A Two-stage Floating Raft Isolation System Featuring Electrorheological Damper with Semi-active Static Output Feedback Variable Structure Control

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ABSTRACT: In this study, a semi-active static output feedback variable structure control (VSC) strategy is presented to control a two-stage electrorheological (ER) floating raft isolation system. A continuous semi-active static output feedback VSC controller is designed and a bypass ER damper is applied to achieve the best control effect more rapidly and accurately. The sliding surface parameters are determined in terms of the Routh–Hurwitz stability criterion. The optimal vibration attenuation of the intermediate mass is guaranteed due to the control rule that damping force only dissipates the vibration energy of the intermediate mass. The robustness of the control method with respect to parameter variations and the effectiveness of vibration isolation are demonstrated by numerical simulation results. It shows that the designed semi-active static output feedback VSC strategy realized by the ER damper can achieve better performance than that of optimally passive damping even if system parameter uncertainties exist. In addition, the experiment was carried out to demonstrate the practical effectiveness of the presented control scheme in this study. The experimental results are satisfied.

Key Words: semi-active, static output feedback, variable structure control, two-stage floating raft isolation system, electrorheological damper.

INTRODUCTION

T HREE main types of vibration control methods have been proposed, namely passive method, semi-active method, and active method. Conventionally, the passive damping technique provides design simplicity, system reliability, and cost effectiveness. However, performance limitations are inevitable due to fixed damping parameters (Rakheja et al., 1994). On the other hand, active damping forces provide high control performance in a wide frequency range at the cost of high power supplies (Krtolica and Hrovat, 1992). The semi-active damping system (Karnopp et al., 1974) is very attractive because it offers a desirable performance generally enhanced in the active model without requiring large power sources and expensive hardware. Especially, when some controllable dampers, such as electrorheological (ER) dampers and magnetorheological (MR) dampers, are available in practice recently, the semi-active control method is more practical than ever in engineering realization. With the development of the ER fluid, the ER damper was found to shorten the control time and increase the reliability of the whole control system dramatically.

Two-stage isolation systems can achieve better vibration cancellation than one-stage isolation systems, especially at high frequencies (Harris and Crede, 1961; Goyder and White, 1980). Based on two-stage isolation systems, floating raft systems, a type of special vibration isolation structures, were developed about twenty years ago especially for ships and submarines. They can isolate vibration of hosts and auxiliary machines and reduce the structural noise of ships and submarines effectively. They can also protect the equipments and instruments in ships and submarines from being damaged, and make them to be operated properly when ships and submarines are subjected to external loads and sudden shocks. As isolation of floating raft systems is a key technique for ships, in particular submarines, it has received much research attention in recent years (Nilson and Seland, 1977; Nelson, 1982; Pan and Hansen, 1993, 1994; Pan et al., 1992, 1993; Xiong et al., 1996; Howard et al., 1997; Li et al., 1997; Sun and Song, 1999; Qu and Chang, 2000; Niu et al., 2001; Xiong et al., 2001; Sun et al., 2002). However, substantial works about floating raft isolation systems are limited to passive and active systems. Niu et al. (2005) present a novel analytical active–passive
model of floating rafts. They obtain the solution of the power flow transmitted into the foundation. Power transmission characteristics of the system are investigated under different control types when minimization of total power flow strategy is applied. Through numerical simulations, the control efficiencies of the different control types (machine control, raft control, and full control) are compared, illustrating the efficiency of the presented model, obtaining some valuable results, and presenting some general design principles of the active floating raft isolation systems. Xiong et al. (2001) derive the generalized mobility/impedance matrices of a three-dimensional rigid/elastic structure with various configurations. The equivalent mobility matrix (EMM) and equivalent impedance matrix (EIM) methods are introduced by them. The example of a flexible raft vibration isolation system demonstrates the application of the proposed generalized approaches. Li et al. (1997) complete the power flow analysis for a floating raft isolation system consisting of constrained damped beams by using the Green function. An analytical method takes place of the traditional mobility approach. They calculate the transmitted power flow from a harmonic force excitation to the foundation beams via an isolator-raft beam-isolator system. Some important structural parameters that influence the transmitted power flow are discussed.

It is known that the semi-active control method can achieve desirable performance than the passive method and consume much less power than the active method. In addition, employing full state in ER damping vibration control system may be impractical or overly complicated to implement for a practical two-stage floating raft isolation system subject to external disturbances. Based on this perspective, a semi-active static output feedback variable structure control (VSC) method is presented to make the best of the continuously and rapidly damping performance of ER damper for vibration isolation in this study. The sliding surface parameters are determined in terms of the Routh–Hurwitz stability criterion. The optimal vibration attenuation of the intermediate mass is guaranteed due to the control rule that damping force only dissipates the vibration energy of the intermediate mass. It is shown that the proposed semi-active static output feedback VSC strategy realized by the ER damper is more effective in the two-stage floating raft isolation system than that of optimally passive damping even if system parameter uncertainties exist. In addition, the experiment was carried out to demonstrate the practical effectiveness of the presented control scheme in this study. The experimental results are satisfied. The structure and behavior of the ER damper, the structure and experimental platform of the semi-active two-stage floating raft isolation system, a continuous output feedback VSC controller, all are presented in various sections in this study. The numerical simulation and experiment results to demonstrate the effectiveness of the designed controller are also presented.

**PROPERTIES OF ER DAMPER**

Most researches in the application of the ER fluids are focused on structural vibration control and flow power system. Stanway et al. (1996) made a comprehensive survey in the state of the ER fluids and the application of the ER fluids in mechanical engineering. The classification of the modes of ER fluid operation and their potential engineering applications in vibration control were reported. Petek (1992) proposed a monotube-type ER damper and demonstrated its superiority over the conventional damper by showing that the damping force of the ER damper could be increased with respect to the applied electric field regardless of the piston velocity. In addition, he put forth that the ER damper could provide better performance than a conventional one when driving over road and bump profiles. However, these results were obtained by applying only constant electric fields without a feedback controller. Lou et al. (1994) classified the types of ER dampers into three modes: flow mode, shear mode, and mixed mode. They theoretically analyzed the performance of all three modes of ER dampers, focusing on the difference of pressure drop induced by a sinusoidal flow, but experimental work was not undertaken. Sturk et al. (1995) proposed a high voltage supply unit for controlling the voltage applied to the ER damper and experimentally evaluated its performance. In order to do this, they manufactured a small-sized ER damper and applied to a quarter car suspension system. They demonstrated the effectiveness for vibration isolation of the suspension system with the proposed high voltage supply unit. It is evident from previous recent works that numerous research works have been focused only on the effectiveness for vibration isolation of a two-stage vibration isolation system featuring the ER damper. The present study shows how a continuously variable ER damper can be satisfactorily used for the feedback control of desired damping forces.

In this study, the schematic diagram of the bypass ER damper used is shown in Figure 1. The damper consists of a hydraulic cylinder, which is divided into two working chambers by a piston. The bypass, fitted to the side of the hydraulic cylinder, comprises two concentric tubular electrodes and an annulus through which the ER fluids flow. The positive voltage produced by a high voltage supply unit is applied to the inner electrode, while the negative voltage is connected to the outer electrode. In the absence of electric fields, the ER damper produces the damping force only by the
fluid-flowing resistance. However, if a certain level of the electric voltage is supplied to the ER damper, additional damping force due to the yield stress of the ER fluid would be produced. This damping force of the ER damper can be continuously tuned by changing the voltage applied to the damper. Based on the Bingham constitutive model of ER fluids (Gavin et al., 1996a,b; Jason and Norman, 1999), which is sufficiently accurate for design calculation although it does not capture details of the deformation behavior of an actual damper, the approximation of the damping force in the bypass damper is obtained as:

\[
F = c_0 \dot{X}_p + F_{\text{ER}} \text{sgn}(\dot{X}_p),
\]

where

\[
F_{\text{ER}} = \alpha_0 + \alpha_1 U + \alpha_2 U^2,
\]

where \(c_0\) is the zero-voltage damping coefficient, which is determined by the plastic viscosity of ER fluids and the geometry of the manufactured damper, \(F_{\text{ER}}\) is the controllable damping force generated by the applied voltage, \(\alpha_0, \alpha_1,\) and \(\alpha_2\) are the intrinsic parameters of the ER damper and can be experimentally determined; \(U\) is the voltage; \(\dot{X}_p\) is the velocity of piston motion; and \(\text{sgn}()\) is a signum function.

Figure 2 reports the measured damping force with respect to the piston velocity at various voltages. The piston velocity increases from 15 to 100 mm/s gradually, while the excitation amplitude is maintained to be constant at 60 mm. Such a plot is frequently employed to evaluate the level of damping performance in damper manufacturing industry. For the proposed damper, the damping force increased with the applied voltage, as expected. For instance, the damping force increases up to 227 N at a piston velocity of 100 mm/s and...
voltage of 5 kV. This figure agrees fairly well with the presented damping model given by Equation (1).

**SEMI-ACTIVE TWO-STAGE FLOATING RAFT ISOLATION SYSTEM**

**Two-stage Floating Raft Isolation System Model**

Figure 3 shows the model of two-degrees-of-freedom floating raft isolation system modeled by the linear springs $k_1$ and $k_2$, ER damper with zero-voltage damping coefficient $c_0$, and controllable damping force $u$ representing $F_{ER}$, the main mass $m_1$ and intermediate mass $m_2$. The dynamic equations of the system can be written as:

\[
\begin{align*}
    m_1 \ddot{z}_1 &= -k_1(z_1 - z_2) - c_0(\dot{z}_1 - \dot{z}_2) - u + f_{in}, \\
    m_2 \ddot{z}_2 &= k_1(z_1 - z_2) + c_0(\dot{z}_1 - \dot{z}_2) + u - k_2z_2,
\end{align*}
\]

where $z_1$, $\dot{z}_1$, and $\ddot{z}_1$ denote, respectively, the displacement, velocity, and acceleration of the main mass; $z_2$, $\dot{z}_2$, and $\ddot{z}_2$ are the corresponding ones of intermediate mass; $f_{in}$ is a disturbance force acting on the main mass. The output force acting on the fixed base is $k_2z_2$. It is assumed that velocities $\dot{z}_1$ and $\dot{z}_2$ are derived from accelerometers installed on the main mass and intermediate mass, respectively, while relative displacement $z_1 - z_2$ between the main mass and intermediate mass is measured using a linear voltage differential transformer.

After choosing the state variables as

\[
\begin{align*}
    x_1 &= z_1 - z_2, \\
    x_2 &= \dot{z}_1, \\
    x_3 &= z_2, \\
    x_4 &= \dot{z}_2,
\end{align*}
\]

and defining

\[
x = [x_1 \ x_2 \ x_3 \ x_4]^T,
\]

The following state-space equations are obtained

\[
\begin{align*}
    \dot{x} &= Ax + Bu + Ef_{in}, \\
    y &= Cx,
\end{align*}
\]

where

\[
A = \begin{bmatrix}
    0 & 1 & 0 & -1 \\
    -\frac{k_1}{m_1} & -\frac{c_0}{m_1} & 0 & \frac{c_0}{m_1} \\
    0 & 0 & 0 & 1 \\
    0 & 0 & 0 & -\frac{1}{m_1}
\end{bmatrix}, \quad B = \begin{bmatrix}
    0 \\
    -\frac{1}{m_1} \\
    0 \\
    0
\end{bmatrix},
\]

\[
E = \begin{bmatrix}
    0 \\
    0 \\
    \frac{1}{m_1} \\
    0
\end{bmatrix}, \quad C = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}.
\]

**Experimental Platform of Two-stage Floating Raft Isolation System**

Figures 4 and 5 show the experimental platform scheme of two-stage floating raft isolation system and the plan view of vibration exciter, respectively. With the constraint of four gag lever posts A3, the experimental platform which consist of a vibration exciter A1, the linear springs A6 and A8, a top plate A5, and an intermediate plate A7 is vertically installed on the fixed base. The piston rod and the shell of ER damper B3 are mounted, respectively, on the center position of the top plate and the intermediate plate. Vibration exciter is installed on the M-type platform A2 which is mounted on the top plate. The main mass consists of vibration exciter, M-type platform, and the top plate. The intermediate mass consists of the intermediate plate and additional mass which is not shown in Figure 4. In order to reduce friction, bearings A4 are installed on the top plate and also on the intermediate plate.

Many physics information can be measured because various sensors are installed on the experimental platform. Accelerometers B1 and B6 measure, respectively, the acceleration of the main mass and the intermediate mass. The output force of ER damper is measured by force sensor B5. Displacement transducers B2 and B7 measure, respectively, absolute displacement of the main mass and the intermediate mass. Relative displacement between the main mass and intermediate mass is measured using displacement transformer B4.

**Analysis of Exciting Force**

The vibration exciter designed in this study consists of two vibratory motors which synchronistically rotate with the constraint of a pair of gears installed on the
motor axes. There is the same offcenter mass on each terminal of the two vibratory motors. According to the theory of synchronistical vibration, two vibratory motors driven by a frequency transformer not only always synchronistically rotate but also have opposite rotation direction and symmetrical phase. Therefore, the offcenter forces that four rotary offcenter masses generate mutually counteract in the horizontal direction and superpose in the vertical direction. In other words, the vibration exciter provides exciting force only in the vertical direction.

Figure 6 shows the mechanical model of exciting force of the experiment rig. It is assumed that motors driven by the frequency transformer have a uniform angular speed $\omega$. The relative velocity in the vertical direction between offcenter mass $m_0$ and main mass $m_1$ at the moment $t$ is

$$\ddot{x}_0 - \ddot{x}_1 = r\omega \cos \theta = r\omega \cos \omega t,$$  \hspace{1cm} (10)

After $\Delta t$, Equation (10) becomes

$$(x_0 + \Delta x_0) - (x_1 + \Delta x_1) = r\omega \cos \omega (t + \Delta t).$$  \hspace{1cm} (11)

According to Figure 6, one can know

$$\Delta \dot{x}_0 = \dot{x}_0 \cdot \Delta t,$$  \hspace{1cm} (12)

$$\Delta \dot{x}_1 = \dot{x}_1 \cdot \Delta t.$$  \hspace{1cm} (13)

Substituting Equations (10), (12), and (13) into Equation (11), yields

$$r\omega \cos \omega t + (\dot{x}_0 - \dot{x}_1)\Delta t = r\omega \cos \omega t \cdot \cos \omega \Delta t - r\omega \sin \omega t \cdot \sin \omega \Delta t.$$  \hspace{1cm} (14)

When $\Delta t \to 0$, one can obtain $\cos \omega \Delta t \to 1$ and $\sin \omega \Delta t \to \omega \Delta t$. Thus, Equation (14) becomes

$$\ddot{x}_0 = \ddot{x}_1 - r\omega^2 \sin \omega t.$$  \hspace{1cm} (15)

If the weight influence of offcenter masses is neglected, applying Newton’s law, the vertical exciting force $f_{in}(t)$ acting on the main mass $m_1$ is

$$f_{in}(t) = -4m_0 \ddot{x}_0(t) = 4m_0 r\omega^2 \sin \omega t - 4m_0 \ddot{x}_1.$$  \hspace{1cm} (16)

Equation (16) indicates that the vertical exciting force $f_{in}(t)$ virtually equals to vertical component of offcenter force subtracting inertial force of offcenter masses which move together with the main mass.

**FORMULATION OF CONTROL SCHEME**

The VSC strategy is one of the robust control methods which have many good features such as insensitivity to system parameter variations, external disturbance rejection, and good transient performance. In terms of the recent literatures, substantial works
about floating raft isolation systems are limited to passive and active systems. However, the semi-active control method can achieve desirable performance than the passive method and consume much less power than the active method. In addition, utilizing full state in ER damping vibration control system may be impractical or overly complicated to implement for a practical two-stage floating raft isolation system subject to external disturbances. Hence, a semi-active static output feedback VSC strategy is presented to control a two-stage floating raft isolation system. Saturation is incorporated into the control law in terms of the limited capacity of the ER damper. The optimal vibration attenuation of the intermediate mass is guaranteed due to the control rule that damping force only dissipates the vibration energy of the intermediate mass.

Formulation and Design Freedom of Static Output Feedback VSC

Consider the usual linear time-invariant state model

\[ \dot{x} = Ax + Bu, \quad y = Cx, \]  

having state \( x \in \mathbb{R}^n \), control \( u \in \mathbb{R}^m \), and output \( y \in \mathbb{R}^h \). The system is assumed to be completely controllable and completely observable, and \( CB \) has full rank \( m \).

The VSC control is based on output feedback, so the ith component can be written as:

\[ u_i(y) = \begin{cases} u_i^+(y), & S_i(y) \geq 0, \\ u_i^-(y), & S_i(y) < 0. \end{cases}, \quad i = 1, \ldots, m. \]  

The sliding surfaces have the form \( S = Gy \), where \( G \) is an \( m \times h \) real parameter matrix to be designed. Setting \( \dot{S} = 0 \) and solving for the equivalent control yields the equations of motions on the sliding surfaces:

\[ \dot{x} = (A - B(GCB)^{-1}GCA)x, \]  

which exists uniquely if \( GCB \) is nonsingular. The nonsingularity of \( GCB \) depends exclusively on the choice of the sliding surface matrix \( G \). The goal is to choose the sliding surfaces \( \dot{S} = Gy \) and a switched output feedback control form of Equation (19) to stabilize the state dynamics of Equation (20) under the constraint that \( GCB \) is nonsingular. It is required that \( GCB \) is nonsingular, i.e.,

\[ \text{rank}(GCB) = m. \]  

From Equation (21), one can obtain the following restricted condition:

\[ m \leq l \leq n. \]  

Obviously, the number of outputs is at least equal to the number of inputs.

The sliding surfaces in the measurement space are specified by \( S = Gy = 0 \), i.e., they are the null space of \( G \). On the other hand, the sliding surfaces viewed in the state space are the null space of \( GC \). In particular, the null space of \( GC \) is spanned by \( n - m \) n-dimensional vectors, \( n - l \) of which are determined by the null space of \( C \). Hence only \( l - m \) spanning vectors may be chosen freely.

According to linear system theory, due to the controllable matrix pair \( (A, B) \), it is convenient to convert Equations (17) and (18) into the following reduced forms:

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{B}_2 \end{bmatrix} u, \]  

\[ y = \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \]  

where \( \tilde{B}_2 \in \mathbb{R}^{m \times m} \) is nonsingular, \( \tilde{x}_1 \in \mathbb{R}^{n-m} \), \( \tilde{x}_2 \in \mathbb{R}^m \), \( \tilde{A}_{11}, \tilde{A}_{12}, \tilde{A}_{21}, \tilde{A}_{22}, \tilde{C}_1 \) and \( \tilde{C}_2 \) are real matrices of appropriate dimensions. The sliding surfaces in the new coordinates becomes

\[ S = G \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0. \]  

One can obtain that \( G\tilde{C}_2 \) is nonsingular because one has the requirement of \( GCB \) be nonsingular. It was assumed that the system’s state trajectory is initially on the sliding surface, then \( \dot{S} = 0 \) and \( \dot{S} = 0 \). Solving Equation (25) for \( \tilde{x}_2 \) and substituting into Equation (23), the following reduced-order dynamics is obtained:

\[ \dot{x}_1 = (\tilde{A}_{11} - \tilde{A}_{12}(G\tilde{C}_2)^{-1}G\tilde{C}_1)\tilde{x}_1 = (\tilde{A}_{11} - \tilde{A}_{12}H\tilde{C}_1)\tilde{x}_1, \]  

where \( H = (G\tilde{C}_2)^{-1}G \) represents an output feedback gain matrix, \( \tilde{A}_{12} \) and \( \tilde{C}_1 \) appear respectively as input and output matrices of the reduced-order system. \( G \) must be chosen such that \( H = (G\tilde{C}_2)^{-1}G \). A necessary and sufficient (Heck and Ferri, 1989) condition for this is

\[ \text{rank}(\tilde{C}_2H - I) \leq l - m. \]
Design of Sliding Surface

The first step in VSC method is to design the sliding surface on which the response is stable. In the design of the sliding surface, the external disturbance $f_{io}$ in Equation (7) is neglected, however, it is taken into account in the design of the controllers (Yang et al., 1996). Considering Equations (7) and (8), the sliding surface is defined as:

$$S = Gy,$$  

(29)

where

$$G = [g_1 \ g_2 \ g_3].$$  

(30)

is the so-called sliding surface matrix.

Owing to the controllable matrix pair $(A, B)$, Equations (7) and (8) can be converted into the reduced forms of Equations (23) and (24) by the following transformation $\tilde{x} = Qx$,

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & m_2/m_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.  

(31)

is a transformation matrix.

The partitioned coefficient matrices of the reduced forms of the two-stage floating raft isolation system are

$$\tilde{A}_{11} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -k_2/m_1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{A}_{12} = \begin{bmatrix} -1 - m_2/m_1 \\ 0 \\ 1 \end{bmatrix},$$

$$\tilde{A}_{21} = \begin{bmatrix} k_1/m_2 & c_0/m_2 & -k_2/m_2 \end{bmatrix},$$

$$\tilde{A}_{22} = -c_0 \left( \frac{1}{m_1} + \frac{1}{m_2} \right), \quad \tilde{B}_2 = \frac{1}{m_2}, \quad \tilde{C}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{C}_2 = \begin{bmatrix} 0 \\ -m_2/m_1 \\ 1 \end{bmatrix}.  

(32)

The full rank condition of $GCB$ in Equation (21) requires that

$$m_1g_3 - m_2g_2 \neq 0.$$  

(33)

The reduced-order dynamics system is expressed as the form of Equation (26), where $H = [h_1 \ h_2 \ h_3].$

The condition in Equation (28) can be further expressed as:

$$\text{rank} \begin{bmatrix} -1 & 0 & 0 \\ -m_2h_1/m_1 & -1 - m_2h_2/m_1 & -m_2h_3/m_1 \\ h_1 & h_2 & -1 + h_3 \end{bmatrix} \leq 2.$$  

(34)

According to Equation (34), one can obtain

$$h_3 = 1 + \frac{m_2}{m_1}h_2.$$  

(35)

In terms of $H = (G\tilde{C}_2)^{-1}G$, yields

$$g_1 = \frac{m_1h_1}{m_1 + m_2h_2}g_3,$$  

(36)

$$g_2 = \frac{m_1h_2}{m_1 + m_2h_2}g_3.$$  

(37)

Design of Continuous Controller with Saturation

The second step in VSC method is to design controller that forces system's trajectory to and maintains it on the sliding surface. The controller design requires that the reaching condition of sliding mode $S^T\dot{S}<0$ is met. External excitations are incorporated into the designed control law. For practical implementation, the saturation of the controller is also considered.

The characteristic polynomial of Equation (26) is

$$D(\lambda) = \lambda^3 - h_1 \left( \frac{m_2}{m_1} + 1 \right) \lambda^2 - \frac{k_2h_2}{m_1} \lambda - \frac{k_2h_1}{m_1}.$$  

(38)

According to the Routh–Hurwitz stable criterion, the constraints which guarantee the sliding mode dynamics to be stable are:

$$h_1 < 0,$$  

(39)

$$h_2 < -\frac{m_1}{m_1 + m_2}.$$  

(40)

Under the restricted conditions of Equations (39) and (40), it is simply choosen

$$h_1 = h_2 = -1.$$  

(41)
Substituting Equation (41) into Equations (36) and (37) yields
\[ g_1 = g_2 = \frac{m_1}{m_1 - m_2}g_3. \] (42)

For the two-stage floating raft isolation system studied in this article, the relationship between the main mass and the intermediate mass is
\[ m_1 = 2m_2 \] (43)

In this study,
\[ g_3 = -1 \] (44)

Substituting Equations (43) and (44) into Equation (42), one obtains
\[ G = \begin{bmatrix} g_1 & g_2 & g_3 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix}. \] (45)

In terms of Equations (43) and (45), the restricted condition of Equation (33) is satisfied.

The control law \( u \) is chosen as
\[ u = -(GCB)^{-1}(GCANy + GCE_{in}) - k(GCB)^{-1}S \] (46)
where \( k > 0 \).

A Lyapunov function candidate is chosen as
\[ V = \frac{1}{2}S^T S. \] (47)

The gain matrix \( N \) is chosen to satisfy the reaching condition \( S^T S < 0 \) given by
\[ \dot{V} = S^T \dot{S} = x^T C^T GCA(I - NC)x - k|S|^2, \] (48)
where \( k > 0 \). Note that if \( C^T G^T GCA \leq 0 \), then \( N = 0 \) insures that the reaching condition is satisfied.

In terms of Equations (9) and (45), one can obtain
\[ C^T G^T GCA = \begin{bmatrix} -\frac{4k_1}{m_1} - \frac{2k_1}{m_2} & 4 - \frac{4c_0}{m_1} - \frac{2c_0}{m_2} & \frac{2k_2}{m_1} - \frac{4c_0}{m_1} + \frac{2c_0}{m_2} & \frac{2k_3}{m_1} + \frac{2c_0}{m_2} \\ \frac{2k_1}{m_1} - \frac{2k_1}{m_2} & -\frac{4c_0}{m_1} - \frac{2c_0}{m_2} & 2 - \frac{4c_0}{m_1} + \frac{2c_0}{m_2} & \frac{2k_2}{m_1} + \frac{2c_0}{m_2} \\ \frac{2k_1}{m_1} + \frac{k_1}{m_2} & -2 + \frac{2c_0}{m_1} + \frac{c_0}{m_2} & \frac{k_2}{m_1} + \frac{k_2}{m_2} & -\frac{c_0}{m_1} - \frac{c_0}{m_2} \end{bmatrix}. \] (49)

For the two-stage floating raft isolation system studied in this article, \( C^T G^T GCA \) is negative semi-definite. In terms of Equation (48), \( N = 0 \) is chosen. The continuous control law of Equation (46) can be further expressed as:
\[ u = -(GCB)^{-1}GCE_{in} - k(GCB)^{-1}S, \] (50)

where \( k \) is chosen as \( k = k_1/m_2^2 \) in this study.

From Equation (50), it should be noted that the feedforward compensation, i.e., \(-(GCB)^{-1}GCE_{in}\), is accounted for in the design of the controller. The acceleration of the main mass \( \ddot{z}_1 \) and relative displacement \( z_1 - z_2 \) between the main mass and intermediate mass have been measured. For practical floating raft isolation system studied in this article, the damping force of ER damper is measurable and can be derived via force sensor measurement. The vertical exciting force \( f_{in} \) can be known in terms of the expression
\[ f_{in}(t) = m_1\ddot{z}_1 - F_d + k_1(z_1 - z_2) \] (51)
where \( F_d \) is the damping force of ER damper that force sensor measures.

The preceding control scheme Equation (50) which maintains its performance under uncertain disturbances requires control inputs to be high enough. However, the inputs of a system are usually restricted by its physical parameters, which mean that the system inputs have the character of saturation (Liu et al., 2005). The saturated damping devices to weaken the vibration of floating raft isolation system do not lead to the instability of the system because the damping devices in practical vibration isolation system always dissipate the vibration energy. The saturated continuous controllers, corresponding to Equation (50), are
\[ u^* = \begin{cases} u, & |u| < u_{max}, \\ u_{max}\text{sign}(u), & |u| \geq u_{max}, \end{cases} \] (52)
where \( u_{max} \) is the upper bound of \( u \). \( u_{max} \) is specified in terms of the physical devices used in practical applications. The controller presented in Equation (52) are referred to as saturated controllers because the control effort \( u^* \) is saturated at \( u_{max} \). The controllable maximum damping force of the ER damper is 106.9 N because of the breakdown voltage of 5 kV for the ER fluids used.

**Semi-active Controller**

The controller \( u \) given by Equation (50) is designed in an active actuating manner. However, the ER damper applied in the proposed floating raft isolation system, as shown in Figure 3, is desired to be a
semi-active actuator. Thus, the control input force
should be applied in terms of the actuating condition

\[ u' = \begin{cases} u^*, & \dot{z}_2(\dot{z}_1 - \dot{z}_2) < 0, \\ 0, & \dot{z}_2(\dot{z}_1 - \dot{z}_2) \geq 0. \end{cases} \] (53)

This condition physically implies that the actuating of
the controller \( u \) only assures the increment of energy
dissipation of the stable system.

When the control input \( u' \) is determined, from
Equation (2) one can get the control electric voltage
applied to the ER damper. It is given as:

\[ U = -\alpha_1 + \sqrt{\alpha_1^2 - 4\alpha_2\alpha_0 + 4\alpha_2u'} / 2\alpha_2. \] (54)

The proposed control strategy is shown in the control
block diagram of Figure 7.

SIMULATION, EXPERIMENT, AND RESULTS

In this section, the performance of the semi-active
static output feedback VSC controller applied to the
two-stage floating raft isolation system with one
controllable ER damper is evaluated by using a
Runge–Kutta fourth-order method. In addition, the
experiment was carried out in the State Key Laboratory
of Vibration, Shock and Noise. The setup of the
experimental equipment is displayed in Figure 8.
The parameter values are: \( m_0 = 3.2 \text{ kg}, \ m_1 = 32 \text{ kg}, \ m_2 = 16 \text{ kg}, \ k_1 = 33 \text{ kN/m}, \ k_2 = 185 \text{ kN/m}, \ c_0 = 1013.4 \text{ Ns/m}, \ \alpha_0 = 10.23 \text{ N/kV}, \ \alpha_1 = 6.38 \text{ N/kV}, \ \alpha_2 = 2.59 \text{ N/kV}^2, \ U_{\text{max}} = 5 \text{ kV}, \ r = 0.002 \text{ m}, \ k = k_1/m_1^2, \) and \( g_1 = g_2 = 2, \ g_3 = -1. \) The first-order and the second-order natural
frequencies of vibration isolation system are close
to 5 and 16 Hz.

The system performances of vibration isolation are
evaluated under two types of excitations: a sinusoidal
chirp excitation and dual-frequency excitation.

For the purpose of illustration, Figure 9 shows force
transmissibility of floating raft isolation system to one
chirp signal which is described by

\[ f_{\text{excitation}} = \text{chirp}(t, 0, 30, 30), \] (55)

where \( f_{\text{excitation}} \) is not exciting force but an exciting signal.
Chirp time is 30 s. The chirp frequency increases from 0 to
30 Hz. In order to effectively illustrate the vibration
attenuation performance of two-stage floating raft
isolation system controlled using semi-active output
feedback VSC, force transmissibility of minimum damp-
ing and maximum damping systems also are shown in
Figure 9. It can be seen that the vibration attenuation
performance of two-stage floating raft isolation system
controlled using semi-active output feedback VSC is
better than that of optimally passive damping.

According to Equation (16), for two-stage floating raft
isolation system controlled using semi-active output

![Figure 7. Block diagram of semi-active VSC for a two-stage floating raft isolation system with ER damper.](a.png)

![Figure 8. Photograph of the experimental apparatus: (a) testing bench and (b) computerized measure and control system.](b.png)
feedback VSC and optimally passive damping system, the input exciting forces are not identical under the same exciting signal because their accelerations of main masses are different. Therefore, only by simultaneously illustrating input force and output force can the vibration attenuation performances of two different systems be evaluated. Figures 10 and 11 show, respectively, input force and output force of optimal damping system and two-stage floating raft isolation system controlled using semi-active output feedback VSC to dual-frequency exciting signal which is described by

\[ f_{\text{excitation}} = A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) \]  

(56)

where \( A_1 = A_2 = 1 \), \( f_1 \) and \( f_2 \) is, respectively, the first-order natural frequency 5 Hz and the second-order natural frequency 16 Hz of floating raft isolation system. These simulation results indicate that the vibration of floating raft controlled using semi-active static VSC is remarkably reduced in comparison with optimally passive damping system.

In order to illustrate the robustness of the proposed semi-active feedback VSC design in the presence of parametric uncertainties, ±10% variations of main mass \( m_1 \) and stiffness \( k_1, k_2 \) from their nominal values are considered. For optimally passive damping and the proposed semi-active VSC, Figure 12 presents input force and output force with system parameters variations \( \Delta m_1 = 0.1 m_1 \), \( \Delta k_1 = 0.1 k_1 \), and \( \Delta k_2 = 0.1 k_2 \)

![Figure 9. Force transmissibility ratio of floating raft isolation system to chirp signal.](image)

![Figure 10. Input force and output force of optimal damping vibration isolation system to dual-frequency signal.](image)

![Figure 11. Input force and output force of semi-active output feedback VSC floating raft isolation system to dual-frequency signal.](image)

![Figure 12. Input force and output force of vibration isolation system to dual-frequency signal excitation with parameter uncertainties \( \Delta m_1 = 0.1 m_1 \), \( \Delta k_1 = 0.1 k_1 \), and \( \Delta k_2 = 0.1 k_2 \): (a) optimal damping and (b) semi-active output feedback VSC.](image)
under dual-frequency exciting signal described by Equation (56) while Figure 13 is with system parameters variations \( \Delta m_1 = -0.1 m_1, \Delta k_1 = -0.1 k_1, \) and \( \Delta k_2 = -0.1 k_2 \). These simulation results indicate that the performance of the proposed semi-active VSC floating raft isolation system is better than that of optimally passive damping even with respect to system parameter variations.

In order to demonstrate the practical effectiveness of the presented control scheme in this study, the experiment was carried out in the State Key Laboratory of Vibration, Shock and Noise. In the experiment, the vibration exciter is driven by the frequency transformer. The transmissibility of force isolation system is expressed as:

\[
T(\omega) = \frac{F_{\text{out}}(\omega)}{F_{\text{in}}(\omega)} = \frac{F[f_{\text{out}}(t)\text{]}/C1}{F[f_{\text{in}}(t)\text{]}/C0} \tag{57}
\]

where \( F[\cdot] \) is the Fourier transformation, \( f_{\text{out}}(t) \) is the output force acting on the fixed base, \( f_{\text{in}}(t) \) is the input force acting on the main mass, and \( \omega \) is the angular speed.

In the two-stage vibration isolation system, one can obtain

\[
f_{\text{in}}(t) = m_1 \ddot{z}_1(t) + f_d(t) + k_1(z_1(t) - z_2(t))
\]

\[
f_{\text{out}}(t) = k_2 z_2(t)
\]

where \( f_d(t) \) is the damping force of ER damper that force sensor measures.

In order to protect measured results from external disturbance, practical measurement method is given as:

\[
T(\omega) = \frac{S_{f_{\text{out}},f_{\text{in}}}(\omega)}{S_{f_{\text{in}},f_{\text{in}}}(\omega)} \tag{59}
\]

where \( S_{f_{\text{out}},f_{\text{in}}}(\omega) \) is the cross power spectrum function of \( f_{\text{in}}(t) \) and \( f_{\text{out}}(t) \), \( S_{f_{\text{in}},f_{\text{in}}}(\omega) \) is the auto power spectrum function of \( f_{\text{in}}(t) \). In the experiment, PULSE acoustics

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**Figure 13.** Input force and output force of vibration isolation system to dual-frequency signal excitation with parameter uncertainties \( \Delta m_1 = -0.1 m_1, \Delta k_1 = -0.1 k_1, \) and \( \Delta k_2 = -0.1 k_2 \): (a) optimal damping and (b) semi-active output feedback VSC.

**Figure 14.** Experimental frequency response plot of force transmissibility \( T \) of force isolation system.
and vibration analysis system software made by Denmark B&K Corporation is used.

Figure 14 shows experimental frequency response plot of force transmissibility $T$ of force isolation system. It presents that the measurement method is reliable and the practical effectiveness of the presented control scheme in this study is satisfied.

**CONCLUSIONS**

This study successfully demonstrates the application of a semi-active static output feedback VSC controller to a two-stage ER floating raft isolation system which is frequently subjected to parameter uncertainties and external disturbances. The design procedure of the semi-active static output feedback VSC controller is described in detail. Moreover, the simulation results illustrate that the proposed semi-active static output feedback VSC method is better in enhancing the performance of the two-stage ER floating raft isolation system in comparison with optimally passive damping. Moreover, the experimental results present that the measurement method is reliable and the practical effectiveness of the presented control scheme in this study is satisfied. Four merits of the control system are concluded as follows:

1. A semi-active static output feedback VSC controller is developed based on the measurements from a limited number of sensors installed at strategic locations.
2. The sliding surface parameters are determined in terms of the Routh–Hurwitz stability criterion.
3. The two-stage floating raft isolation system is combined with the ER damper, the control effect of the whole system becomes faster and more reliable by using the character of the ER fluids.
4. The robust performance of the two-stage floating raft isolation system is obtained by the proposed controller to parametric uncertainties. The experiment was carried out to demonstrate the practical effectiveness of the presented control scheme in this study. The experiment results present that the measurement method is reliable and the practical effectiveness of the presented control scheme in this study is satisfied.

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