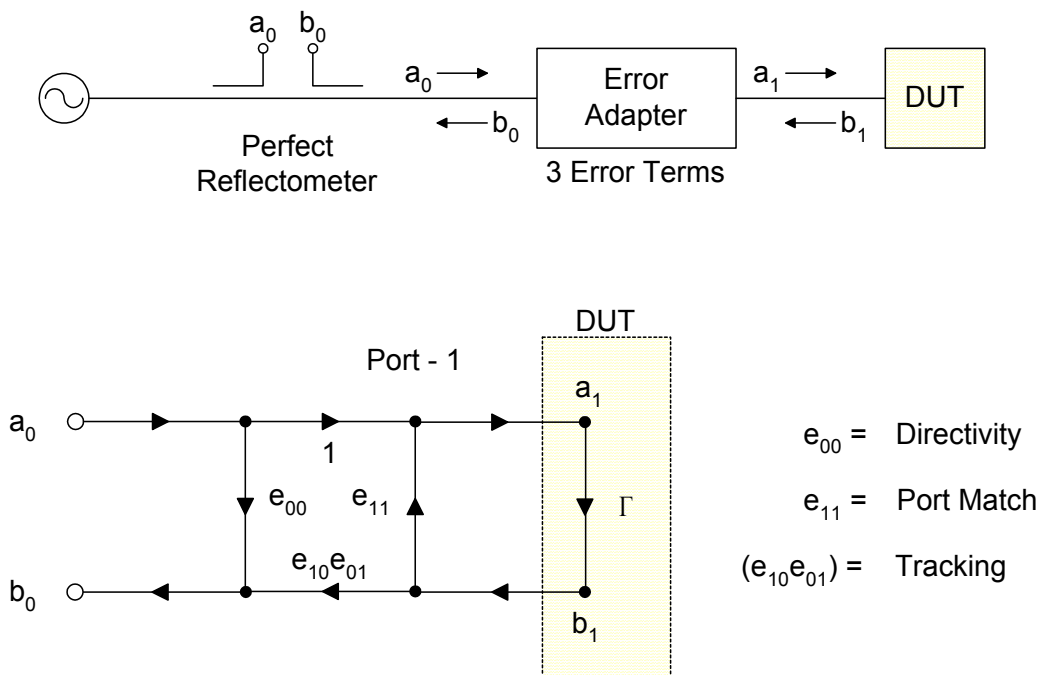
# One-Port Error Model and Calibration



---

The one-port model will be first developed. This will then be used to further develop the two-port model.

# One-Port, 3-Term Error Model



The one-port calibration procedure will now be described. The 12-term model described earlier simplifies considerably when a one-port device is being measured. The model simplifies to just the terms describing the directivity, port match, and tracking errors at each port.

The errors can be lumped into a fictitious error adapter that modifies the actual DUT reflection coefficient which is then measured by a 'perfect' reflectometer.

# One-Port Calibration Method

$$\begin{array}{cc} \text{Measured} & \text{Actual} \\ \Gamma_M = \frac{b_0}{a_0} = \frac{e_{00} - \Delta_e \Gamma}{1 - e_{11} \Gamma} & \Gamma = \frac{\Gamma_M - e_{00}}{\Gamma_M e_{11} - \Delta_e} \end{array}$$

$$\Delta_e = e_{00} e_{11} - (e_{10} e_{01})$$

For ratio measurements there are 3 error terms  
The equation can be written in the linear form

$$e_{00} + \Gamma \Gamma_M e_{11} - \Gamma \Delta_e = \Gamma_M$$

With 3 different known  $\Gamma$ , measure the resultant 3  $\Gamma_M$   
This yields 3 equations to solve for  $e_{00}$ ,  $e_{11}$ , and  $\Delta_e$

$$e_{00} + \Gamma_1 \Gamma_{M1} e_{11} - \Gamma_1 \Delta_e = \Gamma_{M1}$$

$$e_{00} + \Gamma_2 \Gamma_{M2} e_{11} - \Gamma_2 \Delta_e = \Gamma_{M2}$$

$$e_{00} + \Gamma_3 \Gamma_{M3} e_{11} - \Gamma_3 \Delta_e = \Gamma_{M3}$$

Any 3 independent measurements can be used

Solving the one-port flow graph yields a bilinear relationship between the actual and measured reflection coefficient. The actual reflection coefficient is ‘mapped’ or modified by the three error terms to the measured result. This equation can be inverted to solve for the actual reflection coefficient knowing the measured result and the three error terms.

The three error terms can be determined by measuring three known standards (such as an open, short and load) that yield three simultaneous equations. These three equations can then be solved for the three error terms.

---

# Two-Port Error Models and Calibration

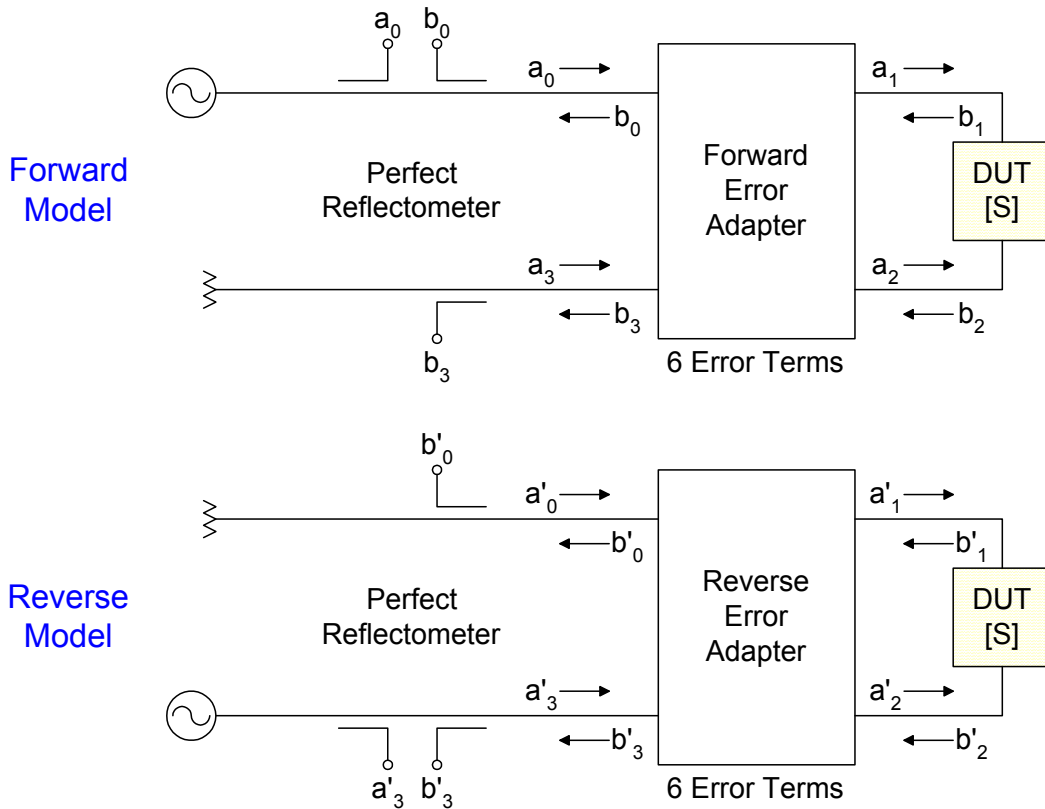
12-Term Method  
8-Term Method  
16-Term Method



---

The classic 12-term model will be developed first. Then the more recent 8-term and 16-term models will be described.

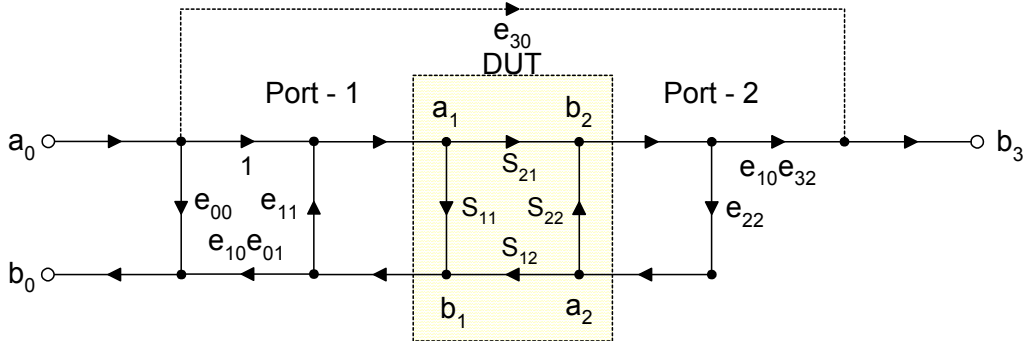
# 12-Term Error Model



The two-port case can be modeled in the same manner as the one-port. A fictitious error adapter is placed between the two-port DUT and the 'perfect reflectometer' measurement ports. This error adapter contains the 6 error terms for the forward direction. A similar 6 term model is used in the reverse direction.

# 12-Term Error Model

## FORWARD MODEL



$e_{00}$  = Directivity

$e_{11}$  = Port-1 Match

$(e_{10}e_{01})$  = Reflection Tracking

$(e_{10}e_{32})$  = Transmission Tracking

$e_{22}$  = Port-2 Match

$e_{30}$  = Leakage

$$S_{11M} = \frac{b_0}{a_0} = e_{00} + (e_{10}e_{01}) \frac{S_{11} - e_{22}\Delta_S}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\Delta_S}$$

$$S_{21M} = \frac{b_3}{a_0} = e_{30} + (e_{10}e_{32}) \frac{S_{21}}{1 - e_{11}S_{11} - e_{22}S_{22} + e_{11}e_{22}\Delta_S}$$

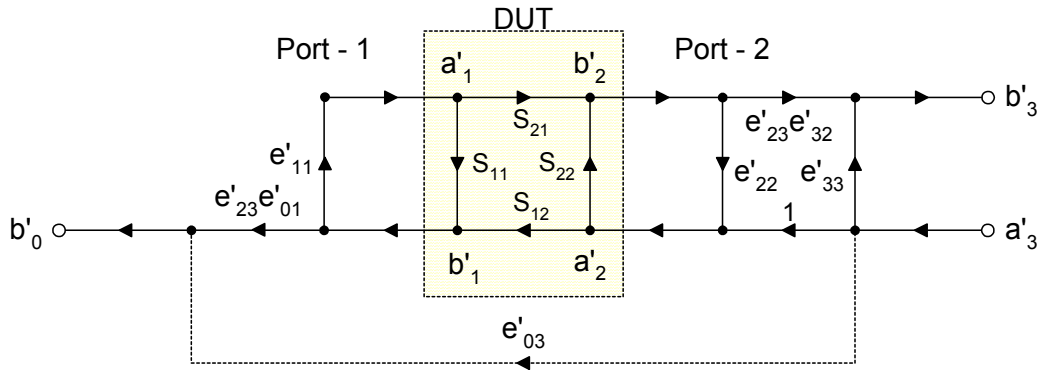
$$\Delta_S = S_{11}S_{22} - S_{21}S_{12}$$

Solving the forward flow graph yields measurements  $S_{11M}$  and  $S_{21M}$ . These two equations contain all four actual s-parameters of the DUT and the six forward error terms.



# 12-Term Error Model

## REVERSE MODEL



$e'_{33}$  = Directivity

$e'_{11}$  = Port-1 Match

$(e'_{23}e'_{32})$  = Reflection Tracking

$(e'_{23}e'_{01})$  = Transmission Tracking

$e'_{22}$  = Port-2 Match

$e'_{03}$  = Leakage

$$S_{22M} = \frac{b'_3}{a'_3} = e'_{33} + (e'_{23}e'_{32}) \frac{S_{22} - e'_{11} \Delta_S}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22} \Delta_S}$$

$$S_{12M} = \frac{b'_0}{a'_3} = e'_{03} + (e'_{23}e'_{01}) \frac{S_{12}}{1 - e'_{11}S_{11} - e'_{22}S_{22} + e'_{11}e'_{22} \Delta_S}$$

$$\Delta_S = S_{11}S_{22} - S_{21}S_{12}$$

Solving the reverse flow graph yields measurements  $S_{22M}$  and  $S_{12M}$ . These two equations contain all four actual s-parameters of the DUT and the six reverse error terms.

The forward and reverse equations combine to give four equations containing the four actual s-parameters of the DUT and 12 error terms. If the 12 error terms are known these four equations can be solved for the actual s-parameters of the DUT.

# 12-Term Calibration Method

STEP 1: Calibrate Port-1 using One-Port procedure

Solve for  $e_{11}$ ,  $e_{00}$ , &  $(e_{10}e_{01})$ , Calculate  $(e_{10}e_{01})$  from  $\Delta_e$

STEP 2: Connect  $Z_0$  terminations to Ports 1 & 2

Measure  $S_{21M}$  gives  $e_{30}$  directly

STEP 3: Connect Ports 1 & 2 together

$$e_{22} = \frac{S_{11M} - e_{00}}{S_{11M}e_{11} - \Delta_e}$$

$$e_{10}e_{32} = (S_{21M} - e_{30})(1 - e_{11}e_{22})$$

Use the same process for the reverse model



The 12 error terms will now be determined. First solve for the 6 terms in the forward direction. Then the same procedure can be used to solve for the 6 reverse terms.

Step one calibrates port-1 of the network analyzer using the same procedure used in the one-port case. This determines the directivity, match, and reflection tracking at port-1 ( $e_{00}$ ,  $e_{11}$ , and  $e_{10}e_{01}$ ).

Step two measures the leakage or crosstalk error ( $e_{30}$ ) from port-1 to port-2 directly by placing loads on each of the ports.

Step three consists of connecting port-1 and port-2 together. Then measure the port-2 match ( $e_{22}$ ) directly with the calibrated port-1 reflectometer. Then with the ports connected, measure the transmitted signal and calculate the transmission tracking ( $e_{10}e_{32}$ ).

## 12-Term Calibration Method

$$S_{11} = \frac{\left(\frac{S_{11M} - e_{00}}{e_{10} e_{01}}\right) \left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23} e'_{32}}\right) e'_{22}\right] - e_{22} \left(\frac{S_{21M} - e_{30}}{e_{10} e_{32}}\right) \left(\frac{S_{12M} - e'_{03}}{e'_{23} e'_{01}}\right)}{D}$$

$$S_{21} = \frac{\left(\frac{S_{21M} - e_{30}}{e_{10} e_{32}}\right) \left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23} e'_{32}}\right) (e'_{22} - e_{22})\right]}{D}$$

$$S_{22} = \frac{\left(\frac{S_{22M} - e'_{33}}{e'_{23} e'_{32}}\right) \left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10} e_{01}}\right) e_{11}\right] - e'_{11} \left(\frac{S_{21M} - e_{30}}{e_{10} e_{32}}\right) \left(\frac{S_{12M} - e'_{03}}{e'_{23} e'_{01}}\right)}{D}$$

$$S_{12} = \frac{\left(\frac{S_{12M} - e'_{03}}{e'_{23} e'_{01}}\right) \left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10} e_{01}}\right) (e_{11} - e'_{11})\right]}{D}$$

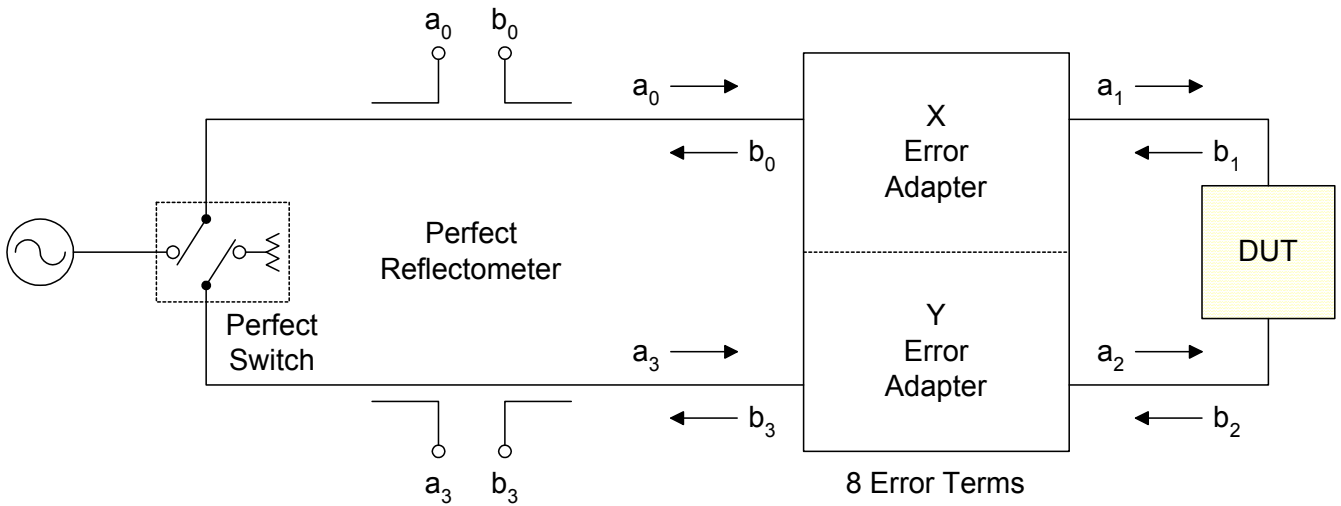
$$D = \left[1 + \left(\frac{S_{11M} - e_{00}}{e_{10} e_{01}}\right) e_{11}\right] \left[1 + \left(\frac{S_{22M} - e'_{33}}{e'_{23} e'_{32}}\right) e'_{22}\right] - \left(\frac{S_{21M} - e_{30}}{e_{10} e_{32}}\right) \left(\frac{S_{12M} - e'_{03}}{e'_{23} e'_{01}}\right) e_{22} e'_{11}$$



This is the result of solving the four simultaneous measured s-parameter equations. Note that each actual s-parameter calculated requires measuring all four s-parameters as well as knowing the 12 error terms.

Refer to references [1] and [2] for more details.

# 8-Term Error Model

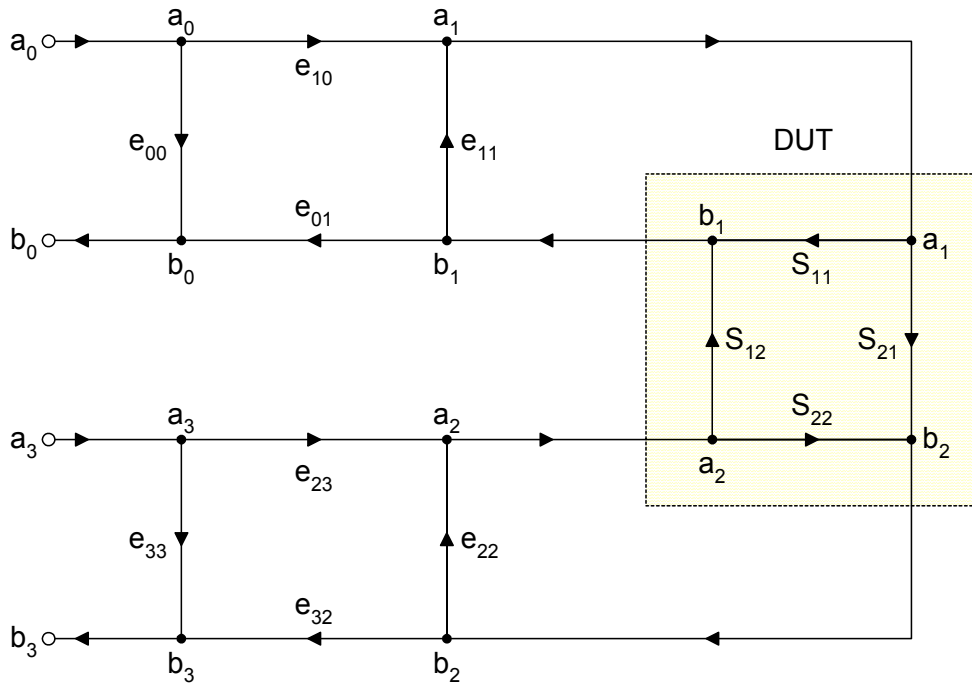


To remove the effects of an imperfect switch, use the procedure described later.

The 8-term model can be derived from the 12-term model. First assume that the crosstalk leakage term is zero. Or that it can be determined in a separate calibration step. Then assume that the switch is perfect and does not change the port match of the network analyzer as it is switched from forward to reverse. This assumption is valid if there are 4 measurement channels that are all on the DUT side of the switch. Then it is possible to mathematically ratio out the switch. This mathematical approach will be explained at the end of this paper.

The same error adapter approach can now be used to describe the 8 error terms.

# 8-Term Error Model



One of the 8 error terms can be normalized to yield 7 error terms

The flow graph consists of an error adapter at the input and output of the DUT. For ratio measurements of s-parameters, the number of error terms is reduced to 7 since the error terms can be normalized.

# 8-Term Error Model

Using the cascade parameters in matrix form yields

MEASURED

$$\mathbf{T}_M = \mathbf{T}_X \mathbf{T} \mathbf{T}_Y$$

$$\mathbf{T} = \frac{1}{S_{21}} \begin{bmatrix} -\Delta_S & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

$$\Delta_S = S_{11}S_{22} - S_{12}S_{21}$$

$$\mathbf{T}_X = \frac{1}{e_{10}} \begin{bmatrix} -\Delta_X & e_{00} \\ -e_{11} & 1 \end{bmatrix}$$

$$\Delta_X = e_{00}e_{11} - e_{10}e_{01}$$

$$\mathbf{T}_M = \frac{1}{(e_{10}e_{32})} \begin{bmatrix} -\Delta_X & e_{00} \\ -e_{11} & 1 \end{bmatrix} \mathbf{T} \begin{bmatrix} -\Delta_Y & e_{22} \\ -e_{33} & 1 \end{bmatrix} = \frac{1}{(e_{10}e_{32})} \mathbf{ATB}$$

ACTUAL

$$\mathbf{T} = \mathbf{T}_X^{-1} \mathbf{T}_M \mathbf{T}_Y^{-1}$$

$$\mathbf{T}_M = \frac{1}{S_{21M}} \begin{bmatrix} -\Delta_M & S_{11M} \\ -S_{22M} & 1 \end{bmatrix}$$

$$\Delta_M = S_{11M}S_{22M} - S_{12M}S_{21M}$$

$$\mathbf{T}_Y = \frac{1}{e_{32}} \begin{bmatrix} -\Delta_Y & e_{22} \\ -e_{33} & 1 \end{bmatrix}$$

$$\Delta_Y = e_{22}e_{33} - e_{32}e_{23}$$



Note that the flow graph is a cascade of the input error box (X), the DUT, and the output error box (Y). The measured result of this cascade is most easily calculated by using the cascade matrix definition (t-parameters).

This formulation was used by Engen and Hoer in their classic TRL development for the six-port network analyzer [3]. And is the same approach used in the HP 8510 network analyzer.

From the last equation in the slide above, the 7 error terms are easily identified. There are 3 at port-1 ( $\Delta_X$ ,  $e_{00}$ , and  $e_{11}$ ) and 3 at port-2 ( $\Delta_Y$ ,  $e_{22}$ , and  $e_{33}$ ) and one transmission term ( $e_{10}e_{32}$ ).

The calibration approach require enough calibration standards to allow at least 7 independent observations of the measurement system.

Refer to reference [4] for more details.

# 8-Term Error Model

Forming the Equations Differently Yields:

$$\begin{bmatrix} b_0 \\ b_3 \\ a_0 \\ a_3 \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{T}_2 \\ \mathbf{T}_3 & \mathbf{T}_4 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{bmatrix}$$

$$\mathbf{T}_1 = \begin{bmatrix} -\Delta_x & 0 \\ 0 & -k\Delta_y \end{bmatrix}$$

$$\mathbf{T}_2 = \begin{bmatrix} e_{00} & 0 \\ 0 & ke_{33} \end{bmatrix}$$

$$\mathbf{T}_3 = \begin{bmatrix} -e_{11} & 0 \\ 0 & -ke_{22} \end{bmatrix}$$

$$\mathbf{T}_4 = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$$

$$k = \frac{e_{10}}{e_{23}}$$



There is another mathematical formulation for the 8 term error model. Consider the error adapter as just one adapter between the perfect measurement system and the DUT. Then model this error adapter using the cascade t-parameters. This t-parameter matrix (T) can be partitioned into the four sub matrixes  $\mathbf{T}_1$ ,  $\mathbf{T}_2$ ,  $\mathbf{T}_3$ , and  $\mathbf{T}_4$ . The 7 error terms are now defined as  $\Delta_x$ ,  $k\Delta_y$ ,  $e_{00}$ ,  $ke_{33}$ ,  $e_{11}$ ,  $ke_{22}$ , and  $k$ .

# 8-Term Error Model

Measured S-Parameters

$$\mathbf{S}_M = (\mathbf{T}_1 \mathbf{S} + \mathbf{T}_2)(\mathbf{T}_3 \mathbf{S} + \mathbf{T}_4)^{-1}$$

Actual S-Parameters

$$\mathbf{S} = (\mathbf{T}_1 - \mathbf{S}_M \mathbf{T}_3)^{-1}(\mathbf{S}_M \mathbf{T}_4 - \mathbf{T}_2)$$

Linear-in-T Form

$$\mathbf{T}_1 \mathbf{S} + \mathbf{T}_2 - \mathbf{S}_M \mathbf{T}_3 \mathbf{S} - \mathbf{S}_M \mathbf{T}_4 = \mathbf{0}$$

Expanding Yields:

$$\begin{array}{cccccccc}
 e_{00} & + S_{11} S_{11M} e_{11} & - S_{11} \Delta_X & + 0 & + S_{21} S_{12M} (k e_{22}) & + 0 & + 0 & = S_{11M} \\
 0 & + S_{12} S_{11M} e_{11} & - S_{12} \Delta_X & + 0 & + S_{22} S_{12M} (k e_{22}) & + 0 & - S_{12M} k & = 0 \\
 0 & + S_{11} S_{21M} e_{11} & + 0 & + 0 & + S_{21} S_{22M} (k e_{22}) & - S_{21} (k \Delta_Y) & + 0 & = S_{21M} \\
 0 & + S_{12} S_{21M} e_{11} & + 0 & + (k e_{33}) & + S_{22} S_{22M} (k e_{22}) & - S_{22} (k \Delta_Y) & - S_{22M} k & = 0
 \end{array}$$

Using this approach the measured s-parameters formulation is a ‘bilinear matrix equation.’ It looks much the same as the one-port bilinear transformation described earlier. The equation can be easily ‘inverted’ to solve for the actual s-parameters. And most important the relationship can be put in linear form. Expanding this matrix equation for the two-port case yields 4 equations with 4 measured s-parameters, 4 actual s-parameters, and 7 error terms. Note that these 4 equations are linear with regards to the 7 error terms.

This approach is particularly attractive for multi-port measurement systems. The matrix formulation does not change at all as additional ports are added.

Refer to references [5] [6] [7] for more details.



# 8-Term Error Model

Seven or more independent known conditions must be measured  
 A known impedance ( $Z_0$ ) and a port-1 to port-2 connection are required

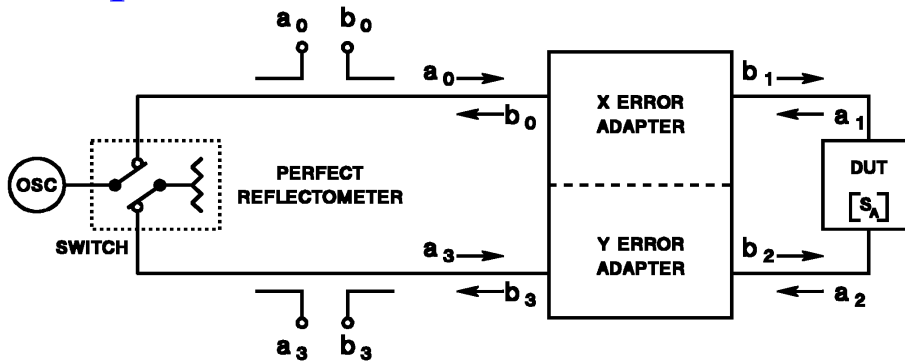
TRL & LRL	Thru (T) or Line (L) with known S-parameters [4 conditions]	Unknown equal Reflect (R) on port-1 and port-2 [1 condition]	Line (L) with known $S_{11}$ and $S_{22}$ [2 conditions]
TRM & LRM	Thru (T) or Line (L) with known S-parameters [4 conditions]	Unknown equal Reflect (R) on port-1 and port-2 [1 condition]	Known Match (M) on port-1 and port-2 [2 conditions]
TXYZ & LXYZ	Thru (T) or Line (L) with known S-parameters [4 conditions]	3 known Reflects (XYZ) on port-1 or port-2 [3 conditions]	
TXYX & LXYY	Thru (T) or Line (L) with known S-parameters [4 conditions]	2 known Reflects (XY) on port-1 [2 conditions]	One known Reflect (X) on port-2 [1 condition]
LRRM	Line (L) with known S-parameters [4 conditions]	2 unknown equal Reflects (RR) on port-1 and port-2 [2 conditions]	Known match (M) on port-1 [1 condition]
UXYZ	Unknown Line (U) with $S_{12} = S_{21}$ [1 condition]	3 known Reflects (XYZ) on port-1 [3 conditions]	3 known Reflects (XYZ) on port-2 [3 conditions]

Using either of the two formulations described, there is a number of calibration techniques that have been developed. Seven or more independent conditions must be measured. There must be a known impedance standard termination or a known transmission line. And port-1 and port-2 must be connected for one of the measurements.

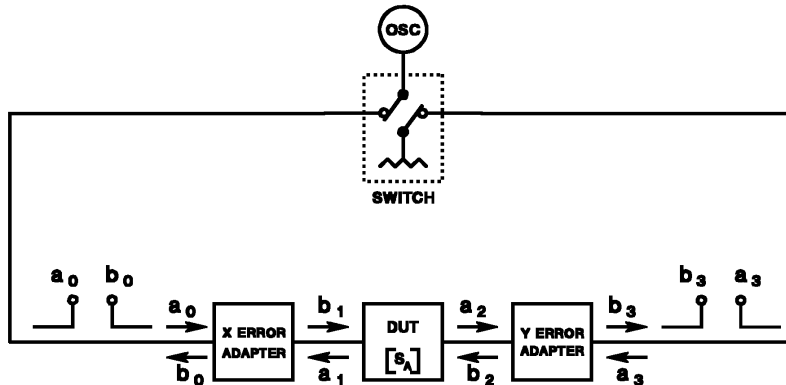
The list of calibration approaches can be much longer than the ones shown above. And there continues to be new and novel ways to solve for the seven error terms and calibrate the system.

The 8 term error model approach has yielded more accurate calibration methods as well as simplified the calibration process. TRL and LRL provide the best accuracy. The other methods simplify the calibration steps compared to the older 12 term model. In one case (UXYZ above) the thru standard does not need to be known as long as it is passive.

# Example: TRL



After removing leakage terms 8 error terms remain



The best know calibration method using the 8-term model is TRL. We will now review this calibration method. The math nomenclature is slightly different in this review.

The first step involves separating the system into a perfect reflectometer followed by a 4-port error adapter. This error adapter represents all the errors in the system that can be corrected. It can be split into two 2-port error adapters, X (at port-1) and Y (at port-2), after removing the leakage (crosstalk) terms as a first step in the calibration. Since X and Y are 2-ports it would appear there are 8 unknowns to find, however since all measurements are made as ratios of the b's and a's, there are actually only 7 error terms to calculate. This means that only 7 characteristics of the calibration standards are required to be known. If a thru (4 known characteristics) is used as one of the standards, only 3 additional characteristics of the standards are needed.

## Example: TRL

(1)  $M = X A Y$  , measured DUT

---

(2)  $M_1 = X C_1 Y$  , measured 2-port cal std #1

(3)  $M_2 = X C_2 Y$  , measured 2-port cal std #2

(4)  $M_3 = X C_3 Y$  , measured 2-port cal std #3



It is convenient to use t-parameters because it allows one to represent the overall measurement,  $M$ , of the DUT,  $A$ , as corrupted by the error adapters as a simple product of the matrixes,

$$M = XAY.$$

In a similar manner, each measurement of three 2-port standards,  $C_1$ ,  $C_2$ , and  $C_3$  can be represented as  $M_1$ ,  $M_2$ , and  $M_3$ .

$$M_1 = XC_1Y$$

$$M_2 = XC_2Y$$

$$M_3 = XC_3Y$$

## Example: TRL

**Measurements of the 3 two-port standards yields 12 independent equations.**

**Only 7 equations are needed to calibrate the system.**

**Equations (2), (3), and (4) can be solved for X.**

**Also 5 terms of the three two-port calibration standards can be determined.**



While there are 7 unknowns, measuring three 2-port standards yields a set of 12 equations. Due to this redundancy, it is not necessary to know all the parameters of all the standards. X and Y can be solved for directly plus 5 characteristics of the calibration standards.

## Example: TRL

- C<sub>1</sub> Must be totally known.**
- C<sub>2</sub> Can have 2 unknown transmission terms.  
The 2 reflection coef must be known.**
- C<sub>3</sub> Can have 3 unknowns.  
If  $S_{11} = S_{22}$ , no other terms are needed.  
Best if highly reflective.**

**The standards must be independent from each other.**



All 4 parameters of  $C_1$  must be known but only 2 parameters for  $C_2$  and none for  $C_3$  if  $S_{11} = S_{22}$ . The simplest of all standards is a through line, so let  $C_1$  be a thru and  $C_2$  a  $Z_0$  matched device. If needed, impedance renormalization can be used to shift to a different impedance base. The other parameters of  $C_2$  and  $C_3$  can be solved from the data.

For this calibration method there are several combinations of standards that fit the requirements. However, there are also choices that generate ill-conditioned solutions or singularities. In choosing appropriate standards, one standard needs to be  $Z_0$  based, one needs to present a high mismatch reflection, and one needs to connect port-1 to port-2. In addition, all three standards need to be sufficiently different to create three independent measurements.

## Example: TRL

### Possible Combinations of Two-Port Standards Must know 7 characteristics

Cal Type	Std C <sub>1</sub>	Std C <sub>3</sub>	Std C <sub>2</sub>
	[S]	$S_{11} = S_{22}$	$S_{11}$ & $S_{22}$
TRL	Thru	Reflect	Line
TRM	Thru	Reflect	Match
TRA	Thru	Reflect	Attenuator
LRL	Line	Reflect	Line
LRM	Line	Reflect	Match
LRA	Line	Reflect	Attenuator



There are several possible strategies in choosing standards. For the first standard ( $C_1$ ), the use of a zero length thru is an obvious selection. But a non-zero length thru is also acceptable if its characteristics are known or the desired reference plane is in the center of the non-zero length thru. This standard will determine 4 of the error terms.

The second standard ( $C_2$ ) needs to provide a  $Z_0$  reference. In this solution, only the match of this standard needs to be of concern. Its  $S_{21}$  and  $S_{12}$  can be any value and do not need to be known. In fact, they will be found during the calibration process. This opens up the choices to a wide range of 2-port components, such as a transmission line, pair of matched loads, or an attenuator. This standard will determine 2 of the error terms.

For the final standard ( $C_3$ ) only one piece of information is needed. This could be an unknown reflection value for the same reflection connected to each port ( $S_{11} = S_{22}$ ). Since the other standards have been well matched, this standard should have a higher reflection. This standard determines the last error term.

The table shows a partial list of possible calibration configurations with appropriate three letter acronyms.

## Example: TRL

Now to determine A, given X is known.

From  $M = X A Y$ , solve for A.

$$A = X^{-1} M Y^{-1}$$

From  $M_1 = X C_1 Y$ , solve for  $Y^{-1}$ .

$Y^{-1} = M_1^{-1} X C_1$ , then finally solve for A.

$$A = X^{-1} M M_1^{-1} X C_1$$



The unknown device characteristics can be easily calculated by knowing the parameters of the X error adapter, the known standard  $C_1$ , and the measured data of the test device and measured data for  $C_1$ .

## Example: Unknown $\mathbf{T}$ , Known $\mathbf{A}$ & $\mathbf{B}$

$$\mathbf{T}_M = \frac{1}{(\mathbf{e}_{10}\mathbf{e}_{32})} \begin{bmatrix} -\Delta_x & \mathbf{e}_{00} \\ -\mathbf{e}_{11} & 1 \end{bmatrix} \mathbf{T} \begin{bmatrix} -\Delta_y & \mathbf{e}_{22} \\ -\mathbf{e}_{33} & 1 \end{bmatrix} = \frac{1}{(\mathbf{e}_{10}\mathbf{e}_{32})} \mathbf{ATB}$$

$$(\mathbf{e}_{10}\mathbf{e}_{32})\mathbf{T}_M = \mathbf{ATB}$$

$$\det[(\mathbf{e}_{10}\mathbf{e}_{32})\mathbf{T}_M] = \det[\mathbf{ATB}]$$

$$(\mathbf{e}_{10}\mathbf{e}_{32})^2 \det\mathbf{T}_M = \det[\mathbf{AB}], \text{ since } \det\mathbf{T} = 1, \text{ because } S_{21} = S_{12}$$

Therefore

$$(\mathbf{e}_{10}\mathbf{e}_{32}) = \pm \sqrt{\frac{\det\mathbf{A}\det\mathbf{B}}{\det\mathbf{T}_M}}$$



This example is for the unknown thru calibration method (UXYZ). This is most easily developed using the cascaded t-parameter formulation. With 3 known standards at port-1 and port-2, 6 conditions are provided. The thru standard with  $S_{21} = S_{12}$  provides the 7th required condition. The key to solving this approach is that the determinant of  $\mathbf{T}$  is unity for the passive thru calibration connection ( $S_{21} = S_{12}$ ).

Refer to reference [8] for more detail.



## Example: Unknown **B**, Known **A** & **T**

$$\mathbf{T}_M = \frac{1}{(\mathbf{e}_{10}\mathbf{e}_{32})} \begin{bmatrix} -\Delta_x & \mathbf{e}_{00} \\ -\mathbf{e}_{11} & 1 \end{bmatrix} \mathbf{T} \begin{bmatrix} -\Delta_y & \mathbf{e}_{22} \\ -\mathbf{e}_{33} & 1 \end{bmatrix} = \frac{1}{(\mathbf{e}_{10}\mathbf{e}_{32})} \mathbf{A}\mathbf{T}\mathbf{B}$$

$$\frac{1}{(\mathbf{e}_{10}\mathbf{e}_{32})} \mathbf{B} = \mathbf{T}^{-1}\mathbf{A}^{-1}\mathbf{T}_M = \mathbf{D}, \text{ and } \mathbf{D} \text{ is completely known}$$

$$\begin{bmatrix} -\frac{\Delta_y}{(\mathbf{e}_{10}\mathbf{e}_{32})} & \frac{\mathbf{e}_{22}}{(\mathbf{e}_{10}\mathbf{e}_{32})} \\ -\frac{\mathbf{e}_{33}}{(\mathbf{e}_{10}\mathbf{e}_{32})} & \frac{1}{(\mathbf{e}_{10}\mathbf{e}_{32})} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

Therefore

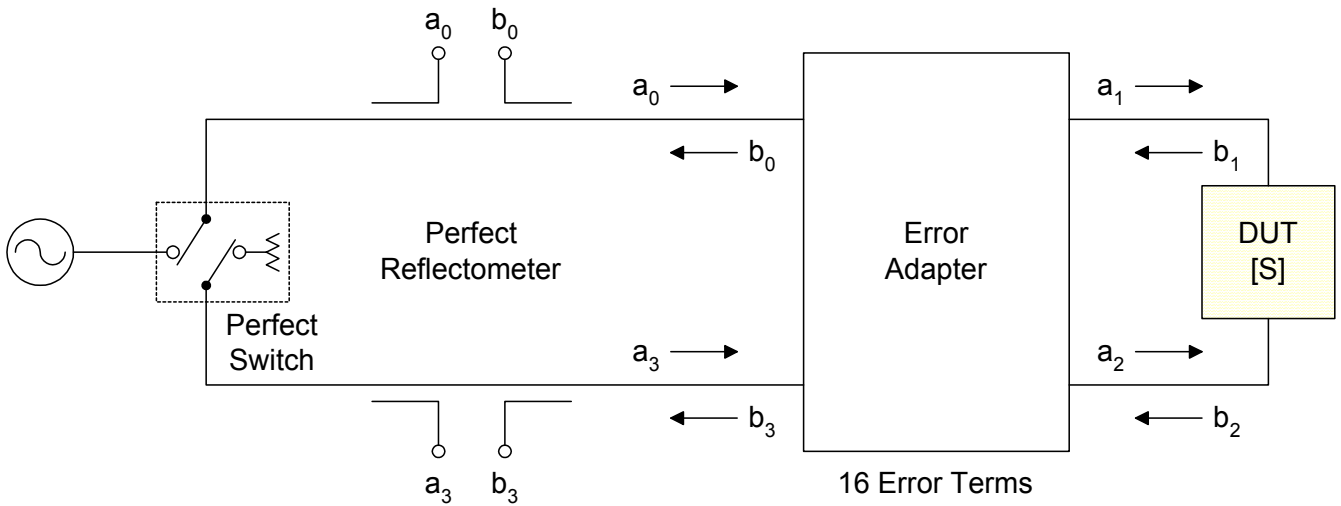
$$(\mathbf{e}_{10}\mathbf{e}_{32}) = \frac{1}{D_{22}} \quad \mathbf{e}_{22} = \frac{D_{12}}{D_{22}} \quad \mathbf{e}_{33} = -\frac{D_{21}}{D_{22}} \quad \Delta_y = -\frac{D_{11}}{D_{22}}$$



This example calibration approach (TXYZ) requires 3 known standards on port-1 but none on port-2 as long as the thru connection is known. The known thru connection provides 4 observations. The 3 known reflection standards connected to port-1 complete the required 7 known conditions.

Refer to references [9] [10] for more details.

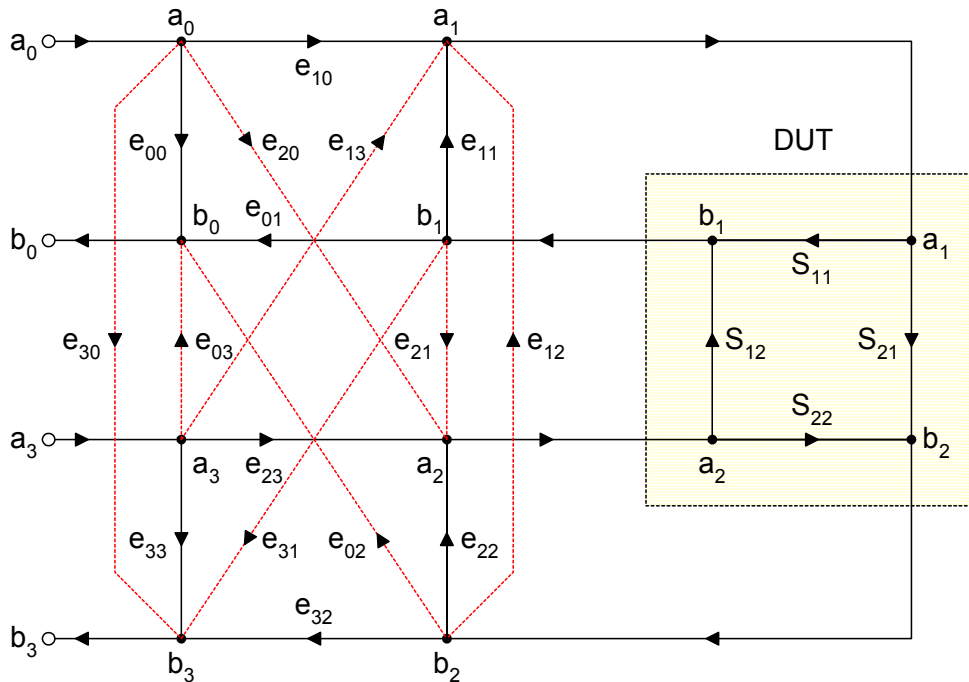
# 16-Term Error Model



To remove the effects of an imperfect switch, use the procedure described later.

With the 8-term model we assumed that there was no leakage between the various measurement ports. The 16-term model makes no assumptions about leakage.

# 16-Term Error Model



One of the 16 error terms can be normalized to yield 15 error terms

These leakage terms add 8 additional error terms to the model. Not only is the traditional crosstalk term included, but switch leakage, signals reflecting from the DUT and leaking to the transmission port, common mode inductance, and other leakage paths are included. In a coaxial or waveguide system, assuming the switch has high isolation, these errors are small. But in a fixtured or wafer probe environment these errors can be much larger.

In a wafer probe environment it is important that the error terms do not change as the probes are moved around the circuit. If the error terms change, the 16-term model then changes and the accuracy will reduce.

Again the error terms can be normalized so that for ratio measurements there are only 15 error terms.

# 16-Term Error Model

Error Model

$$\begin{bmatrix} b_0 \\ b_3 \\ a_0 \\ a_3 \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{T}_2 \\ \mathbf{T}_3 & \mathbf{T}_4 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{bmatrix}$$

Measured S-Parameters  $\mathbf{S}_M = (\mathbf{T}_1 \mathbf{S} + \mathbf{T}_2)(\mathbf{T}_3 \mathbf{S} + \mathbf{T}_4)^{-1}$

Actual S-Parameters  $\mathbf{S} = (\mathbf{T}_1 - \mathbf{S}_M \mathbf{T}_3)^{-1}(\mathbf{S}_M \mathbf{T}_4 - \mathbf{T}_2)$

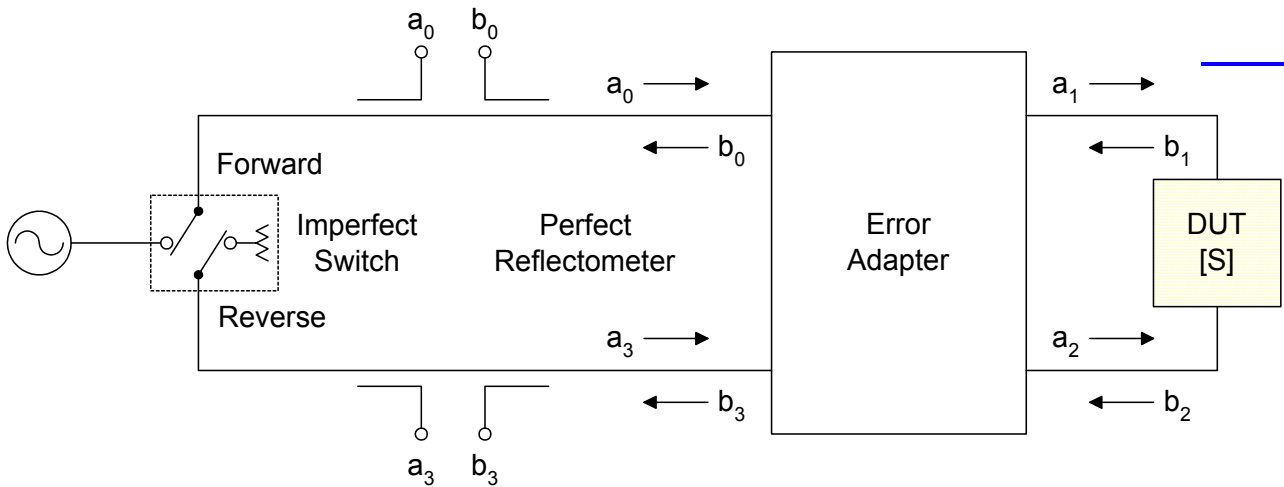
Linear-in-T Form  $\mathbf{T}_1 \mathbf{S} + \mathbf{T}_2 - \mathbf{S}_M \mathbf{T}_3 \mathbf{S} - \mathbf{S}_M \mathbf{T}_4 = \mathbf{0}$

With 15 or more independent observations the linear matrix equation can be solved. TRL as well as OSLT calibration methods are possible.

Solving the 16-term model can be done the same way that the 8-term model was solved. In the 8-term model there were 8 zero terms in the T matrix with the assumption of no leakage. For the 16-term model all terms are present. Of course the set of resultant linear equations contain more error terms, but the approach is the same.

Refer to references [11] [12] [13] for more details.

# Removing Effects of Imperfect Switch



Forward

$$\begin{aligned} b_0 &= S_{11M} a_0 + S_{21M} a_3 \\ b_3 &= S_{21M} a_0 + S_{22M} a_3 \end{aligned}$$

Reverse

$$\begin{aligned} b'_0 &= S_{11M} a'_0 + S_{21M} a'_3 \\ b'_3 &= S_{21M} a'_0 + S_{22M} a'_3 \end{aligned}$$

As stated earlier there is a technique to ratio out or remove the effects of an imperfect switch. Note that the measurement channels are all on the DUT side of the switch. This allows the measurements of incident and reflected signals from the switch. Then we can use the reflectometers to measure the reflection and transmission of the switch and effectively 'remove' the switch from the system.

Mathematically the s-parameters of the system generate 4 equations. Two in the forward direction and two in the reverse direction. These 4 equations can then be solved for the 4 measured s-parameters. This general way of measuring s-parameters does not require the DUT to be terminated in a  $Z_0$  environment.

# Removing Effects of Imperfect Switch

## For S-parameters

Solving the 4 equations yields

$$S_{11M} = \frac{b_0 - b'_0 a_3}{a_0 a'_3 a_0} d$$

$$S_{12M} = \frac{b'_0 - b_0 a'_0}{a'_3 a_0 a'_3} d$$

$$S_{21M} = \frac{b_3 - b'_3 a_3}{a_0 a'_3 a_0} d$$

$$S_{22M} = \frac{b'_3 - b_3 a'_0}{a'_3 a_0 a'_3} d$$

$$d = 1 - \frac{a_3 a'_0}{a_0 a'_3}$$



Solving the 4 previous equations yield the above results. Note that the equations are written to allow ratio measurements by the network analyzer. Typically the network analyzer is more accurate making measurements this way. Noise and other common mode errors are reduced.

Using this method for measuring the 4 s-parameters requires 6 ratio measurements. The additional two measurements are required to remove the effects of the switch.

# Removing Effects of Imperfect Switch

## For S-parameters

Or by defining

$$\Gamma_1 = \frac{a'_0}{b'_0} \quad \text{and} \quad \Gamma_2 = \frac{a_3}{b_3}$$

$$S_{11M} = \frac{\frac{b_0}{a_0} - \frac{b'_0}{a'_3} \frac{b_3}{a_0} \Gamma_2}{d}$$

$$S_{12M} = \frac{\frac{b'_0}{a'_3} - \frac{b_0}{a_0} \frac{b'_0}{a'_3} \Gamma_1}{d}$$

$$S_{21M} = \frac{\frac{b_3}{a_0} - \frac{b'_3}{a'_3} \frac{b_3}{a_0} \Gamma_2}{d}$$

$$S_{22M} = \frac{\frac{b'_3}{a'_3} - \frac{b_3}{a_0} \frac{b'_0}{a'_3} \Gamma_1}{d}$$

$$d = 1 - \frac{b_3}{a_0} \frac{b'_0}{a'_3} \Gamma_1 \Gamma_2$$



The equations can be written in a slightly different form. Two of the ratios that are measuring the switch match do not change when the DUT is changed. So these two ratios can be measured once and stored as  $\Gamma_1$  and  $\Gamma_2$ . Then when making DUT measurements these just become fixed constants in the calculations and reduce the needed ratio measurements from six to four. Of course the switch needs to be stable for this approach to work. If the switch characteristics are changing then these switch effects can be removed by measuring all 6 ratios.

# Removing Effects of Imperfect Switch

For T-parameters

$$T_{11M} = \frac{\frac{b_3}{a_0} \frac{b'_0}{a'_3} - \frac{b_0}{a_0} \frac{b'_3}{a'_3}}{d}$$

$$T_{12M} = \frac{\frac{b_0}{a_0} - \frac{a_3}{a_0} \frac{b'_0}{a'_3}}{d}$$

$$T_{21M} = \frac{-\frac{b'_3}{a'_3} + \frac{b_3}{a_0} \frac{a'_0}{a'_3}}{d}$$

$$T_{22M} = \frac{1 - \frac{a_3}{a_0} \frac{a'_0}{a'_3}}{d}$$

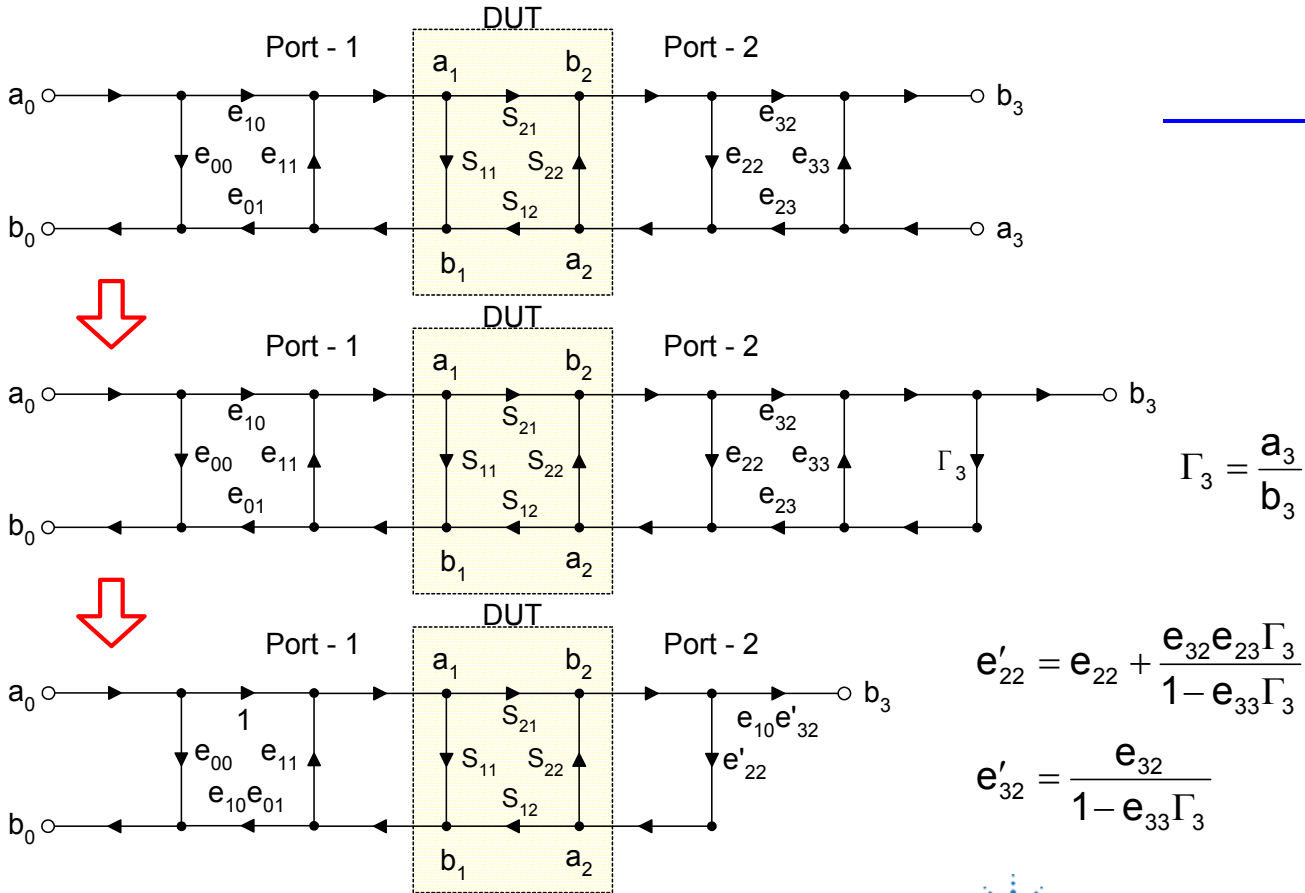
$$d = \frac{b_3}{a_0} - \frac{a_3}{a_0} \frac{b'_3}{a'_3}$$



The same approach can be used for measuring the t-parameters. This will also remove the effects of the switch and requires 6 ratio measurements.

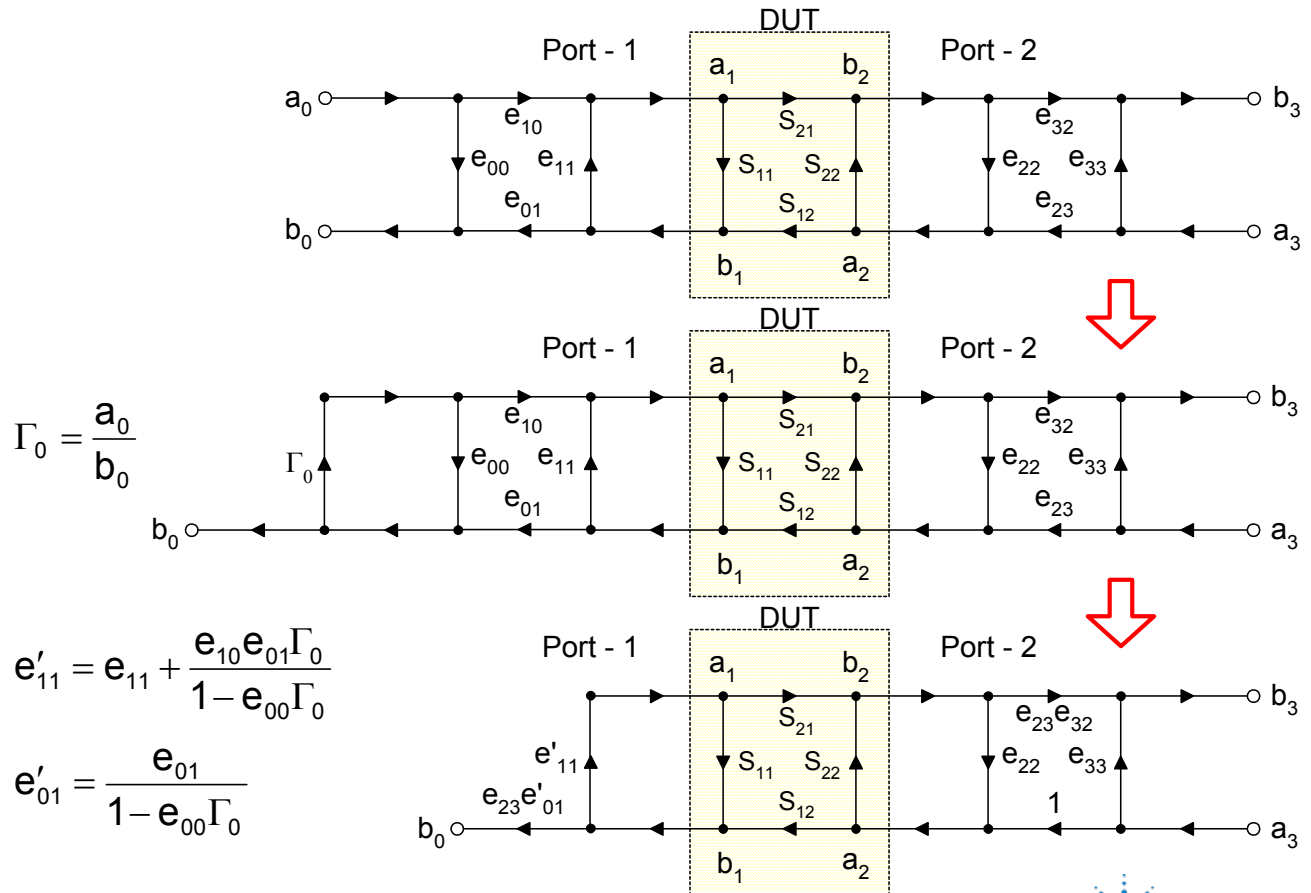


# 8-Term to 10-Term Conversion - Forward



The 12-term and 8-term models describe the same system. So there must be a relationship between them. First let's reduce the 12 term model to 10 terms by removing the crosstalk terms which can easily be measured in a separate step. Then the 8-term model can be modified as shown above. First the 4th measurement channel ( $a_3$ ) is removed by defining the 'switch match' as  $\Gamma_3$ . Then the forward model error terms for port match and transmission tracking ( $e'_{22}$  and  $e'_{32}$ ) can be calculated. This gives the standard forward model if we form the two products  $e_{10}e_{01}$  and  $e_{10}e'_{32}$  and normalize  $e_{10}$  to 1.

# 8-Term to 10-Term Conversion - Reverse



$$\Gamma_0 = \frac{a_0}{b_0}$$

$$e'_{11} = e_{11} + \frac{e_{10}e_{01}\Gamma_0}{1 - e_{00}\Gamma_0}$$

$$e'_{01} = \frac{e_{01}}{1 - e_{00}\Gamma_0}$$

The same procedure can be used for the reverse model. There is a constraining relationship for the 8-term and 10-term models. This is the same as saying that the 8-term model can reduce to 7 terms when making ratio measurements. And the 10-term model can reduce to 9 terms.

8-term:  $(e_{10}e_{01})(e_{23}e_{32}) = (e_{10}e_{32})(e_{23}e_{01})$

10-term:  $[e_{10}e_{01} + e_{00}(e'_{11} - e_{11})][e_{23}e_{32} + e_{33}(e'_{22} - e_{22})] = (e_{10}e'_{32})(e_{23}e'_{01})$

Refer to reference [14] for more details.

## References

- [1] S. Rehnmark, "On the Calibration Process of Automatic Network Analyzer Systems," *IEEE Trans. on Microwave Theory and Techniques*, April 1974, pp 457-458
- [2] J. Fitzpatrick, "Error Models for Systems Measurement," *Microwave Journal*, May 1978, pp 63-66
- [3] G. F. Engen and C. A. Hoer, "Thru-Reflect-Line: An Improved Technique for Calibrating the Dual 6-Port Automatic Network Analyzer," *IEEE Trans. on Microwave Theory and Techniques*, MTT- 27-12, Dec. 1979, pp 983 - 987
- [4] H. J. Eul and B. Schiek, "A Generalized Theory and New Calibration Procedures for Network Analyzer Self-Calibration," *IEEE Trans. on Microwave Theory & Techniques*, vol. 39, April 1991, pp 724-731
- [5] R. A. Speciale, "A Generation of the TSD Network-Analyzer Calibration Procedure, Covering N-Port Scattering-Parameter Measurements, Affected by Leakage Errors," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-25, December 1977, pp 1100-1115
- [6] Kimmo J. Silvonen, "A General Approach to Network Analyzer Calibration," *IEEE Trans. on Microwave Theory & Techniques*, Vol 40, April 1992
- [7] Andrea Ferrero, Ferdinando Sanpietro and Umberto Pisani, "Accurate Coaxial Standard Verification by Multiport Vector Network Analyzer," *1994 IEEE MTT-S Digest*, pp 1365-1367
- [8] Andrea Ferrero and Umberto Pisani, "Two-Port Network Analyzer Calibration Using an Unknown 'Thru'," *IEEE Microwave and Guided Wave Letters*, Vol. 2, No. 12, December 1992, pp 505-506
- [9] Andrea Ferrero and Umberto Pisani, "QSOLT: A new Fast Calibration Algorithm for Two Port S Parameter Measurements," *38th ARFTG Conference Digest*, Winter 1991, pp 15-24
- [10] H. J. Eul and B. Scheik, "Reducing the Number of Calibration Standards for Network Analyzer Calibration," *IEEE Trans. Instrumentation Measurement*, vol 40, August 1991, pp 732-735
- [11] Holger Heuermann and Burkhard Schiek, "Results of Network Analyzer Measurements with Leakage Errors Corrected with the TMS-15-Term Procedure," *Proceedings of the IEEE MTT-S International Microwave Symposium*, San Diego, 1994, pp 1361-1364
- [12] Hugo Van hamme and Marc Vanden Bossche, "Flexible Vector Network Analyzer Calibration With Accuracy Bounds Using an 8-Term or a 16-Term Error Correction Model," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 42, No. 6, June 1994, pp 976-987
- [13] A. Ferrero and F. Sanpietro, "A simplified Algorithm for Leaky Network Analyzer Calibration," *IEEE Microwave and Guided Wave Letters*, Vol. 5, No. 4, April 1995, pp 119-121
- [14] Roger B. Marks, "Formulations of the Basic Vector Network Analyzer Error Model Including Switch Terms," *50th ARFTG Conference Digest*, Fall 1997, pp 115-126