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# Ladislav Svante Rieger and His Algebraic Work

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**Abstract.** This paper introduces the personality of the Czech mathematician of the first half of the 20th century, Ladislav Svante Rieger. Rieger's professional activities were devoted to three main mathematical areas; algebra, mathematical logic and axiomatic set theory. First, we describe his personal and professional life and give a brief overview of his mathematical activities. Second, his major results reached in the domain of algebra are presented. At the end, a list of Rieger's most significant scientific publications is provided.

## Introduction

Ladislav Svante Rieger (1916–1963) was a prominent Czech mathematician of the 20th century. However, he is not well known to the mathematical public, mainly because of his untimely death. In his less-than-20-year mathematical career, L.S. Rieger significantly contributed to the research in algebra, mathematical logic and axiomatic set theory, and his results were appreciated worldwide. Nevertheless, his major contribution is to the study and development of the Czechoslovak mathematics. Up to the present day, L.S. Rieger is regarded by Czech and Slovak mathematicians as a founder of mathematical logic and axiomatic set theory in our countries.

In the following text Rieger's family background, his personal life, and his pedagogical and scientific activities will be presented in more detail. Attention will be also paid to Rieger's major contribution in his chronologically first domain of interest – algebra. In this area L.S. Rieger focused on ordered and cyclically ordered groups and on lattice theory, especially on Boolean algebras. His principal works will be briefly described and evaluated in the context of Rieger's period.

## Ladislav Svante Rieger's family

Ladislav Svante Rieger was a member of the famous Rieger family. Let us thus start with his family background.

L.S. Rieger's great-grand father was a well known and influential figure of the era of the Czech National Revival, František Ladislav Rieger (1818–1903). F.L. Rieger was one of the leading characters of the Czech patriotic movement. In the revolution years 1848–1849 he had a role of a spokesman of the Czech political representation. Together with František Palacký (1796–1876) he was in the lead of the Czech politics for 30 years. F.L. Rieger was also an editor of the first Czech universal encyclopedia *Riegrův slovník naučný* and played a significant role in the foundation of the Czech National Theatre.

F.L. Rieger's son and L.S. Rieger's grand father, Bohuslav Rieger (1857–1907), was a prominent lawyer and historian. In 1899, he was appointed a full professor of Austrian history at the Czech university in Prague. In his scientific and pedagogical activity B. Rieger focused namely on constitutional and court law. Among others, he was a founder and editor of the journal *Sborník věd státních a správních*.

L.S. Rieger's father, Ladislav Rieger (1890–1958), was one of eminent Czech philosophers of his period. He was a representative of Marxist philosophy – dialectical materialism. L. Rieger originally studied physics and chemistry, from 1920 he fully devoted himself to philosophy. From 1945 he lectured at the Faculty of Arts, Charles University and in the same year was appointed a full professor of philosophy. From 1952 L. Rieger was the head of the division of philosophy *Kabinet pro filosofii* (from 1957 bearing the name the Institute of Philosophy) of the Czechoslovak Academy of Sciences. Apart from other things, he was the first editor of the philosophical journal *Filosofický časopis*.

## Childhood and Studies

L.S. Rieger's mother, Alžběta Jarešová (1890–1951), had worked as a teacher before she married L. Rieger in 1915. The married couple moved to Sweden very shortly after their wedding. Ladislav Svante Rieger was born on June 25, 1916 in Malmö. After the end of the First World War, the family moved back to Prague where Alžběta gave birth to the daugter Věra (*1919).[1] L.S. Rieger's childhood was, we dare say, quite dramatic. He had to cope with unpleasant atmosphere caused by financial problems of the family, parents' disputes which finally ended in divorce, and changes of housing. Having attended several schools, in June 1935, L.S. Rieger graduated from the state grammar school *Státní československé reformní reálné gymnasium* in Prague XIX.

L.S. Rieger was very active in public as well as in political life already as a grammar school student. He became a member of several student and other politically left-wing oriented organizations. Ideas of ISAV[2] appealed to him probably most of all. His inclination to left-wing politics grew even stronger during the Second World War when L.S. Rieger actively participated in illegal communist activities. In 1945, he joined the Communist Party of Czechoslovakia.

Besides mathematics, L.S. Rieger was also concerned with philosophy (perhaps under influence of his father), first with dialectical materialism. In this period, he published his first works, all of them philosophically oriented. In the subsequent years, L.S. Rieger was intrigued by ideas of a group of scholars, the so-called *Vienna Circle*.[3] The philosophical movement associated with this group has been called logical positivism. At this time, L.S. Rieger started studying logic intensively.

In October 1935, Ladislav Svante Rieger became a student of mathematics and physics at the Faculty of Science, Charles University. His lecturers were, above all, Vojtěch Jarník (1897–1970), Vladimír Kořínek (1899–1981), Karel Petr (1868–1950) and František Záviška (1879–1945). In February 1938, L.S. Rieger passed the first state exam in both the subjects. Unfortunately he did not have an opportunity to take the second one until 1945 due to the wartime difficulties.

## Professional Life

Very shortly after the end of the Second World War, in September 1945, L.S. Rieger passed the second state exam which enabled him to teach at Czechoslovak secondary schools. As early as March 1946, Rieger submitted his dissertation, passed two rigorosum examinations, and was conferred the degree Doctor of Natural Sciences (RNDr.). One year later, Ladislav Svante Rieger married neurologist Helena Holingerová (*1918). In their marriage two daughters were born, Jitka (*1950) and Alena (*1956).[4]

Already in August 1945, in fact before finishing studies at the university, L.S. Rieger accepted a position of an assistant at the Mathematical Institute (with František Rádl (1876–1957) as the head) of the Czech Technical University in Prague. In October 1951, he was appointed an associate professor and in the subsequent academic years 1951/52–1957/58 was

[1]Věra specialized herself in German studies and became an associate professor ("docent") at the Faculty of Arts, Charles University. Since her retirement she has been translating (especially prose) from German to Czech.

[2]Abbr. for *Internacia socialista asocio vivo*, International Socialistic Association of Life.

[3]The Vienna Circle was founded by M. Schlick (1882–1936), among its members were e.g. R. Carnap (1891–1970), P. Frank (1884-1966) and O. Neurath (1882–1945).

[4]Helena Riegrová spent most of her professional career in the *Institute for further education of physicians and pharmacists*, in 1968, she received the degree associate professor. Jitka is working as a clinical psychologist with specialization in psychotherapy. Alena Riegrová studied special pedagogy and presently is individually teaching retarded children.

the head of the *Mathematical Institute of the Faculty of Engeneering*. Rieger's activity at the technical university was mainly of a pedagogical character. In this period, he gave mainly compulsory lectures for the first and second grade students. This activity was, considering a wide range of lectures and a great amount of students, very time-consuming. Besides, he lead some optional courses, e.g. in vector analysis, operator calculus, and statistics.

At the end of 1958, Ladislav Svante Rieger moved to the *Mathematical Institute of the Czechoslovak Academy of Sciences* where he spent the rest of his life. He worked in the division of numerical methods. At the beginning of the following year, Rieger submitted his doctoral dissertation whose opponents were V. Alda, M. Katětov (1918–1995) and A. Mostowski (1913––1975). On the grounds of its successful defence, which took part in December that year, L.S. Rieger was conferred the degree Doctor of Physical and Mathematical Sciences (DrSc.).

Apart from the main pedagogical and research workplaces, L.S. Rieger marginally lectured also at the Charles University. From the academic year 1951/52 (with short breaks) until his death in 1963, he lead some optional and recommended courses, first at the Faculty of Science, and from 1952 at the Faculty of Mathematics and Physics. He usually had one two-hour course each semester, namely lectures and seminars in mathematical logic and axiomatic set theory. Participants of Rieger's seminars became later his successors in these areas. The persons in question were Jiří Bečvář, Petr Vopěnka (*1935) whose seminars followed up with Rieger's seminar after his death, and former Rieger's postgraduate student Petr Hájek (*1940).

In the period 1954–1960, L.S. Rieger was one of editors of the mathematical journal *Časopis pro pěstování matematiky*, and in the following year, he went over to an editorial board of *Czechoslovak Mathematical Journal*. In the years 1959–1962, Rieger also wrote reviews to the international reference journal *Mathematical Reviews*.

Ladislav Svante Rieger died untimely at the age of 46. On February 14, 1963 he succumbed to brain cancer.

## Characteristics of Rieger's Work

The complete list of Rieger's publications comprises 50 papers (6 of them, published before 1948, are of a philosophical character), nearly half of them are original scientific treatises. Predominantly, the publications have a character of journal articles, exceptions are three textbooks, one monograph and one popularizing work. As mentioned above, Rieger's principal papers can be divided into three main domains; algebra, mathematical logic, and axiomatic set theory. Let us now describe evolution of Rieger's mathematical interests.

The first mathematical area which L.S. Rieger devoted himself to was theory of ordered and cyclically ordered groups. He already studied related problems in a difficult wartime period 1941–1944 and consulted them by means of correspondence with V. Kořínek. At the end of 1945, Rieger submitted his results in the three-part dissertation *On ordered and cyclically ordered groups I, II, III* which was subsequently published in a form of three papers [R1] (1946), [R2] (1947), and [R3] (1948). For outstanding qualities of the dissertation L.S. Rieger obtained an award by the *Royal Czech Scientific Society*.

There are several references to Rieger's results from this field in scientific treatises and also in the most significant monographs on ordered algebraic structures. Many of them are in works published after 1990. Rieger's research in cyclically ordered groups was followed especially by eminent Slovak mathematician Ján Jakubík (*1923).

Thereafter (1949–1951) L.S. Rieger focused on problems of lattice theory, especially of Boolean algebras (works [R4]–[R7]). Perhaps most of all he was inspired by M.H. Stone (1903––1989) and G. Birkhoff (1911–1996). In a range of works from this field application to logic is typical.

In 1950, Rieger spent six months on a study visit in Warsaw, Poland. This stay, especially cooperation with prominent Polish logician Andrzej Mostowski (1913–1975), was of a great importance for his further scientific work. Despite having been intensively concerned with study of mathematical logic before, in Warsaw L.S. Rieger probably reached a decision to pursue a continual research in this area.

Rieger's algebraic approach to predicate calculus, which he presented at Mostowski's seminar, gained a favourable response and some Polish mathematicians continued in his investigation (e.g. H. Rasiowa (1914–?) and R. Sikorski (1920–1983)).

It is hard to draw a line between algebra and logic in Rieger's works. Papers [R8], [R10], [R11] (1951–1955) are on the boundary of abstract algebra and mathematical logic, their common and essential feature is algebraization of mathematical logic.

Publications from these two areas belong to the most cited Rieger's works (including citations in several significant monographs on lattice theory as well as on mathematical logic).

Already in 1950, L.S. Rieger started working on his only but substantial monograph about algebraic methods of mathematical logic. His aim was to summarize relevant results from algebraic logic achieved in that period. In fact, the monograph itself followed two Rieger's teaching texts from the years 1951 and 1961. Rieger also made use of his results summed up in the manuscript which was published after his death as [R20]. However, he was not able to finish his book. Thanks to help of P. Hájek the monograph was published in 1967 as [R21].

Around 1954, L.S. Rieger started the study of specific problems of axiomatic set theory, considering Gödel's axiomatics. He was primarily concerned with Gödel's theory of finite sets to which works [R12] (1957), [R14] (1959), and [R17] (1963) are devoted. They are of a great importance in Rieger's research in set theory. The first two parts, *A contribution to Gödel's axiomatic set theory I, II*, formed his doctoral dissertation. Among other works from this area are papers [R15] (1959), and [R16] (1960). In [R19] (1963) Rieger gave a new proof of consistency of the axiom of choice and the generalized continuum hypothesis.[5]

Apart from his research in Gödel's axiomatic set theory, in a few last years L.S. Rieger worked at new trends on the margin of mathematical logic associated with development of the first computers (publications [R13] (1958) and [R18] (1963)).

## Ordered and Cyclically Ordered Groups

A group is called *ordered* if the set of its elements is linearly ordered in such a way that it is possible to multiply the ordering relation from both the left and the right side. Similarly, if a trinomial relation of cyclical ordering is given in the set of elements of a group such that it can be multiplied from both the sides, the group is called *cyclically ordered*.[6]

The aim of the three papers [R1], [R2], [R3] was to investigate how the fact that a group can be (cyclically) ordered determines the algebraic structure of the group, and in what manner the structure is determined if the group is (cyclically) ordered in a specific way. The first paper is devoted exlusively to ordered groups, in the second one cyclically ordered groups are introduced. In the third work these groups are studied by a full use of topological means.

Let us remark that before Rieger only commutative ordered groups were systematically studied, namely in *Hahn* [1907].

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[5] This result was first proved by K. Gödel in 1940.

[6] A cyclically ordered group $\mathcal{G}$ is defined by the following axioms:
for every $x, y, z \in \mathcal{G}, x \neq y \neq z \neq x$, and every $v \in \mathcal{G}$

1. $< x, y, z < \Rightarrow < y, z, x <$
2. one of the following possibilities is satisfied

$$< x, y, z < \text{ or } < y, x, z <$$

3. $(< x, y, z < \ \& \ < x, z, v <) \Rightarrow < x, y, v <$
4. $< x, y, z < \Rightarrow < vx, vy, vz < \ \& \ < xv, yv, zv < .$

One of the principal results given in the work [R1] is a (purely algebraic) *necessary and sufficient condition for a group to be able to be ordered.* For that purpose L.S. Rieger introduced the notion of *magnitude subgroups.* This result was later stated by other mathematicians by means of convex subgroups. The notion of magnitude subgroups is one of the central terms in the work and Rieger addressed some other problems and features related to it.

Further, L.S. Rieger stated *a necessary and sufficient condition for an ordered group to be commutative.* Thereby he gave a connection between general and commutative groups, and integrated Hahn's results (especially those concerning a lexicographical product of ordered groups) into general theory of ordered groups.

In the paper [R2] cyclically ordered groups are studied. The relation of cyclical ordering was first introduced by Eduard Čech (1893–1960) in *Čech* [1937]. Ladislav Svante Rieger was probably the first who was intensively working in the field of cyclically ordered groups and who obtained substantial results.

Ordered groups are (in a certain sense) a special case of cyclically ordered groups. It is easy to realize that arbitrary ordered group can form a cyclically ordered one, simply by putting (for $x \neq y \neq z \neq y$) $< x, y, z <$ if either $x < y < z$ or $y < z < x$ or $z < x < y$.

Further, L.S. Rieger proved the "inverse" statement that *every cyclically ordered group can be represented as a factor group of a certain ordered group.*

The last treatise [R3] contains applications partly of results derived in the previous two works, and partly from general theory of topological groups. Ordered and cyclically ordered groups belong among well known *topological groups.*[7] First, Rieger introduced topologies to ordered and cyclically ordered groups and called them "natural".

Further, he stated several algebraic consequences derived from properties of natural topologies of both types of groups. The main theorem can be formulated as follows:

*Every cyclically ordered group which is compact in its natural topology is isomorphic with a multiplicative group of complex numbers with an absolute value equal to one.*

### ***On Groups and Lattices* [R9]**

Let us now mention Rieger's book *On groups and lattices* [R9] published in 1952. It has quite an unusual position in his works for its popularizing character. This book was meant for common readers, especially for secondary school students, and its aim was to present fundamental notions of group theory and lattice theory to wider public. The first part (*On lattices*) was slightly altered and republished in 1974.

As far as the situation in algebraic textbooks is concerned, only one textbook on groups had been in use in Czechoslovakia before Rieger's work [R9] was published. It was O. Borůvka's *Introduction to group theory* (*Borůvka* [1944]). This book was intended for university students and in comparison with [R9] it was far more difficult for understanding. As for lattice theory, with respect to a very short period of existence of this discipline (less than 20 years), it is apparent that no other textbook had been published in Czechoslovakia before.

## Lattice Theory

In the following we will describe the most significant Rieger's works from lattice theory in more detail.

***A note on topological representations of distributive lattices* [R4].** This work is a continuation of core investigation of M.H. Stone in topological representation of distributive lattices (*Stone* [1937]).

A distributive lattice $\mathcal{L}$ is said to be *topologically represented* in a topological $T_0$-space $S(\mathcal{L})$ if there exists an isomorphism of $\mathcal{L}$ onto a set-ring $\mathcal{R}$ of certain open subsets of $S(\mathcal{L})$ such that

[7] I.e. groups whose set of elements has such topology that multiplication as well as assignment of an inverse element are continuous mappings.

$\mathcal{R}$ forms an open basis of $S(\mathcal{L})$.

M.H. Stone described a "universal" space $\bar{S}(\mathcal{L})$ which contains every representation space $S(\mathcal{L})$ as a dense subset. $\bar{S}(\mathcal{L})$ is the space of all prime filters of $\mathcal{L}$. L.S. Rieger presented another characterisation of $\bar{S}(\mathcal{L})$ for distributive lattices with zero and unit. As a consequence he obtained the assertion that *any distributive lattice with zero in which all prime filters are maximal is a generalized Boolean algebra and any distributive lattice with zero and unit in which all prime filters are maximal is a Boolean algebra.*

***On the lattice theory of Brouwerian propositional logic* [R5].** The purpose of this paper was to show that by means of the notion of a special residuated lattice with zero and unit (which Rieger called *sdruz-lattice*) lattice theory can work as an efficient mathematical tool for both the syntax and the semantics of a language using Brouwerian logic.[8] First who studied lattice-theoretical interpretation of the Heyting calculus was G. Birkhoff in *Birkhoff* [1940].

However, timing of Rieger's work [R5] was rather unfortunate because a year before the paper *McKinsey and Tarski* [1948] was published which deals with similar problems. L.S. Rieger did not have access to this treatise and proved several result independently.

One of the primary Rieger's results is *characterization of the Heyting propositional calculus as a free sdruz-lattice with countable infinity of generators*, and thus its semantical interpretation by means of countable infinity of "logical values". He showed that the same sdruz-lattice can be constructed in various ways within four areas.

By simple algebraic considerations, L.S. Rieger obtained some theorems of K. Gödel (1906–1948) (*Gödel* [1933]) and others. He also gave a relatively simple and elementary computational method for a decision problem of the Heyting calculus which can be applied to general topology and abstract algebra. Furthermore, Rieger presented the algebraic essence of relations between the classical and Heyting propositional calculus.

***Some remarks on automorphisms of Boolean algebras* [R6].** The main subject of the work [R6] is construction of a Boolean algebra admitting no proper homomorphic mapping onto itself. L.S. Rieger found the solution by transferring the problem onto the topological problem of *existence of a zero dimensional compact space without proper homeomorphic transformations onto any of its subspaces.* Thus he gave the (negative) answer to Birkhoff's Problem 74, one of the problems stated in the monograph *Lattice theory* (*Birkhoff* [1948]). Rieger concluded the paper with some remarks on Birkhoff's Problem 75.

***On free $\aleph_\xi$-complete Boolean algebras* [R7].** Boolean algebra $\mathcal{A}$ is called *$\aleph_\xi$-complete* if any of its subsets whose power does not exceed $\aleph_\xi$ has a supremum and an infimum in $\mathcal{A}$.

In the first and main part of the paper [R7], L.S. Rieger investigated general properties of free $\aleph_\xi$-complete Boolean algebras with a special attention paid to $\aleph_0$-complete Boolean algebras (shortly $\sigma$-algebras) due to their numerous applications. The second part comprises of applications of obtained results to the domain of mathematical logic.

Initially, L.S. Rieger provided a constructive proof of *existence of a free $\aleph_\xi$-complete Boolean algebra* and addressed its uniquess and "universality" as well. Further, he arrived at the conclusion that *a free $\sigma$-algebra can be $\sigma$-isomorphically represented by a $\sigma$-field of subsets of a set of its $\sigma$-prime filters.* He also proved that this assertion does not hold for free $\aleph_\xi$-complete Boolean algebras in general.

As a consequence Rieger obtained the following statement: *a free $\sigma$-algebra with $m$ generators can be $\sigma$-isomorphically represented by the minimal $\sigma$-field of Borel subsets of a generalized Cantor discontinuum $C_m$.* By obtained results L.S. Rieger found solutions of Problems 78, 79, and 80 from the monograph *Birkhoff* [1948].

In the second part, as mentioned above, L.S. Rieger applied achieved results to mathemat-

[8] *Intuitionistic (Brouwerian) logic* differs from the classical logic by replacing the "non-constructive" postulate $A \vee nonA$ or the law of double negation $non(nonA) \rightarrow A$ by the law of contradiction $(A \wedge nonA) \rightarrow B$. Its formal system is called the *Heyting calculus.*

ical logic, namely to the *Tarski-Lindenbaum algebra*[9] of a lower predicate calculus. Thus he obtained an algebraic proof of the famous Gödel's completeness theorem (translated into the language of theory of Boolean algebras).

**Other partly algebraic works.** To conclude this treatise on Rieger's contribution in abstract algebra, let us make a few remarks on his works on the boundary of algebra and mathematical logic.

In the paper [R8] L.S. Rieger introduced a generalization of the notion of $\sigma$-algebra in the sense that countably infinite joins and meets are performed on members of certain multiple sequencies (called *marked*). Rieger showed that *if the set of marked sequencies is countable then a countable generalized $\sigma$-algebra can be isomorphically represented by a set field.* The main aim of the paper is the same as in the second part of [R7]; by application of the results to the Tarski-Lindenbaum algebra of a lower predicate calculus, L.S. Rieger immediately obtained a new proof of Gödel's completeness theorem.

Work [R10] is also devoted to the similar problems. Rieger proved here in a new way one principal theorem on generalized $\sigma$-algebras. At the end various proofs of Gödel's completeness theorem known by that time are compared. In [R11] more general *Suslin algebras* are introduced and used for description of predicate variables.

## Conclusion

Ladislav Svante Rieger was an oustanding personality of the Czech and Slovak mathematics. His contribution should be known to wider mathematical public. This is the main aim of our research.

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## List of Publications of Ladislav Svante Rieger

[R1] O uspořádaných a cyklicky uspořádaných grupách I, *Věstník Král. české spol. nauk*, tř. mat.--přír., 1946, č. 6, 1–31.

[R2] O uspořádaných a cyklicky uspořádaných grupách II, *Věstník Král. české spol. nauk*, tř. mat.--přír., 1947, č. 1, 1–33.

[R3] O uspořádaných a cyklicky uspořádaných grupách III, *Věstník Král. české spol. nauk*, tř. mat.--přír., 1948, č. 1, 1–26.

[R4] A note on topological representations of distributive lattices, *Čas. pro pěst. mat. a fys.* **74**(1949), 55–61.

[R5] On the lattice theory of Brouwerian propositional logic, *Acta Facultatis Rerum Naturalium Universitatis Carolinae* **189**(1949), 3–40.

[R6] Some remarks on automorphisms in Boolean algebras, *Fundamenta Mathematicae* **38**(1951), 209–216.

[R7] On free $\aleph_\xi$-complete Boolean algebras, *Fundamenta Mathematicae* **38**(1951), 35–52.

[R8] On countable generalized $\sigma$-algebras, with a new proof of Gödel's completeness theorem, *Czechoslovak Math. Journal* **1(76)**(1951), 29–40.

[R9] O grupách a svazech, *Cesta k vědění*, sv. 65, Přírodovědecké vydavatelství, Praha, 1952.

[R10] O jedné základní větě matematické logiky, *Čas. pro pěst. mat.* **80**(1955), 217–231.

[R11] Ob algebrach Suslina ($S$-algebrach) i ich predstavlenii, *Czechoslovak Math. Journal* **5(80)**(1955), 99–142.

[R12] A contribution to Gödel's axiomatic set theory I, *Czechoslovak Math. Journal* **7(82)**(1957), 323–357.

[R13] O teorii neuronových sítí, *Aplikace matematiky* **3**(1958), 243–274, 483.

[9] I.e. a set of classes of formulas when two formulas belong to the same class if they are logically equivalent.

[R14] A contribution to Gödel's axiomatic set theory II, *Czechoslovak Math. Journal* **9(84)**(1959), 1–49.

[R15] Sur le problème des nombres naturels, *Infinitistic methods, Proceedings of the Symposium on the Foundations of Mathematics*, Warsaw, 1959, 225–233.

[R16] Problém tzv. absolutně nerozhodnutelných vět teorie čísel, *Čas. pro pěst. mat.* **85**(1960), 1–13.

[R17] A contribution to Gödel's axiomatic set theory III, *Czechoslovak Math. Journal* **13(88)**(1963), 51–88.

[R18] Ke Kleeneho normální formě strojově vyčíslitelných funkcí, *Čas. pro pěst. mat.* **88**(1963), 349–363.

[R19] On the consistency of the generalized continuum hypothesis, *Rozprawy matematyczne* **31**(1963), 1–45.

[R20] Zu den Strukturen der klassischen Prädikatenlogik, *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* **10**(1964), 121–138.

[R21] Algebraic methods of mathematical logic, Academia, Praha, 1967.

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