Mysticism and mathematics: Brouwer, Gödel, and the common core thesis[1]

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David Hilbert opened 'Axiomatic Thought' [15] with the observation that 'the most important bearers of mathematical thought,' for 'the benefit of mathematics itself have always [...] cultivated the relations to the domains of physics and the [philosophical] theory of knowledge.' We have in L.E.J. Brouwer[3] and Kurt Gödel[4] two of those 'most important bearers of mathematical thought' who cultivated the relations to philosophy for the benefit of mathematics (though not only for that). And both went beyond philosophy, cultivating relations to mysticism for the benefit of mathematics (though not for that alone).

There is a basic conception of mysticism that is singularly relevant here. ('Mysticism' labels that.) That corresponds to a basic conception of philosophy ('Philosophy'), also singularly relevant here. Both Mystic and Philosopher begin in a condition of seriously unpleasant, existential unease, and aim at a condition of abiding ease. For Mystic and Philosopher the way to that ease is through being enlightened about the real and true good of all things. Thus Mysticism and Philosophy are triply optimistic: there is a real, true good of all things, the Philosopher and Mystic can become enlightened about it, and being thus enlightened would give them ease.

That Enlightenment sought comes from some sort of cognitive or intelligent engagement with what we will here call 'the Good'. Some use 'the Absolute' when it seems important to emphasize that 'the Good' is unconditioned—there is nothing behind it, nothing above it. Others use 'the One'; still others, 'God'. It is natural to regard the Good as somehow mind-like, or like something (permanently) in mind. It should in either case be in some way homogeneous with, or in sympathy with, our minds, for the Good must attract and support the intelligent engagement of it by our minds. In that way it can enlighten us.

The distinction between Philosophy and Mysticism is a matter of degree. Philosophy is dominated by the intention to articulate and rationally proof all

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[3]1881–1966. For his biography, see [19], [8], and [9].

[4]1906–1978. For his biography, see [21], [22], and [10].

claims and insights. Mysticism is not so dominated. But nevertheless, perhaps at some point close to the Good, where every step so far has been rationally proofed, the Philosopher could well have the final and most sublime enlightenment, but find that it is beyond his power or interest to articulate and rationally proof the content of that. It could be beyond his interest because the massive insight is so bright and sharp, so ineluctably clear and certain, that any rational proof at its best could only yield something comparatively darker and less compelling. Gödel attributed such an experience to various philosophers:

> I myself never had such an experience. For me there is no absolute knowledge: everything goes only by probability. Both Descartes and Schelling explicitly reported an experience of sudden illumination when they began to see everything in a different light. [22, p.170]

Where we encounter a Philosopher making claims from out of such a moment, but without any successful rational proofing of those claims, then we can regard what he has given us as Mysticism. In what follows, we will think of the practice of Mysticism as trying to find ways to experience this Good directly. The practice of Philosophy is the attempt to describe this Good intelligently.

Below, we will describe how Brouwer and Gödel each relate mysticism and mathematics, and make a comparison. On the basis of that, we then present a partial argument against what is known as 'the common (or universal) core thesis' (CCT). CCT says that the various mysticisms in the end all are just different ways to express the same core of truths. It seems to us that the common core thesis can be analysed into two propositions:

(a) Mysticism holds that Reality is Good. Mystical practice aims to perceive this Good.

(b) This Good is objective, i.e., the same for all varieties of mysticism.

Of course (a) is only a minimal characterisation of mysticism. It leaves out most aspects of mysticism (e.g. feelings of bliss), but it seems extensionally adequate. We take it to be empirically adequate. 'Reality' with a capital 'R' has a different meaning than 'all-inclusive reality'. The latter surely is one, in the tautological sense that there can be nothing outside of it. But the Good known from mystical traditions has more meaning attached to it.

What makes CCT prima facie implausible is that among the various mystical traditions, and even within each tradition, we find so much disagreement on explicit doctrine and methodology. However, the interest of the common core thesis *depends* on the existence of such disagreements, for in the absence of that it would be almost trivially true.

There is a somewhat analogous case in the philosophy of science: scientific realists hold to a common core thesis with respect to scientific theories through the ages. These theories show massive disagreements; still, the realist holds, they all try to express the same objective reality.

One is reminded of a metaphor that Leibniz used: The same city may look very different depending on what direction you approach it from.

Of course, neither the analogy to scientific realism nor Leibniz's metaphor *adds* support to CCT. They just suggest how the thesis *might* be true in spite of prima facie evidence against it. The argument we want to suggest aims to weaken the case for CCT. It attempts to show that the references of Gödel's and Brouwer's terms for the Good cannot possibly be the same (Gödel speaks of 'the Absolute', Brouwer does not have a term but speaks of a return of consciousness to 'its deepest home'). This leaves open the possibility that at least one of them even does not refer at all. The Good as conceived of by Brouwer may not exist, and the Good as conceived of by Gödel may not exist. One can intend, but not establish, reference to something that doesn't exist. So an argument from the assumption that at least one of these does not exist to the conclusion that Gödel and Brouwer cannot be referring to the same thing is trivial. The case that remains is to assume that both do exist and see if you can then also reach the conclusion that they cannot in fact be the same. Therefore we will consider the latter case.

1 Brouwer's Mysticism

Brouwer thought that there was a 'deepest home' of consciousness [5]. In the deepest home, our experience oscillates between stillness and having sensations. There is no subject-object distinction there. This state Brouwer identifies with wisdom (compare [3, p.108] and [5, p.1240]). Our awareness of objects and other people arises in various stages on what he calls an 'exodus' of consciousness from the deepest home. The first step of this exodus is the result of a free-will act that introduces an awareness of time. In fact, time consciousness is a prerequisite for the awareness of objects and people (including oneself as an embodied person) and everything else in the exterior world. It is time awareness that introduces a distance between the experiencing 'I' and what is experienced. The latter recedes into the past, as a memory, while the former remains in the streaming 'now'. This is the genesis of intentionality (a word that Brouwer does not use).

Brouwer calls consciousness in so far as it exhibits intentionality 'mind' (we will, after the discussion of this particular aspect of Brouwer's philosophy, not use that word in his technical sense). Once that is in place, the mind further develops with consciousness indulging itself in organising sensations into complexes, in particular into 'causal chains' and 'things': the former being vehicles for empowering the will to control the latter. Whatever hold all of these mind-particularizing contents have on the particular self, it is a hold that self maintains; the self could in principle and in practice free itself from that hold by as it were disclaiming all the relevant sensation complexes, for those complexes were adopted on the foundation of absolutely free will intrinsic to consciousness:

> Everyone can have the inner experience, that he can at will dream himself to be without time awareness and without the separation of the *I* and the world of perceptions, or bring about this latter separation by his own effort [4, p.154] [our translation]

Mathematics, Brouwer says, is also built up from our experience of time, as in Kant—hence the name 'intuitionism' for Brouwer's philosophy of mathmatics, referring to the pure intuition of time. The discrete (the natural numbers) arises from our awareness of successive 'nows', the continuous (e.g. the straight line) from our awareness that time is a flow and hence there is something 'in between' the discrete 'nows'. In what Brouwer calls the unfolding of this basic intuition, all of mathematics is created. On this picture, mathematics is a creation of the individual mind. It does not describe an independent reality. It comes into being in an act of the will. In formalizing mathematics, on the other hand, any possible volitional elements are precisely shut out. This is why Brouwer kept clear from formalizing intuitionistic logic (as his student Heyting did), and from setting up a formal theory of the role of the subject in mathematics. As regards the latter endeavour, Stanley Rosen has aptly remarked that

> [A]nalytical philosophy [...] objectifies the subject, or overlooks the presence of the subject in the structure of the proposition [...] This tendency is illustrated in the attempt by Kreisel and others to mathematize Brouwer's conception of the creative subject as expressing the force of mathematics, a force that cannot itself be expressed in mathematical terms. [17, p.186]

According to Brouwer, if you look at it from the philosophical and not merely technical point of view, engaging in mathematics is one of the first things that lead consciousness out of its deepest home. Consciousness builds up its world by starting an 'exodus from the deepest home. We saw that he thinks of these building processes as operating on sequences of sensations, and that is where mathematics comes in. It is not only when doing technical work, but when just constructing 1 and 2 that you are on the wrong track, according to Brouwer! In his notebooks in which he conceived his 1907 thesis, there are many astonishing remarks on how destructive he thinks mathematics is. For example, one there finds gems such as 'One could see as the goal of one's life: abolition and delivery from all mathematics' [8, p.83]. And he meant it: in writings all through his career, Brouwer comments on how mathematics (and, based on that, the natural sciences) introduces great unhappiness in our lives and keeps us away from attaining wisdom again (by returning to the deepest home). Without time awareness, there can be no mathematics. But to be free of mathematics is exactly what we should aim for in our pursuit of the deepest home. And there is even a chance of using language to indicate mystical experiences; but not in the form of analytical (i.e., mathematically structured) prose:

> Perhaps the greatest merit of *mysticism* is its use of language independent of mathematical systems of human collusion, independent also of the direct animal emotions of fear and desire. If it expresses itself in such a way that these two kinds of representations cannot be detected, then the contemplative thoughts—whose mathematical restriction appears as the only live element in the mathematical system—may perhaps again come through without obscurity, since

> there is no mathematical system that distorts them.[Our modification of Van Stigt's translation [19, p.409]; original emphasis]

> The mystical writer will even be careful to avoid anything that smacks of mathematics or logic: weak minds might otherwise be easily made to believe and act mathematically outside the domain where this is required either by the community or their own struggle for life and end up in all kinds of follies. [19, pp.409–410]

> Nowhere in mysticism is there a thread or appropiate sequence; every sentence stands by itself and does not need another to precede or follow it [1, p.76] (trl. [19, p.122])

As examples of such language, Brouwer quotes, in 1905, from Meister Eckehart and Jacob Boehme [1]; and in 1948, from the Bhagavad-Gita [5]. The intellect has nothing to do with it. Access to the Good is only possible when the intellect is switched off. In a review (1915) of a book called 'Geometry and Mysticism', Brouwer wrote:

> As the making and observing of mathematical forms in the *Anschauungs*-world is a preparation for, and a consequence of, the *intellectual* self-preservation of man, and since theoretical mathematics can only be defined as the activity of the *intellect in isolation*, and since furthermore, mystical vision only begins after the intellect has gone to sleep, practical nor theoretical geometry can have anything to do with mysticism. [8, p.287] [original emphases]

We note that for Brouwer, mystical practice was a serious and solitary affair. When visiting Krishnamurti, Brouwer said to a friend: 'Oh my, this is the baby room of philosophy'. [9, p.324] [our trl.]

For Brouwer the intellect plays a negative role in spiritual life. Mathematics is a necessary step away from apprehending mystical truth to apprehending the outside world, one's own body, one's fellows.

2 Gödel's Mysticism

Rudy Rucker has reported on his conversations on mysticism with Gödel [18, pp.182–183]. Gödel's philosophy of mathematics is called platonism. He held that mathematical objects are part of an objective reality, and that what the mathematician has to do is perceive and describe them. Gödel once published some very brief remarks on how we have a perception of the abstract objects of mathematics in a way that is analogous to our perception of concrete objects [12]. Rucker, seeking elucidation of these remarks, asked Gödel 'how best to perceive pure abstract possibility'. Gödel says that, first, you have to close off the other senses, for instance, by lying down in a quiet place, and, second, you have to seek actively. Finally,

> The ultimate goal of such thought, and of all philosophy, is the perception of the Absolute [...] When Plato could fully perceive the Good, his philosophy ended.[5]

Therefore, according to Gödel, doing mathematics is one way to get into contact with that Absolute. Not so much studying mathematics as such, but studying it in a particular frame of mind. This is how we interpret Gödel's remark about Plato. There is, then, no break between mathematical and mystical practice. The one is part of the other, and the good of mathematics is part of the Good.

Gödel also talked about his interest in perceiving the Absolute with his Eckermann, Hao Wang. Wang reports:

> One of Gödel's recurrent themes was the importance of experiencing a sudden illumination—like a religious conversion—in philosophy. (This theme, by the way, reminds me of the teachings of Hui Neng's 'sudden school' of Zen (Chan) Buddhism in China.) In particular, Gödel believed that Husserl had such an experience at some point during the transition between his early and later philosophy.' [22, pp.169–170]

In the late 50's, Gödel began to develop an interest in Edmund Husserl's phenomenology. Besides a specific application of phenomenology to the foundations of mathematics, Gödel had a broader interest. This is again related to the Absolute. To Wang he said,

> At some time between 1906 and 1910 Husserl had a psychological crisis. He doubted whether he had accomplished anything, and his wife was very sick. At some point in this period, everything suddenly became clear to Husserl, and he did arrive at some absolute knowledge. But one cannot transfer absolute knowledge to somebody else; therefore, one cannot publish it. A lecture on the nature of time also came from this period, when Husserl's experience of seeing absolute knowledge took place. I myself never had such an experience. For me there is no absolute knowledge: everything goes only by probability. Both Descartes and Schelling explicitly reported an experience of sudden illumination when they began to see everything in a different light. [22, pp.169–170]

and, as we saw above,

> Later, Husserl was more like Plato and Descartes. It is possible to attain a state of mind to see the world differently. One fundamental idea is this: true philosophy is [arrived at by] something like a religious conversion. [22, p.293]

[5] The original incorrectly has 'Plautus' instead of 'Plato', but Rucker confirmed to us that this is a misprint.

It is likely that Gödel tried to experience such an illumination or conversion. In this connection, we mention that besides books on Christianity and Islam, introductions to Buddhism, Watchtower publications, works on theosophy and some on spiritism, Gödel's personal library also contained

> Wallace, R.K. (1973) The physiological effects of transcendental meditation, 3rd ed., MIU Press.

Rudy Rucker asked Gödel if he believed that there is a single Mind behind all the various appearances and activities of the world [18, 183]. Gödel assented: 'yes, the Mind is the thing that is structured, but the Mind exists independently of its individual properties'. When Rucker then went on to ask Gödel if he believed that the Mind is everywhere, as opposed to being localized in the brains of people, Gödel again assented, saying, 'Of course. This is the basic mystic teaching'.

Gödel was convinced that 'the world is rational', and that this rationality can be grasped by the mind: 'There is a scientific (exact) philosophy and theology, which deals with concepts of the highest abstractness; an this is also most highly fruitful for science' [22, p.316]. For Gödel, then, the intellect has a positive role to play in spiritual life.

3 Comparison of Brouwer and Gödel: mathematics and the Good

We have seen that both Gödel and Brouwer were looking for mystical experiences, in which an openness of the mind to the Absolute is operative. What is disclosed in such experiences has the air of being something imparted to the person. The imparting is preceded by a preparation or transformation of the person. The self must be brought into a condition to receive, support, and appreciate what is to be disclosed. This preparation we see mentioned by both Brouwer (the abandonment of mathematics) and Gödel (closing off the senses, etc.)

However, they made very different claims as to how what is disclosed in such experience is related to mathematics. What strikes us is how the bond between mathematics and mysticism is equally tight in Gödel and Brouwer, but that the signs are different so to speak. According to both, mathematics relates individual thought to ultimate reality, but Gödel thinks of a positive relation and Brouwer of a negative one.

For Gödel, doing mathematics is a way of *accessing* the Absolute. For Brouwer, doing mathematics precisely *prohibits* access to the Absolute.

Put differently, according to Gödel, mathematical experience *reveals* (part of) Reality; according to Brouwer, mathematical experience *conceals* Reality.

A mystical disclosure in the relevant sense has about it the phenomenological character of being a form of knowing or enlightened understanding; it discloses the Good, the significant, the important, fundamental values. Therefore, we

may try to formulate the difference between Brouwer and Gödel in terms of the Good. What do they think about the relation of the Good and mathematics, and what do they think is the good of mathematics itself?[6]

Let us call historical mathematics, mathematics as it is now and has been standardly practiced, H-mathematics. We can speak of the good of the good of such hammers as the one that we happen to find in our toolbox. But for that good, this hammer may not be the best we could make. Similarly, it could very well happen that *the* good of mathematics is not best served by any H-mathematics. That is to say, mathematics at its best (given what is the good of mathematics) may be rather different than H-mathematics.

Brouwer's intuitionistic mathematics is often construed as merely an epistemological or semantical affair. But it might be better understood as a reform of H-mathematics in the direction of better serving its good. Brouwer tried to to realize a mathematics which is at once a creation of the free will for the sake of the fullest, most free, and most concrete exercise of the will. Our will and inner time are coeval, and inner time is where the will meets causality. Definitely controlling the structuring of time is the finest possible preparation for the will to exercise itself on causality through those temporal structures, which are the structures of intuitionistic mathematics. The good of mathematics, on this picture, is that it facilitates our will to power. Brouwer's program shows how a particular understanding of the good of mathematics can have a revisionary effect on mathematical practice.

Notice also that we can speak of the good of something without that good being ultimately beneficial to us, and in that way not part of the Good. This is how Brouwer speaks of the good of mathematics. It facilitates our will to power, but thereby collaborates in furthering our Fallen Condition, and in that sense is an evil. The good of mathematics does not coincide with any absolute good. As Brouwer wrote in the notebooks already mentioned, 'That mathematics and its applications are sinful follows from the intuition of time, which is immediately felt as sinful' [8, p.83]. What Brouwer means is that it is the move of time which leads to the way out of the deepest home; anything that keeps you from returning there is defined as 'sinful'. However, Brouwer did acknowledge a more intrinsic yet limited or conditional goodness of parts of mathematics. For example, about classical logic he says,

> Fortunately classical algebra of logic has its merits quite apart from the question of its applicability to mathematics. Not only as a formal image of the technique of common-sensical thinking has it reached a high degree of perfection, but also in itself, as an edifice of thought, it is a thing of exceptional harmony and beauty. Indeed, its successor, the sumptuous symbolic logic of the twentieth century which at present is continually raising the most captivating problems and making the most surprising and penetrating discoveries, likewise is

[6] G.H. Hardy [14] aims at evaluating the good of mathematics and the good of mathematics in relation to himself. But he certainly is straining to avoid mystic ways. This gives a contrast between mystical and non-mystical evaluations.

> for a great part cultivated for its own sake. [6, p.116]

A yet higher beauty is that found in intuitionistic mathemaics:

> But the fullest constructional beauty is the introspective beauty of mathematics, where [...] the basic intuition is left to free unfolding. This unfolding is not bound to the exterior world, and threby to finiteness and responsibility; consequently its introspective harmonies can attain any degree of richness and clearness. [5, p.1239]

But neither classical nor intuitionistic mathematics has a share in the ultimate or highest beauty. The best would be to abandon logic and mathematics in order to return to the deepest home: 'In wisdom, there is no logic' [3] [trl. CW 110]. For Brouwer, the worth of philosophical investigation of mathematics is shown by its disclosing the relationships between the good of intuitionistic and classical mathematics, and between the good of mathematics and the Good. '[R]esearch in foundations of mathematics is inner inquiry with revealing and liberating consequences, also in non-mathematical domains of thought' [5, p.1249].

We saw that for Gödel, on the other hand, the good of mathematics is part of the Good. This allows him to form, as projections from mathematical knowledge, expectations about the Good:

> One uses inductive evidence. It is surprising that in some parts of mathematics we get complete developments (such as some work by Gauss in number theory). Mathematics has a form of perfection. In mathematics one attains knowledge once for all. We may expect that the conceptual world is perfect and, furthermore, that objective reality is beautiful, good, and perfect. [22, p.316]

Such an induction would have been unacceptable to Brouwer.

In Brouwer, then, mathematics is action for volition, while in Gödel, mathematics is contemplation. Hence Brouwer's disinterest in theoretical values in mathematics (values furthering contemplative knowledge, understanding), and hence Godel's obsession with the theoretical (contemplative) form of mathematics.[7] The contrast between the two attitudes is well illustrated by comparing this quote from Brouwer,

> Strictly speaking the construction of intuitive mathematics in itself is an *action* and not a *science*; it only becomes a science [...] in a mathematics of the second order, which consists of the *mathematical consideration of mathematics* or *of the language of mathematics* [2, p.99] [trl. CW p.61; original emphasis)

with Plato's statement in the Republic, 527a6-b1,

[7]Incidentally, this distinction between views of mathematics pegged on contemplation and on volition also bears on the old issue whether mathematics is an 'art' or a 'science'. The former correlates mathematics with activities, actions, controlled volitions, cunningly skilled doings; the latter correlates mathematics with demonstration, exhibition, insightful seeing and understanding.

> They [i.e., geometers] speak in a way which is ridiculous and compulsory; for they are always talking about squaring and applying and adding as if they were doing things and were developing all their propositions for the sake of action; but, in fact, the whole subject is pursued for the sake of understanding.

Brouwer sees the good of mathematics in its power to facilitate the action of the will in 'the world', as only—given his conception of the world as something we have constructed from organized sensations, his 'unbound by concept' world, and on a higher level, one step up, mathematics reveals the freedom of the will and its power over all presumed logical apriori. The free becoming of mathematics instructs us in the power of free will. It can shake the supposedly logic a priori, get around even such supposed universal and necessary laws. Brouwer's debt to Schopenhauer is fully manifest [16]. For both, Will is prior to Intellect. The Will in its freedom can slay the 'brain children' of intellect, it can slay the very laws of logic.

Gödel sees the good of doing mathematics conceptually. It reveals the power of the logical apriori, its universality. It pervades all of Reality, and therefore Mind cannot free itself from it. In view of this, and in stark contrast to Brouwer, Gödel plays down the freedom of the will considerably, in the following sense. To Rucker he said,

> It should be possible to form a complete theory of human behavior, i.e., to predict from the hereditary and environmental givens what a person will do. However, if a mischievous person learns of this theory, he can act in such a way so as to negate it. Hence I conclude that such a theory exists, but that no mischievous person will learn of it. [...] The *a priori* is greatly neglected. Logic is very powerful. [18, p.181]

This is most revealing about the depth of the Goodness of things. It means that even though there are in principle deep freedoms, they are kept from those who would use them for evil purposes or mischief.

Brouwer and Gödel would agree that the Good is to be sought; but they would disagree on the role that mathematics could play in that search.

4 A partial argument against CCT

We now suggest that, if you believe strongly enough in the stability of mathematics to recognize that in spite of their differences—the differences between classical and intuitionistic mathematics—, Gödel and Brouwer are both dealing with the same subject matter (i.e. mathematics), then their two cases taken together function as an argument against the common core thesis; for what is there left for a common core of truths if according to Gödel, mathematics leads you to the Absolute, whereas according to Brouwer, the same thing leads you away from it? What gives one access to the Good according to Gödel, denies

this access according to Brouwer. But if both are speaking the truth, which we assumed for the sake of argument, then this must mean that by 'the Good' they mean different things. Therefore, Brouwer and Gödel cannot be referring to the same when they speak about the Good.

Note that the argument does not show that CCT is false; but, if correct, it shows the following: as long as we don't know that Gödel's or Brouwer's position is false, there is no argument for CCT.

So we hold that an argument for CCT would have to show that the positions of Gödel and Brouwer cannot both be true. To establish that would actually be a stronger conclusion than our one, which it implies, but is not implied by. However, it seems much simpler to point out a difference in methods of access such that it precludes sameness of reference, than directly to establish the truth or falsity of these mystical positions.

We are thus trying to say something about CCT while avoiding having to make doctrinal comments about what the Absolute is really like. We refrain from that (at the cost of not being able to say something about the truth of CCT directly) and focus on methods of access to the Absolute. Given that in history there has been much doctrinal as well as methodological disagreement, we see no reason why *in general* a method of access argument should fare better than a doctrinal argument. The relations between alternative (alleged) methods may be so unclear or loose as to yield no argument. What makes the Gödel/Brouwer case different is that their particular implicit disagreement on method admits formulation as a sharp antithesis, and what they are disagreeing about, mathematics, is itself something very stable.

5 Closing remarks

We would like to end by making the following two remarks.

First, of course one could, and usually does, engage in mathematics for its own sake, without any interest in relating it, be it positively or negatively, to mysticism. From Gödel's and Brouwer's point of view, that would probably be not unlike the possibility to perform a hymn for its own sake, without any interest in the religious meaning it may have.

The second remark is related to the first. In spite of the incommensurability of Brouwer's and Gödel's positions, their respective *motivations* to take the mystical turn may have much in common. Both were disgruntled with the materialistic and formalistic philosophies prevalent at their times; both thought that these philosophies could not do justice to the Good.

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