Linear and nonlinear model large-eddy simulations of a plane jet

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Abstract

The mixed-nonlinear Kosović, Leray and LANS-α subgrid scale models are contrasted with linear Smagorinsky and Yoshizawa large-eddy simulations for a Re = 4000 plane jet simulation. Comparisons are made with direct numerical simulation data and reliable measurements. Global properties of the jet such as the spreading and centerline velocity decay rates are investigated. The mean-flow and turbulence parameters in the self-similar region are also studied. Mixed models developed for boundary layers compare well with the benchmark jet data and the linear model predictions. Differences between the models are minor and none showed any clear advantage in all the data comparisons.

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1. Introduction

Turbulent free jets transport heat, scalars and produce noise [1]. They are found in combustion [2], propulsive momentum fluxes from aero engines and many other applications. The dynamics of the shear layers of these jets is relevant to the modeling of flows in the upper air ways and extreme vascular stenosis. With the projected doubling of the world aircraft fleet by 2020 the environmental implications of noise are particularly important.

The free planar jet, where fluid is discharged from a slot into an open space, is a useful tool for research into the physics of jets and a benchmark for models intended to predict their dynamics. Experimental studies [3–7] demonstrate that all turbulent free jet flows have similar structures i.e. the presence of vortex rollup in two shear layers (having vorticity of opposite sign) which are separated by a potential core region within which the flow eventually reaches a fully developed self-similar (or self-preserving) state.

Numerical approaches for studying jets range from direct numerical simulation (DNS) where all turbulence scales are numerically resolved to Reynolds Averaged Navier–Stokes RANS where all turbulence scales are modeled. DNS has the highest predictive accuracy at low Reynolds numbers and greatest computational cost whereas RANS typically has the opposite traits for equivalent Reynolds numbers. However, RANS can be applied to high Reynolds numbers that DNS cannot reach. For both accuracy and cost Large-Eddy Simulation LES lies between these two extremes and is becoming a popular compromise for simulating jet flows. In LES, only the larger energy-containing eddies are resolved and the effects of the smaller eddies are modeled using a SGS (Subgrid Scale) model (see later).

Stanley et al. [8] use DNS to study a plane jet with a weak (virtually negligible) co-flow. As would be expected, predictions of the jet spreading rate, mean velocity and Reynolds stresses match a range of measurements well. For the same case, Ribault et al. [9] conduct LES using the Smagorinsky, dynamic Smagorinsky [10] and a dynamic mixed SGS model. Mixed models, where a dissipative component is used along with potentially anti-dissipative elements, are not unusual. In Ribault et al., the scale similarity model of Bardina et al. [11] is combined with the dynamic Smagorinsky model. Ribault et al. find that the Smagorinsky model is too dissipative, with the mixed and dynamic Smagorinsky models giving the best agreement.
with both DNS [8] and measurements. Klein et al. [12] perform plane jet flow DNS for a range of Reynolds numbers (Re). Results show that when Re $\lesssim 6000$ (based on the slot width, $D$, and maximum velocity at the nozzle exit, $U_0$), Re has a significant influence on the jet development.

In LES, SGS models are designed to simulate energy transfer between the resolved larger eddies and the unresolved, modeled smaller ones (the forward cascade process). This transfer could involve nonlinear interactions between the resolved and unresolved scales. Since the forward cascade process is usually dominant and the backward cascade process (backscatter) is usually weak, backscatter is neglected in most SGS models. However, it has become clear that to make accurate predictions of real flows, an SGS model with the ability to capture backscatter of energy, could be important [13].

Most SGS models are linear and isotropic in the sense that there is a scalar eddy viscosity multiplying the linear stress tensor. Examples include the Smagorinsky model and those based upon kinetic energy transport such as the Yoshizawa model [14]. A newer approach is to use models classified as nonlinear and anisotropic. While more complex, these models could reflect turbulent flow physics such as backwards energy cascade better. As with RANS models, nonlinear LES models can be created by adding nonlinear terms to the usual linear Boussinesq approximation. Although not phrased in this context, Clark et al. [15] (in [16] it is argued Leonard [17] should be given credit for the Clark terms) introduced the first nonlinear SGS model. Later Kosović [13] developed a phenomenological nonlinear SGS model using nonlinear RANS modeling inspired by constitutive relationships for non-Newtonian fluids. Relative to the Smagorinsky model, the Kosović model when applied to a high Reynolds number, shear driven, boundary layer gave improved shear and low-order statistics. Abe [18] has also found encouraging, fully developed channel flow, results when a nonlinear RANS model is used near walls and LES is used further away from them. The latter is based on the same nonlinear constitutive relation as Kosović (this point is not noted by Abe). Finally, Wang and Bergstrom [19] propose a dynamic nonlinear SGS model based upon the quadratic nonlinear relation of Speziale [20].

Another approach to determining nonlinear LES terms [22,23] is based upon the regularization work of Leray [21]. Besides the nonlinear Leray regularization, Domaradzki and Holm [24] have recast the Lagrangian averaged Navier–Stokes (LANS)-$\alpha$ model as a nonlinear model. These SGS models are mathematically asymmetric and nonlinear, relying on estimating $u'$ as $-\partial Q \nabla \tilde{U}$ where $\tilde{U}$ is the resolved velocity and $x$ is the filter width. Recently, Kerr and Tucker [25] proposed a mixed Leray model that combines Yoshizawa’s linear with a portion of the nonlinear term from the Leray model. When the flow aligned tensor terms are used, the model gives surprisingly good agreement with a benchmark ‘law of the wall’ for plane channel flow. Winckelmans et al. [16] present a mixed Clark (or Leonard) model which includes the dynamic Smagorinsky model. While a priori tests for homogenous decaying turbulence were encouraging, results for more realistic flows are similar to those for the dynamic Smagorinsky model. Nonetheless, Meneveau and Katz [26], in their review paper, conclude ‘a priori tests in a variety of flow conditions show that mixed models are, on the whole, superior to pure eddy viscosity models’. Hence, they seem worthy of further study here. As noted by Geurts and Holm [22,27], relative to the dynamic Smagorinsky model, the simpler to implement Leray and LANS-$\alpha$ models are significantly more computationally efficient.

With one exception [9], all of the LES for plane jets reviewed above use linear SGS models. For free jets, the shear/mixing layers generated between the emitted and ambient fluids are unstable and anisotropic (see [8]), making this a good candidate for using nonlinear SGS models to capture the turbulent anisotropy and energy backscatter. As shown in the context of a temporal mixing/shear layer [22,27], the non-mixed Leray and LANS-$\alpha$ models capture turbulent mixing better than the dynamic Smagorinsky model. This justifies the use of the mixed modeling approach considered here. Our objective is to evaluate whether the mixed-nonlinear Kosović [13], Leray [25] and LANS-$\alpha$ [22] models improve predictions for a plane jet compared with those from the linear Yoshizawa and Smagorinsky models. For this purpose, the jet at Re = 4000 investigated by Klein et al. [12] using DNS is revisited using a grid density comparable to that in the LES work of Ribault et al. [9]. For this grid density, Ribault et al. observe substantial differences between results for different LES models. Our general criteria for judging the SGS models shall be the centerline velocity decay rate, the jet spreading rate, mean velocity distributions and turbulence intensity distributions. This analysis could be used for future studies exploring more complex jets including co-flow and heating, and for explorations of turbulence-induced sound.

2. Governing equations

2.1. Continuity and momentum equations

The flow will be represented by the spatially filtered, isothermal, incompressible Navier–Stokes equations:

$$\rho \frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial (\tilde{\pi} \tilde{x}_i)}{\partial \tilde{x}_j} = -\frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \frac{\partial}{\partial \tilde{x}_j} \left[ \mu \frac{\partial \tilde{u}_i}{\partial \tilde{x}_j} \right] \frac{\partial \tau_{ij}}{\partial \tilde{x}_i} \frac{\partial \tau_{ij}}{\partial \tilde{x}_j} = 0$$

(1)

Variables with overbars represent filtered values. For the Smagorinsky, Yoshizawa and Kosović models top hat filtering is used. A discrete approximation to a Gaussian filter is used instead of Fourier Helmholtz filtering for the mixed-nonlinear Leray and $\alpha$ models. In (1), $\tilde{\pi}$ is a fluid velocity component ($i = 1, 2, 3$ corresponding to the $x, y$ and $z$ directions, respectively), $\rho$ is the fluid density, $\mu$ is dynamic viscosity, $\tilde{p}$ is the filtered static pressure and $t$ is time. To close the above equations, the subgrid scale stress tensor $\tau_{ij}$ in (1) needs to be modeled.
2.2. Subgrid scale models (SGS)

The stress \( \tau_{ij} \) can generally be expressed as

\[
\tau_{ij} = L + NL
\]

where \( L \) and \( NL \) represent linear and nonlinear terms, respectively. For eddy viscosity based models, \( L = \tau_{ij} \delta_{ij} / 3 - 2\mu \delta_{ij} \), where \( \delta_{ij} = 0.5(\partial \xi_j / \partial x_i + \partial \xi_i / \partial x_j) \) is the strain rate tensor and \( \mu \), the eddy viscosity. For the Smagorinsky model

\[
\mu_i = \rho C_s^2 \kappa^{1/2}
\]

where \( C_s \) is the Smagorinsky constant and \( \kappa \) denotes the filter width. That is the characteristic length scale of the largest subgrid scale eddies. In this study, \( \kappa \) is based upon the cell volume filter size \( L = (\Delta x \Delta y \Delta z)^{1/3} \). Our mixed-nonlinear Kosovic, Leray and LANS-\( \alpha \) models will use the \( k \)-based Yoshizawa model for \( \mu_i \):

\[
\mu_i = \rho C_s^2 \kappa^{1/2}
\]

For the linear Smagorinsky and Yoshizawa models, in (2) \( NL = 0 \). Following Kerr and Tucker [25], \( NL \) for the mixed-nonlinear SGS models can be expressed as shown in Table 1 where there is implicit summation over \( I \).

As noted in the introduction, the Leray and LANS-\( \alpha \) nonlinear terms are equivalent to creating nonlinear terms out of the following estimate of the subgrid scale velocity fluctuations:

\[
u' = -\nabla' \nabla^3 \Pi
\]

where \( \zeta = \Lambda \). This is a practical and easy estimate of \( \nu' \) that avoids the double filtering needed in the dynamic Smagorinsky model [10]. In the LANS-\( \alpha \) model the NL terms arise from the convection of SGS vorticity by the resolved scales using \( \partial \tau_{ij}^{NL} / \partial x_k = -\zeta \times \zeta' \) where \( \zeta = \nabla \times \nu' \). The Leray model is based on modeling the convective term \( \zeta \times \nu' \). The transformation of these terms into the nonlinear Leray and LANS-\( \alpha \) models in Table 1 can be found in [24,27].

The transport equation for \( k \) needed in (4) takes the following form:

\[
\frac{\partial k}{\partial t} + \rho \frac{\partial (k \nu_i)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ (\mu + \mu_i) \left( \frac{\partial k}{\partial x_j} \right) \right] + P_k - \epsilon
\]

where \( P_k = 2\mu_i S_{ij} S_{ij} \) and \( \epsilon = C_k \kappa \nu_i / \Lambda \) are production and dissipation terms, respectively. The value of the constant \( C_k \) is 1.05. The standard \( C_k \) values for the Yoshizawa, mixed Leray [25] and Kosovic models are 0.07, 0.05 and 0.11, respectively. Here we take the approximate average of these values with \( C_k = 0.07 \). In this way the influence of the nonlinear components to the linear contributions can more easily be assessed. As will be seen later, this \( C_k \) value also gives reasonable results for homogenous decaying turbulence. For the Smagorinsky jet simulations \( C_s = 0.1 \) while for Smagorinsky homogenous decaying simulations \( C_s = 0.2 \).

3. Numerical details

In this study, the computational domain of the jet is \( 40D(x) \times D(y) \times D(z) \) with, unless otherwise stated, \( 130 \times 221 \times 31 \) \((x,y,z)\) nodes. Fig. 1 gives an \( x - y \) plane grid view. As noted earlier, this LES grid was chosen based on the information given by Ribault et al. [9] and Klein et al. [12]. As can be seen from Fig. 1, the center of the jet inlet with width \( D \), is located at \( x = 0 \) and \( y_c = 20D \). A uniform grid is used in the \( z \)-direction. In the \( y \)-direction, an equally spaced grid is used around the jet inlet i.e. for \( 18D < y < 22D \), and then the grid is stretched on both sides of the inlet. In the \( x \)-direction, the grid is uniform between \( x = 0 \) and \( x = 2D \) and then stretched.

For the homogenous decaying turbulence case, the domain used is a cubic box with sides of length \( 2\pi \). Two meshes with \( 32^3 \) and \( 64^3 \) nodes are tested. Similar tests using a spectral code and a Smagorinsky, not \( k \)-type, eddy viscosity in the mixed models were used for model validation by Kerr and Tucker [25].

The finite volume program described in [28] is used. For spatial discretization, second-order center differences are used and Crank–Nicolson is used for the temporal terms. When discretizing the governing equations, nonlinear terms in (2) are treated as source terms. Results are time
averaged over 850 dimensionless time units (based on $D$ and $U_0$) and also spatially averaged in $z$. For the mixed Kosovič, Leray and LANS-$\alpha$ models, NL is filtered using a discrete approximation to a Gaussian filter. For the Kosovič model this is just a numerical stability aid. For the latter two, this action is necessary for preserving energy-like invariants [24,27]. For further numerical stability enhancement, the magnitude of the nonlinear stresses $j_{NL}$ are limited to be some fraction $\beta$ of the linear ($L$).

Hence, the magnitude of the actual nonlinear contribution $j_{NL}$ used is as below

$$|j_{NL}| = \min(\beta|L|, |NL|)$$  \hspace{1cm} (7)

For the mixed Leray and LANS-$\alpha$ models, which have smaller nonlinear terms than the Kosovič model, $\beta = 1$. For the Kosovič model $\beta = 0.7$.

3.1. Boundary and starting conditions

For the plane jet flow studied here, periodic boundary conditions are employed in the $z$-direction. It is well known that for LES the setting of far field boundary conditions is of particular importance. In LES mode, three types of boundary treatments for the outflow, top and bottom boundaries were tested. These being simple Neumann (derivative with normal velocity components set to conserve mass), convective [29] and sponges [30]. It is found that the Neumann and convective methods yield iterative convergence problems. Therefore only the sponges are used.

As shown in Fig. 2, the widths of the sponge layers are taken as $L = 10D$, $5D$ and $5D$ at the outflow, the top and bottom boundaries, respectively. Following Andersson et al. [31] results from high Reynolds number $k$-$\epsilon$ model solutions are used as target values in the layers. To calculate instantaneous $F^{IU}(l)$ flow quantities within the layer, the following formula is implemented

$$F^{IU}(l) = (1 - f(l))F(l) + f(l)F_{target}(l)$$  \hspace{1cm} (8)

where $F$ is an instantaneous LES solution at a point, $F_{target}$ is the corresponding RANS target solution and following [32] the function $f(l) = (l/L)^2$ for $0 < l < L$. In this study the power used in the sponge blending function $\lambda$ is 3.

Following Klein et al. [12], for the jet outlet region ($|y - y_c| < 0.5D$ and $x = 0$), a top-hat like profile is used for the mean $U$ (streamwise) velocity.

$$U = \frac{U_0}{2} + \frac{U_0}{2} \tanh\left(\frac{|y - y_c| + 0.5D}{\theta}\right)$$  \hspace{1cm} (9)

where $\theta = D/20$ is the momentum thickness. Also, $V = W = u = v = w = 0$. For the $k$-$\epsilon$ RANS solution, an inlet turbulent intensity of 2% is used.

For the homogenous decaying turbulence case, at all boundaries, periodic conditions are used. Also, the initial velocity field is obtained using Fourier modes based on energy spectra measurements [33]. For the jet case the initial velocity field is zero.

4. Results and discussion

4.1. Nonlinear model validation

To validate and verify coding of the mixed-nonlinear Leray (see [25]), LANS-$\alpha$ and Kosovič models the temporal decay of homogenous isotropic turbulence is considered. Comparison is made with the measurements of Comte-Bellot and Corsin [33]. Fig. 3 shows the scaled three-dimensional energy spectra for all the SGS models at $r^* = 2$ (which corresponds to the third location in the experiment). As can be seen, for the 64$^3$ grid, the predicted energy
spectra for the models follow a $-5/3$ slope reasonably well. This is also the case for a Smagorinsky model result on a coarser $32^3$ grid given by the dashed dot dot line. It should be expected that the Smagorinsky model would perform well in this case because it was originally tuned around 1970 to one of the earliest $32^3$ decay simulations. It is worth noting that for the Leray and LANS-$\alpha$ models to give the correct energy spectra the dissipative linear element is necessary. Without this, energy levels at higher wave numbers are too large. Also, consistent with the observations of Geurts and Holm [22] the LANS-$\alpha$ model is relatively unstable. All of these conclusions are consistent with the homogeneous, decaying validation tests of Kerr and Tucker [25].

4.2. Plane jet results

In this discussion, most of the velocity profiles are normalized by the local centerline velocity, $U_{cl}$, the streamwise coordinate $x$ is normalized by $D$ and the transverse coordinate $y$, by the local jet half-width, $y_{1/2}$. The vertical coordinate is with respect to the jet axis, $y' = y - y_c$. For the near field of the jet, measurements are either from Browne et al. [3] ($Re = 3000$), Thomas and Chu [4] ($Re = 8300$), Gutmark and Wygnanski (GW) [5] ($Re = 30000$) or Namer and Otugen (NO) [6] ($Re = 4000$). For the far field, just the GW and NO data is used. Both Browne et al. and Thomas and Chu (TC) do not provide full data for this region.

![Fig. 4. Contours of mid $x$-$y$ plane instantaneous spanwise vorticity: (a) mixed Leray model and (b) Yoshizawa model.](image)

![Fig. 5. Mean vertical $U$ distributions at different axial locations: (a) Kosović model and (b) Yoshizawa model.](image)
4.3. Jet development

Fig. 4 gives instantaneous contours of $z$-component of vorticity for the mixed Leray (Frame (a)) and Yoshizawa (Frame (b)) models. As can be seen, these models (like the other models not shown) predict similar structures. The two mixing layers at the jet edges ($y/D = 19.5$ and $20.5$) start merging at around $x/D = 5$. In this region, relatively large, coherent structures can be observed. Further downstream, these break down into finer scales. All the models reproduce the qualitative experimental findings noted in the introduction.

Fig. 5 gives vertical distributions of normalized mean longitudinal velocities ($U/U_{cl}$) for the Kosovic model (Frame (a)) and Yoshizawa (Frame (b)) models at different axial locations. It can be seen that after $x/D > 8$ that profiles reach self-similarity. In the self-preserving region, predicted longitudinal velocities show encouraging agreement with the measurements.

Fig. 6 shows the mean vertical or transverse velocity, $V$, profiles for the Kosovic model. Again, after $x/D > 8$, a tendency to self-similarity can be observed. However, as with the DNS of Klein et al. [12] there are small deviations in $V$. The other models (Smagorinsky, Yoshizawa, mixed Leray and mixed LANS-$\alpha$) give similar results for $U$ and $V$ to those described above.

Fig. 7 compares the streamwise decay in centerline velocity, $U_{cl}$, for the five LES models with the measurements of TC, Browne et al. and the DNS of Klein et al. [12]. Three DNS data sets of Klein et al. [12] based on different inflow boundary conditions are included. DNS0 denotes a laminar inflow condition, and DNS1 and DNS2 denote turbulent inflow conditions with different turbulence length scales. The latter uses a larger length

Table 2

<table>
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<th>$K_1$</th>
<th>$K_2$</th>
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</tr>
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<td>Namer and Otugen ($Re = 6000$) [6]</td>
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<td>Thomas and Goldschmidt ($Re = 6000$) [7]</td>
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<td>DNS of Klein et al. ($Re = 4000$) [12]</td>
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Fig. 8. Comparison of the evolution of the jet half-width.

Please cite this article in press as: Liu Y et al., Linear and nonlinear model large-eddy simulations of a plane jet, Comput Fluids (2007), doi:10.1016/j.compfluid.2007.02.005
From the DNS data, it can be seen that the jet development is sensitive to the inflow boundary conditions. Compared with the DNS and measurements, none of the present LES results look out of place. The Kosović model appears to give the best prediction. The mixed LANS-$\alpha$ model delays transition slightly more than the other LES models, creating the longest potential core region.

Fig. 8 plots the growth of the jet half-width, $y_{1/2}$. The Kosović model prediction matches the measurements best, while the other models are similar.

In the self-similar region the following relationships hold

$$\frac{y_{1/2}}{D} = K_1 \left( \frac{x}{D} + C_1 \right), \quad \left( \frac{U_0}{U_{cl}} \right)^2 = K_2 \left( \frac{x}{D} + C_2 \right)$$

where $K_1$ and $K_2$ are the jet spreading and centerline velocity decay rates, respectively. As shown by Figs. 7 and 8, for similarity $x/D > 8$. For $8 < x/D < 12$, the $K$ coefficients in (10) are estimated. Values are summarized in Table 2 for the models tested here along with measurements including those of Thomas and Goldschmidt (TG) [7].

As found by Stanley et al. [8], the measurements of the jet spreading rate ($K_1$) at different Reynolds numbers are relatively consistent with a magnitude of around 0.1.
However, as can be seen from Table 2, the measurements for the jet centerline velocity decay rate differ substantially from one study to another. Our LES results give $K_1 = 0.12$ which is slightly larger than the measured values. However, the predictions of the centerline velocity decay rate ($K_2$) lie in the (wide) range of measured data.

4.3.1. Self-similar profile comparisons

The vertical (cross-stream) distribution of normalized streamwise mean velocity $U$ for different SGS models is shown in Fig. 9. Frames (a) and (b) are for $x/D = 12.5$ and $x/D = 15$, respectively. All, the LES predictions match the DNS data of Klein et al. [12] and measurements of GW [5] well. As would be expected, the $U$ profiles at the two locations in the self-preserving region are virtually identical. Fig. 10 gives the vertical mean velocity, $V$ distribution. Observations for this are similar to those for Fig. 9.

Figs. 11–13 give the three resolved velocity fluctuation distributions, $u'$, $v'$ and $w'$, respectively, at the two locations. It should be mentioned that Namer and Otugen (NO) [6] measurements for $v'$ and $w'$ (and shear stress) are not available. As noted by Klein et al. [12], the GW [5] data has some uncertainties. All the LES models give similar results. However, for $u'$, the Kosović model clearly performs the best with the distributions being on average closest to the DNS data. It should be noted, even the DNS data is not perfectly self-similar for $u'$, $v'$ and $w'$.

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![Fig. 11. Streamwise fluctuation distributions at: (a) x/D = 12.5 and (b) x/D = 15.](image)

![Fig. 12. Lateral fluctuation distributions at: (a) x/D = 12.5 and (b) x/D = 15.](image)
Fig. 14 gives the turbulent shear stress distributions for the LES models together with the DNS data of Klein et al. [12] and the measurements of GW [12]. As can be seen, like the normal stress fluctuations, all the LES show reasonable agreement with the data. However, on balance, perhaps the worst shear stress results are for the Smagorinsky LES. It is worth noting that reducing $C_l$ to 0.05 for the mixed Leray model decreases the model’s predictive accuracy. This demonstrates the importance of the dissipative element. Comparisons, not shown here, for the mixed Leray and Yoshizawa models on grids where the number of cross-stream nodes is halved are slightly inconclusive. On balance the mixed Leray results show marginal accuracy improvements. For the mixed LANS-$\alpha$ and Leray models Geurts and Holm [22] found that for best performance the filter width should be at least greater than four times the grid spacing. The sensitivity of results to this ratio might be worth exploring as future work. As a further point, the grid density used in the lower Reynolds number LES simulations of Ribault et al. [9] using a fourth-order compact scheme are similar to that used here. However, in this work more substantial differences can be observed between predictions for different LES models.

5. Conclusions

The mixed-nonlinear Kosovic, Leray and LANS-$\alpha$ SGS models were contrasted with linear Smagorinsky and Yoshizawa LES simulations for a $Re = 4000$ plane jet with homogenous isotropic decaying turbulence used for model validation. All but one of the mixed-nonlinear models gave similar, encouraging results. The linear dissipative element was found to be important for both accuracy and numerical stability.

For the jet, comparisons were made with DNS data and measurements. Spreading and centerline velocity decay rates were found in accord with measurements. In the self-similar region all models generally gave encouraging agreement with the DNS data and reliable measurements. Solution differences for the models were relatively minor with no model giving clear improvements for all data comparisons.

For second-order discretizations, there is a body of literature that suggests that unless the filter spans many cells, it can be difficult to separate the influence of discretization from that of the subgrid scale model. It is difficult to determine whether this is occurring here and could be masking other modeling issues.

It should also be stressed that the current flow is not wall bounded. It is generally understood that for wall bounded flows an eddy viscosity LES will not produce turbulent physical fluctuations unless the modeling coefficient is variable or wall modeling used. The issue this paper addresses is what improvement can be achieved in a shear far from boundaries when simple, nonlinear subgrid scale contributions are added. The extra terms chosen are all simple quadratic combinations of the velocity derivatives and can be implemented at a computational cost comparable to the existing nonlinear terms. Our conclusion is that except for a model that suppresses small-scale vortex stretching (the LANS-$\alpha$ model), these methods might provide a small robust improvement to a basic LES. However, the promise of a significant improvement suggested by the specialized case of Kerr and Tucker [25] has not been realized. The evidence is that some aspects of the small-scale vortical structure needs to be included, which is the next step in our program. Currently, of the LES models tested, our recommendation would be to choose one that is numerically well-conditioned, efficient, and easy implement. Except for the LANS-$\alpha$ model, which was found to be relatively unstable, all the models considered would pass these criteria. For this non-wall bounded flow, the sophistication of the LES
model seems less important than a careful implementation of the sponge boundary conditions.

Acknowledgements

The authors are grateful for the kind help of Dr. Klein who provided DNS and some experimental data. Financial support from the UK Engineering and Physical Sciences Research Council (EPSRC) under grant number GR/T06629/01 is greatly appreciated. We would also like to thank M. Shur for supplying coding and data necessary to run the homogenous decaying turbulence case. This project was enabled with the assistance of IBM Deep Computing.

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