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Outage Diversity of MIMO Block-Fading Channels with Causal Channel State Information

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Abstract—We study the outage diversity of the multiple-input multiple-output (MIMO) Rayleigh block-fading channel when causal channel state information (CSI) is available at the transmitter (CSIT). Within this setting, we consider the optimal power allocation for blocks $b = 1, \ldots, B$ given perfect CSIT for blocks $1, \ldots, b - u$ only, subject to a long-term power constraint. The parameter $0 \leq u \leq B$ is a fixed arbitrary integer that determines the delay in acquiring perfect knowledge of the CSI at the transmitter. Without explicitly solving the optimal power allocation problem, we derive the outage diversity of the system. For general $0 \leq u \leq B$, we derive a simple recursive expression for computing the outage diversity. For the special case $u = 0$, it is shown that the outage diversity is infinite, coinciding with previously known results. For $1 \leq u \leq B$, the outage diversity becomes finite and for the special case of $u = 1, 2$ it can be expressed in simple closed form.

I. INTRODUCTION

The mitigation of fading is a particularly challenging aspect in the design of reliable and efficient wireless communication systems [1]. Methods that attempt to deal with this impairment depend on many factors, e.g., the behaviour of the fading (coherence time/frequency) and system constraints (delay/power). For systems with no delay constraints or fast fading channels, the channel can be considered *ergodic*. In this case, long-interleaved fixed-rate codes not exceeding the channel capacity can be employed to ensure an arbitrarily low probability of error [2, 3]. On the other hand, for slowly fading channels with delay constraints, the codeword may only experience a small finite number of independent fading realisations and hence the channel is *non-ergodic*.

The *block-fading* channel [2, 4] is a simple model that captures the essence of non-ergodic channels. Here, each codeword comprises a finite number of blocks, where each block experiences an independent fading realisation, which remains constant within a given block. In this case, the instantaneous input-output *mutual information* is a random variable dependent on the underlying fading distribution. For most fading statistics, the channel capacity is zero in the strict Shannon sense as there is a non-zero *outage probability* that a fixed information rate cannot be supported [2, 4]. The outage

probability is the lowest achievable word error probability of codes with sufficiently long block length [5]. As such, a rate-reliability tradeoff exists, whereby for a fixed number of blocks, a high rate is penalised by a large error probability.

Most works that study the block-fading channel focus on adaptive transmission techniques in which the power and/or rate is adapted to the channel conditions subject to system constraints (see [6] for a recent review). Adaptation, however, requires a certain degree of knowledge of the channel fades, also referred to as *channel state information* (CSI), at the transmitter and receiver. A large body of works consider perfect *a-causal* CSI at the transmitter (CSIT), i.e., the transmitter knows exact values of the fades on all blocks, which enables adaptation using full CSIT. This approach has practical relevance for systems exhibiting a set of instantaneous parallel channels, such as Orthogonal Frequency Division Multiplexing (OFDM) systems. However, the assumption is unrealistic for slowly time-varying channels where there is a delay in acquiring the CSIT, e.g., free-space optical channels [7]. For these types of systems it is only realistic to assume that only causal CSI, i.e., past channel fades, are available at the transmitter.

Several works have analysed the block-fading channel with causal CSIT [8–10], where power adaptation algorithms based on *dynamic programming* are proposed. However, in [8–10] it is assumed that perfect CSIT is available up to and including the current block to be transmitted. In practical systems there can be additional latency in the transmitter acquiring the CSIT, e.g. propagation and processing delays. Such additional delays are our primary motivation in this paper. In particular, we consider the multiple-input multiple-output (MIMO) block-fading channel with discrete-input constellations, where there is an arbitrary fixed delay in the availability of perfect CSI at the transmitter. The transmitter employs optimal power allocation that minimises the outage probability subject to a long-term power constraint. The optimal power allocation rule can be obtained as an extension of the dynamic programs in [8–10], however, the algorithms may grow prohibitively complex with increasing delay in acquiring CSIT. Dynamic programming therefore provides little insight in the outage performance of systems with causal CSI. Using recent results from [6, 11], we derive the *outage diversity* without explicitly solving the optimal power allocation problem. We show that the

outage diversity requires the solution to a linear homogeneous recurrence relation (difference equation) [12], which admits no closed form in general, but can be easily computed numerically. For the special case of a single block delay, the outage diversity can be solved in closed form and coincides with recent results on incremental redundancy, automatic repeat-request (INR-ARQ) systems [13, 14]. For a two-block delay, the difference equation can be solved analytically, yielding a closed-form expression for the outage diversity. Furthermore, the outage diversity of the single-input single-output (SISO) case can be simply characterised from the Fibonacci series. In general, our results completely characterise the loss in outage diversity for an arbitrary fixed delay in obtaining CSIT. Furthermore, it is also shown that only CSIT with sufficiently small delay is useful in terms of outage diversity.

II. SYSTEM MODEL

In this paper, we consider a discrete-input MIMO block-fading channel, with N_t and N_r transmit and receive antennae, respectively. Binary data is encoded with a code of rate R bits per channel use, constructed over an alphabet $\mathcal{X} \subseteq \mathbb{C}$. The resulting transmitted codeword consists of B blocks, where each block comprises of L vector channel uses of size $N_t \times 1$. We denote $\boldsymbol{x}_b[l] \in \mathcal{X}^{N_t \times 1}$ as the lth transmitted symbol vector of block b, for $l = 1, \ldots, L$ and $b = 1, \ldots, B$. The symbols are assumed to be drawn from $\mathcal{X}$ with unit average energy, i.e. $\mathbb{E}\left[\boldsymbol{x}_b[l]\boldsymbol{x}_b^\dagger[l]\right] = \boldsymbol{I}_{N_t}$, where $(\cdot)^\dagger$ denotes the Hermitian transpose and $\boldsymbol{I}_N$ the $N \times N$ identity matrix.

We denote by $\boldsymbol{H}_b$ the $N_r \times N_t$ complex channel matrix for block $b = 1, \ldots, B$. These matrices are drawn independently for each block, and remain fixed for the corresponding L channel uses. In addition, we assume the elements of the channel matrix are i.i.d. Gaussian random variables (Rayleigh fading model [1]). At the transmission of block b, we assume the transmitter only has knowledge of $\boldsymbol{H}^{(b-u)} = (\boldsymbol{H}_1, \ldots, \boldsymbol{H}_{b-u})$, where $0 \leq u \leq B$ is an arbitrary fixed integer (causal-CSIT). The parameter u models the delay in the transmitter obtaining the CSIT, due to e.g., propagation and processing delays.

We assume the receiver has perfect knowledge of $\boldsymbol{H}^{(b)}$ at transmission block b, and the signal at each receive antenna is corrupted by independent, zero-mean unit-variance additive white Gaussian noise (AWGN). Hence, under these assumptions, the bth block of $N_r \times L$ received noisy symbols is

$$\boldsymbol{Y}_b = \boldsymbol{H}_b \boldsymbol{P}_b^{\frac{1}{2}}\left(\boldsymbol{H}^{(b-u)}\right)\boldsymbol{X}_b + \boldsymbol{W}_b, \qquad b = 1, \ldots, B, \quad (1)$$

where: $\boldsymbol{X}_b \in \mathcal{X}^{N_t \times L}$; $\boldsymbol{H}_b \in \mathbb{C}^{N_r \times N_t}$; $\boldsymbol{Y}_b \in \mathbb{C}^{N_r \times L}$; $\boldsymbol{W}_b \in \mathbb{C}^{N_r \times L}$ whose elements are i.i.d zero-mean unit-variance white Gaussian noise; and $\boldsymbol{P}_b\left(\boldsymbol{H}^{(b-u)}\right) \in \mathbb{R}_+^{N_t \times N_t}$ is a diagonal matrix whose ith diagonal element denotes the power allocated to transmit antenna i of block b. The power allocation is subject to the long-term power constraint,

$$\mathbb{E}\left[\frac{1}{B}\sum_{b=1}^{B} \text{tr}\left(\boldsymbol{P}_b\left(\boldsymbol{H}^{(b-u)}\right)\right)\right] \leq P. \qquad (2)$$

III. PRELIMINARIES

The channel described by (1) under the quasi-static assumption is not information stable [15] and therefore, the capacity in the strict Shannon sense is zero. We therefore study the information outage probability,

$$\mathsf{P}_{\text{out}}(P, R) = \text{Pr}\left\{\frac{1}{B}\sum_{b=1}^{B} I_\mathcal{X}\left(\boldsymbol{H}_b \boldsymbol{P}_b^{\frac{1}{2}}\left(\boldsymbol{H}^{(b-u)}\right)\right) < R\right\}, \qquad (3)$$

which lower bounds the codeword error probability of any coding scheme [2, 4]. In (3), $I_\mathcal{X}(\boldsymbol{S})$ denotes the input-output mutual information of a MIMO block-fading channel with input constellation $\mathcal{X}$ and channel matrix $\boldsymbol{S}$. With the optimal Gaussian input constellation,

$$I_\mathcal{X}(\boldsymbol{S}) = \log_2 \det(\boldsymbol{I}_{N_r} + \boldsymbol{S}\boldsymbol{S}^\dagger); \qquad (4)$$

while with a uniform discrete constellation $\mathcal{X}$ of size 2^M,

$$I_\mathcal{X}(\boldsymbol{S}) = MN_t -$$

$$\frac{1}{2^{MN_t}}\sum_{\boldsymbol{x}\in\mathcal{X}^{N_t}} \mathbb{E}_{\boldsymbol{z}}\left[\log_2\left(\sum_{\boldsymbol{x}'\in\mathcal{X}^{N_t}} e^{-\|\boldsymbol{S}(\boldsymbol{x}-\boldsymbol{x}')+\boldsymbol{z}\|^2 + \|\boldsymbol{z}\|^2}\right)\right] \qquad (5)$$

Given perfect knowledge of $\boldsymbol{H}^{(b-u)}$, $\boldsymbol{P}_b(\boldsymbol{H}^{(b-u)})$ is the solution to the minimisation problem

$$\begin{cases} \text{minimise} & \mathsf{P}_{\text{out}}(P, R) \\ \text{subject to:} & \mathbb{E}\left[\frac{1}{B}\sum_{b=1}^{B}\text{tr}\left(\boldsymbol{P}_b\left(\boldsymbol{H}^{(b-u)}\right)\right)\right] \leq P. \end{cases} \qquad (6)$$

For $u = 0$, (6) can be solved via dynamic programming [8, 10]. The extension to $u > 0$ is also possible, although the problem becomes exceedingly difficult as u increases. As we shall see, it is possible to examine the asymptotic behaviour of $\mathsf{P}_{\text{out}}(P, R)$ without explicitly solving (6). Particularly, we study the outage diversity $d(R)$,

$$d(R) \triangleq \lim_{P\to\infty} \frac{-\log\mathsf{P}_{\text{out}}(P, R)}{\log P}. \qquad (7)$$

For systems with uniform power allocation, the outage diversity is $d^{\text{uni}}(R) = BN_tN_r$, achieved with Gaussian input constellation. With a discrete constellation $\mathcal{X}$ of size 2^M, the outage diversity is given by the Singleton bound [6, 16]

$$d^{\text{uni}}(R) = d_S(R) = N_r\left(1 + \left\lfloor B\left(N_t - \frac{R}{M}\right)\right\rfloor\right), \qquad (8)$$

with $\lfloor x \rfloor$ being the largest integer not greater than x. Note that $d^{\text{uni}}(R)$ is also the outage diversity of systems with short-term power constraint $\sum_{b=1}^{B}\text{tr}\left(\boldsymbol{P}_b\left(\boldsymbol{H}^{((b-u))}\right)\right) \leq BP$.

IV. MAIN RESULTS

For systems with causal CSIT, the outage diversity in both Gaussian and discrete input cases is a function of $d^{\text{uni}}(R)$, as given in the following Theorem.

Theorem 1: Consider transmission at rate R over the MIMO block-fading channel in (1) with Rayleigh fading. With

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long-term power constraint in (2) and CSIT delay $0 < u \leq B$, the optimal outage diversity is

$$d(R) = N_t N_r \sum_{b=1}^{\hat{b}} a_b + \left(d^{\mathrm{uni}}(R) - \hat{b} N_t N_r \right) a_{\hat{b}+1} \quad (9)$$

where $\hat{b} = \left\lfloor \frac{d^{\mathrm{uni}}(R)}{N_t N_r} \right\rfloor$, $d^{\mathrm{uni}}(R)$ is given in (8) and

$$a_b = \begin{cases} 1, & 1 \leq b \leq u \\ a_{b-1} + N_t N_r a_{b-u}, & u < b \leq \hat{b} + 1. \end{cases} \quad (10)$$

Proof: Due to space constraints, only a proof sketch for the discrete input case is given in the Appendix. ∎

Theorem 1 characterises the tradeoff between outage diversity, transmission rate and delay u for the MIMO block-fading channel. When $u = 0$, it can be shown that $d(R) = \infty$, and when $u = B$, Theorem 1 yields the outage diversity for the uniform power allocation case (8). For systems with Gaussian input constellations, the outage diversity $d(R)$ is independent of R, given by

$$d(R) = N_t N_r \sum_{b=1}^{B} a_b, \quad (11)$$

where a_b's are defined in (10). The tradeoff between outage diversity and CSIT delay is reflected through the recursive function of a_b in (10). For systems with discrete input constellation, additional tradeoff between transmission rate R and outage diversity is observed. Specifically, increasing R reduces $d^{\mathrm{uni}}(R)$ and $\hat{b}$ in (9), thus significantly reducing $d(R)$. When R is sufficiently large, a_b's in (10) take value 1, and thus the outage diversity $d(R)$ is the same as that of the uniform power allocation case. The rate threshold for zero-gain in outage diversity is given in the following.

Corollary 1: Consider a MIMO block-fading channel with causal CSIT and optimal power adaption (6). The outage diversity is equal to that of uniform power allocation, i.e., $d(R) = d^{\mathrm{uni}}(R)$, if

$$\frac{d^{\mathrm{uni}}(R)}{N_t N_r} \leq u. \quad (12)$$

Corollary 1 provides an important rule-of-thumb for power adaptation with causal CSIT. Whenever (12) is satisfied, CSIT is useless in terms of improving the outage diversity. Hence, uniform power allocation yields the optimal diversity.

For $0 < u < \frac{d^{\mathrm{uni}}(R)}{N_t N_r}$, (9) and (11) requires the solution to (10), a linear homogeneous difference equation, which cannot be solved in its general form, but for specific u can be solved using standard techniques [12]. Special cases of interest are considered in the following.

Corollary 2: Suppose $u = 1$ in Theorem 1, then

$$d(R) = (1 + N_t N_r)^{\hat{b}} \left(1 + \left(d^{\mathrm{uni}}(R) - \hat{b} N_t N_r \right) \right) - 1, \quad (13)$$

where $\hat{b} = \left\lfloor \frac{d^{\mathrm{uni}}(R)}{N_t N_r} \right\rfloor$.

Proof: With $u = 1$, it follows from (10) that

$$a_b = (1 + N_t N_r) a_{b-1} = (1 + N_t N_r)^{b-1}, \quad b = 2, \ldots, \hat{b} + 1.$$

The proof then follows from (9). ∎

Noting that with discrete input constellation $\mathcal{X}$ of size 2^M, $u = 1$ and $R \in \left(0, \frac{M N_t}{B}\right)$, the outage probability of the system is equivalent to that of an ARQ system with transmission rate BR in the first ARQ round, infinite feedback and a delay constraint of B ARQ rounds, where each round is subject to an i.i.d. Rayleigh flat fading channel matrix. Then, Corollary 2 is a special case of the results in [6, 13], which derives the optimal outage diversity of ARQ transmission with multi-bit feedback over MIMO block-fading channels.

Corollary 3: Suppose $u = 2$ in Theorem 1, then

$$d(R) = -1 + \frac{1}{\Delta} \left[\left(\frac{1+\Delta}{2} \right)^{\hat{b}+2} - \left(\frac{1-\Delta}{2} \right)^{\hat{b}+2} \right]$$
$$+ \frac{d^{\mathrm{uni}}(R) - \hat{b} N_t N_r}{\Delta} \left[\left(\frac{1+\Delta}{2} \right)^{\hat{b}+1} - \left(\frac{1-\Delta}{2} \right)^{\hat{b}+1} \right], \quad (14)$$

where $\hat{b} = \left\lfloor \frac{d^{\mathrm{uni}}(R)}{N_t N_r} \right\rfloor$ and $\Delta = \sqrt{1 + 4 N_t N_r}$.

Proof: With $u = 2$, the solution to (10) (which becomes a 2nd order difference equation) is

$$a_b = \frac{1}{\Delta} \left[\left(\frac{1+\Delta}{2} \right)^b - \left(\frac{1-\Delta}{2} \right)^b \right], b = 1, 2, \ldots \quad (15)$$

Then, $d(R)$ is readily obtained from (9). ∎

Corollary 3 gives a closed-form expression to the outage diversity for the special case $u = 2$. The SISO case $N_t = N_r = 1$ simplifies further, where a_b in (10) forms a Fibonacci series [17, p.381]. Therefore, noting that $\hat{b} = d^{\mathrm{uni}}(R)$ in SISO channels, the outage diversity in (9) reduces to

$$d(R) = \sum_{b=1}^{d^{\mathrm{uni}}(R)} \xi_b = \xi_{d^{\mathrm{uni}}(R)+2} - 1, \quad (16)$$

where ξ_n is the nth Fibonacci number.

Fig. 1 illustrates how u affects the outage diversity for a $M = 4$ bits/symbol modulation scheme with $B = 8$ blocks per codeword. For systems with Gaussian input and a given delay u, the maximum diversity is achieved for all rates. For discrete input constellation, rate-diversity tradeoff is observed. For the SISO case, Fig. 1 (left), we see that significant gains over uniform power allocation are possible for low code rates, but these gains rapidly decrease as the rate increases. For $u = 4$ corresponding to half a codeword delay in the transmitter obtaining the CSI, there is no improvement in diversity for rates above 2 bits per channel use. For the 2×2 MIMO case, Fig. 1 (right), we see that although the outage diversity is significantly affected by u (e.g. we see several orders of magnitude difference between $u = 1$ and $u = 2$ cases), it is so large that for all intents and purposes may be considered infinite. Even when the the delay is half a codeword ($u = 4$), for low code rates, power adaptation yields very large gains over uniform allocation. Fig. 1 also shows that for a given delay u, there is a threshold R at which power adaptation gives no improvement in outage diversity over uniform power allocation, as derived in Corollary 1.
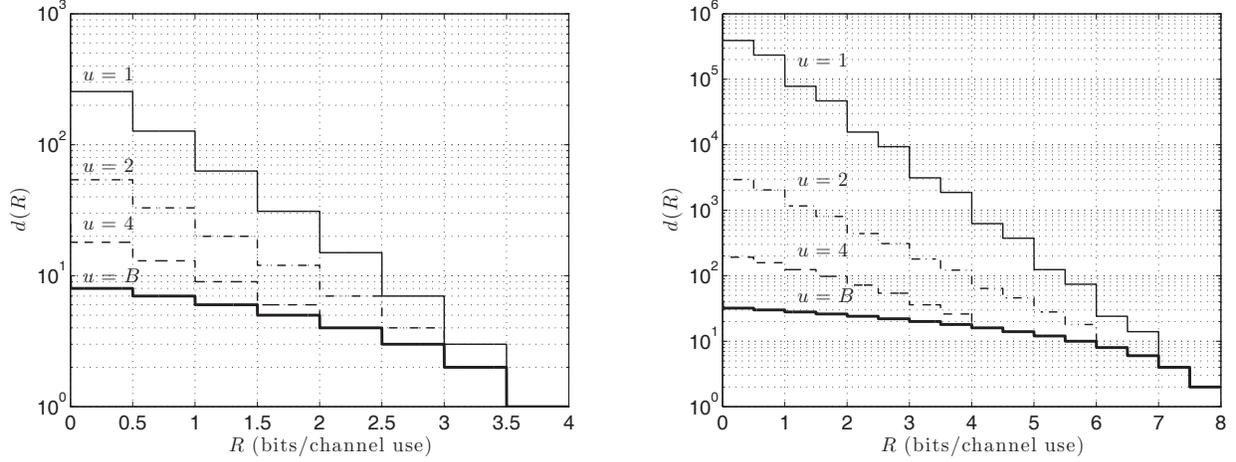
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Fig. 1. The effect of u on outage diversity for an $M = 4$, $B = 8$ system: (left) SISO channel; (right) 2×2 MIMO channel.

V. CONCLUSIONS

In this paper we considered the MIMO block-fading channel where the transmitter only has causal CSI. In particular, we considered generalised causality, whereby at block b, the transmitter only has knowledge of the channel fades of blocks $1, \ldots, (b - u)$ for a fixed integer $0 \le u \le B$. We derived a general expression for obtaining the outage diversity of the system, which uses optimal power adaptation subject to a long-term power constraint. Our results generalise previous works on block-fading channels with causal CSIT, which are contained as special cases. In addition, we showed that for a fixed rate, constellation size and number of codeword blocks, there is a threshold CSIT delay at which power adaptation no longer improves the outage diversity compared to uniform power allocation. For SISO channels this threshold turns out to be the Singleton bound (which is the outage diversity of the system with no CSIT), i.e. for CSIT delays larger than the Singleton bound, power adaptation gives no improvement in outage diversity. Our results give new insight and important design criteria for delay-limited systems where CSIT delay is also a practical system constraint.

APPENDIX

The power constraint in (2) is written as

$$\int_{\boldsymbol{H}^{(B)}} \sum_{b=1}^{B} \operatorname{tr}\left(\boldsymbol{P}_b\left(\boldsymbol{H}^{(b-u)}\right)\right) f_{\boldsymbol{H}^{(B)}}\left(\boldsymbol{H}^{(B)}\right) d\boldsymbol{H}^{(B)} \le P.$$

Following the analysis in [18], define the normalised fading gains $\omega_{b,t,r} = -\frac{\log |h_{b,t,r}|^2}{\log P}$, where $h_{b,t,r} = |h_{b,t,r}|e^{i\theta_{b,t,r}}$ are the entries of $\boldsymbol{H}_b$. The fading gains $\omega_{b,t,r}$ are collected into a matrix $\boldsymbol{\Omega}_b \in \mathbb{R}^{N_r \times N_t}, b = 1, \ldots, B$. We also define $\boldsymbol{\Omega}^{(b)} = \operatorname{diag}\left(\boldsymbol{\Omega}_1, \ldots, \boldsymbol{\Omega}_b\right)$. Further define $\pi_b\left(\boldsymbol{\Omega}^{(b-u)}\right) \equiv \pi_b\left(\boldsymbol{H}^{(b-u)}\right) \triangleq \frac{\log \operatorname{tr}\left(\boldsymbol{P}_b\left(\boldsymbol{H}^{(b-u)}\right)\right)}{\log P}$. By changing the variable from $\boldsymbol{H}^{(B)}$ to $\boldsymbol{\Omega}^{(B)}$,

$$\sum_{b=1}^{B} \int_{\boldsymbol{\Omega}^{(b-u)} \in \mathcal{A}} P^{\pi_b(\boldsymbol{\Omega}^{(b-u)})} P^{-\sum_{b'=1,t,r}^{b-u} \omega_{b',t,r}} d\boldsymbol{\Omega}^{(b-u)} \dot{\le} P,$$

where $\mathcal{A} = \mathbb{R}_{+}^{(b-u)N_r \times (b-u)N_t}$. Applying Varadhan's lemma [19, Sec. 4.3], $\pi_b\left(\boldsymbol{\Omega}^{(b-u)}\right)$ such that[1]

$$\sup_{\boldsymbol{\Omega}^{(b-u)}} \left\{ \pi_b\left(\boldsymbol{\Omega}^{(b-u)}\right) - \sum_{b'=1,t,r}^{b-u} \omega_{b',t,r} \right\} \le 1, \quad b = 1, \ldots, B$$

asymptotically satisfies the power constraint. Since the outage probability decreases with the transmit power, the optimal power allocation rule satisfies

$$\pi_b(\boldsymbol{\Omega}^{(b-u)}) = 1 + \sum_{b'=1,t,r}^{b-u} \omega_{b',t,r}, \quad b = 1, \ldots, B. \quad (17)$$

Hence, for large P, the optimal power allocation rule is

$$\boldsymbol{P}_b\left(\boldsymbol{H}^{(b-u)}\right) \doteq P^{\pi_b(\boldsymbol{\Omega}^{(b-u)})} \mathbf{I}_{N_t}, \quad (18)$$

where $\mathbf{I}_{N_t}$ is an identity matrix of size $N_t \times N_t$. Therefore, in the remainder of the proof, we consider

$$\boldsymbol{P}_b\left(\boldsymbol{H}^{(b-u)}\right) = P_b\left(\boldsymbol{H}^{(b-u)}\right) \mathbf{I}_{N_t} \doteq P^{\pi_b(\boldsymbol{\Omega}^{(b-u)})} \mathbf{I}_{N_t}. \quad (19)$$

The proof is now divided into two parts. First, we prove that the outage diversity $d(R)$ is lower bounded by (9). Then, we prove that (9) also upper bounds $d(R)$.

A. Lower bound on outage diversity

From (5), for a given channel realization $\boldsymbol{H}_b$ in block b,

$$I_{\mathcal{X}}\left(\sqrt{P_b\left(\boldsymbol{\Omega}^{(b-u)}\right)} \boldsymbol{H}_b\right) \doteq MN_t -$$
$$\frac{1}{2^{MN_t}} \sum_{\boldsymbol{x} \in \mathcal{X}^{N_t}} \mathbb{E}_{\boldsymbol{z}}\left[\log_2\left(\sum_{\boldsymbol{x}' \in \mathcal{X}^{N_t}} e^{T(\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{z})}\right)\right], \quad (20)$$

---

[1] For compact notation, $\sum_{b'=1,t,r}^{b-u} \omega_{b',t,r}$ is used to denote $\sum_{b'=1}^{b-u} \sum_{t=1}^{N_t} \sum_{r=1}^{N_r} \omega_{b',t,r}$.

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where

$$T(\boldsymbol{x}, \boldsymbol{x}', \boldsymbol{z}) =$$
$$\sum_{r=1}^{N_r} \left\{ - \left| \sum_{t=1}^{N_t} P^{\frac{\pi_b - \omega_{b,t,r}}{2}} e^{i\theta_{b,t,r}} (x_t - x'_t) + z_r \right|^2 + |z_r|^2 \right\}.$$

Let $\omega_{b,t} = \min_{r=1,\dots,N_r} \{\omega_{b,t,r}\}$ and define

$$\mathcal{S}_b^{(\epsilon)} \triangleq \left\{ t \in \{1, \dots, N_t\} : \pi_b(\boldsymbol{\Omega}^{(b-u)}) - \omega_{b,t} > \epsilon \right\}. \quad (21)$$

For any $\epsilon > 0$, in the limit for large P [6],

$$I_\mathcal{X} \left(\sqrt{P_b \left(\boldsymbol{\Omega}^{(b-u)} \right)} \boldsymbol{H}_b \right) \geq M |\mathcal{S}_b^{(\epsilon)}|. \quad (22)$$

The outage probability therefore satisfies

$$\mathsf{P}_{\text{out}}(P, R) \dot{\leq} \Pr \left\{ \boldsymbol{\Omega}^{(B)} \in \mathcal{O}^{(\epsilon)} \right\}, \quad (23)$$

where, by letting $\overline{\mathcal{S}}_b^{(\epsilon)}$ denotes the complement of $\mathcal{S}_b^{(\epsilon)}$,

$$\mathcal{O}^{(\epsilon)} = \left\{ \boldsymbol{\Omega}^{(B)} \in \mathbb{R}^{BN_tN_r} : \sum_{b=1}^{B} \left| \overline{\mathcal{S}}_b^{(\epsilon)} \right| \geq \frac{d^{\text{uni}}(R)}{N_r} \right\}. \quad (24)$$

From Varadhan's lemma [19], the outage diversity satisfies

$$d(R) \geq \inf_{\boldsymbol{\Omega}^{(B)} \in \mathcal{O}^{(\epsilon)} \cap \mathbb{R}_+^{BN_tN_r}} \left\{ \sum_{b,t,r} \omega_{b,t,r} \right\}. \quad (25)$$

Since the argument of the infimum in (25) is increasing with $\omega_{b,t,r}$, it follows from (24) that the infimum is attained with

$$\omega_{b,t,r} = \begin{cases} \pi_b \left(\boldsymbol{\Omega}^{(b-u)} \right) - \epsilon, & (b-1)N_t + t \leq \frac{d^{\text{uni}}(R)}{N_r} \\ 0, & \text{otherwise.} \end{cases}$$
$$= \begin{cases} 1 - \epsilon, & b \leq u \\ \omega_{b-1,1,1} + N_t N_r \omega_{b-u,1,1}, & b > u; (b-1)N_t + t \leq \frac{d^{\text{uni}}(R)}{N_r} \\ 0, & \text{otherwise.} \end{cases}$$

Let $\hat{b} = \left\lfloor \frac{d^{\text{uni}}(R)}{N_t N_r} \right\rfloor$ and $a_b = \omega_{b,1,r}$, we have from (25) that

$$d(R) \geq N_t N_r \sum_{b=1}^{\hat{b}} a_b + N_r (d^{\text{uni}}(R) - \hat{b} N_t) a_{\hat{b}+1}, \quad (26)$$

where

$$a_b = \begin{cases} 1 - \epsilon, & 1 \leq b \leq u \\ a_{b-1} + N_t N_r a_{b-u}, & u < b \leq \hat{b} + 1. \end{cases} \quad (27)$$

By letting $\epsilon \downarrow 0$, the outage diversity is lower bounded by (9).

B. Upper bound on outage diversity

The input-output mutual information of a MIMO channel is upper bounded by that of a genie aided receiver, where the interference caused by multiple transmit antenna is completely removed at the receiver. The resulting channel therefore consists of N_t parallel channels, each component channel consists of one transmit and N_r received antenna. Therefore,

$$I_\mathcal{X} \left(\sqrt{P_b(\boldsymbol{H}^{(b-u)})} \boldsymbol{H}_b \right) \leq \sum_{t=1}^{N_t} I_\mathcal{X} \left(\sqrt{P_b \left(\boldsymbol{H}^{(b-u)} \right) \xi_{b,t}} \right),$$

where $\xi_{b,t} = \sum_{r=1}^{N_r} |h_{b,t,r}|^2$. Letting $\omega_{b,t} = \min\{\omega_{b,t,r}, r = 1, \dots, N_r\}$, it follows that $\xi_{b,t} \dot{=} P^{-\omega_{b,t}}$. Thus, for any $\epsilon > 0$, in the limit for large P,

$$I_\mathcal{X} \left(\sqrt{P_b(\boldsymbol{H}^{(b-u)})} \boldsymbol{H}_b \right) \leq \sum_{t=1}^{N_t} I_\mathcal{X} \left(P^{\frac{\pi_b\left(\boldsymbol{\Omega}^{(b-u)}\right) - \omega_{b,t}}{2}} \right)$$
$$\leq M |\mathcal{S}_b^{(-\epsilon)}|,$$

where $\mathcal{S}_b^{(-\epsilon)}$ is defined by (21). By letting $\epsilon \downarrow 0$, it then follows from part A of the proof that the outage diversity is upper bounded by (9).

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