

# Fair Coexistence MAC Protocol for Contention-Based Heterogeneous Networks

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**This paper proposes a contention-based fair coexistence mechanism among heterogeneous networks that have different transmission power and/or coverage. First, we show that the existing carrier sensing multiple access (CSMA) mechanism, that is a prevailing contention-based protocol, results in significant unfairness in channel access when heterogeneous networks coexist; a system with lower transmission power hardly occupies the shared channel due to interference from a system with higher transmission power. We analyze the causes of unfairness in terms of (i) the asymmetry of carrier sensing and (ii) the blindness of binary exponential backoff mechanism and the link adaptation mechanism, and we derive an analytical model of per-system throughput to investigate the effects of these causes. To resolve this problem, we propose a fair coexistence CSMA protocol consisting of *access etiquette* and *interference-aware backoff*. The former adaptively controls the contention window size so that the high-power system allows transmission opportunities to the low-power system in a fair and efficient manner. The latter differentiates between the response to transmission failure caused by collision and the response to failure caused by interference. The simulation results confirm that the proposed scheme effectively mitigates the unfairness of channel sharing while attaining high spectral efficiency.**

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## 1. INTRODUCTION

The Internet is continuously expanding to include many new emerging services and portable devices, and users want to access the Internet anywhere, anytime and with any device. To satisfy the diverse requirements of these services and user demands, wireless communication systems have evolved to support high capacity and quality of service and a new advanced communication system has appeared. Therefore, the demand for wide bandwidth is continuously increasing, but the frequency spectrum is essentially a limited resource. Thus, it is imperative to effectively manage and allocate the frequency spectrum.

Recently, Federal Communications Commission released the 3.65 GHz spectrum for license-exempt non-exclusive coexistence among heterogeneous networks and mandated the contention-based protocol to assure fair sharing among them. To satisfy these regulatory requirements, the standards of IEEE 802.16h and IEEE 802.11y have been established with

coexistence mechanisms for license-exempt operation [1, 2]. The studies in [3, 4] investigated the coexistence issue of IEEE 802.16 systems and IEEE 802.11 systems in a license-exempt shared frequency band.

This paper deals with coexistence issues arising when heterogeneous networks that have different transmission power and/or coverage employ the contention-based protocol for fair channel sharing. First, we show that the existing carrier sensing multiple access (CSMA) mechanism, which is a prevailing contention-based protocol, results in significant unfairness of channel access. We analyze the cause of unfair channel access from the viewpoints of (i) the asymmetry of carrier sensing ability and (ii) the blindness of the binary exponential backoff (BEB) mechanism and the link adaptation mechanism. The high-power system may fail to detect the ongoing transmission of the low-power system, and thus the low-power system suffers from frequent transmission failures due to interference caused by the high-power system. In addition, the BEB

mechanism and the link adaptation mechanism, which controls the contention window size and transmission rate depending on the transmission status and channel status, respectively, exacerbate the unfair channel access because they are unaware of the interference-driven transmission failure. We also derive an analytical model for per-system throughput to investigate the effects of the contention window and the transmission rate on the fairness and efficiency of channel sharing.

To resolve this problem, we propose the fair coexistence CSMA protocol, which makes a small and simple change to the existing CSMA mechanism. The proposed mechanism consists of two schemes: *access etiquette* and *interference-aware backoff*. The former reduces the effect of interference caused by the high-power system on the low-power system, while the latter enables the low-power system not to blindly defer the channel access in response to transmission failure due to interference. The access etiquette scheme is applied to the high-power system and controls its contention window size based on the feedback information about per-system throughput to maximize the given objective function that accounts for the overall efficiency and fairness of channel sharing. On the other hand, the interference-aware backoff scheme is applied to the low-power system. The receiver of the low-power system determines the cause of transmission failure: intra-system collision, or inter-system interference, and it informs the corresponding sender of the excessive interference. Then the sender does not unnecessarily increase the contention window on the interference-driven transmission failure. The extensive simulation results confirm that the proposed approach significantly reduces the unfairness of channel sharing, while attaining high efficiency under various network configurations.

The coexistence issue has been widely studied in the literature, especially for wireless local area network (WLAN) and Bluetooth in the unlicensed band [5–7]. These studies are different from this work from the viewpoints of channel access mechanism and network deployment. As channel access mechanism, the WLAN uses CSMA while Bluetooth uses frequency-hopping spread spectrum. Moreover, the coexistence of the WLAN and Bluetooth is irrelevant to the issue of spatial reuse, which should be carefully considered in the multi-cell networks. Recently, the IEEE 802.22 working group has been developing a standard for wireless regional area networks, aiming to provide broadband access in rural areas by allowing sharing of geographically unused spectrum allocated to the television broadcast service on a non-interfering basis using cognitive radio technology [8]. Also, several approaches have been proposed to reduce interference and/or collision and to improve spatial reuse in contention-based multi-rate multi-hop 802.11 WLANs [9–13]. These approaches focus on the trade-off between interference mitigation and spatial reuse in setting the physical carrier sensing threshold (CSTH) and/or the transmit power. These works derived the optimal value of the physical CSTH and proposed the adaptive control of the CSTH, transmission power and/or transmission data rate.

Compared with the previous studies, this work makes the following contributions:

- (i) This work focuses on the fair and efficient contention-based coexistence of heterogeneous networks that are asymmetric in terms of transmission power and coverage. We identify several problems and their causes arising in this situation.
- (ii) Unlike the previous approaches that may be infeasible or undesirable in heterogeneous networks, the proposed scheme adaptively controls the contention window for fair channel sharing without degrading the efficiency of channel sharing.
- (iii) The proposed MAC protocol is novel in that it can control the trade-off between the fairness and efficiency of channel sharing by using a control parameter, and it differentiates the response to transmission failure due to collision from the response to failure due to interference.

The rest of this paper is organized as follows. Section 2 presents the motivation of the study; we state the problem and cause of unfair channel sharing among heterogeneous networks. Section 3 analyzes the feasibility of CSTH control and derives the analysis model for per-system throughput. Section 4 proposes the access etiquette and interference-aware backoff schemes for achieving fair and efficient channel sharing. Section 5 presents the simulation results to evaluate the performance of the proposed scheme. The conclusions follow in Section 6.

## 2. PROBLEM STATEMENT

We consider two different wireless networks, the wireless metro area network (WMAN) and the WLAN, which coexist and operate in the same frequency band. As illustrated in Fig. 1, the WMANs are deployed as cellular networks with the frequency reuse factor of 1/3, and they have relatively higher transmission power ( $P_{tx,1}$ ) and coverage ( $D_{tx,1}$ )<sup>1</sup>, compared with the WLAN. The WLAN is deployed within the coverage of the central WMAN,  $WMAN_0$ , and the other six WMANs surrounding  $WMAN_0$ , denoted as  $WMAN_i$ , are considered as the first-tier interfering cells. As interference from  $WMAN_i$ s is dominant, we neglect the interference from the second-tier or further-away interfering cells. In order to focus on the performance anomaly in the contention-based coexistence among heterogeneous networks, we consider a general coexistence scenario where both the WMAN and the WLAN deploy the CSMA protocol for channel access.<sup>2</sup>

<sup>1</sup>For the simplicity of notation, we use subscript '1' and '2' for WMAN and WLAN, respectively.

<sup>2</sup>According to IEEE 802.16h [1], the contention-based protocol, which is similar to the CSMA of the WLAN, is used in the specified frames, called as *coordinated coexistence contention-based interval*, to allow non-exclusive coexistence with non-WiMAX systems (e.g. WLAN).

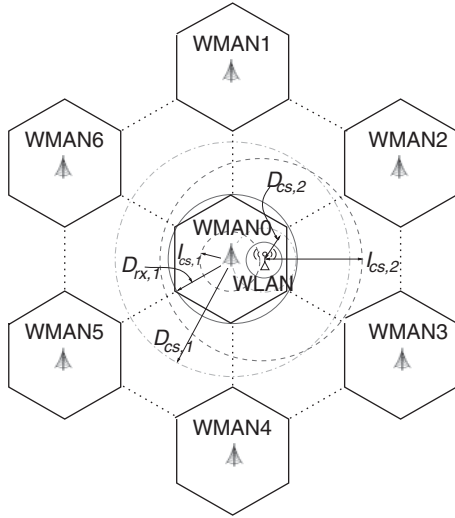


FIGURE 1. Coexistence scenario of the WMAN and the WLAN (WMANs are deployed with a frequency reuse factor of 1/3).

2.1. Asymmetry of carrier sensing

Consider that WMAN\_0 and WLAN have the physical CSTDs,  $P_{cs,1}$  and  $P_{cs,2}$ , respectively, and that they have different transmission power, that is,  $P_{tx,1} > P_{tx,2}$ . Note that there exist the regulatory restrictions on the CSTD and transmission power for non-exclusive coexistence. We consider the energy detection approach to be used as physical carrier sensing. Let us define  $D_{cs}$  and  $I_{cs}$  as the carrier sensing range within which the transmitter can detect a busy channel due to transmission by the homogeneous system and by the heterogeneous system, respectively. Accordingly,  $D_{cs}$  and  $I_{cs}$  are related to intra-system collision and inter-system interference, respectively. The maximum transmission range with the most robust transmission rate,  $D_{tx}$ , can be determined by the minimum receiver sensitivity,  $P_{min}$ , and the transmission power. Considering the path-loss propagation model with the path-loss exponent of  $\alpha$ , which is the most dominant factor for the channel model, these three ranges for WMAN\_0 can be represented as

$$\begin{aligned}
 D_{cs,1} &= \left( \frac{G_1 P_{tx,1}}{P_{cs,1}} \right)^{1/\alpha}, \\
 I_{cs,1} &= \left( \frac{G_2 P_{tx,2}}{P_{cs,1}} \right)^{1/\alpha}, \\
 D_{tx,1} &= \left( \frac{G_1 P_{tx,1}}{P_{min,1}} \right)^{1/\alpha},
 \end{aligned} \tag{1}$$

where  $G_1$  and  $G_2$  are the channel gain for WMAN\_0 and WLAN, respectively. Similarly, for WLAN,  $D_{cs,2}$ ,  $I_{cs,2}$  and  $D_{tx,2}$  can be obtained using the path-loss model. It is known that  $D_{cs} > D_{tx}$  since carrier sensing is available with the weaker signal required for successful decoding. As indicated in Fig. 1, we assume that

the cells of the WMAN are well planned so that the spatial reuse is available among WMANs. From (1), it can be shown that

$$\begin{aligned}
 \text{if } P_{tx,1} > P_{tx,2} &\rightarrow \begin{cases} \text{WMAN: } D_{cs,1} > I_{cs,1}, \\ \text{WLAN: } D_{cs,2} < I_{cs,2}. \end{cases} \\
 \text{if } P_{tx,1} > P_{tx,2} \text{ and } P_{cs,1} = P_{cs,2} &\rightarrow D_{cs,1} = I_{cs,2} > D_{cs,2} = I_{cs,1}.
 \end{aligned} \tag{2}$$

This asymmetry gives priority to WMAN\_0 over WLAN in terms of channel access. Depending on the values of transmission power and the locations of WMAN/WLAN nodes, it is possible that (i)  $I_{cs,1}$  is small enough not to cover the transmission range of WLAN at all, that is, the transmitter of WMAN\_0 hardly detects the channel occupation by WLAN (*WLAN becomes a hidden terminal to WMAN\_0*) and (ii)  $I_{cs,2}$  is large enough to cover the whole transmission range of WMAN\_0, i.e. the transmitter of WLAN defers its transmission whenever WMAN\_0 occupies the channel (*WMAN\_0 becomes an exposed terminal to WLAN*). Accordingly, there occurs a severe unfair channel access between WMAN\_0 and WLAN; WMAN\_0 can access the channel regardless of whether or not WLAN occupies the channel, while WLAN is significantly liable to be interfered by WMAN\_0.

2.2. BEB and link adaptation due to interference

The BEB mechanism adopted in the IEEE 802.11 WLAN is helpful to alleviate the intra-system collision by doubling the contention window size on detecting a transmission failure. However, it degrades the fairness of channel sharing among the WMAN and the WLAN due to the inter-system interference. As shown in Fig. 2, the BEB mechanism of the WMAN and the WLAN works in an asymmetric way, responding to the inter-system interference as follows:

- (i) WMAN: The WMAN sender cannot detect the ongoing packet transmission by the WLAN due to the weak WLAN signal, and starts transmitting its packet. However, the WMAN receiver can tolerate the interference by the WLAN since the interference signal strength is relatively small. Thus, the WMAN receiver can successfully receive the packet and the contention window of the WMAN sender does not increase.

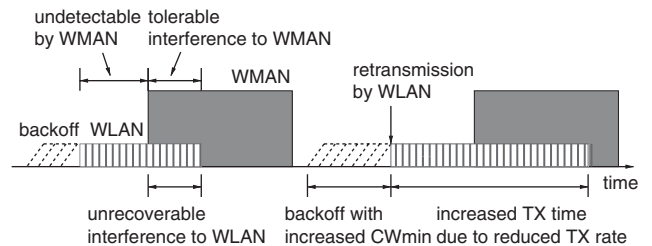


FIGURE 2. Responses of WMAN and WLAN to inter-system interference.

- (ii) WLAN: The high interference signal from the WMAN may corrupt the WLAN packet transmission, and the WLAN sender may retransmit the packet with the increased contention window according to the BEB mechanism.

Therefore, the BEB mechanism, which cannot distinguish collision-driven failure from interference-driven failure, reduces the channel access opportunity of WLAN and deteriorates fair channel sharing.

Similar to the BEB mechanism, the link adaptation mechanism may also worsen the problem. The automatic rate fallback (ARF) [14], the most common link adaptation algorithm, adjusts the transmission rate by estimating the channel condition based on the numbers of successive transmission successes and failures. If the packet transmission fails consecutively (regardless of the cause of failure, i.e. intra-system collision, inter-system interference, temporary degrade of channel quality), then the ARF decreases the transmission rate to the next lower one among the available sets (e.g. 54, 48, 36, 24, 18, 12, 9, and 6 Mb/s in the case of IEEE 802.11a/g). Then the receiver can correctly decode the packet with higher probability because the more robust modulation and coding scheme is used in the reduced transmission rate. In a sense, the link adaptation mechanism is beneficial in mitigating interference; however, it has an adverse effect in the case where the WMAN and the WLAN coexist with different transmission power. It is important to note that the transmission time required for transmitting a constant-size packet is inversely proportional to the transmission rate. The reduced transmission rate of the WLAN increases its transmission time, and the probability that WLAN packet transmission is interfered by WMAN increases accordingly (Fig. 2), because the WMAN sender is unaware of the packet transmission of the WLAN, regardless of the transmission rate of the WLAN. In this case, the reduced transmission rate makes the WLAN more vulnerable to interference from the WMAN. Moreover, the decrease of transmission rate reduces the throughput accordingly. This logic is still applicable to the link adaptation mechanism based on the signal to interference and noise ratio (SINR).

### 2.3. Preliminary simulation

We perform preliminary simulations to identify the problem of unfair channel sharing and to observe the effect of Csth on the achievable throughput. We implement a simulator using C programming language. The parameters and their values used in the simulations are listed in Table 1, and they are carefully determined considering regulatory restrictions and realistic deployment. The wireless channel is modeled by taking path loss, shadowing and multi-path fading into account. Considering the urban environment, the path-loss exponent is set to 3.7, and the Rayleigh fading model is used. The shadowing is modeled as a log-normal random variable with zero mean

TABLE 1. Parameters used in the simulations.

Parameter	Value	
	WMAN	WLAN
Transmission power ( $P_{Tx}$ )	1000 mW	50 mW
minimum receiver sensitivity ( $P_{min}$ )	-80 dBm	-80 dBm
cell radius ( $D_{rx}$ )	750 m	100 m
number of users per cell ( $N$ )	10	10
path-loss exponent, $\alpha$		3.7
distance between the centers of WMAN_0 and WLAN, $D_{int-sys}$		300 m

TABLE 2. Minimum threshold values of SINR for each data rate.

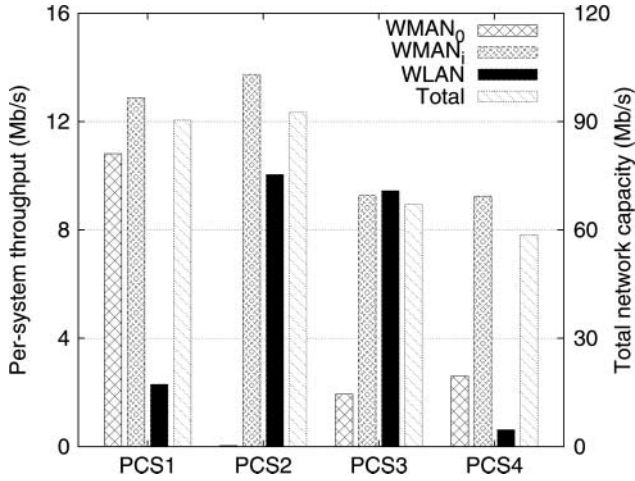
Modulation and coding rate	Data rate (Mb/s)	Minimum SINR (dB)
64QAM 3/4	56	24.56
64QAM 2/3	48	24.05
16QAM 3/4	36	18.80
16QAM 1/2	24	17.04
QPSK 3/4	18	10.79
QPSK 1/2	12	9.03
BPSK 3/4	9	7.78
BPSK 1/2	6	6.02

and standard deviation of 8 dB. The thermal noise power is set to -100 dBm. The SINR-based link adaptation mechanism is implemented in the simulation and the corresponding minimum value of the SINR for each data rate is given in Table 2 [15]. The packet error is modeled according to [16]. The IEEE 802.11a MAC/PHY parameters are used in the simulation. Ten users are uniformly distributed per cell, and the simulation does not consider node mobility. For each user, a 1000-byte packet is randomly generated such that its inter-arrival time follows a Poisson random variable with a mean value of 5 ms. This packet generation rate is high enough to investigate saturated network capacity. Users send/receive packets to/from the base station (BS) of the WMAN and the access point (AP) of WLAN, and so both uplink and downlink communications exist.

Figure 3 shows per-system throughput and total network capacity for the following four cases with different values of Csth. Here,  $P_{cs,i}$  is defined as the Csth for WMAN<sub>i</sub>:

- (i) PCS1:  $(P_{cs,1}, P_{cs,i}, P_{cs,2}) = (-90, -90, -90)$  dBm;
- (ii) PCS2:  $(P_{cs,1}, P_{cs,i}, P_{cs,2}) = (-100, -90, -90)$  dBm;
- (iii) PCS3:  $(P_{cs,1}, P_{cs,i}, P_{cs,2}) = (-100, -100, -90)$  dBm;
- (iv) PCS4:  $(P_{cs,1}, P_{cs,i}, P_{cs,2}) = (-100, -100, -100)$  dBm.

In Fig. 3, the throughput of WMAN<sub>i</sub> is represented as an average value, while the total network capacity is calculated



**FIGURE 3.** Per-system throughput and aggregate network capacity for four cases with different values of the CSTH.

as the throughput sum of WMAN<sub>0</sub>, WLAN and six WMAN<sub>i</sub>s. The case of PCS1 is set as a baseline scenario where we can evaluate the degree of unfairness. In this case,  $D_{cs,2}$  ( $= I_{cs,2}$ ) and  $D_{cs,1}$  ( $= I_{cs,1}$ ) are  $\sim 1400$  and  $186$  m, respectively, and there occurs a severe unfairness problem; the throughput of WMAN<sub>0</sub> is about five times higher than that of WLAN. As a naive solution to this problem, we consider the case of PCS2 where  $P_{cs,1}$  is decreased to detect the channel occupation by WLAN, i.e.  $P_{cs,1}$  ( $= -100$  dBm)  $< (G_2 P_{tx,2}) / (D_{int-sys}^\alpha)$ . In this case,  $I_{cs,1}$  ( $= 347$  m) becomes larger than  $D_{int-sys}$  and the transmitter of WMAN<sub>0</sub> can detect a busy channel due to WLAN transmission. Note that all the senders of WMAN<sub>0</sub> cannot completely detect WLAN's packet transmission. Here,  $P_{cs,i}$  and  $P_{cs,2}$  are not changed from  $-90$  dBm. Figure 3 shows that this approach cannot improve the fairness of channel sharing at all; the throughput of WMAN<sub>0</sub> becomes almost zero while that of WLAN is increased to about 10 Mb/s. The reason is as follows: the decrease of  $P_{cs,1}$  increases  $D_{cs,1}$  ( $\approx 2600$  m) as well as  $I_{cs,1}$ , then the senders of WMAN<sub>0</sub> are likely to defer transmission during WMAN<sub>i</sub>'s packet transmission (WMAN<sub>i</sub>s are less interfered by WMAN<sub>0</sub>) and WMAN<sub>0</sub>'s transmission will encounter more interference by WMAN<sub>i</sub>s. Next, we consider the case of PCS3 where  $P_{cs,1} = P_{cs,i} = -100$  dBm for all WMANs, but  $P_{cs,2}$  for WLAN remains unchanged from  $-90$  dBm. In this configuration, the throughput of WLAN is about five times higher than that of WMAN<sub>0</sub>. The unfair channel sharing between WMAN<sub>0</sub> and WLAN appears in the reverse aspect, compared with the case of PCS1. Meanwhile, the decrease of  $P_{cs,i}$  creates another problem; the overall network capacity is reduced by about 25% compared with those of the PCS1 and PCS2 cases, because the spatial reuse cannot be fully employed. Lastly, in the case of PCS4,  $P_{cs}$  for all the systems are set to  $-100$  dBm. As shown in Fig. 3, the total capacity is further decreased (by more than 35% compared with those in the cases of PCS1 and PCS2), the throughput of WLAN is less than that of WMAN<sub>0</sub> by more than

four times, and the cell where both WMAN<sub>0</sub> and WLAN coexist has about four times lower throughput than that in the case of PCS1. None of these four cases achieves fair channel sharing, and the performance is quite sensitive to the value of the CSTH.

### 3. ANALYSIS OF UNFAIR CHANNEL SHARING

In this section, we first focus on the feasibility of CSTH control and obtain the upper and lower bounds of the CSTH from the requirements of fair channel sharing and spatial reuse. Next, we derive a simple analytical per-system throughput model to numerically evaluate the degree of unfair channel sharing and to observe the effect of several parameters on the fairness and efficiency of channel sharing.

#### 3.1. Feasible condition of CSTH control

To achieve fair and efficient channel sharing, the value of  $P_{cs,1}$  should satisfy two requirements of *mutual detection* and *spatial reuse*: (i) it should be small enough for the WMAN<sub>0</sub> sender to detect the transmission of WLAN and (ii) at the same time, it should be large enough for the WMAN<sub>0</sub> sender not to defer its channel access during the transmission of WMAN<sub>i</sub>. Let us denote  $d_{tx1-tx2}$  and  $d_{tx1-txi}$  as the distance between transmitters of WMAN<sub>0</sub> and WLAN and that between transmitters of WMAN<sub>0</sub> and WMAN<sub>i</sub>, respectively, and define  $K_1$  and  $K_2$  as the relative distance of  $d_{tx1-tx2}$  and  $d_{tx1-txi}$ , respectively, normalized by  $D_{rx,1}$ , i.e.  $K_1 = d_{tx1-tx2} / D_{rx,1}$  and  $K_2 = d_{tx1-txi} / D_{rx,1}$ . Note that  $0 \leq K_1 \leq 2$  under the condition of overlapping deployment of WMAN<sub>0</sub> and WLAN and  $1 \leq K_2 \leq 5$  considering the deployment of WMANs with the frequency reuse factor of 1/3. These two requirements can be represented in terms of  $I_{cs,1}$  and  $D_{cs,1}$  as

$$\begin{aligned} I_{cs,1} &> K_1 D_{rx,1}, \\ D_{cs,1} &< K_2 D_{rx,1}. \end{aligned} \quad (3)$$

Assuming  $P_{min,1} = P_{min,2}$ , we can derive the upper/lower bounds on  $P_{cs,1}$ , defined as  $\bar{P}_{cs,1}$  and  $\underline{P}_{cs,1}$ , respectively, from (1) and (3);

$$\begin{aligned} \bar{P}_{cs,1}(\text{dBm}) &= P_{min,1}(\text{dBm}) - 10\alpha \log_{10}(K_1/K_a), \\ \underline{P}_{cs,1}(\text{dBm}) &= P_{min,1}(\text{dBm}) - 10\alpha \log_{10}(K_2). \end{aligned} \quad (4)$$

Here,  $K_a$  is defined as the coverage asymmetry factor, i.e.  $K_a = D_{rx,2} / D_{rx,1}$ .

It is worthwhile to note that  $P_{cs,1}$  satisfying (4) does not always exist; it only exists with the following condition on  $K_1$ ,  $K_2$ , and  $K_a$ :

$$\frac{K_1}{K_2} < K_a = \frac{D_{rx,2}}{D_{rx,1}}. \quad (5)$$

This analysis means that the proper range of the CSTH depends on the node placement and cell deployment and that it is not always possible to find the proper range of the CSTH

achieving mutual detection and spatial reuse at the same time. Thus, if a WLAN coexists with WMANs that are deployed as cellular networks and there exists an asymmetry in the transmission power, then the approach of adjusting the CSTDH (or transmission power) is neither desirable nor feasible to assure fair sharing between the WMAN and the WLAN while attaining spatial reuse.

### 3.2. Derivation of per-system throughput model

There are several well-known throughput models for the CSMA protocol (e.g. distributed coordination function of IEEE 802.11) that consider the dynamics of the contention window and/or link adaptation [17–19]. However, they cannot be straightforwardly applied to this study where the transmission failure mainly results from inter-system interference, instead of collision or channel error. The purpose of deriving an analytic throughput model is to evaluate the effect of several parameters (e.g. contention window, the number of nodes, transmission rate and transmission coverage) on the achievable throughput and to find a clue to solving the problem of unfair channel sharing. For this reason, we do not intend to capture the dynamic characteristics of contention window control and link adaptation, i.e. the contention window size and the transmission rate are intentionally set to be constant.

Consider that there are  $N_1$  WMAN senders within the transmission coverage of WMAN and  $N_2$  WLAN senders coexist with the WMAN nodes. We define  $CW_1$  and  $CW_2$  as the contention window size of the WMAN and WLAN senders, respectively, and define *transmission round* as the time interval between two consecutive packet transmissions. We make the following assumptions.<sup>3</sup>

- (A1) All WMAN and WLAN senders always have data to send and compete for channel access.
- (A2)  $P_{cs,1}$  is configured to satisfy the requirement of spatial reuse, i.e.  $P_{cs,1} > \underline{P}_{cs,1}$ .
- (A3) The WMAN senders can detect the WLAN's packet transmission with a probability of  $p_d$ , while the WLAN senders can detect the WMAN's packet transmission with a probability of 1.
- (A4) At an initial time slot of transmission round, WMAN and WLAN nodes select their own random backoff counters,  $b_1$  and  $b_2$ , respectively.

<sup>3</sup>The detection probability  $p_d$  depends on several parameters including the CSTDH, transmission power, channel model and location of WMAN and WLAN transmitters. In this study, we consider that it is constant and approximate it as  $(I_{cs,1}/D_{rx,1})^2$ . Moreover, the backoff counter is either selected randomly at the beginning of transmission round or decreased from the one selected in the previous transmission round, depending on the transmission status. The assumptions (A3) and (A4) are made for the tractability of analysis but dropped in the simulation. The comparison between the analysis results and simulation results in the next subsection validates these assumptions.

The backoff probability of the WMAN sender, denoted as  $p_{bo,1}$ , becomes

$$p_{bo,1} \triangleq \Pr\{b_1 = k\} = \frac{1}{CW_1}, \quad 0 \leq k \leq CW_1 - 1. \quad (6)$$

Similarly,  $p_{bo,2}$  can be represented with  $CW_2$  as (6). Let us define  $n_{bo,1}$  and  $n_{bo,2}$  as the minimum value of backoff counters among  $N_1$  WMAN senders and  $N_2$  WLAN senders, respectively. Depending on the values of  $n_{bo,1}$  and  $n_{bo,2}$ , a WMAN or WLAN sender can start packet transmission. We consider the following two cases:

- (i) CASE 1:  $n_{bo,1} < n_{bo,2}$  (interference-free channel occupation by WMAN)
- (ii) CASE 2:  $n_{bo,1} \geq n_{bo,2}$  (interference-prone channel occupation by WLAN)

First, we focus on CASE 1 where a WMAN sender starts to transmit a packet at the  $(n_{bo,1} + 1)$ th time slot, while a WLAN sender defers its transmission. Let us denote  $p_a^{(1)}(k_1)$  as the probability that a WMAN sender accesses the channel without intra-system collision and inter-system interference after  $n_{bo,1} = k_1$  backoff time. Then,  $p_a^{(1)}(k_1)$  is represented as

$$p_a^{(1)}(k_1) = N_1 p_{bo,1} (1 - (k_1 + 1) p_{bo,1})^{N_1 - 1} (1 - (k_1 + 1) p_{bo,2})^{N_2}, \quad (7)$$

and the average throughput achieved by WMAN nodes becomes

$$TH_1^{(1)} = \sum_{k_1=0}^{CW_m-1} p_a^{(1)}(k_1) (1 - \text{ber}_1)^L \frac{L}{k_1 t_s + L/R_1 + t_{oh}}, \quad (8)$$

where  $CW_m = \min(CW_1, CW_2)$ ,  $L$  denotes the packet size whose unit is bits.  $R_1$  is the transmission rate of the WMAN nodes,  $t_{oh}$  accounts for several overhead time (e.g. inter-frame spaces, transmission time of PHY/MAC header and ACK frame) and  $t_s$  is the slot time. The bit error rate (BER),  $\text{ber}_1$  in (8), can be obtained with a path-loss model and a geometric model specifying the locations of nodes and the assumption of additive white Gaussian noise channel [20]. It is noteworthy that in CASE 1 ( $n_{bo,1} < n_{bo,2}$ ), WLAN defers the channel access, and thus its throughput is zero.

Next, we consider the CASE 2 where  $n_{bo,1} \geq n_{bo,2}$ , i.e. the WLAN sender makes the channel access attempt before the WMAN sender does. In this case, we need to distinguish the case where the WMAN sender detects WLAN's transmission from the case where the WMAN sender does not (there occurs inter-system interference). Let us define  $TH_{2,n}^{(2)}$  and  $TH_{2,i}^{(2)}$  as the WLAN throughput in the former case and in the latter case, respectively. Similarly,  $TH_{1,n}^{(2)}$  and  $TH_{1,i}^{(2)}$  denote the throughput achieved by the WMAN node for these two cases. Note that  $TH_{1,n}^{(2)}$  is zero. Then the throughputs of the WMAN and the WLAN in CASE 2 become

$$\begin{aligned} \text{WMAN} : TH_1^{(2)} &= (1 - p_d) TH_{1,i}^{(2)}, \\ \text{WLAN} : TH_2^{(2)} &= p_d TH_{2,n}^{(2)} + (1 - p_d) TH_{2,i}^{(2)}. \end{aligned} \quad (9)$$

The value of  $\text{TH}_{2,n}^{(2)}$ , i.e. the WLAN throughput without interference by the WMAN, can be calculated similarly to (8). Now, we derive  $\text{TH}_{1,i}^{(2)}$  and  $\text{TH}_{2,i}^{(2)}$ , the throughput of the WMAN and the WLAN with the inter-system interference. The channel access probability of WLAN and WMAN senders when  $n_{\text{bo},1} = k_1$  and  $n_{\text{bo},2} = k_2 (\leq k_1)$  is

$$p_a^{(2)}(k_1, k_2) = N_1 p_{\text{bo},1} (1 - (k_1 + 1) p_{\text{bo},1})^{N_1 - 1} N_2 p_{\text{bo},2} (1 - (k_2 + 1) p_{\text{bo},2})^{N_2 - 1}, \quad (10)$$

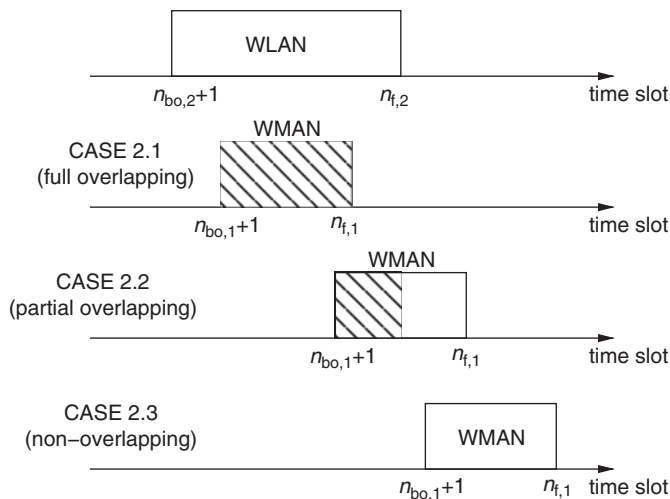
because  $n_{\text{bo},1}$  and  $n_{\text{bo},2}$  are independent. We define  $n_{f,1}$  as the number of time slots elapsed from the start of transmission round to the end of WMAN's packet transmission:

$$n_{f,1} = (k_1 t_s + L/R_1 + t_{\text{oh}})/t_s. \quad (11)$$

In the same way,  $n_{f,2}$  can be represented. Depending on the values of  $n_{\text{bo},1}$ ,  $n_{f,1}$  and  $n_{f,2}$ , there exist the following three sub-cases as depicted in Fig. 4:

- (i) CASE 2.1 (full overlapping):  $n_{f,1} \leq n_{f,2}$
- (ii) CASE 2.2 (partial overlapping):  $n_{f,1} > n_{f,2}$  and  $n_{\text{bo},1} < n_{f,2}$
- (iii) CASE 2.3 (non-overlapping):  $n_{f,1} > n_{f,2}$  and  $n_{\text{bo},1} \geq n_{f,2}$

Note that CASE 2.1 may occur if  $R_1 > R_2$ . For these three sub-cases, we can calculate the number of time slots with inter-system interference,  $s_i$ , during which both WMAN and WLAN senders transmit packets. Also, we can obtain the total number of time slots occupied by the WMAN and WLAN senders without interference, which are defined as  $s_{1,n}$  and  $s_{2,n}$ , respectively. We assume that the BERs of the WMAN during the  $s_i$  and the  $s_{1,n}$  intervals do not significantly change from  $\text{ber}_{1,i}$  and  $\text{ber}_{1,n}$ , respectively. Similarly, we denote  $\text{ber}_{2,i}$  and  $\text{ber}_{2,n}$  as the BERs



**FIGURE 4.** Three sub-cases of inter-system interference when  $n_{\text{bo},1} \geq n_{\text{bo},2}$ .

of WLAN during  $s_i$  and  $s_{2,n}$  intervals. The effective packet error rates (PERs) of WMAN and WLAN can be represented as

$$\begin{aligned} \text{per}_1^{(2)} &= 1 - (1 - \text{ber}_{1,n})^{s_{1,n} t_s R_1} (1 - \text{ber}_{1,i})^{s_i t_s R_1}, \\ \text{per}_2^{(2)} &= 1 - (1 - \text{ber}_{2,n})^{s_{2,n} t_s R_2} (1 - \text{ber}_{2,i})^{s_i t_s R_2}. \end{aligned} \quad (12)$$

Then,  $\text{TH}_{1,i}^{(2)}$  and  $\text{TH}_{2,i}^{(2)}$  are represented as

$$\begin{aligned} \text{TH}_{1,i}^{(2)} &= \sum_{k_2=0}^{CW_m-1} \sum_{k_1=k_2}^{CW_1-1} p_a^{(2)}(k_1, k_2) (1 - \text{per}_1^{(2)}) \frac{L}{n_{f,1} t_s}, \\ \text{TH}_{2,i}^{(2)} &= \sum_{k_2=0}^{CW_m-1} \sum_{k_1=k_2}^{CW_1-1} p_a^{(2)}(k_1, k_2) (1 - \text{per}_2^{(2)}) \frac{L}{n_{f,\text{max}} t_s}, \end{aligned} \quad (13)$$

where  $n_{f,\text{max}} = \max(n_{f,1}, n_{f,2})$ . Finally, the per-system throughput of WMAN and WLAN can be written as

$$\begin{aligned} \text{TH}_1 &= \text{TH}_1^{(1)} + \text{TH}_1^{(2)}, \\ \text{TH}_2 &= \text{TH}_2^{(2)}, \end{aligned} \quad (14)$$

which can be obtained from (8), (9) and (13).

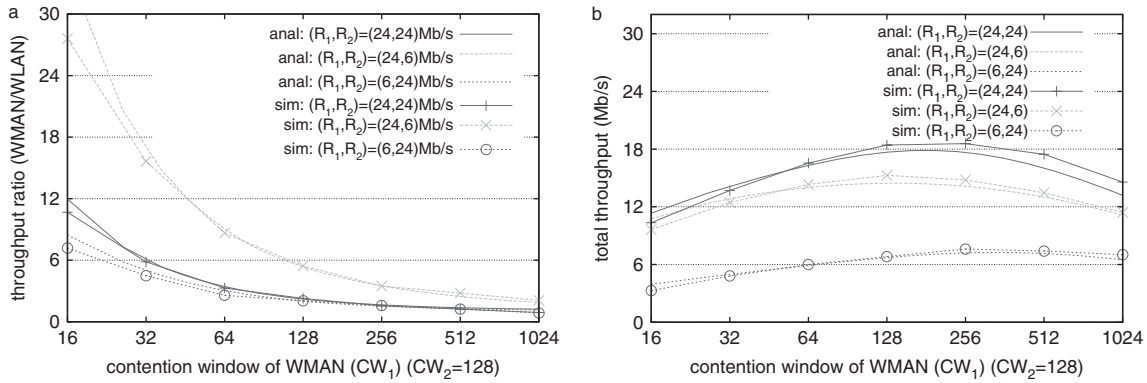
### 3.3. Effect of contention window and transmission rate

Using the derived throughput model, we investigate the effect of the contention window and the transmission rate on fair channel sharing. We consider a simple geometry model, where the distance between the transmitter and receiver of the WMAN is 100 m and that of the WLAN is 10 m and the distance between WMAN and WLAN transmitters is 300 m, as shown in Fig. 1. The other configurations are the same as those in Table 1.

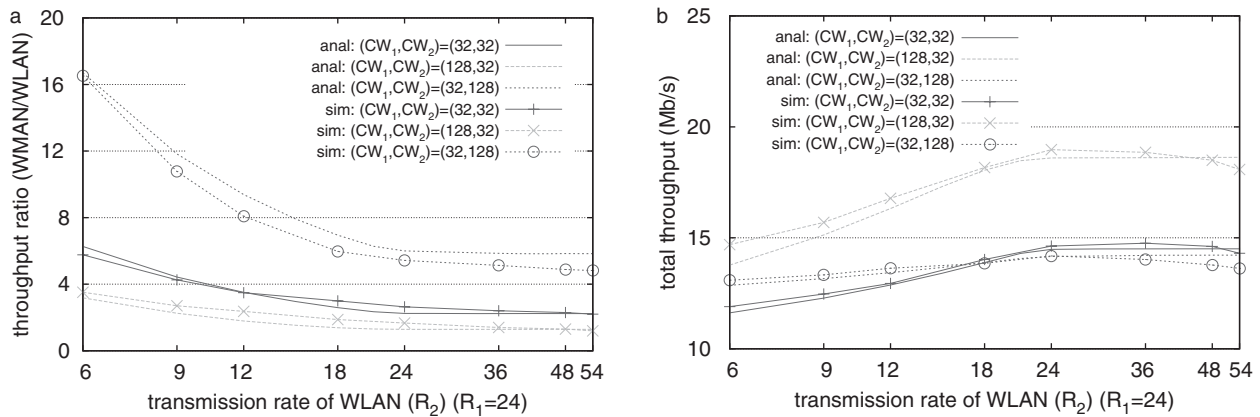
The contention window and the transmission rate are controlled by the BEB mechanism and the link adaptation mechanism, which are reasons of unfair channel sharing as addressed in Section 2.2. To evaluate the performance in terms of fairness and efficiency of channel sharing, we establish two performance indices, throughput ratio ( $\gamma_{\text{ratio}}$ ) and total throughput ( $\eta_{\text{eff}}$ ); the former is defined as the ratio of the average throughput achieved by a WMAN node to that achieved by a WLAN node and the latter is defined as the total throughput achieved by all the nodes, i.e.

$$\begin{aligned} \gamma_{\text{ratio}} &= \frac{\text{TH}_1/N_1}{\text{TH}_2/N_2}, \\ \eta_{\text{eff}} &= \text{TH}_1 + \text{TH}_2. \end{aligned} \quad (15)$$

Figure 5 represents  $\gamma_{\text{ratio}}$  and  $\eta_{\text{eff}}$  for several pairs of  $R_1$  and  $R_2$  that were derived from the analysis in Section 3.2, along with those obtained from the simulation. Here, we set  $CW_2$  to 128 and change  $CW_1$  from 16 to 1024, in order to observe the effect of the contention window size. In general,  $CW_2$  becomes relatively higher than  $CW_1$  because the WLAN suffers more transmission failures than the WMAN, which triggers the BEB



**FIGURE 5.** Effect of the contention window on the fairness and efficiency of channel sharing. (a) Throughput ratio ( $\gamma_{\text{ratio}}$ ) and (b) total throughput ( $\eta_{\text{eff}}$ ).



**FIGURE 6.** Effect of the transmission rate on the fairness and efficiency of channel sharing. (a) Throughput ratio ( $\gamma_{\text{ratio}}$ ) and (b) total throughput ( $\eta_{\text{eff}}$ ).

mechanism and doubles the contention window size. Figure 5a shows that  $\gamma_{\text{ratio}}$  decreases toward 1 as  $CW_1$  increases, regardless of the values of  $R_1$  and  $R_2$ . The increase of  $CW_1$  contributes to improving fairness since the WLAN gets more channel access opportunities as  $CW_1$  increases. However, fairness is significantly degraded when  $R_2 < R_1$  and  $CW_1$  is small, e.g.  $\gamma_{\text{ratio}}$  is about 30 when  $(R_1, R_2) = (24, 6)$  Mb/s, and  $(CW_1, CW_2) = (16, 128)$ . From Fig. 5b, we observe the effect of the contention window size on  $\eta_{\text{eff}}$ . As  $CW_1$  increases up to a certain value,  $\eta_{\text{eff}}$  increases accordingly, which results from the decreased probability of transmission failure due to collision or interference. However, if  $CW_1$  exceeds a certain value (e.g. 180 when  $R_1 = R_2 = 24$  Mb/s),  $\eta_{\text{eff}}$  decreases because of the increased average backoff time. This result confirms the trade-off between collision probability and overhead time related to the contention window size. Also, we observe that  $\eta_{\text{eff}}$  in the case of  $(R_1, R_2) = (6, 24)$  Mb/s is much lower than that in the case of  $(R_1, R_2) = (24, 6)$  Mb/s. This means that the WLAN makes a minor contribution to  $\eta_{\text{eff}}$ .

The effect of the transmission rate on  $\gamma_{\text{ratio}}$  and  $\eta_{\text{eff}}$  can be observed in Fig. 6, where  $R_1$  is set to 24 Mb/s and  $R_2$  changes from 6 to 54 Mb/s. As shown in Fig. 6a,  $\gamma_{\text{ratio}}$  decreases (i.e. fairness is improved) as  $R_2$  increases. Although the robust modulation and coding scheme can reduce the BER at the cost of a decreased transmission rate, the decrease in the transmission rate rather increases the packet transmission time and causes more interference to the WLAN, as explained in Section 2.2. Therefore, as  $R_2$  decreases, the throughput of the WLAN decreases and  $\gamma_{\text{ratio}}$  increases accordingly. This result implies that the link adaptation mechanism is not effective to deal with the inter-system interference; it may rather worsen the fairness. Also, Fig. 6a shows that  $\gamma_{\text{ratio}}$  is almost immune to the value of  $R_2$  as long as  $R_2 > R_1 = 24$  Mb/s. On the other hand, we observe from Fig. 6b that  $\eta_{\text{eff}}$  increases as  $R_2$  increases up to  $R_1 (= 24)$  Mb/s. The increase of  $R_2$  decreases the probability of the interference-driven transmission failure, as well as decreases the transmission time, and so it contributes to the increase of  $\eta_{\text{eff}}$ . However, this positive effect is balanced with the negative effect



(i.e. increased BER);  $\eta_{\text{eff}}$  no longer increases once  $R_2 > R_1$ , as is shown in Fig. 6b. By comparing three cases with different values of  $CW_1$  and  $CW_2$  in Fig. 6, we observe that the case of  $(CW_1, CW_2) = (128, 32)$  outperforms the other cases in terms of fairness and efficiency;  $\gamma_{\text{ratio}}$  is closer to 1 and  $\eta_{\text{eff}}$  has a higher value.

Moreover, Figs 5 and 6 show that there is no significant difference between the analysis results and the simulation results, which confirms the validity of the analysis. Finally, we can make the following conclusions from the results in Figs 5 and 6, which provide a clue to the solution for achieving fair and efficient channel sharing.

- (i) Increasing the contention window size of the interferer (WMAN),  $CW_1$ , improves fairness since it provides more interference-free channel access opportunities to the victim (WLAN). Moreover, the total throughput increases as  $CW_1$  increases up to a certain value.
- (ii) Reducing the transmission rate of the victim,  $R_2$ , not only worsens fairness but also reduces the total throughput. Maintaining  $R_2$  comparable to  $R_1$  contributes to improving both fairness and efficiency.

#### 4. FAIR COEXISTENCE CSMA PROTOCOL

Considering the causes of unfair channel sharing discussed in Section 2 and the analysis results in Section 3, this section proposes the fair coexistence CSMA protocol consisting of two schemes: *access etiquette* and *interference-aware backoff*. The key idea of the proposed mechanism is 2-fold. First, the access etiquette scheme is designed to alleviate the unfairness of channel sharing and to mitigate interference by decreasing the channel access attempt of the interferer (WMAN) and allowing more channel access opportunities to the victim (WLAN) accordingly. This idea can be realized by the dynamic control of WMAN's contention window size. Next, the interference-aware backoff scheme is intended to remove the blindness of the BEB mechanism, which plays as one cause of unfair channel sharing as discussed in Section 2.2. This scheme recognizes the reason of transmission failure, and disables the BEB mechanism of the WLAN, i.e. the channel access attempt of the WLAN is not necessarily decreased, in response to the interference-driven transmission failure. The details are described in the following subsections.

##### 4.1. Access etiquette

The objective of the access etiquette is to assure fair channel sharing between the WMAN and the WLAN while attaining high efficiency of channel usage. The access etiquette scheme is applied to WMAN senders and controls their contention window size in order for the WLAN to occupy the channel in a fair manner without interference or with minimal interference. The analysis results in Section 3.3 (Fig. 5) indicate that we

can control the trade-off between fairness and efficiency by adjusting the size of the WMAN's contention window,  $CW_1$ , i.e. the large value of  $CW_1$  improves fairness by giving more channel access opportunities to WLAN, but it reduces efficiency due to the increased backoff time, and that there exists an optimal value of  $CW_1$  that strikes a balance between fairness and efficiency.

Under this rationale, we establish an objective function for controlling  $CW_1$  as a linear combination of  $\eta_{\text{eff}}/C_{\text{max}}$  and  $\tau_{\text{fair}}$ , i.e.

$$\mathcal{F}(CW_1) = w \frac{\eta_{\text{eff}}}{C_{\text{max}}} + (1 - w)\mu_{\text{fair}}, \quad (16)$$

where  $w$  is a weight factor ( $0 \leq w \leq 1$ ),  $C_{\text{max}}$  is the ideal maximum capacity and  $\mu_{\text{fair}}$  is Jain's fairness index [21] considering the average per-node throughput of the WMAN and the WLAN,

$$\mu_{\text{fair}} = \frac{(\text{TH}_1/N_1 + \text{TH}_2/N_2)^2}{2((\text{TH}_1/N_1)^2 + (\text{TH}_2/N_2)^2)}. \quad (17)$$

The value of  $C_{\text{max}}$  can be calculated under the assumption that there is neither collision nor interference so that the contention window has the minimum value of  $CW_{\text{min}}$  and the transmission rate has the maximum value of  $R_{\text{max}}$ , i.e.

$$C_{\text{max}} = \frac{L}{t_{\text{oh}} + ((CW_{\text{min}} - 1)/2)t_s + L/R_{\text{max}}}. \quad (18)$$

The first term in the right-hand side of (16),  $\eta_{\text{eff}}/C_{\text{max}}$ , represents the normalized efficiency of channel sharing and is less than 1. The second term  $\mu_{\text{fair}}$  in (16) is used to consider the fairness of channel sharing;<sup>4</sup> it has the largest value of 1 when both the WMAN and the WLAN have the same per-node throughput, i.e.  $\text{TH}_1/N_1 = \text{TH}_2/N_2$  (best case), and the minimum value of 1/2 when either the WMAN or the WLAN cannot occupy the channel at all, i.e.  $\text{TH}_1$  or  $\text{TH}_2$  is zero (worst case). The problem can be formulated as an optimization problem, i.e.

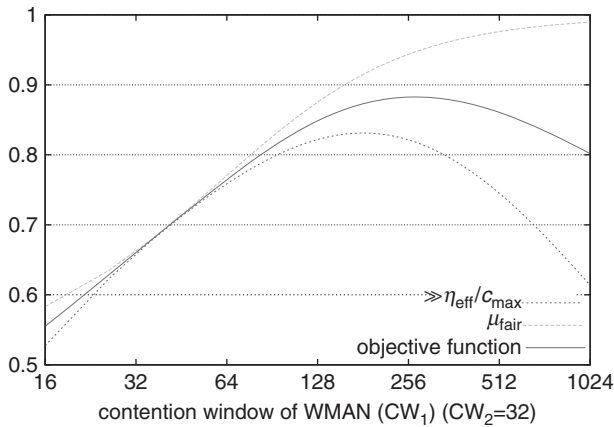
$$\begin{aligned} & \text{maximize}_{CW_1} \quad \mathcal{F}(CW_1) \\ & \text{subject to} \quad CW_1 \in \Omega, \end{aligned} \quad (19)$$

where  $\Omega = \{CW_1 | CW_{\text{min}} \leq CW_1 \leq CW_{\text{max}}\}$ ,  $CW_{\text{min}}$  and  $CW_{\text{max}}$  are the minimum and maximum values of contention window, respectively, and the optimal value of  $CW_1$  is

$$CW_1^* = \text{argmax}_{CW_1 \in \Omega} \mathcal{F}(CW_1). \quad (20)$$

Figure 7 shows an example of  $\mathcal{F}(CW_1)$  with  $w = 0.5$ , which is obtained from the analysis in Section 3.3 with  $CW_2 = 128$ ,

<sup>4</sup>In Section 3.3, we have already introduced  $\gamma_{\text{ratio}}$  as an intuitive measure of fairness, however, we make use of another fairness index in the objective function of (16), whose range is comparable to that of  $\eta_{\text{eff}}/C_{\text{max}}$  and whose value increases as the fairness improves.



**FIGURE 7.** Objective function (weighted sum of  $\eta_{\text{eff}}/C_{\text{max}}$  and  $\mu_{\text{fair}}$ ) versus  $CW_1$ .

$R_1 = R_2 = 24 \text{ Mb/s}$  and  $N_1 = N_2 = 10$ . From Fig. 7, we observe that the objective function  $\mathcal{F}(CW_1)$  is a *unimodal and concave* function with respect to  $CW_1$  so that there exists a *unique optimal* value of  $CW_1$  (e.g. 270 in this case) that maximizes  $\mathcal{F}$ .<sup>5</sup>

Next, we propose a control rule of  $CW_1$  to maximize the objective function. For every monitoring interval of  $T_{\text{ae}}$ , the AP of WLAN measures its aggregate throughput  $\text{TH}_2$  and keeps track of the number of associated nodes  $N_2$ . We assume that a signaling path is established between coexisting BS and AP for the license-exempt coexistence according to the legal requirements. The AP sends a control message for resource management, which contains  $\text{TH}_2$  and  $N_2$ , to the BS of the WMAN. Similarly, the BS measures the aggregate throughput and manages the number of WMAN nodes. Using this information, BS estimates the objective function  $\mathcal{F}(CW_1)$  from (16) to (18) for every  $T_{\text{ae}}$ . On the basis of the measurement of  $\mathcal{F}(CW_1)$ , the value of  $CW_1$  is updated toward  $CW_1^*$  using the golden section search algorithm [22], which is a simple algorithm for finding the local minimum (or maximum) of a one-dimensional unimodal function in a closed interval without requiring the derivative of function. The pseudo code for the update rule of  $CW_1$  is given in Fig. 8. Initially, the value of  $\mathcal{F}$  is evaluated at an intermediate point of  $\text{cw\_int} (= CW_{\text{min}} + \rho(CW_{\text{max}} - CW_{\text{min}}))$ , and the next probe point is set as  $\text{cw\_opt}$ . At every interval of  $T_{\text{ae}}$ , the value of  $\mathcal{F}$  is evaluated at the point of  $\text{cw\_opt}$ , and  $\text{cw\_opt} (= CW_1)$  is updated within the narrower search space of  $[\text{min}(\text{cw\_bound1}, \text{cw\_bound2}), \text{max}(\text{cw\_bound1}, \text{cw\_bound2})]$  to be converged to  $CW_1^*$ . The value of  $\text{cw\_opt}$  is broadcasted via a control message so that all the WMAN senders set their contention window size as  $\text{cw\_opt}$  until the next update interval of  $T_{\text{ae}}$ .

<sup>5</sup>We assume that the objective function is unimodal and concave from the intuition of the trade-off between fairness and efficiency as well as from the numerical analysis results in Figs 5 and 7, even though it is hard to prove it mathematically.

**REMARK 1.** The proposed access etiquette scheme is flexible in that (i) the trade-off between fairness and efficiency can be controlled by assigning the value of  $w$  ( $0 \leq w \leq 1$ ) in the objective function of (16), i.e. emphasis can be put on the efficiency or on fairness, as  $w$  is close to 1 or 0, respectively, and (ii) the weighted fairness can also be supported, i.e.  $\gamma_{\text{ratio}}$  can be attained close to an arbitrary value of  $K$  by replacing  $\text{TH}_2$  with  $K\text{TH}_2$  in (17).

**REMARK 2.** The only requirement for finding the (local) optimal value of  $CW_1$  with the golden section search algorithm is that  $\mathcal{F}$  is unimodal in the given interval. If this requirement is met, the value of  $CW_1$  converges to its optimal value  $CW_1^*$  at the rate of  $(1 - \rho)$ , i.e. the search space is reduced by  $(1 - \rho)$  at every stage [22]. Therefore, the error between  $CW_1$  and  $CW_1^*$  becomes less than  $\epsilon$ , i.e.  $|CW_1 - CW_1^*| < \epsilon$ , as long as the number of iterations exceeds  $N_c$ ,

$$N_c = \frac{\log(\epsilon/(CW_{\text{max}} - CW_{\text{min}}))}{\log(1 - \rho)}. \quad (21)$$

For example, when  $CW_{\text{max}} = 1024$ ,  $CW_{\text{min}} = 16$  and  $\epsilon = 1$ ,  $N_c$  becomes 14.37.

**REMARK 3.** The update rule of  $CW_1$  in Fig. 8 requires that the objective function should be evaluated at every iteration, which can be obtained based on the measurement of per-node throughput from (16) to (18). Therefore, the proposed method does not resort to a specific system model and it is free from the system modeling error. However, we need to cope with the case where the measurement error is not negligible so that the value of  $CW_1$  may be converged to a non-optimal value. For this purpose, we consider three points. First, the update period  $T_{\text{ae}}$  should be long enough so that the change in  $CW_1$  can be reflected in the objective function.<sup>6</sup> Next, the per-system throughputs  $\text{TH}_1$  and  $\text{TH}_2$  are averaged in the form of an exponentially weighted moving average when evaluating  $\mathcal{F}$ , so that the temporary abnormal change or the measurement error can be relieved. Lastly, in order for the value of  $CW_1$  to escape from the non-optimal value, the procedure of updating  $CW_1$  needs to be periodically re-initialized or re-initialized upon the crucial change of system configuration.

## 4.2. Interference-aware backoff

The interference-aware backoff mechanism is applied to the WLAN senders, and it differentiates the response to transmission failure due to the inter-system interference from the response to that due to the intra-system collision. It freezes the contention window size of the WLAN,  $CW_2$ , i.e. not

<sup>6</sup>As  $T_{\text{ae}}$  increases, the signaling overhead decreases and the stability can be improved at the cost of convergence time. The purpose of the access etiquette scheme is to provide long-term fairness, instead of short-term fairness, while attaining high efficiency. Therefore, we consider that the proper value of  $T_{\text{ae}}$  is the time required for transmitting several hundreds of packets.

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state variables and constants	
cw_bound1, cw_bound2	two bounds of search space
cw_int	intermediate point in the given interval
cw_opt	optimal point of $CW_1$ updated periodically
obj_prv / obj_cur	objective function evaluated previously / currently
$\rho$	constant of the golden section search algorithm, $(3-\sqrt{5})/2$

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Initialization

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```

cw_bound1 = CW_min, cw_bound2 = CW_max;
cw_int = cw_bound1 +  $\rho$  * (cw_bound2 - cw_bound1); // initial intermediate point
obj_prv = calc_obj_function(TH1, N1, TH2, N2); // initial measure of  $\mathcal{F}(cw\_int)$ 
cw_opt = cw_int +  $\rho$  * (cw_bound2 - cw_int); // initial probe point of  $CW_1$ 

```

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At every update interval of  $T_{ae}$

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```

obj_cur = calc_obj_function(TH1, N1, TH2, N2); // periodic measure of  $\mathcal{F}(cw\_opt)$ 

if ( obj_cur > obj_prv ) {
    cw_bound1 ← cw_int;
    cw_int ← cw_opt;
    obj_prv ← obj_cur; }

else {
    cw_bound2 ← cw_bound1;
    cw_bound1 ← cw_opt; }

cw_opt = cw_int +  $\rho$  * (cw_bound2 - cw_int); // set  $CW_1$  as cw_opt

```

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**FIGURE 8.** Pseudo code of access etiquette scheme based on the golden section search algorithm.

increasing  $CW_2$  unnecessarily, in the case of the interference-driven transmission failure. For this purpose, it is essential to determine the cause of transmission failure. Note that there are two independent error detection codes in the WLAN [15]; one in the PHY header [e.g. 16-bit cyclic redundancy check (CRC) code in the physical layer convergence procedure (PLCP) header] and the other one in the MAC protocol data unit (MPDU) (e.g. 32-bit CRC in frame control sequence field). If the receiver of the WLAN successfully receives the PLCP header but fails to decode the MPDU correctly, it determines that the failure is due to interference. Otherwise, if neither the PLCP

header nor MPDU is received successfully, then the failure is considered to be due to collision. The underlying rationale is that the WMAN sender usually starts to cause interference in the middle of the WLAN's transmission rather than at the start of the WLAN's transmission (Fig. 2), so the WLAN receiver possibly decodes the PHY header, in spite of interference by the WMAN. We consider that the transmission failure due to the temporary degradation of channel quality is insignificant if a proper link adaptation mechanism along with the proposed mechanism is employed. To inform the WLAN senders of an interference-driven transmission failure, we introduce a binary

control flag, called the severe interference notification (SIN) flag, which needs to be newly added in the acknowledgement (ACK) frame. If a WLAN node fails to receive packets due to interference, then it transmits a negative ACK frame where the SIN flag is set. The transmission of such a negative ACK frame is deferred until the channel becomes idle to avoid transmission failure due to interference by the WMAN. As the conventional CSMA mechanism, the WLAN sender doubles the contention window if it does not receive any positive ACK frame until retransmission time-out is reached. However, if the WLAN sender receives a negative ACK frame with the SIN flag, then it reduces  $CW_2$  by half. With the aid of explicit interference notification, the WLAN sender disables the BEB mechanism in the case of interference-driven failure, but it is only activated in the case of collision-driven failure.

## 5. SIMULATION

In this section, we conduct extensive simulations to validate the performance of the proposed mechanism. First, we focus on the stability and controllability of the proposed mechanism; we observe how the access etiquette algorithm converges the contention window size and how it controls the trade-off between fairness and efficiency in channel sharing. Next, we compare the performance of the proposed mechanism with those of several approaches in relation to the variations in certain system configurations, such as the degree of overlapping deployment between the WMAN and the WLAN, the asymmetry of transmission power/coverage, and the number of nodes. We consider the following four approaches for fair channel sharing and interference mitigation, as well as the proposed mechanism:

- (i) BASE: This does not employ any coexistence mechanism, and it is considered as a baseline mechanism to compare the performance of other approaches.
- (ii) CS1-: Compared with BASE, this approach reduces the physical CSTD of coexisting WMAN ( $w_{\text{WMAN}_0}$ ) to increase the probability of detecting the WLAN's packet transmission:  $P_{\text{cs},1} = -97.65$  dBm. Note that the value of the CSTD of the first-tier interfering WMANs ( $w_{\text{WMAN}_i}$ ),  $P_{\text{cs},i}$ , is not changed from  $-90$  dBm. With this configuration, the carrier sensing range of  $w_{\text{WMAN}_0}$  for detecting the inter-system interference,  $I_{\text{cs},1}$ , increases up to 300 m, which is the default value of  $D_{\text{int,sys}}$ , while that for detecting the intra-system collision,  $D_{\text{cs},1}$ , also increases to 2250 m.
- (iii) TR2-: In this approach, the WLAN uses the most robust modulation and coding scheme to overcome interference from WMANs, i.e.  $R_2$  is fixed to 6 Mb/s.
- (iv) CW1+: By increasing the minimum contention window of  $w_{\text{WMAN}_0}$ , the channel access probability of the WLAN can be increased and interference to the WLAN can also be alleviated. This approach sets the minimum

contention windows of  $w_{\text{WMAN}_0}$  to the maximum value of 1024, i.e.  $CW_{\text{min}} = CW_{\text{max}} = 1024$  for  $w_{\text{WMAN}_0}$ .

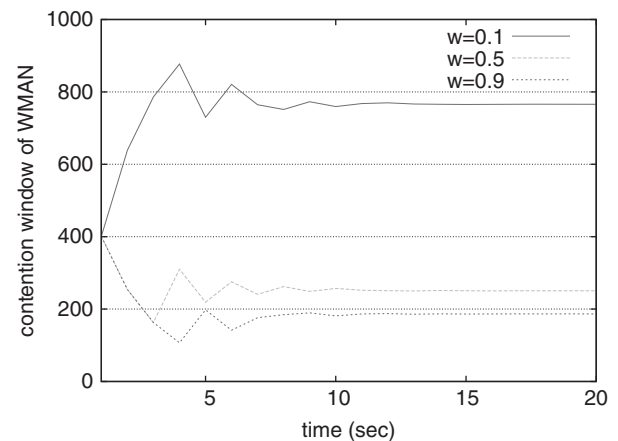
- (v) AE-IB: This is the proposed mechanism deploying the access etiquette algorithm to  $w_{\text{WMAN}_0}$  and the interference-aware backoff algorithm to WLAN. The design parameters of the access etiquette algorithm are set as;  $T_{\text{ae}} = 1$  s,  $w = 0.5$  and  $\epsilon = 1$ .

We use the same simulation configuration in Section 2.3, unless otherwise stated. To evaluate the performance in terms of fairness and efficiency, two performance indices are used, respectively;  $\gamma_{\text{ratio}} = (\text{TH}_1/N_1)/(\text{TH}_2/N_2)$  and  $\eta_{\text{eff}} = \text{TH}_1 + \text{TH}_2$ . We do not represent the throughput of the six first-tier interfering  $w_{\text{WMAN}_i}$ s, because there was no notable difference among the different approaches, except for CS1-. The simulation time is set to 10 million time slots, and the simulation results are averaged over ten simulation runs, each of which has the random placement of nodes.

### 5.1. Stability and controllability of the proposed mechanism

The objective of the first simulation is to validate the stability and controllability of the access etiquette scheme, which controls  $CW_1$  to maximize the objective function according to the golden section search algorithm. Figure 9 shows how  $CW_1$  is updated for several cases that have different values of weight in the objective function,  $w$ . Recall that we can control the trade-off between fairness and efficiency by assigning the value of  $w$  ( $0 \leq w \leq 1$ ) in (16); as the value of  $w$  is close to 0, fairness is much attainable at the cost of efficiency; otherwise, the efficiency can be improved by setting  $w$  close to 1.

As shown in Fig. 9,  $CW_1$  converges to the value  $CW_1^*$  for each case:  $CW_1^* \approx 765, 251$  and 187 when  $w = 0.1, 0.5$  and 0.9, respectively. This simulation result agrees with the intuition that



**FIGURE 9.** Convergence of WMAN's contention window size control according to the access etiquette scheme with different values of weight in the objective function.

**TABLE 3.** Several performance indices of the proposed mechanism with different values of weight in the objective function.

Weight	Per-system throughput (Mb/s)		Total throughput (Mb/s)	Throughput ratio
	WMAN	WLAN		
0.1 (fairness-centric)	7.31	4.62	11.93	1.58
0.5 (balanced)	9.15	4.36	13.51	2.10
0.9 (efficiency-centric)	12.09	3.04	15.13	3.98
Baseline	10.91	2.33	13.24	4.68

$CW_1^*$  becomes higher as  $w$  becomes smaller (i.e. higher priority to fairness). On the other hand, Fig. 9 shows that the convergence time is independent of  $w$ , as already stated in Section 4.1. Table 3 lists the performance indices of the proposed mechanism for the three cases of  $w = 0.1, 0.5$  and  $0.9$ , along with those of BASE. In the case when  $w = 0.1$  the throughput ratio ( $\gamma_{\text{ratio}}$ ) is 1.58, which is smaller than the case when  $w = 0.9$  by more than 2.5 times, but the total throughput ( $\eta_{\text{eff}}$ ) is smaller than the case when  $w = 0.9$  by about 21%. Compared with BASE, the proposed mechanism can improve fairness (e.g.  $\gamma_{\text{ratio}}$  decreases by about three times when  $w = 0.1$ ), or it can enhance efficiency (e.g.  $\eta_{\text{eff}}$  increases by about 15% when  $w = 0.9$ ), depending on the value of  $w$ . These results confirm that  $CW_1$  is controlled and converged to maximize the given objective function and that the proposed mechanism can control the trade-off between the fairness and efficiency of channel sharing by assigning a proper value to  $w$ .

## 5.2. Effect of the degree of overlapping deployment between WMAN and WLAN

This simulation compares the performance of several approaches under different degree of overlapping deployment between the WMAN and the WLAN;  $D_{\text{int-sys}}$  ranges from 0 to 750 m (the cell radius of WMAN). Here, the other simulation parameters are unchanged from those in Table 1.

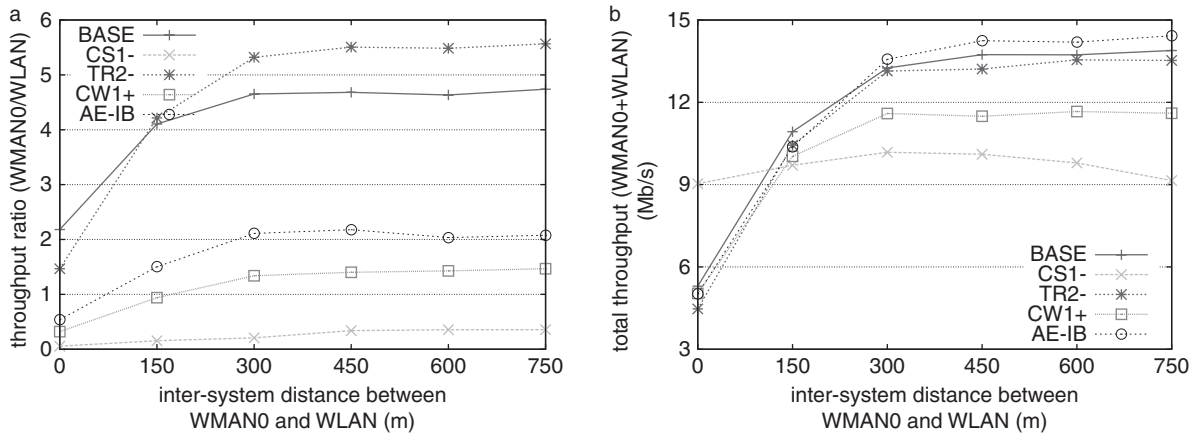
Figure 10 shows  $\gamma_{\text{ratio}}$  and  $\eta_{\text{eff}}$  for various values of  $D_{\text{int-sys}}$ . Except for the case of fully overlapping deployment ( $D_{\text{int-sys}} = 0$ ),  $CW1+$  shows the best performance in terms of fairness;  $\gamma_{\text{ratio}}$  lies between 0.94 and 1.47 when  $D_{\text{int-sys}} \geq 150$  m; however,  $CW1+$  improves fairness at the cost of efficiency;  $\eta_{\text{eff}}$  of  $CW1+$  is smaller than that of BASE by up to 20%. In the cases of BASE and TR2-, as  $D_{\text{int-sys}}$  increases from 0 to 750 m,  $\gamma_{\text{ratio}}$  increases from 2.2 and 1.5 to 4.7 and 5.6, respectively. Except for the case of  $D_{\text{int-sys}} = 0$ ,  $\gamma_{\text{ratio}}$  of TR2- is higher than that of BASE, but  $\eta_{\text{eff}}$  of TR2- is lower than that of BASE, implying that reducing the transmission rate of the WLAN is not effective for fairness and efficiency. These results conform to the analysis results in Fig. 6 of Section 3.3. Contrary to the other approaches, CS1- gives the preference of channel sharing to the WLAN, rather than to the WMAN;

$\gamma_{\text{ratio}}$  is not greater than 0.36. Moreover,  $\eta_{\text{eff}}$  of CS1- is less affected by  $D_{\text{int-sys}}$  compared with the other approaches, and it is quite larger than the others when  $D_{\text{int-sys}} = 0$ , but lower than the others when  $D_{\text{int-sys}} \geq 150$  m. These results can be explained as follows: CS1- cannot fully employ spatial reuse among WMANs, i.e. the sender of WMAN\_0 unnecessarily defers its channel access when the senders of WMAN\_i occupy the channel. Therefore, the approach of reducing the CSTH not only fails to assure fairness but also reduces efficiency. However, AE-IB succeeds in alleviating the unfair sharing of channel without impairing the efficiency;  $\gamma_{\text{ratio}}$  is maintained between 0.54 and 2.17 for the entire range of  $D_{\text{int-sys}}$ , which is smaller than that of BASE by up to 2.7 times, and at the same time,  $\eta_{\text{eff}}$  of AE-IB is higher than that of  $CW1+$  by about 25%. The outstanding performance of AE-IB stems from (i) the adaptive control of contention window to enforce fairness and to improve efficiency (due to the access etiquette scheme) and (ii) the prevention of unnecessary increase of contention window in response to the interference-driven transmission failure (due to the interference-aware backoff scheme).

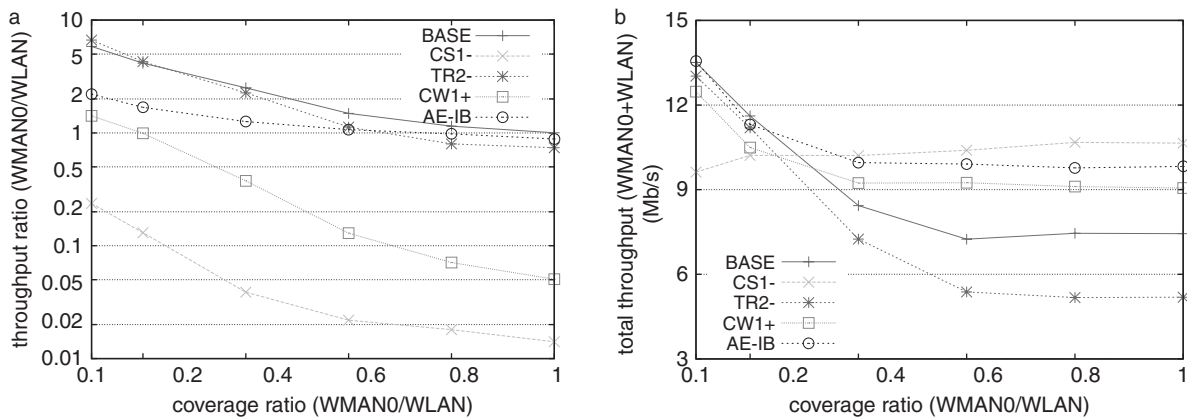
## 5.3. Effect of power/coverage asymmetry between WMAN and WLAN

We investigate the effect of the asymmetry of transmission power or coverage between the WMAN and the WLAN on coexistence performance. Recall that the coverage asymmetry factor  $K_a$  was defined in Section 3.1 as  $K_a = D_{\text{rx},2}/D_{\text{rx},1}$ , which is also related to the asymmetry of transmission power. We fix  $D_{\text{rx},1}$  to 750 m and change  $P_{\text{tx},2}$  such that  $D_{\text{rx},2}$  ranges from 75 to 750 m, i.e.  $K_a$  changes from 0.1 to 1.0. Here,  $D_{\text{int-sys}}$  is set to 300 m.

Figure 11 shows the throughput ratio and total throughput for various values of  $K_a$ . As was expected, the problem of unfair sharing becomes alleviated as the degree of asymmetry decreases, i.e. in the cases of BASE and TR2-,  $\gamma_{\text{ratio}}$  monotonically decreases to 1 as  $K_a$  increases to 1; however,  $\eta_{\text{eff}}$  decreases until  $K_a$  increases to 0.6 and slightly changes once  $K_a > 0.6$ . The decrease of  $\eta_{\text{eff}}$  results from the increased interference power of the WLAN, which decreases



**FIGURE 10.** Performance comparison of several approaches with different degree of overlapping deployment between WMAN0 and the WLAN. (a) Throughput ratio ( $\gamma_{\text{ratio}}$ ) and (b) total throughput ( $\eta_{\text{eff}}$ ).



**FIGURE 11.** Performance comparison of several approaches with different values of the WLAN's coverage. (a) Throughput ratio ( $\gamma_{\text{ratio}}$ ) and (b) total throughput ( $\eta_{\text{eff}}$ ).

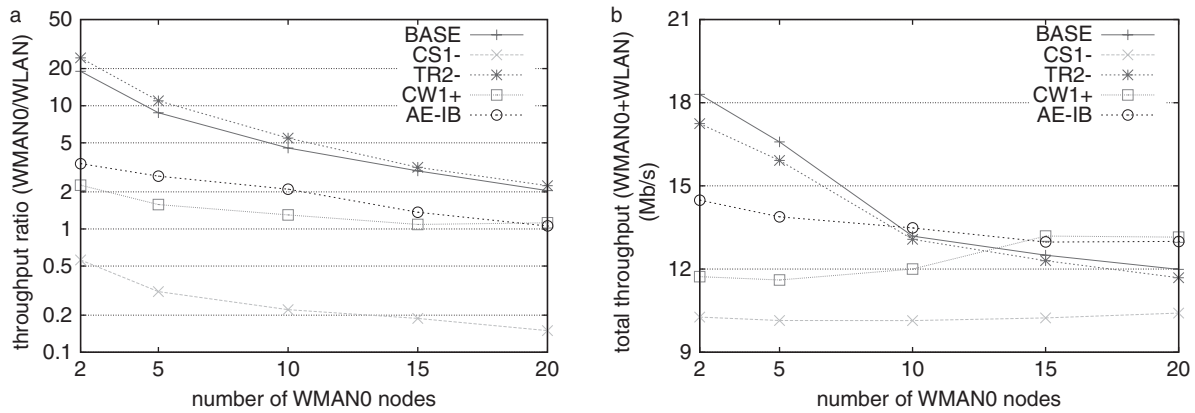
the WMAN transmission rate or increases its transmission failure accordingly. Among the five approaches, TR2- suffers remarkable decreases of  $\eta_{\text{eff}}$ , e.g., it is lower than those of BASE and CS1- by about 40% and 100%, respectively, when  $K_a = 1.0$ . As  $K_a$  increases, the channel occupation by the WLAN can be detected by the WMAN, and the WLAN can transmit packets without interference from the WMAN. These results reconfirm that, compared with BASE, the approach of TR2- does not achieve any gain, but decreases the throughput. On the other hand, in the cases of CS1- and CW1+, as  $K_a$  increases,  $\gamma_{\text{ratio}}$  significantly decreases from 0.24 and 1.40 to 0.014 and 0.05, respectively. Unlike the other approaches,  $\eta_{\text{eff}}$  of CS1- is almost immune to  $K_a$ , because the throughput of the WLAN is mostly unrelated to its transmission power or coverage, and its contribution to  $\eta_{\text{eff}}$  is dominant. The proposed AE-IB mechanism outperforms the other mechanisms; compared with BASE,  $\gamma_{\text{ratio}}$  decreases from

5.84 to 2.20 when  $K_a = 0.1$ , and  $\eta_{\text{eff}}$  increases by up to 36% when  $K_a = 1.0$ .

#### 5.4. Effect of the number of nodes

The number of nodes is one of the most important factors affecting performance, since it is related to the degree of intra-system collision and inter-system interference. We observe its effect on  $\gamma_{\text{ratio}}$  and  $\eta_{\text{eff}}$  from Fig. 12. Here, we change the number of WMAN nodes ( $N_1$ ) from 2 to 20, while that of WLAN nodes ( $N_2$ ) is fixed at 10, and  $D_{\text{int-sys}} = 300$  m,  $D_{\text{rx},1} = 750$  m and  $D_{\text{rx},2} = 100$  m. Note that the number of nodes is considered in evaluating  $\gamma_{\text{ratio}}$  as (15).

As shown in Fig. 12a, expect for CS1-, fairness is quite impaired when  $N_1$  is small, e.g. in the cases of BASE and TR2-,  $\gamma_{\text{ratio}}$  is about 19 and 24 when  $N_1 = 2$ , respectively, but it is improved as  $N_1$  increases. However, fairness achieved by the



**FIGURE 12.** Performance comparison of several approaches with different values of the number of WMAN nodes. (a) Throughput ratio ( $\gamma_{\text{ratio}}$ ) and (b) total throughput ( $\eta_{\text{eff}}$ ).

CS1- mechanism is deteriorated as  $N_1$  increases because the channel access is biased against WMAN in CS1-, as already observed in Figs. 10 and 11. As  $N_1$  increases from 2 to 20,  $\gamma_{\text{ratio}}$  of CW1+ and AE-IB decreases from 2.26 and 3.38 to 1.12 and 1.06, respectively.

Fig. 12b shows  $\eta_{\text{eff}}$  when  $N_1$  has different values. In the BASE and TR2- mechanisms,  $\eta_{\text{eff}}$  decreases as  $N_1$  increases, which results from the increase of intra-system collision of the WMAN. Similarly,  $\eta_{\text{eff}}$  of AE-IB decreases as  $N_1$  increases. When  $N_1 < 10$ , AE-IB achieves a lower total throughput than BASE to enforce fairness; however, when  $N_1 \geq 10$ , its total throughput becomes higher than those of BASE and TR2-. The total throughput of CS1- is insensitive to the change of  $N_1$ , as already observed and explained in the previous simulations. As  $N_1$  increases, CW1+ improves both fairness and efficiency, from which we can infer that the size of  $CW_1$  used in CW1+ is close to the optimal value. It is important to note that the value of  $CW_1^*$  depends on many design parameters and network configurations, so it is quite difficult to determine either by analysis or by experiment. However, the proposed AE-IB mechanism updates the value of  $CW_1$  to maximize the given objective function represented in terms of fairness and efficiency, only by measuring the objective function without requiring the exact information about network configurations. Therefore, the proposed mechanism is effective for a wide range of network configurations, as confirmed in Figs 10–12.

## 6. CONCLUSION

In this study, we have addressed the issues of coexistence arising when heterogeneous networks with different transmission power employ the contention-based MAC protocol for non-exclusive coexistence. Since the high-power system fails to detect the ongoing packet transmission by the lower-power system, the high-power system attempts to access the channel even when the lower-power system has already

occupied the shared channel, creating interference that causes the low-power system to suffer from frequent transmission failures. Therefore, there occurs a severe unfairness of channel sharing between the two systems. We have shown that, in addition to the asymmetry of carrier sensing ability, the BEB mechanism and the link adaptation mechanism exacerbate the fair channel sharing. As a solution to mitigate interference and achieve fair channel sharing, we have proposed the fair coexistence CSMA mechanism, which consists of two schemes, the access etiquette and interference-aware backoff schemes. The proposed mechanism controls the contention window size to maximize the given objective function, which is represented in terms of fairness and efficiency, based on the feedback information about per-system throughput. Moreover, it differentiates the response to collision-driven transmission failure from that to interference-driven failure. In this way, the proposed mechanism assures the fairness of channel sharing without impairing efficiency, which is confirmed in this paper by extensive simulations performed under various network configurations.

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