A Foundational Principle for Quantum Mechanics[1]

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In contrast to the theories of relativity, quantum mechanics is not yet based on a generally accepted conceptual foundation. It is proposed here that the missing principle may be identified through the observation that all knowledge in physics has to be expressed in propositions and that therefore the most elementary system represents the truth value of one proposition, i.e., it carries just one bit of information. Therefore an elementary system can only give a definite result in one specific measurement. The irreducible randomness in other measurements is then a necessary consequence. For composite systems entanglement results if all possible information is exhausted in specifying joint properties of the constituents.

1. INTRODUCTION

Quantum mechanics is magic.

Daniel M. Greenberger[1]

Physics in the 20th century is signified by the invention of the theories of special and general relativity and of quantum theory. Of these, both the special and the general theory of relativity are based on firm foundational principles, while quantum mechanics[3] lacks such a principle to this day. By such a principle, I do not mean an axiomatic formalization of the mathematical foundations of quantum mechanics, but a foundational conceptual principle. In the case of the special theory, it is the Principle of Relativity,

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[1] This paper is dedicated to Daniel M. Greenberger on the occasion of his 65th birthday.

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[3] Here "quantum theory," "quantum mechanics," and "quantum physics" are used interchangeably, all in a very broad sense.

stating that all laws of physics must be the same in all inertial reference frames, independent of their state of relative motion. In the case of the theory of general relativity, we have the Principle of Equivalences:[2] "In a gravitational field (of small spatial extension), things behave as they do in a space free of gravitation, if one introduces, in place of an "inertial system," a reference system that is accelerated relative to an inertial system." Both foundational principles are very simple and intuitively clear.

On these principles the two theories of relativity are built, which then lead to some surprising and in part even counterintuitive consequences, even as the theories themselves are based on such intuitively nearly obvious principles. I submit that it is because of the very existence of these fundamental principles and their general acceptance in the physics community that, at present, we do not have a significant debate on the interpretation of the theories of relativity. Indeed, the implications of relativity theory for our basic notions of space and time are broadly accepted.

In quantum mechanics, to the contrary, we do observe the presence of a broad discussion about the interpretation of the theory. In fact, we have a number of coexisting interpretations utilizing mutually contradictory concepts.[3] Possibly the coexistence of such a large number of philosophically quite different interpretations in itself contains an important message. I suggest that the message is that a generally accepted foundational principle for quantum mechanics has not yet been identified.

A few remarks are essential here in order to clarify what I mean by interpretation. As I analyzed earlier,[3] there exist at least two different levels of interpretation of a theory. On the first, basic, level, interpretation tells us how to verify the theoretical predictions. A huge set of operational and experimental rules and concepts connects the symbols used in the theory with observation. In the case of quantum mechanics, an essential ingredient at the basic level is the interpretation of the absolute square of the amplitude as a probability or probability density. On the second, the meta-level, less operational but conceptually more significant, interpretation means an analysis of what the theory implies for our general view of the world ("Weltbild"). It implies questions as to the meaning of the theory in a deeper sense.

It is my understanding that on the first, the operational, level, all interpretations of quantum mechanics essentially agree. They lead to the same experimental predictions. Suggestions actually to change quantum mechanics[4] are not just interpretations but are really alternative theories. In view of the extremely high precision with which the theory has been experimentally confirmed, and in view of its superb mathematical beauty and symmetry, I consider a final success of such attempts to be extremely unlikely.

On the second level of interpretation, where we deal with questions of the meaning of the theory, the situation is complicated. Of the many interpretations, a very incomplete list includes the original Copenhagen Interpretation,[5] the Many-Worlds Interpretation,[6] the Transactional Interpretation,[7] Bohm's interpretation[8] in terms of a quantum potential, and, most recently, Mermin's Ithaca interpretation.[9] As I analyzed earlier,[3] these interpretations imply very different ideas about Nature, about the world, or about our position in the world. While I personally prefer the Copenhagen interpretation because of its extreme austerity and clarity, the purpose of my present paper is not to compare and analyze these interpretations but to attempt to go significantly beyond them. In fact, I wish to suggest ideas for a foundational principle for quantum mechanics.[4]

2. RANDOMNESS

> *Die Schwäche der Theorie liegt ... darin, dass sie Zeit und Richtung des Elementarprozesses dem "Zufall" überläßt.*[5]

> Albert Einstein[11]

Our physical description of the world is represented by propositions. Any physical object can be described by a set of true propositions. A complete description of an object in general is a very long list of propositions. In everyday life and in classical physics, one regards these propositions as describing properties the object actually possesses, usually prior to and independent of observation. We now ask ourselves two questions. First, how do we arrive at such propositions and, second, how would we verify them? To answer the first question, we note that any such proposition is obtained through earlier observation. This need not be a single observation, and it need not be observations at the same time or the same place. To answer the second question, we note that any such proposition can be verified through future observation. We thus note that any properties we might assign to an object are arrived at only by observation and are tenable only as long as they do not contradict any further observation. In fact, the object therefore is a useful construct connecting observations.

We have knowledge, i.e., information, of an object only through observation. Thus, any concept of an existing reality has to be based on observations. Yet this does not imply—as tempting as such a conclusion might

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[4] A first, somewhat implicit, use of the principle was made in an analysis of two-photon entanglement and of quantum teleportation (see Ref. 10).

[5] The weakness of the theory lies ... in the fact, that it leaves time and direction of the elementary process to "chance." (translation by A.Z.)

be—that reality is no more than a pure subjective human construct. From our observations we might mentally construct objects of reality. Predictions based on any such specific model of reality may then be checked by any-one. As a result we may arrive at intersubjective agreement on the model, thus lending a sense of objectivity to the mentally constructed objects.

What, then, is the role of physics? Using previously obtained informa-tion we wish to make predictions about the future. Again, our predictions might be formulated as predictions about some future properties of a system or object. Clearly, these predictions have implications for and indeed are propositions about specific future observations. It is, then, an important, though perhaps not the only, role of physics to connect past observation with future observation: or, more precisely, to make specific. but in general probabilistic, statements about results of future observations based on past observations. The connection might be very complicated. To express regularities and generalities in such connections is the point of laws of physics.

In quantum mechanics this expression is exactly what is achieved by the Schrödinger equation. The initial state $\psi(\vec{r}, t_i)$ at time t_i represents all our information as obtained by earlier observation, observation of any rele-vant features of our experimental setup. Using the Schrödinger equation, we can derive a time-evolved final state $\psi(\vec{r}, t_f)$ at some future time t_f. That state is just a short-hand way of representing the outcomes of all possible future observations. In general, those outcomes are probabilistic. By obser-vations, we always mean observations of properties of our classical apparatus. It is not necessary to assume that the future properties of the classical apparatus can be predicted with certainty. Indeed, in general, quantum physics just gives the probabilities of observing specific future properties of the classical apparatus.

According to the standard understanding of quantum mechanics, the specific result is objectively random unless the quantum state is in an eigenstate of the projection operator describing the measurement. To illustrate that point, let us consider the state $|\psi\rangle$, which is an eigenstate of the projection operator $P_{op} = |\psi\rangle\langle\psi|$ with eigenvalue 1, that is, $|\psi\rangle = |\psi\rangle\langle\psi|\psi\rangle$. This simply means that the quantum system described by the state will be found with certainty to be in the state $|\psi\rangle$ if it is measured with the appropriate apparatus. What about other measurements?

Let us consider the specific case of a spin-1/2 particle with spin up along the z-axis, i.e., in the state $|\psi\rangle = |+z\rangle$. Then it follows immediately that the probability to find the particle's spin along a general direction at an angle ϑ with respect to the $+z$ direction is $P = \cos^2(\vartheta/2)$. Thus, specifically, for $\vartheta = 90°$, we obtain $P = 1/2$, that is, the answer the experiment gives when we measure along that direction is completely random. Quantum mechanics

does not provide any reason why in a specific run of the experiment the specific result observed is actually obtained. In essence, this is the famous measurement problem. Bell[12] has expressed most clearly the misgivings of many about the measurement problem. His goal or hope was finally to "explain why events happen."

Here we turn the argumentation around. We will see that Bell's program is unachievable if we accept some very natural principles about the connection between information and elementary systems. In consequence, we will obtain new insight into the foundations of quantum mechanics.

In order to analyze the information content of elementary systems, we now decompose a system which may be represented by numerous propositions into constituent systems. It is natural to assume that each such constituent system will be represented by fewer propositions. How far, then, can this process of subdividing a system go? It is obvious that the limit is reached when an individual system finally represents the truth value to one single proposition only. Such a system we call an elementary system. We thus suggest a principle of quantization of information as follows.

An elementary system represents the truth value of one proposition.

To turn the principle around, the opposite would be absurd, namely, that the information content of a system would not scale with its size. We now note that the truth value of a proposition can be represented by one bit of information with "true" being identified with the bit value "1" and "false" being identified with the bit value 0. Thus, our principle becomes simply:

An elementary system carries 1 bit of information.

We remark that this might also be interpreted as a definition of what is the most elementary system. We stress, again, that by proposition we mean something which can be verified directly by experiment. In order to avoid misconceptions, I would like to underline that notions such as that a system "represents" the truth value of a proposition or that it "carries" one bit of information only implies a statement concerning what can be said about possible measurement results.

Let us again come back to our example above: the spin of a spin-1/2 particle represents the truth value of only one proposition.[6] In our case the

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[6] Clearly, the state of an elementary particle is also characterized by other quantum numbers, it is in general an elementary system in more than one property. The cases of Hilbert spaces of higher dimension deserve separate analysis. E.g., a three-state system represents 1 trit of information. Here we restrict our analysis to two-state systems.

true proposition is, "A measurement of spin along the z-axis will definitely give the result $+$." The spin of the particle carries the answer to one question only, namely, the question, What is its spin along the z-axis? Only if we actually perform a measurement in the z-basis can the measurement result be definite. Since this is the only information the spin carries, measurement along any other direction must necessarily contain an element of randomness. We remark that this kind of randomness must then be irreducible, that is, it cannot be reduced to hidden properties of the system, otherwise the system would carry more than a single bit of information. We have thus found a reason for the irreducible randomness in quantum measurement. It is the simple fact that an elementary system cannot carry enough information to provide definite answers to all questions that could be asked experimentally.

As discussed above, we know that in the case of a spin measurement, the degree of randomness depends on the relative orientation between the measurement direction and the direction along which our system is in an eigenstate. Clearly, from symmetry, the probability of finding a given spin value along the specified measurement direction must depend only on the angle between the measurement direction and the eigenstate direction. In a separate paper, it will be argued[13] that the most natural function describing this behavior consistent with the principle that the quantum system carries only one bit of information is the well-known sinusoidal dependence.

The extreme case is when the measurement direction is orthogonal to the eigenstate direction. Then, for the new measurement situation the system does not carry any information whatsoever, and the result is completely random. Or, in other words, the result is completely random because in such a measurement the elementary system carries no information whatsoever about the measurement result. We note that, most importantly, after the measurement the system is found in a new definite state. The information carried now by the system is not in any way determined by the information it carried before the measurement. Thus we conclude that the new information the system now represents has been spontaneously created in the measurement itself.

We finally remark that the viewpoint just presented lends natural support to Bohr's notion of complementarity. This notion is well-known, for example, for position and momentum or for the interference pattern and the path taken in a two-slit experiment; precise knowledge of one quantity excludes any knowledge of the other complementary quantity. In our case, measurements of a particle's spin along orthogonal directions are complementary, and the reason is, again, the fact that an elementary system carries only one bit of information.

3. ENTANGLEMENT

> *It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature.*
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> Niels Bohr[14]

Another fundamental feature of quantum mechanics is entanglement.[15] We now argue that entanglement follows from a slight generalization of our principle presented above. To do this we analyze how much information is contained in more complex objects, consisting of N elementary systems. Evidently, there are many ways in which the total information represented by a system can increase with its size. Here I argue for one specific solution to the question. Consider N elementary systems, which, by our principle above, therefore represent N independent individual propositions—evidently each system just one. Let us assume that these systems are completely separated initially. By complete separation I mean that we have no interaction between individual elementary systems and no additional information is contained in how the systems relate to each other. Then we have our principle of quantization of information generalized to:

> N *elementary systems represent the truth values of* N *propositions*.
> N *elementary systems carry* N *bits.*

Now let the initially separated systems interact with each other. It is then suggestive to assume that the information represented jointly by the N systems is conserved during the interaction process if there is no information exchange with the environment. That is, the interaction can neither increase the total amount of information represented by the total system nor reduce it. We remark that our principle does not make any statement of how the information contained in the N propositions (the N bits of information) is distributed over the N systems. After the interaction the N bits might still be represented by the N systems individually or, alternatively, they might all be represented by the N systems in a joint way, in the extreme with no individual system carrying any information on its own. In the latter case we have complete entanglement.

In order to analyze entanglement in view of the ideas just proposed above, let us consider two elementary systems carrying therefore two bits of information, i.e., representing the truth value of two propositions. For reason of simplicity, we consider two spin-1/2 particles. Which two propositions are possible to describe completely the system of our two particles?

A most simple case would be propositions which describe each one of the two particles separately. If, without loss of generality, we consider measurement of spin along the z-axis, then proposition 1 could simply be

a statement about the spin of particle 1 along that axis, and proposition 2 could be a statement about the spin of particle 2 along that axis. Then four possibilities result:

$$|\psi\rangle_1 = |+z\rangle_1 |+z\rangle_2$$
$$|\psi\rangle_2 = |+z\rangle_1 |-z\rangle_2$$
$$|\psi\rangle_3 = |-z\rangle_1 |+z\rangle_2 \qquad (1)$$
$$|\psi\rangle_4 = |-z\rangle_1 |-z\rangle_2$$

where, e.g., $|+z\rangle_1$ represents the state of particle 1 along the $+z$-axis. Evidently, the four resulting possibilities are rather trivial and are also present in classical mechanics. In our new language this is the case where each spin itself represents one proposition on its own. Actually, the four states (1) are the representation of the four possible two-bit combinations (true–true, true–false, false–true, false–false) of the truth values of the propositions, "The spin of particle 1 is up along z" and "The spin of particle 2 is up along z."

Instead of choosing propositions which describe the individual members of the system, we could alternatively choose propositions which describe results of joint observations. Consider, e.g., the proposition "The two spins are the same along z." Then, clearly, we have two possibilities: the two spins could be either both up along z or both down along z, i.e.,

$$\text{either} \qquad |+z\rangle_1 |+z\rangle_2 \qquad \text{or} \qquad |-z\rangle_1 |-z\rangle_2 \qquad (2)$$

If this is all we know,[7] then the system is incompletely described. This is necessarily the case because we have exhausted only one possible proposition, i.e., one bit of information. A trivial way to describe the system completely is also to specify the spin af an individual member of the system, i.e., to assume that the system represents the truth value of a proposition like, "Spin 1 is up along the z-axis." Then we have two propositions and, actually, another proposition immediately follows, namely, "Spin 2 is up along the z-axis" and this case reduces to the one just discussed.

Yet we still have another, very different, possibility to complete the description of the system as started using the above proposition, "The two particles have the same spin along the z-axis." Instead of choosing as the second proposition one about the properties of an individual, we could

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[7] Just to stress our point again: By "are the same along z," we mean something like "Should they be measured along z, they would be found to be identical," and analogously for propositions about individual systems. This does not imply that the system necessarily "has" the measured property before the measurement.

choose another proposition also describing joint properties of the system. This could, e.g., be a proposition stating, for some other chosen direction, that the two spins are also equal along the new direction. Consider specifically the proposition, "The two spins are equal along the x-axis." Then we know that either both are up along x or both are down along x should they be measured along x. Quantum mechanically, the situation is

$$\text{either} \qquad |+ x\rangle_1 |+ x\rangle_2 \qquad \text{or} \qquad |- x\rangle_1 |- x\rangle_2 \tag{3}$$

How can both (2) and (3) be true for the same two particles? They can if we note that these two propositions together, namely, that the two spins are equal along the z-axis and they are equal along the x-axis, now uniquely (up to a trivial phase factor) determine the entangled quantum state

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|+ z\rangle_1 |+ z\rangle_2 + |- z\rangle_1 |- z\rangle_2)$$

$$= \frac{1}{\sqrt{2}}(|+ x\rangle_1 |+ x\rangle_2 + |- x\rangle_1 |- x\rangle_2) \tag{4}$$

That state does not contain any information about the individuals; all information is contained in joint properties. In fact, now there cannot be any information carried by the individuals because the two bits of information are exhausted by defining that maximally entangled state, and no further possibility exists also to encode information in individuals. As an example to exhibit the richness of our approach, let us consider an alternative choice for the second proposition, the first one still being equality along the z-direction. Let us assume that the second proposition is now, "The two spins are different along x." Then the two spins are

$$\text{either} \qquad |+ z\rangle_1 |- x\rangle_2 \qquad \text{or} \qquad |- x\rangle_1 |+ x\rangle_2 \tag{5}$$

It can easily be seen that now the entangled quantum state representing the situation is

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|+ z\rangle_1 |+ z\rangle_2 - |- z\rangle_1 |- z\rangle_2)$$

$$= \frac{1}{\sqrt{2}}(|+ x\rangle_1 |- x\rangle_2 + |- x\rangle_1 |+ x\rangle_2) \tag{6}$$

This means that our first proposition determines which of the two terms appear in the entanglement when represented in the z-basis, and the second proposition fixes their relative phase.[10] As above, where the two propositions were used to determine properties of the individuals, we again obtain four orthogonal states

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|+z\rangle_1|+z\rangle_2 + |-z\rangle_1|-z\rangle_2)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|+z\rangle_1|+z\rangle_2 - |-z\rangle_1|-z\rangle_2)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|+z\rangle_1|-z\rangle_2 + |-z\rangle_1|+z\rangle_2)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|+z\rangle_1|-z\rangle_2 - |-z\rangle_1|+z\rangle_2)$$

$$(7)$$

These four Bell states[16] are now the representation of the four possible two-bit combinations (true–true, true–false, false–true, false–false) of the thruth values of the propositions, "The two spins are equal along z" and "The two spins are equal along x."

Note that we have sketched a natural understanding of quantum entanglement as a consequence of our fundamental principle. Also note that we do not make any statement as to the relative time ordering of the observations on the two systems, their relative space arrangement, and the like. Thus nonlocality comes in naturally.

We might finally remark that, from our viewpoint, quantum teleportation[17] also obtains a very natural interpretation. All that changes by Alice's observation is the set of propositions describing possible results without any information actually transmitted to Bob as a consequence of her measurement alone!

It might amuse Dan Greenberger that this procedure can be continued to more and more elementary quantum systems. As a very specific result, three-particle entangled states, so-called GHZ states,[18] can be described by three elementary propositions. For example, consider the eight possible three-particle GHZ states first introduced by Mermin,[19]

$$\frac{1}{\sqrt{2}}(|+++\rangle + |---\rangle)$$

$$\frac{1}{\sqrt{2}}(|+++\rangle - |---\rangle)$$

$$\frac{1}{\sqrt{2}}(|++-\rangle + |--+\rangle)$$

$$\frac{1}{\sqrt{2}}(|++-\rangle - |--+\rangle)$$

$$\frac{1}{\sqrt{2}}(|+-+\rangle + |-+-\rangle)$$

$$\frac{1}{\sqrt{2}}(|+-+\rangle - |-+-\rangle)$$

$$\frac{1}{\sqrt{2}}(|+--\rangle + |-++\rangle)$$

$$\frac{1}{\sqrt{2}}(|+--\rangle - |-++\rangle) \tag{8}$$

where we use the abbreviation $|+++\rangle = |+z\rangle_1 |+z\rangle_2 |+z\rangle_3$, and similarly for the other terms.

It is clear that in order to define all states uniquely, we need all eight combinations of the three propositions from true–true–true to false–false–false. It is also obvious that we cannot start as simply as before in the case of two particles by just taking as proposition 1, "The three spins are equal along z." This is because such a statement distinguishes only the first two states from the remaining six states, the latter not being distinguishable by just two bits, i.e., by the truth values of just two propositions.

Here I leave it as a puzzle what these three propositions are. I hope that Danny Greenberger, who is a passionate solver of Sunday crossword puzzles, will enjoy solving this small birthday present puzzle. And I am sure not only that he will have the solution in no time but also that he will immediately obtain many possible generalizations to more particles.

4. COMMENTS

The principle given above is basic and elementary enough that it actually can serve as a foundational principle of quantum mechanics, that is, that it finally helps to answer the question, "Why the quantum?"[20] This optimism is supported by the observation presented above that the principle carries in its heart two elementary notions of quantum mechanics, namely, the randomness of individual events and entanglement. It is clear that it may be a matter of taste whether or not one accepts the suggested

principle as self-evident, as I do. If not, then I simply propose to turn the reasoning around and, based on our known features of quantum physics, argue for the validity of the principle.

The most fundamental viewpoint here is that the quantum is a consequence of what can be said about the world. Since what can be said has to be expressed in propositions and since the most elementary statement is a single proposition, quantization follows if the most elementary system represents just a single proposition.

While I have given here, only in a very sketchy way, a few points of a new view of quantum mechanics, a number of other fundamental concepts follow and will be elaborated upon in future papers.[21] This will also include a more detailed analysis of philosophical and interpretational consequences. Suffice it to say here that, in my view, the principle naturally supports and extends the Copenhagen interpretation of quantum mechanics. It is evident that one of the immediate consequences is that in physics we cannot talk about reality independent of what can be said about reality. Likewise it does not make sense to reduce the task of physics to just making subjective statements, because any statements about the physical world must ultimately be subject to experiment. Therefore, while in a classical worldview, reality is a primary concept prior to and independent of observation with all its properties, in the emerging view of quantum mechanics the notions of reality and of information are on an equal footing. One implies the other and neither one is sufficient to obtain a complete understanding of the world.

About 20 years ago, I first met Daniel M. Greenberger. For me, this was one of the most important encounters of my life. Not only have we become personal friends, but his openness and ready acceptance of unusual views were very crucial in forming my own view of science. Danny Greenberger is one of the few living physicists who considers it not only possible but highly likely that our present worldview of physics may be overthrown in the not-too-distant future. This is a very healthy attitude against becoming too complacent. In his mind, one of the most useless ideas is that a final Theory of Everything is just around the next corner. I do hope that my suggestion presented above is met by Danny's approval: Not necessarily approval of its contents but, hopefully, approval of the fact that it tries to open up a new avenue for the understanding of quantum mechanics.

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