Goal Programming via Multidimensional Scaling Applied to Senegalese Subsistence Farms

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A methodology is developed to estimate empirically the weights for a multiple-goal objective function of Senegalese subsistence farmers. The methodology includes a farmer-oriented goal preference survey and an application of a multidimensional scaling technique to the survey data. A comparison of model performance under the multiple-goal objective function with a profit-maximization objective function does not indicate that there are distinct advantages to using either function.

Key words: goal programming, multidimensional scaling, multiple objectives, preference survey, profit maximization, ratio scaling, Senegal, subsistence farmers.

Decision makers are often hypothesized to be motivated by multiple objectives rather than a single one, like profit maximization. Many discussions of this topic have appeared in agricultural economics literature (Willis andPerlack). The most common multiobjective approaches have involved profit, risk, and subsistence (e.g., Brink and McCarl, Calkins). However, more general approaches are possible. Ideally, multiobjective approaches allow a more accurate portrayal of the decision makers' utility function. Thus better decisions can be made, or better predictions can be made of decision makers' actions.

Multiple objective research can be characterized as "descriptive," "operational," or "combined." The descriptive approach concerns whether or not decision makers possess multiple objectives, developing relative rankings of objectives (Harman et al., Smith and Capstick, Gasson, Patrick and Blake). The operational approach utilizes hypothesized objective weights, examining their impact upon a decision model (Wheeler and Russell, Eilon, Dobbins). The combined approach embodies an attempt, first, to discover objectives and their weights, and then to utilize them in a decision model. Combined approaches may attempt this simultaneously (Candler and Boehlje or Willis and Perlack) or iteratively, as done in this paper. The combined approach to multiobjective analysis is the least common in agricultural economics. The primary purpose of this paper is to suggest and appraise a combined approach method.

The setting is subsistence farming in Senegal. Subsistence farming is particularly well suited to multiobjective programming. Subsistence farmers are quite frequently said to possess multiple, conflicting decision-making objectives. (These include profit maximization, risk avoidance, maintenance of minimum food consumption, and meeting social obligations.) In addition, accurate prediction of subsistence farmer actions is also important. A factor in subsistence-farming development is technology improvement. An important task is to insure that improved technology will be adopted (Valdes, Scobie, Dillon).

The study was done in the Sine Saloum region of Senegal during the 1977 crop season. This region, located in the southern part of Senegal's peanut basin, has an annual rainfall of 700 to 900 millimeters (mm). Numerous crops are grown, including peanuts, millet, sorghum, cotton, and corn. Farms in the re-
gion vary from 3 to 12 hectares, use little or no fertilizer, employ 3–6 workers, and include animal traction. The household system provides responsibilities and purchased inputs (Hopkins, Kleene). The chef de ménage, the focus of this study, has responsibility for the provision of food and allocation of land and other inputs. In turn, the chef de ménage receives labor from other family members.

Multiobjective Programming

Multiobjective, or goal, programming first developed by Charnes and Cooper and later used extensively by Lee, is an optimization theory incorporating multiple objective functions. Multiobjective programming generally involves a composite objective function. The idea is to minimize deviations from specified levels of two or more goals. The composite objective function is usually stated in one of two ways: (a) goal satisfaction occurs in a stated sequential order, goal A, then goal B, then goal C—a lexicographic utility ordering, see Lee; or (b) goal satisfaction may be traded off using relative ‘‘cost’’ weights on deviations from target levels—an indifference surface approach. In this paper, the indifference surface approach will be used.

Mathematically, the goal problem may then be expressed as follows:

Minimize

$$\sum_i (W_i^+ \cdot d_i^+ + W_i^- \cdot d_i^-),$$

subject to

$$\sum_j G_{ij} X_j - d_i^+ + d_i^- = g_i \quad \text{for all } i,$$

$$\sum_k a_{kj} X_j \leq b_k \quad \text{for all } k,$$

$$x_j, d_i^+, d_i^- \geq 0 \quad \text{for all } j \text{ and } i,$$

where $d_i^+$ refers to the amount of positive deviation, or overproduction of the $i$th objective target level ($g_i$); $d_i^-$ refers to the amount of negative deviation, or shortage of objective satisfaction; $W_i^+$, $W_i^-$ are the weights, or relative importance, attached to the deviation from targets, with $W_i^+$ reflecting the return to oversatisfaction, $W_i^-$ the return to undersatisfaction. The weights reflect the slope of an individual’s indifference surface for objective satisfaction. $G_{ij}$ are the coefficients of objective achievement—the marginal achievement of objective $i$ due to the production of $X_j$; $a_{kj}$ are the per-unit, resources-usage coefficients for production of activities $X_j$; and $b_k$ are the resource endowments.

Difficulties in Goal Specification

Numerical specification of linear programming problems is often difficult. In addition, the multiobjective-programming model adds another set of key parameters, goal weights, and targets. Ideally, both should be supplied by the decision maker, but determining their values can be complicated (Willis and Perlack).

Four methods have been proposed for finding the weights. The first is to choose arbitrarily an initial set of weights, adjusting them until the output satisfactorily resembles the decision maker’s actual behavior (Candler and Boehlje). The second is a ‘‘revealed preference’’ approach, inferring the decision maker’s goal weights from past activities. In each of these approaches, the analysis ends when a ‘‘satisfactory’’ solution is obtained. This permits the right answer to be obtained for the wrong reasons (weights may be discovered which produce the optimal solution, but the weights may not be unique and may not correspond to true preferences). In the third method objective weights are not sought; rather, a dominant set of solutions is generated and presented to the decision maker for choice (Willis and Perlack). This method is difficult to use without extensive direct decision maker contact. The fourth method, used here, involves direct elicitation of preferences through a survey. The principal difficulties with this approach involve (a) proper specification of the survey so that usable data are gathered and (b) translation of the data into a form useable in the programming model.

The model used here also requires specification of objective targets. Ideally, targets should be specified by the decision maker, reflecting his preferences. However, in this analysis we specified the targets independently using farm records because we (a) were

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1 The approaches also differ in that the lexicographic approach requires a modified simplex code, whereas the indifference surface approach utilizes the traditional simplex algorithm. One should also note that the problems are dual to each other and conceptually, with proper weights, should be able to function identically.

2 This method was chosen because of the authors’ lack of direct contact with decision makers and because of their desire to simulate the actions of ‘‘representative’’ decision makers.
dealing with a representative farm rather than a specific individual and (b) had farm records data pertinent to the goals of interest. (A more satisfactory, but much more complicated, technique would utilize survey questions involving objectives and objective target levels simultaneously. This is desirable because the precise target levels and preferences would be interrelated. However, goal weight estimation would have become increasingly complex, as weights would be needed for different target levels of the same goal.)

Goal Specification and Measurement

Using pilot surveys with farmers, discussions with local agricultural economists, previous studies (particularly Hopkins, Kleene), and the authors’ field observations, five potential goals were selected:

(a) Produce a sufficient amount of food to feed the entire family even if the season is not good.
(b) Spend less on inputs (includes annual installments on equipment, fertilizer, and seed) and get lower yields.
(c) Earn more income to buy animals.
(d) Organize the work to have more leisure.
(e) Obtain higher yields by spending more money on inputs.\(^3\)

Formation of Goal Weights

The multiobjective-programming objective function requires ratio-scaled data or, equivalently, a cardinal utility function (cf., Coombs). The most commonly used approach to measure farmer goal preferences (e.g., Harman et al., Smith and Capstick) is paired comparisons—“confusion” scaling. Beginning with the seminal work of Thurstone in 1927, many studies have applied this approach to preference measurement (Bock and Jones). This approach, however, yields data inappropriate for the objective function. At best it yields interval-scaled weights which reflect the degree to which one objective is preferred to other objectives along a scale whose units are equidistant. Preference weights are computed from an arbitrary origin, usually the least-preferred goal. Because the scale has no true reference point, meaningful ratios among goal weights cannot be estimated (Torgerson).

Fortunately, multiple-dimensional scaling (MDS) allows the data to be transformed to applicable ratio objective weights. Three steps were used in doing this. First, respondents were presented with all possible pairs of goals and asked to select the preferred goal in each pair. This provided ordinally scaled data. Second, the ratio scale underlying the ordinally scaled data was estimated utilizing the Schoneman and Wang procedure. Principal assumptions underlying this procedure are: (a) each individual’s paired comparison choices reflect an underlying stable ratio scale for the decisions modeled; (b) the set of individual preferences can be synthesized into representative group preferences (see Bradley and Terry for elaboration); (c) the probability \(P(i > j)\) of an individual choosing one objective \(i\) over another \(j\) is a function of the individual’s preference-scale values \(a_i\), where \(P(i > j) = a_i / (a_i + a_j)\), as developed in Bradley and Terry; (d) each individual surveyed possesses an ideal point in an \(m\)-dimensional utility space (Coombs); (e) each individual’s preference-scale value for an objective is related by a negative exponential utility function of the Euclidian distance \(d\), such that the objective satisfaction level falls symmetrically from the ideal point, where \(a_i = e^{-cd}\) (Schoneman). The distances are ratio-scaled measures of relative preference (Schoneman). This function is fit with minimum squared error, using nonlinear regression (for elaboration, see Moore, Pessimer, Little; or Schoneman and by Schoneman and Wang). Using these assumptions, MDS estimates the underlying ratio scale and provides statistical tests on the goodness of fit of the obtained solution. This is a standard application of metric “ideal point” MDS (Kruskal and Wish, Wierenga, or Zelany), using the algorithmic methods developed by Schoneman and Wang. (For discussion of MDS within agricultural economics see Blake, Schrader, James; Patrick, Whitaker, Blake; or Wierenga.) Third, the ratios among objective preference scores were used as weights for deviations from the objective target level. The highest weighted objective was given a deviation weight of ten. Then, weights for the other objectives were assigned so that the appropriate ratios were preserved. (A footnote to table 1 gives an example.)

\(^3\) Note that goals 2 and 5 are complementary. They constitute two goals within the model: one is related to input spending below the target; the other, to spending above the target.
Table 1. Distance Matrix between the Stimuli and Ideal Points

<table>
<thead>
<tr>
<th></th>
<th>Women</th>
<th>Navetanes</th>
<th>Sourgha</th>
<th>Chef</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Food</td>
<td>1.71</td>
<td>1.85</td>
<td>1.83</td>
<td>1.26b</td>
<td>10.00</td>
</tr>
<tr>
<td>2. Spend less on inputs</td>
<td>1.42</td>
<td>1.89</td>
<td>1.57</td>
<td>1.87</td>
<td>6.74c</td>
</tr>
<tr>
<td>3. Earn income</td>
<td>1.26</td>
<td>0.91</td>
<td>0.82</td>
<td>1.54</td>
<td>8.18</td>
</tr>
<tr>
<td>4. Leisure</td>
<td>1.69</td>
<td>2.01</td>
<td>1.74</td>
<td>2.14</td>
<td>5.89</td>
</tr>
<tr>
<td>5. Spend more on inputs</td>
<td>0.91</td>
<td>1.57</td>
<td>1.20</td>
<td>1.36</td>
<td>9.26</td>
</tr>
<tr>
<td>Number of respondents</td>
<td>10</td>
<td>13</td>
<td>13</td>
<td>43</td>
<td></td>
</tr>
</tbody>
</table>

*a Navetanes and Sourghas are two other Wolof terms for labor groups within the ménage (household). The sourgha is an unmarried son who cultivates the chef de ménage’s field and has some land of his own. The navetane is a hired worker, who works in a close relationship with the chef’s family. See Hopkins and Kleene.

b The smaller the weight (the calculated Euclidian distance between the goal and “ideal point”), the more important the deviation.

c These weights are objective function coefficients. The value 6.74 is defined by taking ten times the distance of the most important goal (food—1.26) divided by the weight of this goal (1.87).

Preference Function Results

A sample of eighty individuals was drawn from a census of farmers in the selected villages at Thysse-Kaymore in Sine Saloum. The individuals were selected from all social strata, thereby allowing comparisons of goal weights.

Each respondent was given each of the ten possible pairs of the five objectives and asked to select the one in each pair which better described his current objectives. The MDS procedure of Schoneman and Wang was then applied to this “paired comparison” data with the respondents grouped as in table 1.

A two-dimensional solution displayed the best fit yielding the distance results shown in table 1. The longer the distance, the greater is the difference between the ideal point for a group and the location of that objective. Thus, the greater the distance, the less important was that objective. These data provided ratio-scaled preferences for all groups. Although the scaling algorithm provided the best-fitting algebraic solution, the compound chi-square test of goodness of fit for the solution (df = 24, chi-square = 139.85, p < .001) indicated that significant deviations existed between the input data matrix and the reproduced matrix. Hence, the data in table 1 should be viewed as best estimates for each status as a whole. However, the results for the chef de ménage were substantially better than the overall results, indicating that the input results were not significantly different from the estimated results. Therefore, we felt confident with the chef’s preference data.

Also, because substantial differences appeared among groups, since the chef group was principally responsible for the decisions being modeled here and because the objective measures of the chef group were acceptable, the chef group objective weights were selected for the GP model. The calculated distances for the chef group can be monotonically transformed without affecting the ratio scale. Thus, by dividing the reciprocal of each distance by 12.6 (1.26 x 10), the distances of each goal from the ideal points were determined and are shown in the extreme right-hand column of table 1.

Formation of the Multiobjective Programming Model

With the weights established, the task is to form the multiobjective programming model as a linear program.

The Constraint Matrix

The linear programming formulation of a Sine Saloum farm developed by Richard, Fall, and Attonaty (RFA) was adopted as a starting point. The RFA model covers production of rain-fed rice, tobacco, forage, peanuts, cotton, sorghum, corn, and millet during ten production periods under horse and oxen traction. The activities for a crop include seeding, land preparation, weeding, harvest, by-product harvest, threshing, product transport, and sale. Other activities deal with crop rotational considerations, family consumption, and input acquisition. The RFA model constraints deal with land, by-product availability, storage, family food requirements, labor availability, animal availability, livestock feed require-
ments, cash availability, and field time availability. Features were added to the RFA model to depict: (a) alternative timing of cultural practices, (b) differential rates of fertilizer use, (c) light or heavy plowing, (d) quarterly storage and marketing/consumption periods throughout the year, and (e) limited credit availability. These modifications resulted in a profit-maximizing, linear-programming matrix with 513 rows and 916 columns.

**The Objective Function**

The objective function then was restructured from profit maximization to the goal objective function using the weights described above for the *chef de ménage* group (assuming that the decisions modeled are made with respect to their goals only). The way in which two of the five goals entered the model is described below. For a detailed discussion of the other three goals, see Barnett. The objectives were entered so that their values ranged between zero and one. This allows direct use of the above goal weights.

The food objective had the largest weight; table 2 shows how it is modeled. Cereal may be either purchased or produced and stored to meet quarterly food requirements. As long as the food consumed can be produced, the food goal will be met. Estimates of annual cereal consumption place this figure at about 220 kilograms (kg)/year (Hopkins). Assuming that ten people are in the *ménage*, the total food target is set at 2,200 kg annually. The variable \( F \) in table 3 is the percentage of the food goal met by purchases. The model assumes that food purchases during any time of the year have equal importance. In order for the objective function coefficient \( F \) to equal zero, all food must be farm grown.

The model also contained a net income objective. The net income goal is to maximize year-end net income. Its target level (100,000 francs) was based on calculated returns per hectare from peanuts cultivated by a better-than-average farmer. Its inclusion in the model is depicted in table 3. If the target of 100,000 francs cannot be met, a nonzero value of \( R \) is present, increasing the value of the objective function. The objective function weight of the net income goal is 8.18.

**Model Experimentation and Results**

A large number of solutions were calculated. These solutions were designed to contribute information toward model verification, comparative performance of the profit versus the multiple-objective formulations, sensitivity of the multiple-objective formulations and development strategies for the region. Because of the methodological nature of this paper, we present information pertinent only to the first three groups of solutions (Barnett presents the Senegalese farming system results and implications at length).

Model verification was accomplished by solving the multiple-objective formulation under base conditions and comparing the results with the farm level data published in studies by Albenque, Hopkins, and Samb. Deviation from the levels of four variables were chosen as the verification criterion. The variables were (a) crop mix (percentage of cash crop in the mix), (b) net income generated, (c) land cultivated, and (d) credit usage. Experiments were done for a number of different farm sizes.

### Table 2. Submatrix Food Goal for Periods 1 and 2

<table>
<thead>
<tr>
<th></th>
<th>Consume Farm-Grown Cereal</th>
<th>Purchase Cereal</th>
<th>Food Deviation</th>
<th>Food Requirement</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function</td>
<td>( P_1^a )</td>
<td>( P_2^a )</td>
<td>( P_1 )</td>
<td>( P_2 )</td>
<td>( 10F )</td>
</tr>
<tr>
<td>Food goal row</td>
<td>1</td>
<td>1</td>
<td>( \leq 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock cereal ( P_1 )</td>
<td>1</td>
<td>( \leq 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock cereal ( P_2 )</td>
<td>( \leq 0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food requirement ( P_1^a )</td>
<td>( \leq 0 )</td>
<td>( \leq 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food requirement ( P_2^a )</td>
<td>( \leq 0 )</td>
<td>( \leq 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. people to feed</td>
<td>( \leq 10 )</td>
<td>( \leq 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash ( P_1 )</td>
<td>( \leq 35 )</td>
<td>( \leq 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash ( P_2 )</td>
<td>( \leq 45 )</td>
<td>( \leq 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Food requirements and food purchase activities for periods \( P_3 \) and \( P_4 \) are not shown.
Table 3. Submatrix: Net Income Goal

<table>
<thead>
<tr>
<th>Sell Crops</th>
<th>Transfer Cash</th>
<th>Transfer Final Cash to Net Income Row</th>
<th>Revenue Goal Deviation</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_1$ to $P_{1+1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Objective function</td>
<td>$8.18R$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crop stock $P_1$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crop stock $P_2$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash $P_1$</td>
<td>$-30$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash $P_2$</td>
<td>$-41.5$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net income row</td>
<td>$-1$</td>
<td></td>
<td>$-100,000R$</td>
<td>$-100,000$</td>
</tr>
</tbody>
</table>

The results of the verification experiments are extensive. Thus, we will overview only the most significant findings. For more detail refer to Barnett. First, the crop-mix results all fell within the range of observed actual crop mixes. Further, the alteration in pattern of crop mix as land size increased was replicated in the results. Second, the amount of land devoted to crops was generally within 2%–3% of the actual amount. Third, the model’s income results are within 5%–8% of income attributed to the chef. Fourth, the model overuses credit, mainly because of a difference between the modelling of equipment-purchase repayment and real-world practices. The model also slightly overused fertilizer. Nevertheless, the overall impression was that the results were satisfactory and further experimentation was merited.

The next task was a comparative investigation of the multiple objective model’s (MO) performance vis-a-vis the expanded RFA profit-maximizing model (PM). Solutions were obtained for two representative farms under both objective function assumptions for changes in land holding and credit availability. Overall, both the PM and MO results were plausible and similar. Each model produced results which were “good enough” in the spirit of the verification section above. A number of observations may be drawn relative to the comparisons of the objective function formulations.

First, the MO solutions, in almost all cases, indicated that all of the objectives, except net income, were at their target levels. Enough cereal was produced to feed the ménage, and enough labor was supplied so that the leisure goal was not violated. The credit goal was also generally satisfied. To a large degree this means that the model behaved on the margin as a profit-maximizing model. However, there were disutilities for insufficient leisure and extra borrowing. Second, the PM model consistently exhibited larger farmed area and greater net income relative to the MO model. This occurred because the PM model did not contain disutility terms with respect to leisure and extra credit. Thus, the PM model, in effect, had a larger resource base.

Each model had both strengths and weaknesses. The PM model performed “better” (more realistically) for one representative farm whereas the MO model performed “better” for another. Thus, we were unable to make an overall judgment as to whether one formulation was superior.

As a last set of experiments, the MO model-objective weights were varied parametrically to investigate sensitivity. The weights for the food goal, credit, and leisure were varied independently from their original values of 10, 5.2, and 5.9, down to a value of 1.0 without meaningful change in output. A different approach was then tried. The model was constrained to increase the satisfaction of the net income goal from an optimal under satisfaction level of 18% to a 100% level. The shadow prices on the revenue goal indicated that this solution would occur when the objective weight was 50. Thus, the model was judged insensitive to major shifts in the goal weights.

Concluding Comments

The objective of this study was to implement and investigate an approach by which Senegalese farmers’ preferences for multiple objectives could be elicited and then used in a multiobjective-decision model. The elicited information was gathered via paired comparisons and transformed using multidimensional scaling into the objective function for the multiobjective model. The resultant model performed satisfactorily, generating results con-
sistent with previously observed Senegalese farmer behavior. However, the multiobjective model did not exhibit superiority over a similarly structured profit-maximizing model. There, the multiple-objective model was relatively insensitive to objective weighting.

The lack of improvement of the multiobjective model over a traditional profit-maximizing model leads to two immediate questions. Why? And what good is the multiobjective model? The lack of improvement using the multiple objective (MO) model arises in this case principally because of the characteristics of the profit-maximizing (PM) version. The PM model basically reproduces observed behavior satisfactorily; thus, the MO model could not greatly improve upon it. A better test of the MO model would involve a case in which a properly structured PM model did not adequately produce plausible results.

One then must wonder if the efforts devoted toward the MO model were worthwhile. The answer within this study is probably not. In general, the MO model leads one to an improved knowledge of the problem and a potentially better model. However, this comes with the costs of objective conceptualization, measurement manipulation (transformation), and inclusion (in the formulation).4

Based on this study, the multiple-objectives-type work should not be abandoned. However, the majority of practitioners should continue to use the "best" assumed model structure (e.g., profit maximization, potentially with risk and subsistence) until other detailed studies determine the need for more involved specifications.

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References


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