The Diversity-Multiplexing Tradeoff for the Multiaccess Relay Channel

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***Abstract*— This paper studies the diversity-multiplexing tradeoff for the multiaccess relay channel (MARC) with static, flat fading. It develops two simple strategies, namely multiaccess amplify-and-forward (MAF) and multiaccess decode-and-forward (MDF), that help the users gain the benefit of cooperative diversity without changes in their devices. Results suggest that in the regime of light system loads, both strategies offer improved performance to each user as if no other users interfere or contend for the relay. However, they are inferior to the optimal dynamic decode forward (DDF) protocol in this regime. In the regime of heavy loads, the MARC with MAF offers better performance, and MARC with MDF degenerates into the multiaccess channel (MAC) without a relay. Moreover, the MAF protocol is optimal in the regime of high multiplexing.**

I. Introduction

Diversity techniques improve the reliability of wireless communication by sending redundant signals through different channels corrupted by independent fading. When time and frequency diversity are unavailable due to delay and bandwidth constraints, and the terminals are limited to a single antenna due to size constraints, one terminal can ask another terminal to relay its information and thereby achieve diversity gain.

A. Related Research

Cooperative diversity has been extensively studied for the single source and single relay case. Assuming the relay cannot transmit and receive simultaneously, [1], [2] develop outage probability analyses for cooperative diversity for several communication protocols. In [3], [4], cooperative diversity is studied from the diversity-multiplexing tradeoff perspective. The results show that, by judicially choosing communication protocols, full diversity gains can be obtained.

Practical communication systems usually involve more than two users. One of the most typical models is the multiaccess channel (MAC). The capacity region of the MAC is well known [5]. In [6], the diversity-multiplexing tradeoff is also developed for the MAC. Cooperative diversity can be extended to the multiple user cases [4], [3], [7]. In [4], multiple users transmit to a common destination and rely on a group of designated relays to gain cooperative diversity. The transmission happens in two phases. In the first phase, the source broadcasts to the destination and relays; in the second phase, the relays either transmit orthogonally or transmit simultaneously by using space-time codes. For the multiaccess channel, [3] considers an amplify-forward strategy, in which each user transmits its own information and its partner's information together in its own time slot. Each user is assigned a different partner periodically so that every user helps other users equally overall. In [7], the achievable diversity-multiplexing tradeoff is studied assuming that, all sources transmit independently in orthogonal time slots and all users do cooperative transmission together for each other in the last slot.

The MAC with a single *shared* relay is called the multiaccess relay channel (MARC) [8]. The achievable rate for the MARC has been studied in [9] employing a partial-decoded-and-forward strategy. However, the capacity region is not yet known. In [10], an upper bound on the diversity-multiplexing tradeoff for the MARC is developed for the two user case. Moreover, a dynamic-decode-forward (DDF) strategy [10] is proposed and shown to meet the upper bound in the regime of low multiplexing.

B. Motivation

Most of the previous work on multi-user cooperative diversity use the channel orthogonally in the sense that, for most of the time, users transmit on a slot-by-slot basis. As is well-known, using the channel orthogonally is not optimal in the general multiaccess in terms of achieving maximum rates. Therefore, this paper focuses on studying nonorthogonal multiaccess schemes. One of the key reasons that the previous works focused on the orthogonal multiaccess schemes is due to the design that all nodes relay for each other and the half-duplex constraint of the terminals, *i.e.*, no terminal can transmit and receive at the same time on the same frequency. In this paper, we consider nonorthogonal multiaccess schemes without violating the half-duplex requirement by adding one *shared* relay into the system.

Moreover, most previous cooperative protocols require partners to explicitly establish cooperation, which might require device changes for all users. In contrast, this paper proposes strategies that does not require users to act as relays; user terminals operate as if in a normal multiaccess channel. Ideally, the users might not even be aware of the existence of a relay. In our proposed schemes, the complexity of implementing cooperative diversity is moved from the users to the service providers, which could provide a smooth transition path from existing systems to new cooperative systems.

II. Channel Model

The scenario of interest is modeled as a multiaccess relay channel (MARC) [11] as shown in Fig. 1. More specifically,

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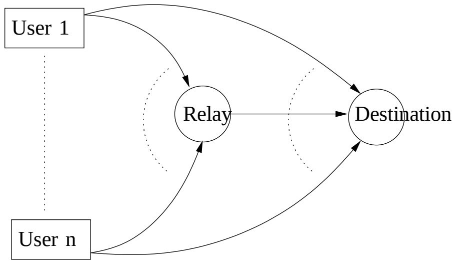

Fig. 1. Multiaccess relay channel (MARC) with one relay.

we consider a network with $n+2$ nodes, in which n users, denoted by $s = 1, ..., n$, send information to a destination, denoted by $n+2$. The relay node is denoted by $n+1$ and is designated to help users transmit information.

We study the asymptotic high SNR behavior of a non-ergodic MARC through the perspective of the diversity-multiplexing tradeoff [12]. Since there is no direct cooperation among users, each user generates their codes independently. Assuming each user has the same transmission power ρ, for each user i, we can consider a family of codes $\mathcal{C}_i(\rho)$ over a single coherent block and indexed by operating point ρ. If $R_i(\rho)$ denotes the data rate in bits per channel use per user, then the multiplexing gain per user is defined as $r_i := R_i(\rho)/\log\rho$. We mainly focus on the symmetric case, *i.e.*, users have the same multiplexing gain requirement such that $r_i = r/n$, where $r \in [0,1]$ is the total multiplexing gain. We assume each user has a diversity gain requirement of d. According to [6], the minimal error probability of the joint ML detector at the destination would yield a diversity gain d for all users.

The wireless links between individual nodes are corrupted by independent multiplicative fading and additive white Gaussian noise (AWGN). In this paper, we assume a frequency non-selective block fading model to gain tractable analysis. The block length l is assumed to be long enough so that instantaneous channel state information (CSI) can be tracked at the appropriate receivers, but CSI is not available to any transmitter. Without loss of generality, the variance of the AWGN is assumed to be unity.

III. Analysis of Different Protocols

In the protocols we consider, all users transmit simultaneously to the destination during all channel uses. The relay listens for the first half of the block and transmits or remains silent in the second half of the block. If the relay listens, the equivalent discrete time channel model can be written as

$$y_{n+2}[j] = \sum_{i=1}^{n} h_{i,n+2} x_i[j] + z_{n+2}[j], \quad (1)$$

$$y_{n+1}[j] = \sum_{i=1}^{n} h_{i,n+1} x_i[j] + z_{n+1}[j], \quad (2)$$

where j is the time index and $j \le l/2$. In the second half of the block, if the relay does not transmit, the channel output is the same as (1). If the relay transmits, the channel output is

$$y_{n+2}[j] = \sum_{i=1}^{n} (h_{i,n+2} x_i[j] + h_{n+1,n+2} x_{n+1}[j]) + z_{n+2}[j], \quad (3)$$

where $l/2 < j \le l$. Here: $h_{i,j}$ denotes the fading coefficient between nodes i and j; x_i denotes the transmitted signal of node i; and y_i is the received signal at node i. The relay transmit signal is correlated with the transmitted signals from the sources, and their relationship will be specified according to different protocols.

A. Multiaccess Amplify-and-Forward

In the multiaccess amplify-and-forward (MAF) protocol we consider, the relay listens for half of the block, amplifies the superposition of signals it received, and broadcasts them to the destination for the remainder of the block. For the single user case, the MAF protocol in this paper is a special case of NAF in [3] that results from choosing the listening time of the relay to be half of the block. In the single user and single relay case, the listening time being half of the block happens to be optimal in terms of the diversity-multiplexing tradeoff. This result might not be true for the multiple user case. Also, in the MARC, there is only one relay for all users, and the MAF we propose lets the relay superimpose all the signals together. In our schemes, the relay assigns equal power to help each user. Optimizing the relay's power assignment and listening time might further improve performance, but is not considered in this paper.

Overall, the transmit signal at the relay is

$$\begin{aligned} x_{n+1}[j] &= \alpha y_{n+1}[j-l/2] \\ &= \alpha\Big(\sum_{i=1}^{n} h_{i,n+1} x_{n+1}[j-l/2] + z_{n+1}[j]\Big), \end{aligned}$$

where $j > l/2$, α is the amplifying coefficient which is chosen to satisfy the average power constraint at the relay,

$$|\alpha|^2 \le \frac{\rho}{\sum_{i=1}^{n} \sigma_{i,n+1}^2 \rho + 1},$$

and $\sigma_{i,j}^2$ is the variance of $h_{i,j}$. The destination is assumed to know CSI between each individual source and the relay.

To facilitate further analysis, we can write the channel model into the matrix form,

$$Y_{n+2}[j] = \sum_{i=1}^{n} H_i X_i[j] + Z[j], \quad (4)$$

where $j \le l/2$ and

$$\begin{aligned} Y_{n+2}[j] &= [y_{n+2}[j], y_{n+2}[j+l/2]]^T, \\ X_i[j] &= [x_i[j], x_i[j+l/2]]^T, \\ Z[j] &= [z_{n+2}[j], z_{n+2}[j+l/2] + \alpha z_{n+1}[j]]^T, \\ H_i &= \begin{bmatrix} h_{i,n+2} & 0 \\ \alpha h_{i,n+1} h_{n+1,n+2} & h_{i,n+2} \end{bmatrix}. \end{aligned}$$

The channel as expressed in (4) can be regarded as a multiaccess channel with multiple transmit and receive antennas. The independent i.i.d complex Gaussian inputs assumed in this paper are due to the requirement that the users not be aware of the existence of relay; however, such inputs need not be optimal in terms of capacity or outage because the channel matrix H is not circular symmetric.

The following theorem provides the diversity-multiplexing tradeoff for the MARC with MAF.

Theorem 1: For a MARC with n users, $n \geq 2$, let the multiplexing gain of each user be r_1. The diversity gain obtained from the multiaccess amplify-and-forward protocol for each user is

$$d_{MAF}(r_1) = \begin{cases} 2-3r_1, & \text{for } 0 \leq r_1 \leq \frac{n-1}{n(n+1)-3} \\ (n+1)(1-nr_1), & \text{for } \frac{n-1}{n(n+1)-3} \leq r_1 \leq \frac{1}{n} \end{cases}. \quad (5)$$

The proof of Theorem 1 follows similar lines as in [6]. By calculating the probability of the outage event and the probability of error conditioned on no outage, it can be shown that that for a large block length l, the typical error event is caused mainly by the channel being in outage.

For $r_1 \leq n-1/[n(n+1)-3]$, the first term in (5) decides the tradeoff curve for MAF. To compare, the diversity-multiplexing tradeoff for a single-user and single-relay cooperative diversity system employing nonorthogonal-amplify-forward [3] is

$$d_{su}(r_{su}) = \begin{cases} 2-3r_{su}, & \text{for } 0 \leq r_1 \leq 1/2 \\ 1-r_{su}, & \text{for } 1/2 \leq r_1 \leq 1 \end{cases}, \quad (6)$$

where the subscript $_{su}$ indicates the single user case. The first term of (5) is the same as that of (6) for $(n-1)/[n(n+1)-3] < 1/2$, $n \geq 2$. This suggests that, in the low multiplexing regime, each user enjoys the diversity benefit from the relay as if there is no interference from other users or contention for the relay.

On the other hand, if $r_1 \geq (n-1)/[n(n+1)-3]$, the second term in (5) decides the tradeoff curve for MAF. The MARC behaves like a system with an $n+1$ transmit antennas and one receive antenna. In the high multiplexing regime, the effect of multiuser interference is the dominant factor of degrading performance. These observations are in line with similar observations for the MAC [6]. The surprising part of our observation is that, in low multiplexing regime, each user behaves as if they each have a dedicated relay. As long as the system operates in the low rate regime, we can add more users into the system without degrading the diversity-multiplexing performance.

For comparison, an upper bound on the diversity-multiplexing tradeoff for the MARC is as follows [10],

$$d_{up}(r_1) = \begin{cases} 2-2r_1, & \text{for } 0 \leq r_1 \leq \frac{1}{n+2} \\ (n+1)(1-nr_1), & \text{for } \frac{1}{n+2} \leq r_1 \leq \frac{1}{n} \end{cases}. \quad (7)$$

Notice that since $1/n+2 < (n-1)/[n(n+1)-3]$ for $n \geq 2$, it is immediately clear that the diversity-multiplexing curve of (5) overlaps with (7) when $(n-1)/[n(n+1)-3] \leq r_1 \leq 1/n$. Therefore, MAF is optimal in the high multiplexing regime. Combining our results with those in [10], we can fully achieve the optimal diversity-multiplexing tradeoff for the MARC. In the low multiplexing regime, a relay using DDF can decode with little degradation from multiaccess interference, and therefore, offers better performance. However, in the high multiplexing regime, there is a high probability of decoding errors at the relay, the relay might have to spend a large portion of time in decoding the mulitaccess channel between the sources and relay, and the relay does not have sufficient channel uses to transmit to the destination. For MAF, because the relay amplifies the noise together with the source signals, performance is degraded in the power limited regime with low multiplexing gain. But for the high multiplexing regime, the main factor limiting performance is multiaccess interference, and the noise amplified by the relay may be negligible.

We can also compare the result (5) to the corresponding result for the MAC without a relay. The diversity-multiplexing tradeoff for an n-user symmetric MAC is [6]

$$d_{mac}(r) = \begin{cases} m(1-r_1), & \text{for } 0 \leq r_1 \leq \frac{1}{n+1} \\ nm(1-nr_1), & \text{for } \frac{1}{n+1} \leq r_1 \leq \frac{1}{n} \end{cases}, \quad (8)$$

where all terminals have m transmit antennas and one receive antenna. It is easy to see that (5) dominates (8) for all $r_1 \in (0, 1/n)$ when $m = 1$. Therefore, if all users are constrained to have one antenna, the MARC is desirable in terms of providing additional diversity. However, the performance of the MAC with two transmit antenna, *i.e.*, $m = 2$, is superior to that of a MARC with MAF. Thus, the use of cooperative diversity with terminals that are equipped with multiple antennas is questionable since higher diversity order brings diminishing return.

B. Multiaccess Decode-and-Forward

We also study a simple multiaccess decode-and-forward (MDF) protocol. In this MDF, the relay listens for the first half of the block, and jointly decodes the signals from all users. If the relay can correctly decode all users' signals, it transmits the superposition of all the codewords to the destination for the rest of block. If the relay cannot decode correctly, it remains silent for the last half block. Our MDF we is different from the DDF protocol [3] in that the relay does not wait until it can successfully decode. Moreover, the relay encodes using the users' codebooks, *i.e.*, repetition coding. MDF might be useful if cost, complexity, or delay prevents implementation of DDF at the relay.

The output signal from the relay is

$$x_{n+1}[j] = \begin{cases} \sqrt{\frac{1}{n}} \sum_{i=1}^{n} x_i[j-l/2], & \text{relay decodes} \\ 0, & \text{relay does not decode} \end{cases}, \quad (9)$$

for $l/2 < j \leq l$. We can also write the channel model into the matrix form (4) with

$$\begin{aligned} Y_{n+2}[j] &= [y_{n+2}[j], y_{n+2}[j+l/2]]^T, \\ X_i[j] &= [x_i[j], x_i[j+l/2]]^T, \\ Z[j] &= [z_{n+2}[j], z_{n+2}[j+l/2]]^T, \\ H_i &= \begin{bmatrix} h_{i,n+2} & 0 \\ a h_{n+1,n+2} & h_{i,n+2} \end{bmatrix}, \end{aligned}$$

where $a = \sqrt{1/n}$ if the relay decodes, and $a = 0$ if the relay does not decode. Before we proceed, we comment on the decoding schemes for MDF. In both the relay and destination, a joint ML detector is used for decoding. An error is declared if one user is not decoded correctly. As in [6], this is sufficient for yielding the diversity-multiplexing tradeoff. The ML detector in general will not be able to detect decoding errors; therefore, an outer CRC code is required in order to detect decoding errors. Moreover, we assume that the destination is informed with knowledge of whether or not the relay is transmitting through information in the protocol headers. This knowledge enables the destination to switch among different detectors conditioned on the relay's status. In most current networks, both mechanisms are readily available. In this paper, we assume that the loss in rate associated with these mechanisms is negligible.

Following [6] and using Bayes rule to condition on the error event at the relay, the diversity-multiplexing tradeoff for the MARC with this MDF is given in the following theorem,

Theorem 2: For an n-user MARC with MDF, let the multiplexing gain of each user be r_1. The diversity gain for each user is given by

$$d_{MDF}(r) = \begin{cases} 2-3r_1, & \text{for } 0 \leq r_1 \leq \frac{1}{2(n+1)} \\ n+1-(2n^2+1)r_1, & \text{for } \frac{1}{2(n+1)} \leq r_1 \leq \frac{1}{2n} \\ 1-r_1, & \text{for } \frac{1}{2n} \leq r_1 \leq \frac{1}{n+1} \\ n(1-nr_1), & \text{for } \frac{1}{n+1} \leq r_1 \leq \frac{1}{n} \end{cases} . \quad (10)$$

An intuitive explanation of (10) is as follows. The channel from sources to the relay is just a MAC, and from (8), the probability of error is of order $\rho^{-(1-2r_1)}$ if $r_1 \leq 1/2(n+1)$ and $\rho^{-n(1-2nr_1)}$ if $r_1 \geq 1/2(n+1)$, where the factor 2 comes from the fact that the relay only listens for half of the block. Given that the relay does not decode, the probability of error at the destination is of order $\rho^{-(1-r_1)}$ if $r_1 \leq 1/(n+1)$ and $\rho^{-n(1-nr_1)}$ if $r_1 \geq 1/(n+1)$. Thus, the probability that both the destination and the relay make errors has exponent

$$\begin{aligned} d_{MDF}(r_1) = \min\{ \quad & (1-r_1)^+ + (1-2r_1)^+, \\ & n(1-2nr_1)^+ + (1-r_1)^+, \\ & n(1-2nr_1)^+ + n(1-nr_1)^+\}, \end{aligned}$$

which results in (10) after algebraic manipulations.

It can be further observed that the first term of (10) is the same as the first term of (5). However, this term has two different operation regimes. For MDF, the first term is the dominant factor until $r_1 \geq 1/2(n+1)$, and for MAF, it is the dominant factor until $r_1 \geq (n-1)/(n(n+1)-3)$. It can be shown that $1/2(n+1) < (n-1)/(n(n+1)-3)$ for $n > 2$. It can also be easily shown that, in other regimes, (5) is larger than (10). Therefore, when $r_1 \leq 1/2(n+1)$, MDF using simple repetition coding is identical to MAF in terms of diversity-multiplexing tradeoff. But, when $1/2(n+1) \leq r_1$, MAF is better than MDF.

Furthermore, when $r_1 \geq 1/2n$, the last two terms of (10) are exactly the same as (8) for $m = 1$. Therefore, in terms of diversity-multiplexing tradeoff, the MARC with MDF degenerates into the normal MAC when the system is highly loaded, *i.e.*, $r_1 \geq 1/2n$.

It can be shown that, if the relay only listens for half of the block, *i.e.*, no dynamic decoding, there is no benefit of using independent encoding compared to repetition coding at the relay in terms of diversity-multiplexing tradeoff. The reason for this is that the bottleneck limiting the performance of MDF is the decoding errors at the relay, and independent encoding at the relay does not help improve the multiaccess channel between the sources and relay.

IV. EXAMPLES

Fig. 2 shows the diversity-multiplexing tradeoff for the MARC with two users, *i.e.*, $n = 2$. As predicted, the curve for MAF overlaps with the upper bound when $r_1 \geq 1/3$. The curve of DDF [10] is also presented and overlaps the upper bound when when $r_1 \leq 1/3$. Combining both schemes, we can fully achieve the optimal diversity-multiplexing tradeoff for the MARC. The curve of the simple MDF overlaps with MAF when $r_1 \leq 1/6$ and then quickly degrades into a MAC with a single antenna per user.

To numerically verify the results of the diversity-multiplexing tradeoff, we can choose the rate R for each user to be a linear function of multiplexing gain r_1, *i.e.*, $R = r_1 \log \rho + R_0$, where R_0 is a constant. For an arbitrary multiplexing gain r_1, we can compute the outage probability $P(R)$ and observe how the outage probability for r_1 scales with SNR. We arbitrarily choose R_0 to be zero. For simplicity of exposition, we consider a two-dimensional network with two users and assume that both users are located at the origin $(0, 0)$. Coordinates of the communication network are normalized by the distance between the sources and destination transceivers, and the positive direction is defined as from the source to the destination. Thus, the destination is located at $(1, 0)$. In general, the coordinates of the relay can be arbitrary. The fading variances $\sigma^2_{i,j}$ are assigned using a path-loss model in the form of $\sigma^2_{i,j} \propto d^{-v}_{i,j}$, where $d_{i,j}$ is the distance from node i to node j, and v is a constant value, chosen to be 4 in our setup. The energy per transmitted bit for each transmit terminal is also normalized to 1.

Fig. 3 provides numerical simulation results for a two-user MARC using DDF and MAF for $r_1 = 0.2$ and $r_1 = 0.4$. This example assumes the relay's coordinates are $(1/2, \sqrt{3}/2)$ such that the fading variances for all channels are unity, *i.e.*, $\sigma^2_{1,n+1} = \sigma^2_{2,n+1} = \sigma^2_{1,n+2} = \sigma^2_{2,n+2} = \sigma^2_{n+1,n+2} = 1$. It is clear that when $r = 0.2$, DDF is better than MAF; and when $r = 0.4$, MAF is better than DDF at high SNR. Moreover, for $r_1 = 0.4$, the curves of MAF and DDF intersect which suggests different diversity gains. These observations are in line with what we predict from the diversity-multiplexing tradeoff.

Although the diversity-multiplexing tradeoff yields insight about performance at asymptotically high SNR, it neglects a coding gain of a protocol. Therefore, one must be careful in the interpretation of implications for system design. To illustrate this point, we consider an example in which the relay is located at $(0.5, 0)$ and show the simulation results in Fig. 4. For $r_1 = 0.2$, DDF shows better performance as predicted. However, the diversity-multiplexing tradeoff falls short of correctly predicting the performance of MAF and DDF for $r_1 = 0.4$. Although MAF has a slightly better

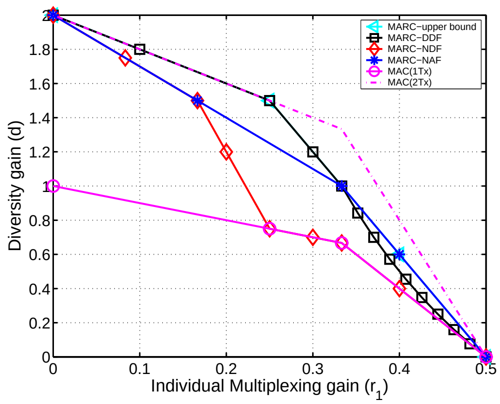

Fig. 2. Diversity-Multiplexing trade off for the Multiaccess relay channel (MARC) with one relay

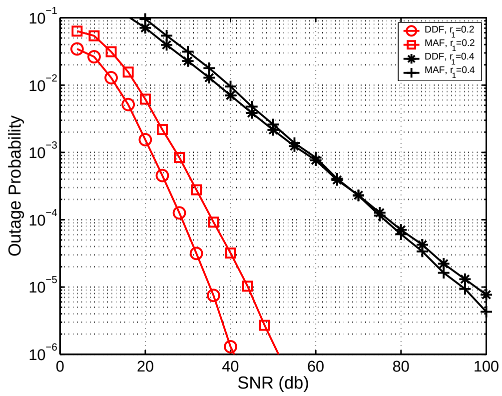

Fig. 3. Outage probability for the MARC

diversity-multiplexing gain in this regime, the performance of DDF is still better than MAF in the SNR regime we simulate. Therefore, one must be cautious in applying the results from diversity-multiplexing to compare performance of different schemes in the regime of finite SNR.

V. Conclusion

This paper studies to what extent users in a MAC will benefit from a single *shared* relay in terms of diversity-multiplexing tradeoff. As the number of users in the system grows, the extra cost of adding one relay can be shared by all users and, therefore might be affordable. Dynamic Decode Forward (DDF) [3] is shown to achieve an upper bound on the diversity-multiplexing tradeoff for the MARC in the regime of low multiplexing. DDF requires a relay capable of dynamic decoding and could lead to a complicated relay and extra protocol overhead. This paper focuses on strategies that are relatively simple in terms of signal processing at the relay. Surprisingly, one of them outperforms DDF in the regime of high multiplexing. More specifically, this paper discusses two protocols, namely, multiaccess amplify-and-forward (MAF) and multiaccess decode-and-forward (MDF), for the multiaccess relay channel (MARC). In both protocols, the users transmit independently as if they are in a normal MAC and are not aware of the existence of a *shared* relay in

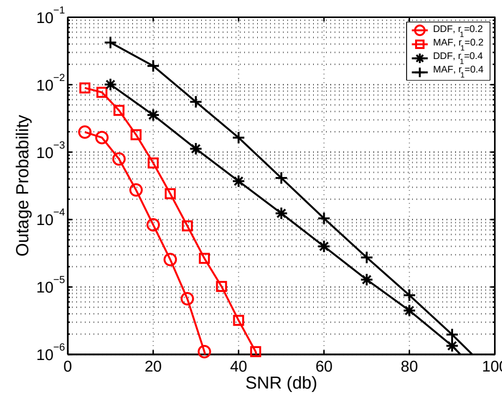

Fig. 4. Outage probability for the MARC with the relay located at (0.5,0).

the network. Analysis of the diversity-multiplexing tradeoff for the MARC with these two protocols suggests that, when the rate requirement is small, each user gains cooperative diversity as if there is no interference from other users and no contention for the relay. As the rate requirement grows, the performance of MAF achieves the upper bound, and is therefore optimal. Combining the MAF and DDF [10], we can fully achieve the optimal diversity-multiplexing tradeoff for the MARC. MAF comes with the requirement of being able to track all CSI at the destination; but the complexity of the relay is simpler compared to that of DDF.

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