

# QCD corrections to excited lepton (pair) production at LHC

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## Abstract

We consider excited lepton ( $\bar{l}^*l$  or  $\bar{l}^*l^*$ ) production in the context of effective theories with being four-fermion contact interaction at LHC. We also consider the two body decay mode of excited lepton ( $l^*$ ) to standard model fermion( $l$ ) and a gauge boson  $V(\equiv \gamma, Z, W)$ . We have performed next-to-leading order (NLO) QCD corrections to this process. In spite of non-renormalizable nature of the interaction, the higher order QCD corrections are possible and meaningful. We have shown that these corrections can be substantial and significant. By considering the issue of scale dependence, it is shown that the scale dependence of the NLO cross sections are greatly reduced as compare to leading order (LO) cross section.

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# 1 Introduction

The recent discovery of a Higgs like scalar particle at LHC may complete the most successful model in particle physics namely the Standard Model (SM). In spite of this huge success, there are other issues like the replication of the fermion families, dark matter, baryogenesis etc. that are still not understood within the framework of the SM and addressing them needs physics beyond the standard model. Some possible candidates are supersymmetry [1], grand unification [2, 3] (with or without supersymmetry), family symmetries (gauged or otherwise) and compositeness for quarks and leptons [4] more suitable than the SM.

The proliferation of fermion generations suggests the possibility of quark-lepton compositeness. In these theories [5, 6], the fundamental constituents, *preons* [7], experience unobserved strong force. At energies far above the certain scale known as composite scale  $\Lambda$ , preons are asymptotically free. At a composite scale  $\Lambda$ , the interaction of preons become very strong to form a bound state (composites) which are to be identified as quarks and leptons compositeness. For energies below the scale  $\Lambda$ , the effective four-fermion Lagrangian is given by [9]

$$\mathcal{L}_{CI} = \frac{2\pi}{\Lambda^2} \sum_{i,j=L,R} \left[ \eta_{ij} (\bar{q}_i \gamma_\mu q_j) (\bar{\ell}_i^* \gamma^\mu \ell_j) + \eta'_{ij} (\bar{q}_i \gamma_\mu q_j) (\bar{\ell}_i^* \gamma^\mu \ell_j^*) + h.c. \right] \quad (1)$$

where  $\ell$  and  $\ell^*$  represents the SM and excited lepton respectively. The Lagrangian of eqn.(1) is not the most general the effective four-fermion Lagrangian. There can be similar operators involving the quarks alone or the leptons alone. However, the Lagrangian of eqn.(1) is enough to serve our purpose. There are other kind of four-fermion like processes mediated by a particle with a mass significantly higher than the energy transfer can be approximated by a contact interaction [9] term. The theories with extended gauge sectors, leptoquarks [11], sfermion exchange in a supersymmetric theory with broken  $R$ -parity [12] etc. are the examples of such type of interactions. In all these cases, the heavy fields with masses  $M_i \gtrsim \Lambda$  [13], have been integrated out and left with a series of such higher-dimensional terms. The terms in eqn.(1) are just the lowest order (in  $\Lambda^{-1}$ ) ones. However, we shall consider only terms involving  $q\bar{q}\bar{\ell}^*\ell$  ( $q\bar{q}\bar{\ell}\ell^*$ ) or  $q\bar{q}\bar{\ell}^*\ell^*$ .

The excited fermions can also be transformed into ordinary SM fermions through the gauge bosons. The effective gauge mediated Lagrangian [9, 10] is given by

$$\mathcal{L}_{GM} = \frac{1}{2\Lambda} \bar{f}_R^* \sigma^{\mu\nu} \left[ g_s f_s \frac{\lambda^a}{2} G_{\mu\nu}^a + g f'' \frac{\tau}{2} \cdot W_{\mu\nu} + g' f' \frac{Y}{2} B_{\mu\nu} \right] f_L + h.c. \quad (2)$$

where  $G_{\mu\nu}^a$ ,  $W_{\mu\nu}$  and  $B_{\mu\nu}$  are the field strength tensor of the  $SU(3)$ , the  $SU(2)$  and the  $U(1)$  gauge fields respectively.  $f^*$  and  $f$  denote the excited fermion and SM fermion respectively.  $f_s$ ,  $f''$  and  $f'$  are parameters of the compositeness. Usually they are taken to be order of 1.

It is evident that these operators may lead to significant phenomenological effects in collider experiments, like  $e^+e^-$  [15],  $eP$  [14] or hadronic [16, 22, 23]. It is quite obvious that the effects would be more pronounced at higher energies for a given higher-dimensional nature of  $\mathcal{L}$ . The best constraints on compositeness came from the Delphi [15] and CDF [16] experiments. More recently the measurement of the  $\bar{\ell}\ell\gamma$  cross section [22, 23] at high invariant masses set the most stringent limits on contact interactions of the type given in eqn.(1).

It is well known the QCD corrections can alter the cross sections quite significantly at hadron colliders. As for example, a simple process like Drell-Yan, the leading order (LO) cross sections have been seriously underestimated. This forced us to incorporate the next-to-leading order (NLO) or next-to-leading log (NLL) [20, 21] results in Monte Carlos codes [20] or event



notation of Ref [24]. For completeness, we have given below

$$\begin{aligned} S &\equiv (p_1 + p_2)^2 & \hat{s} &\equiv (k_1 + k_2)^2 & Q^2 &\equiv (l_1 + l_2)^2 \\ \tau &\equiv \frac{Q^2}{S} & z &\equiv \frac{Q^2}{\hat{s}} & \tau &\equiv z x_1 x_2. \end{aligned} \quad (6)$$

As we argued above, that the current-current structure of the effective lagrangian makes possible the higher order QCD corrections. Although the effective Lagrangian is a nonrenormalizable one, the offending higher order nature can be factored out. Of our particular interest is the leptonic tensor with two massive particle final state, namely

$$\mathcal{L}^{jj' \rightarrow l l'} = \int \prod_i^2 \left( \frac{d^n l_i}{(2\pi)^n} 2\pi \delta^+(l_i^2 - m_i^2) \right) (2\pi)^n \delta^{(n)}(q - l_1 - l_2) |\mathcal{M}^{jj' \rightarrow l l'}|^2, \quad (7)$$

and which leads to

$$\mathcal{L}_{jj' \rightarrow l^* l'} = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2} \right) \mathcal{L}_{l^* l'}(Q^2) \quad (l' = l, l^*) \quad (8)$$

with

$$\mathcal{L}_{l^* l}(Q^2) = \frac{1}{12} \left( Q^2 - \frac{m_1^2 + m_2^2}{2} - \frac{(m_1^2 - m_2^2)^2}{2Q^2} \right). \quad (9)$$

To calculate the  $Q^2$  distribution of the excited lepton pair ( $\bar{l}^* l^*$  or  $\bar{l}^* l$ ), one needs to calculate the hadronic tensor. For this part of our calculation, we have followed the procedure of [24]. We have checked our analytical results with Ref. [24, 26, 27]. The physical hadronic cross section is obtained by convoluting these finite coefficient functions with appropriate parton distribution functions. Finally the inclusive differential cross section is given by

$$\begin{aligned} 2S \frac{d\sigma^{P_1 P_2}}{dQ^2}(\tau, Q^2) &= \sum_q \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz \delta(\tau - z x_1 x_2) \mathcal{F}^{VA} \mathcal{G}_{VA} \\ \mathcal{G}_{VA} &\equiv H_{q\bar{q}}(x_1, x_2, \mu_F^2) \left\{ \Delta_{q\bar{q}}^{(0),VA}(z, Q^2, \mu_F^2) + a_s \Delta_{q\bar{q}}^{(1),VA}(z, Q^2, \mu_F^2) \right\} \\ &+ \left\{ H_{qg}(x_1, x_2, \mu_F^2) + H_{gq}(x_1, x_2, \mu_F^2) \right\} a_s \Delta_{qg}^{(1),VA}(z, \mu_F^2) \end{aligned} \quad (10)$$

where the renormalized parton flux  $H_{ab}(x_1, x_2, \mu_F^2)$  and the finite coefficient functions  $\Delta_{ab}^{(i)}$  are given in Ref. [24, 26, 27]. The constants  $\mathcal{F}^{VA}$  contains information of all couplings, propagators and the massive final state particles which is given by

$$\mathcal{F}^{VA} = \frac{|\eta|^2 \beta Q^2}{12 \Lambda^4} \left\{ 1 - \frac{(m_1^2 + m_2^2)}{2Q^2} - \frac{(m_1^2 - m_2^2)^2}{2Q^4} \right\} \quad (11)$$

$$\beta = \left( 1 + \frac{m_1^4}{Q^4} + \frac{m_2^4}{Q^4} - 2\frac{m_1^2}{Q^2} - 2\frac{m_2^2}{Q^2} - 2\frac{m_1^2 m_2^2}{Q^2 Q^2} \right)^{\frac{1}{2}}. \quad (12)$$

### 3 Results and Discussion

In the previous section, we have calculated the differential distributions with respect to the excited lepton (pair) invariant (either  $\bar{l}^* l(\bar{l}^* l^*)$  or  $\bar{l}^* l^*$ ) mass. For our interest, we have expressed

the aforementioned differential distribution to the total cross section by integrating over  $Q^2$ . For the time being, we have first considered  $\mu_R^2 = \mu_F^2 = Q^2$  and later we have shown the scale dependence of our results. From the eqn.(10), the total cross section is obtained by integrating over  $Q^2$  and is given by

$$\sigma^{P_1 P_2}(M_*^2, S, \Lambda) = \int \frac{d\sigma^{P_1 P_2}(\tau, Q^2)}{dQ^2} dQ^2. \quad (13)$$

For the numerical analysis, we present our results at three different LHC energies  $\sqrt{S} = 7, 8, 14$  TeV. Although the calculation of QCD correction does not depend on the contact interaction scale  $\Lambda$ , for definiteness we have used  $\Lambda = 6$  TeV for each LHC energy unless it is quoted. In presenting our results, we have put all contact interaction coupling strength to be unity. In our numerical analysis, we have used Cteq6Pdf [30] parton distribution functions (PDFs) otherwise mentioned specifically. For Cteq6Pdf, the leading order (LO) hadronic cross section is obtained by convoluting the LO parton distribution (namely Cteq6l1) function with LO partonic cross section and for the NLO hadronic cross section, we have convoluted NLO parton distribution (namely Cteq6m) with NLO partonic cross section. We did our numerical analysis with  $\Lambda_{QCD} = 0.226(0.165)$  GeV for NLO (LO) for  $n_f = 5$ . To start with we will first discuss the NLO corrections of  $\bar{l}^*l$  ( $\bar{l}l^*$ ) and  $\bar{l}^*l^*$  productions in general and later we consider only a particular process  $\bar{l}l\gamma$  production where we have multiplied by the branching fraction of the excited lepton  $l^*$  ( $\bar{l}^*$ ) (decays to  $l\gamma$  ( $\bar{l}\gamma$ )) to the production cross section  $\bar{l}^*l$  ( $\bar{l}l^*$ ). The later process has been analyzed by both CMS [22] and ATLAS [23] collaborations. Although the two body decays of excited lepton  $l^*$  ( $\bar{l}^*$ ) (decays to  $l\gamma$  ( $\bar{l}\gamma$ )) does not have any effect on QCD correction, for definiteness we have done this analysis so that NLO results can be read directly from the figures.

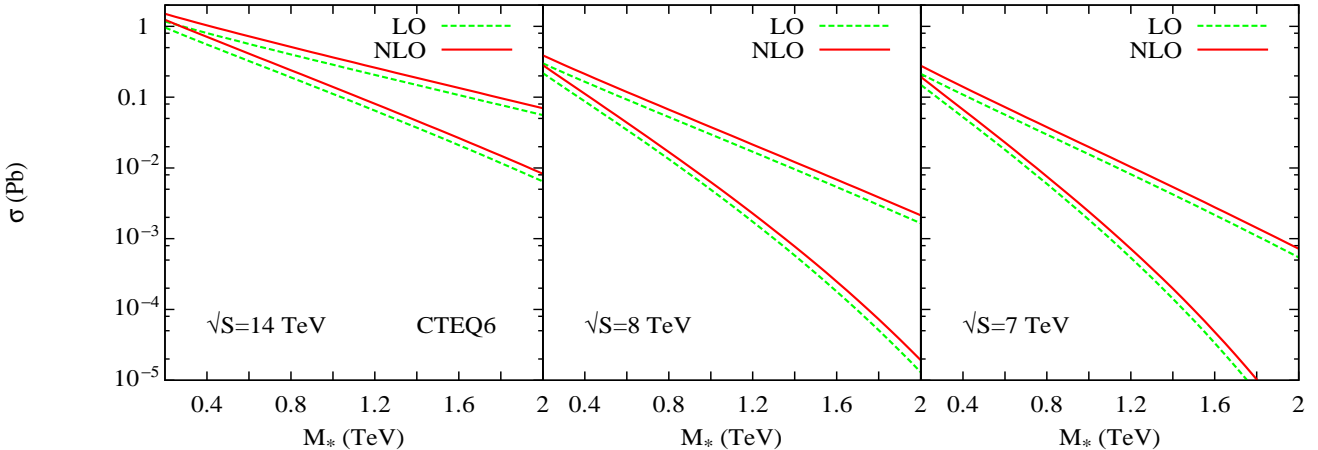


Figure 1: Variation of total cross-section for  $\bar{l}^*l^*$  and  $\bar{l}^*l$  ( $\bar{l}l^*$ ) production with respect to excited lepton mass  $M_*$  at LHC. For each set, the solid (dashed) refer to NLO (LO) cross sections. Upper (lower) set is represents  $\bar{l}^*l$  ( $\bar{l}^*l^*$ ) for  $\Lambda = 6$  TeV only.

In figure 1, we have plotted total cross section of excited lepton pair ( $\bar{l}^*l^*$ ) as well as one excited lepton and a SM lepton ( $\bar{l}^*l$ ,  $\bar{l}l^*$ ) versus the excited lepton mass of  $M_*$  for all the light flavors ( $u, d, s$ -quarks). In figure 2, we have shown the variation of total cross section of one

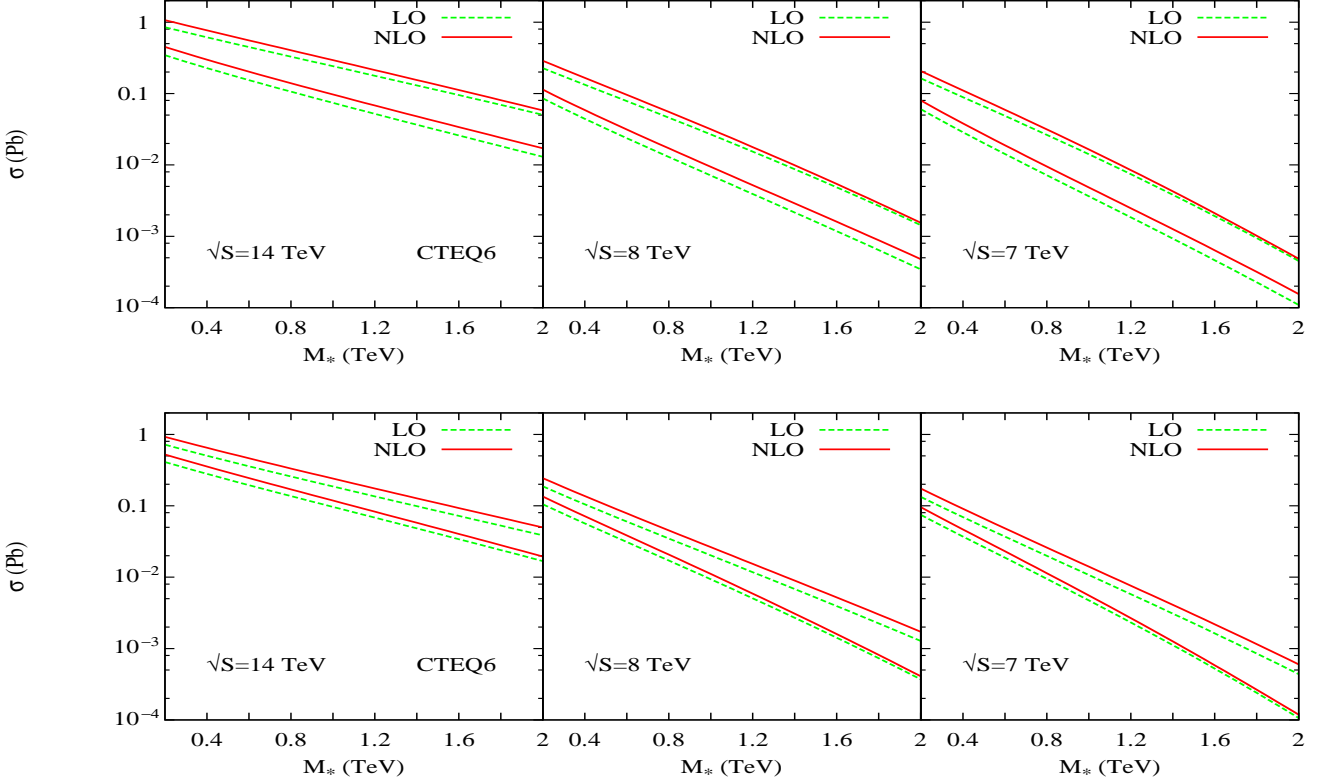


Figure 2: Variation of total cross-section for  $\ell^* \bar{\ell}(\ell \bar{\ell}^*)$  production with respect to excited lepton mass  $M_*$  at LHC. For each set, the solid (dashed) refer to NLO (LO) cross sections. In upper panel, the upper (lower) set represents  $u\bar{u}(d\bar{d})$  initiated process and in lower panel, the upper (lower) set represents  $u\bar{d}(d\bar{u})$  initiated process at born level for  $\Lambda = 6$  TeV only.

excited lepton and a SM lepton ( $\bar{l}^* l$ ,  $\bar{l} l^*$ ) with respect to excited lepton mass  $M_*$  for individual flavor at initial state. We have done this to examine whether it is flavor dependent or not. As we have seen from figures 1, 2 that the cross section decreases as excited lepton mass of  $M_*$  increases due to the fact that parton distribution functions fall at higher momentum fractions. The fall of total cross section is more lower center of mass (c.o.m.) energy  $\sqrt{S}$  ( $\equiv 7, 8$  TeV say) than the higher c.o.m. energy  $\sqrt{S}$  ( $\equiv 14$  TeV). The reason for this is higher momentum fraction  $\tau$  ( $\equiv x$ , the Bjorken scale) and hence we are integrating over small phase space region at lower center of mass energy  $\sqrt{S}$ . As expected, the  $\bar{l}^* l$  production cross section is less and falls faster as compare to  $\bar{l} l$  ( $\bar{l} l^*$ ) production cross section. All the cross sections (figures 1, 2) look similar behaviour except the numerical values. From figure 2, it apparently seems that the cross sections for different flavors at initial state are different i.e. they are flavor dependent. Actually it is not. This difference between the individual cross sections are due the individual flux difference. This implies that contact interaction is flavor blind.

To quantify the enhancement of NLO cross section, we define a variable called  $K$ -factor given by

$$K_i = \frac{\sigma_i^{NLO}}{\sigma_i^{LO}} \quad i = total, u\bar{u}, d\bar{d}, u\bar{d}, d\bar{u} \quad (14)$$

where the LO (NLO) cross sections are computed by convoluting the corresponding parton-level cross sections with the LO (NLO) parton distribution functions.

In figures 3,4 we have shown the variation of  $K$ -factor with respect to the  $M_*$ . The variation of total  $K$ -factor (figure 3) is about 25% – 30% for moderate values of  $M_*$  ( $\leq 1$  TeV) at low

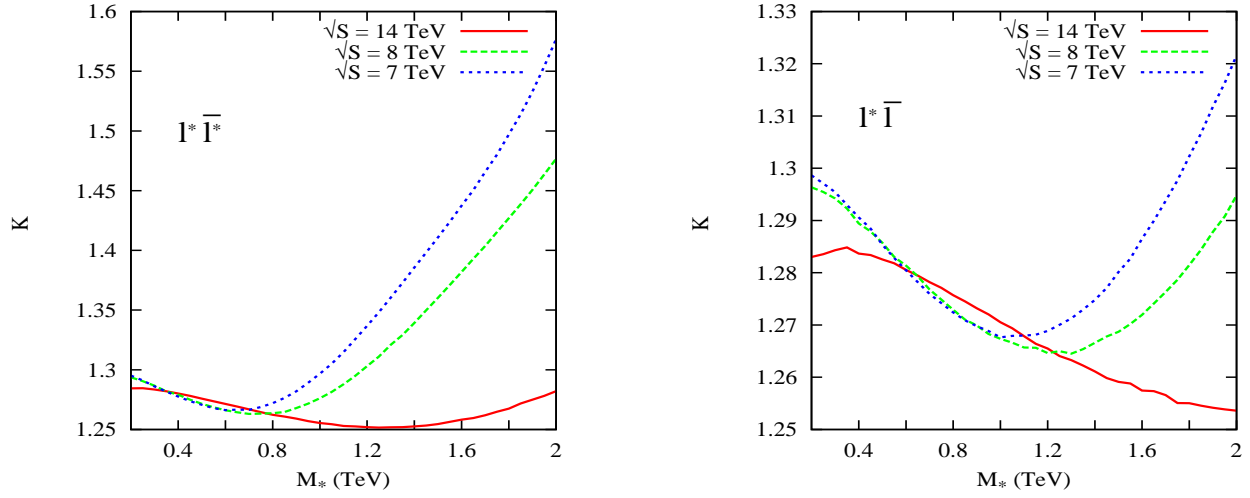


Figure 3: Variation of  $k$ -factor with respect to excited lepton mass  $M_*$  for  $\Lambda = 6$  TeV at LHC.

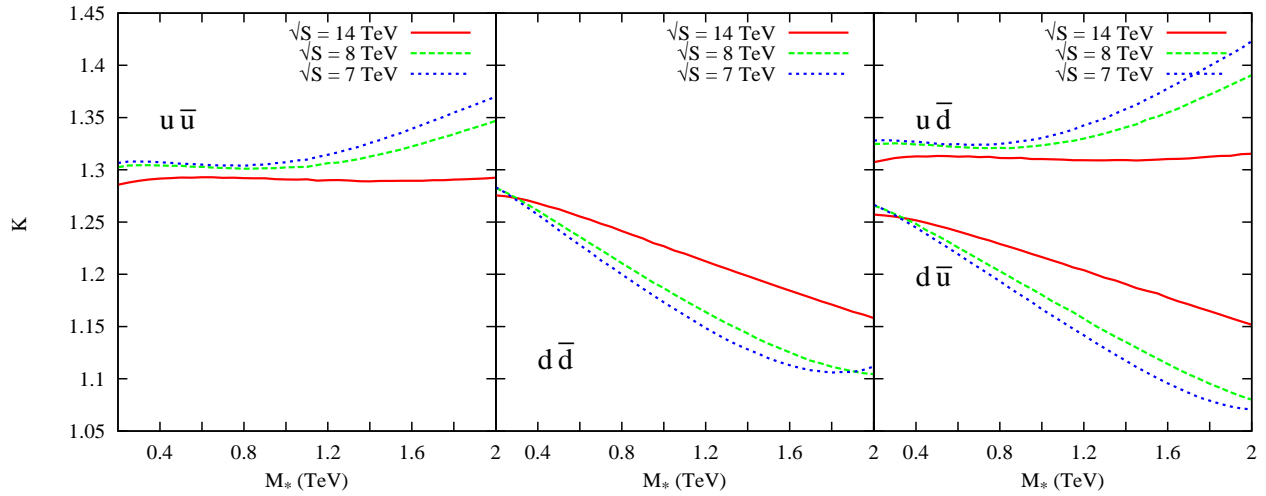


Figure 4: Variation of  $k$ -factor with respect to excited lepton mass  $M_*$  for  $\Lambda = 6$  TeV at LHC.

c.o.m energies ( $\sqrt{S} = 7, 8$  TeV). At larger mass region ( $M_* > 1$  TeV), the  $K$ -factor rises very fast 25% – 60%. At high c.o.m energies ( $\sqrt{S} = 14$  TeV), the variation of  $K$ -factor is about 25% – 30% for reasonably high value of mass range ( $M_* \leq 2$  TeV). In figure 4, we have shown the  $K$ -factor variation for individual flavor only for the  $\bar{l}^* l(\bar{l} l^*)$  production process. In figures (3,4), the rate of fall of  $K$ -factor is much slower at higher c.o.m energy (say  $\sqrt{S} = 14$  TeV) than the lower c.o.m energies because of the fact that at lower c.o.m energy, we are integrating over smaller phase space region and also higher momentum fraction. As the Bjorken  $x$  increases to unity, the parton distribution function falls very steeply and also the fall of LO parton distribution is more than the NLO parton distribution. This is the reason at lower energy, the  $K$ -factor increases very fast as mass  $M_*$  increases towards the center of mass energy. As expected (flavor independent), variation of individual  $K$ -factor is quite similar to the variation of total  $K$ -factor. One can also see from figure 4 the numerical difference between the individual quark  $K$ -factors is due to their respective flux difference.

### 3.1 $\bar{l} l \gamma$ production

The excited heavy lepton will decay into a light SM fermion and a electroweak gauge bosons  $V(\equiv \gamma, Z, W)$  according to Eqn(2). The total NLO cross section of lepton pair ( $\bar{l} l$ ) and a gauge

boson  $V$  can be calculated by multiplying the branching ratio to Eqn.13 as given below

$$\sigma^{P_1 P_2}(M_*^2, S, \Lambda) = BR(l^* \rightarrow lV) \int \frac{d\sigma^{P_1 P_2}(\tau, Q^2)}{dQ^2} dQ^2. \quad (15)$$

The partial decay widths of excited leptons for various electroweak gauge bosons are given by

$$\Gamma(l^* \rightarrow lV) = \frac{1}{8} \alpha f_V^2 \frac{M_*^2}{\Lambda^2} \left(1 - \frac{m_V^2}{M_*^2}\right) \left(2 + \frac{m_V^2}{M_*^2}\right) \quad (16)$$

with

$$f_\gamma = f T_3 + f' \frac{Y}{2} \quad (17)$$

$$f_Z = f T_3 \cot \theta_w - f' \frac{Y}{2} \tan \theta_w \quad (18)$$

$$f_W = \frac{f}{\sqrt{2}} \csc \theta_w \quad (19)$$

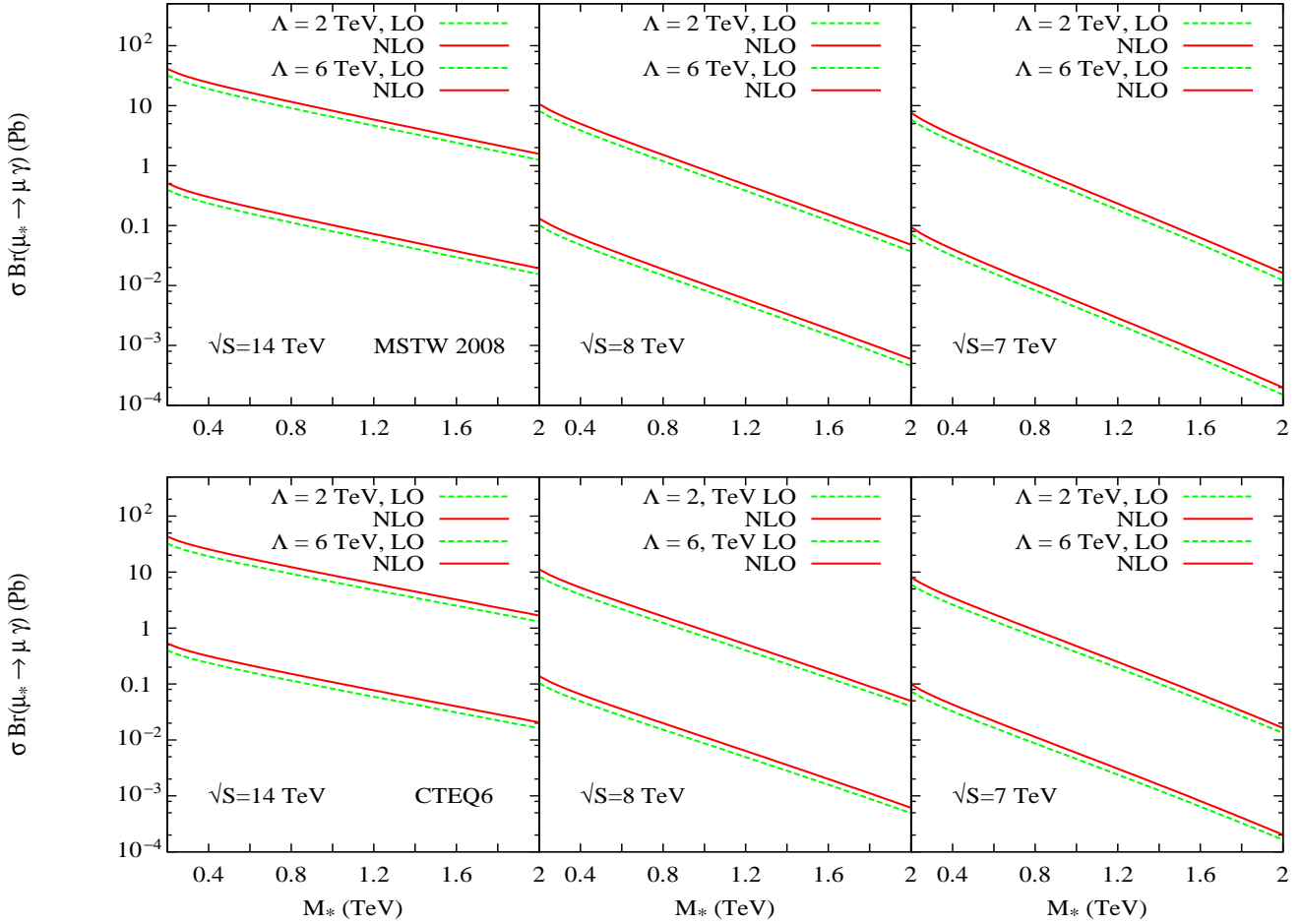


Figure 5: Total cross-section for  $\ell\bar{\ell}\gamma$  production at LHC. For each set, the solid (dashed) refer to NLO (LO) cross sections. Upper (lower) set is for  $\Lambda = 2(6)$  TeV.



In figure 5, we have plotted total cross section versus invariant mass of one lepton and a photon  $M_*$  ( $\equiv M_{\ell\gamma}, M_{\bar{\ell}\gamma}$ ) for two PDFs CTEQ6 and MSTW 2008 [31] all the light flavors ( $u, d, s$ -quarks) for two different contact interaction scale  $\Lambda = 2, 6$  TeV. The cross section decreases as invariant mass  $M_*$  increases due to the fact that parton distribution functions fall at higher momentum fractions as mentioned above. The variation of cross section looks same from figure 5 for two different PDFs. In Actual practice they are not same. This can be found out from figures 6 and has been explained later on. From the figures 5 we see that as the contact interaction scale ( $\Lambda$ ) increases, the cross section (both LO as well as NLO) decreases uniformly as  $\Lambda^{-4}$  as expected (from Lagrangian 1) for a fixed center of mass energy ( $S$ ) and for different values of  $\Lambda$ , the cross section scales accordingly. Therefore, one can obtain the cross section (for both LO as well as NLO) for arbitrary values of  $\Lambda$  by multiplying with an appropriate scale factor to our results.

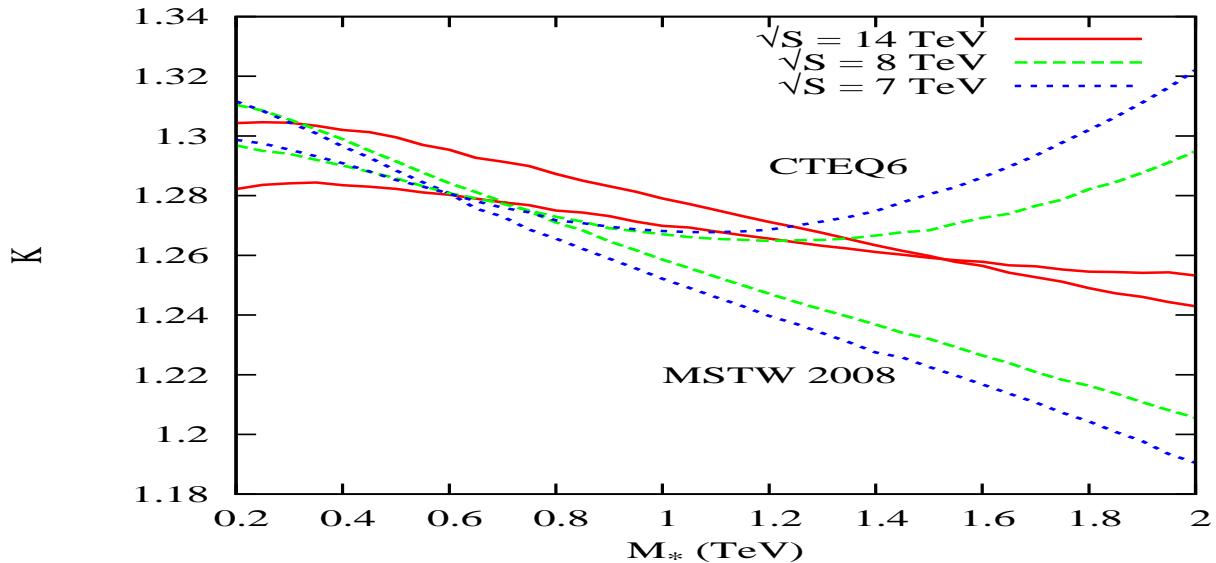


Figure 6:  $K$ -factor for  $\ell\bar{\ell}\gamma$  production at three different energies of LHC. The upper (lower) set is for MSTW 2008 (CTEQ6) parton distribution functions.

In figures 6, we have shown the variation of total  $K$ -factor with respect to the  $M_*$ . The variation of  $K$ -factor is about 25% – 35% for both PDFs CTEQ6 and MSTW 2008. The major difference in  $K$ -factor between the two PDFs (specially at low center of mass energy) is due different parameterizations of their parton distribution functions and the different data sets. In figures 6, the rate of fall of  $K$ -factor is much slower at higher c.o.m energy (say  $\sqrt{S} = 14$  TeV) than the lower c.o.m energy as explained before.

In our above discussion, we consider for simplest case  $\mu_F^2 = \mu_R^2 = Q^2$  where the cross section depends only on physical scales the c.o.m. energy ( $\sqrt{S}$ ) and the masses of final state particles ( $M_*$ ). Now we turn on another scale called factorization scale  $\mu_F^2 (= \mu_R^2$  the renormalization scale). In figure 7, we have shown the factorization scale dependence. We have seen from this figure that the scale dependence reduced greatly at NLO cross section compare to LO cross section. This signifies the necessity of NLO QCD corrections.

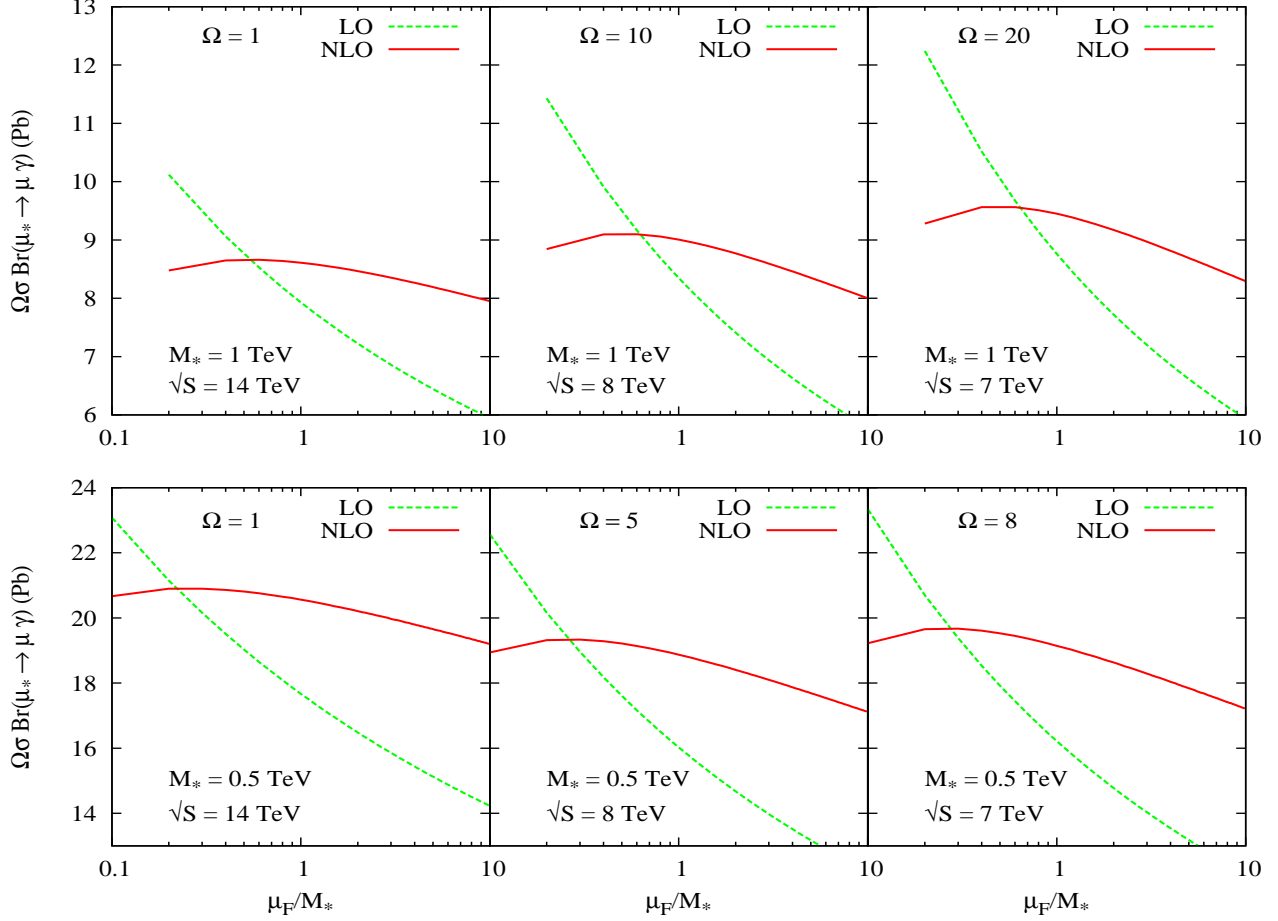


Figure 7: Variation of total cross section with respect to the factorisation scale  $\mu_F$  using CTEQ6 PDFs. Here  $\Omega$  is just a constant scale factor introduced to put all the figures on same frame of scale.

## 4 Conclusions

To conclude, we have systematically calculated the next-to-leading order QCD corrections for the  $V \pm A$  type contact interactions eqn.(1). Opposed to naive expectations, we have showed that the QCD corrections are meaningful and reliable to such non-renormalizable theory.

We have analyzed the variation of cross section with respect to excited lepton mass (and hence the invariant mass of one SM lepton and one SM gauge boson) at the LHC. The enhancement of NLO cross sections over the LO cross sections expectations are found to be quite significant. To quantify the enhancement, we present the corresponding  $K$ -factors in a suitable form for use in experimental analyzes. As it is well known that the predictions on cross sections calculated at leading order in perturbation theory suffer scale uncertainty resulting from arbitrary choice of factorization scales and renormalization scales. These scale uncertainties are due to the absence of higher order contributions to the leading order in perturbation theory. By including more and more higher order contributions to theory, these scale uncertainties reduce gradually and the predictions become more reliable. We have showed these scale dependence of our results. As expected, we have seen that the scale dependences reduced greatly for the case of the NLO results as compared to that for the LO case.

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