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A New Switch-Mode Noise-Constrained Transform Domain NLMS Adaptive Filtering Algorithm

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Abstract—The transform domain normalized least mean squares (TDNLMS) algorithm is an efficient adaptive algorithm, which offers fast convergence speed with a reasonably low arithmetic complexity. However, its convergence speed is usually limited by the fixed step-size so as to achieve a low desired misadjustment. In this paper a new switch-mode noise-constrained TDNLMS (SNC-TDNLMS) algorithm is proposed. It employs a maximum step-size mode in initial convergence and a noise-constrained mode afterwards to improve the convergence speed and steady-state performance. The mean and mean square convergence behaviors of the proposed algorithm are characterized to study their good numerical stability and computational simplicity. In particular, TDNLMS algorithm [3-5] is attractive due to its fast convergence speed and reasonably low arithmetic complexity. It exploits the decorrelation property of transformations, such as the discrete cosine transform (DCT) or the wavelet transform (WT), to approximately prewhiten the input signal to reduce the eigenvalue spread of the input autocorrelation matrix. Consequently, the convergence rate can be improved significantly. In conventional TDNLMS algorithms, the step-size is fixed and therefore the convergence speed is limited by the desired misadjustment. This has motivated considerable interest in designing reliable and efficient variable step-size (VSS) algorithms to overcome this drawback [6-11]. These algorithms aim to employ large step-size to speed up the convergence rate initially and gradually decrease the step-size in order to achieve a low excess mean square error (EMSE). This is often accomplished by varying the step-size values based on a certain measure of convergence status [7-11]. In [6], the modified VSS TDNLMS (MVSS-TDNLMS) algorithm varies the step-size by estimating the noise power.

In this paper, a switch-mode noise-constrained TDNLMS (SNC-TDNLMS) algorithm is proposed. It exploits the prior knowledge of the additive noise variance as in the NCLMS approach [9] and gives rise to a VSS algorithm. Moreover, the improved performance is found to be obtained if maximum step-size is employed at initial convergence while the NC adaptation is more suitable to be used near convergence in order to reduce the steady-state misadjustment. Therefore, the proposed method is extended to include a switch-mode scheme which employs a maximum step-size mode (MSM) during initial convergence and a NC mode (NCM) afterwards so as to simultaneously improve the convergence speed and steady-state performance. The mean and mean squares convergence of the proposed SNC-TDNLMS algorithm is studied and its steady-state EMSE is characterized. Based on the theoretical results, an automatic threshold selection scheme for mode switching is developed. Computer simulations are conducted to show the effectiveness of the proposed algorithm and verify the theoretical results.

I. INTRODUCTION

Adaptive filters are frequently used in system identification and related problems, where the statistics of the underlying signals are either unknown a priori, or slowly-varying. The adaptive filtering algorithms are usually variants of the well known LMS [1] and RLS [12] algorithms. The normalized LMS (NLM) algorithm [2] and the transform domain NLMS (TDNLMS) are also commonly used due to their good numerical stability and computational simplicity. In particular, TDNLMS algorithm [3-5] is attractive due to its fast convergence speed and reasonably low arithmetic complexity. It exploits the decorrelation property of transformations, such as the discrete cosine transform (DCT) or the wavelet transform (WT), to approximately prewhiten the input signal to reduce the eigenvalue spread of the input autocorrelation matrix. Consequently, the convergence rate can be improved significantly. In conventional TDNLMS algorithms, the step-size is fixed and therefore the convergence speed is limited by the desired misadjustment. This has motivated considerable interest in designing reliable and efficient variable step-size (VSS) algorithms to overcome this drawback [6-11]. These algorithms aim to employ large step-size to speed up the convergence rate initially and gradually decrease the step-size in order to achieve a low excess mean square error (EMSE). This is often accomplished by varying the step-size values based on a certain measure of convergence status [7-11]. In [6], the modified VSS TDNLMS (MVSS-TDNLMS) algorithm varies the step-size by estimating the noise power.

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II. THE SNC-TDNLMS ALGORITHM

A. Review of the TDNLMS Algorithm

Consider the identification of a linear time-invariant (LTI) finite impulse response (FIR) system by an adaptive filter with the same length. The impulse response coefficient vector of the system is assumed to be $w^*$ and it is of $L$ taps. The unknown system and adaptive filter are both excited by an input $x(n)$. The measured output of the system is $d(n)$, which is assumed to be corrupted by an additive noise $\eta(n)$, and $d(n)$ is applied to the desired input of the additive filter:

$$d(n) = (w^*)^T x(n) + \eta(n),$$

where $x(n) = [x(n), \cdots, x(n-L+1)]^T$ is the input vector.

The update equations for the TDNLMS algorithm are:
where \( W(n) = Cw(n) = \{W_{c_1}(n), W_{c_2}(n), \cdots, W_{c_N}(n) \}^T \) and \( X_c(n) = \{X_{c_1}(n), X_{c_2}(n), \cdots, X_{c_{c_1}}(n) \}^T \) are the transformed adaptive weight vector and signal vector. \( C \) is an \( L \times L \) transformation matrix such as DFT or DCT. \( \mu \) is the step-size. \( \lambda^i(n) = \text{diag}(\varepsilon^1(n), \varepsilon^2(n), \cdots, \varepsilon^i(n)) \) is an element-wise normalization matrix with \( \varepsilon_i(n) \) being the estimated power of the \( i \)-th signal component after transformation. In this paper, \( \varepsilon_i(n) = \sigma^2 + \alpha X_i^2(n) \) is considered, where \( \alpha_i \) is a positive forgetting factor smaller than one. \( \sigma_i \) is a small positive value which can be chosen as certain prior power estimate of the corresponding component.

**B. The SNC-TDNLMS Algorithm**

In [10], a transformation approach was proposed to derive the NC-NLMS algorithm from its LMS counterpart. This method is also applicable to the TDNLMS algorithm. Hence, the NC-based TDNLMS algorithm can be updated as

\[
\mu(n) = \alpha(n) + \gamma \lambda(n),
\]

(4)

\[
\lambda(n) = (1 - \beta) \lambda(n) + \epsilon \hat{J}(n),
\]

(5)

where \( \alpha, \beta, \gamma \) are constant parameters and \( \hat{J}(n) = e^2(n) - \tilde{\sigma}_v^2 \) is the instantaneous estimate of the EMSE. It can be seen that the convergence measure \( \hat{J}(n) \) is comparatively large during initial convergence, and hence, a larger value of \( \mu(n) \) will be chosen in order to speed up the convergence rate. As the EMSE decreases, \( \mu(n) \) is then gradually decreased to achieve a lower steady-state EMSE.

As suggested in [9], after fixing the nominal step-size \( \alpha \), \( \gamma \) should be chosen as a value large as possible to obtain a fast convergence speed, while \( \beta \) should be chosen as a small value to achieve a desired EMSE. However, the values of \( \gamma \) and \( \alpha \) are still constrained so that the step-size and hence the convergence speed will be significantly limited. From the mean convergence analysis, to be presented in Section III, we found that the mean weight error vector will converge faster if a maximum possible step-size is employed. On the other hand, the NC adaptation should be used when the adaptive filter is nearly converged in order to achieve the desired steady-state EMSE.

Because of the above observations and possible advantages, we propose below a novel switch-mode scheme for the variable step-size. It employs

1) the **maximum step-size mode (MSM)**, where a designed maximum step-size \( \mu_{\text{max}} \) is employed to achieve a faster convergence speed during initial convergence, and

2) the **noise constrained mode (NCM)**, where the step-size is adjusted as in the NC algorithms according to (4) and (5). Thus, the desired EMSE can be achieved after the maximum step-size mode is nearly converged.

Consequently, the corresponding updates for the step-size can be summarized by the following equations

\[
e(n) = d(n) - W^T(n)X_c(n),
\]

\[
W(n + 1) = W(n) + \mu_X X_c(n) e(n),
\]

(3)

where \( W(n) = Cw(n) = \{W_{c_1}(n), W_{c_2}(n), \cdots, W_{c_{c_1}}(n) \}^T \) and \( X_c(n) = \{X_{c_1}(n), X_{c_2}(n), \cdots, X_{c_{c_1}}(n) \}^T \) are the transformed adaptive weight vector and signal vector. \( C \) is an \( L \times L \) transformation matrix such as DFT or DCT. \( \mu \) is the step-size. \( \lambda^i(n) = \text{diag}(\varepsilon^1(n), \varepsilon^2(n), \cdots, \varepsilon^i(n)) \) is an element-wise normalization matrix with \( \varepsilon_i(n) \) being the estimated power of the \( i \)-th signal component after transformation. In this paper, \( \varepsilon_i(n) = \sigma^2 + \alpha X_i^2(n) \) is considered, where \( \alpha_i \) is a positive forgetting factor smaller than one. \( \sigma_i \) is a small positive value which can be chosen as certain prior power estimate of the corresponding component.

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\[ \exp(-ue_\epsilon g(\tilde{\beta}))\eta_n = g(\tilde{\beta}) = (1 + 2\tilde{\beta}R_{x_i,x_i,j}) = \alpha_iu \] and \( R_{x_i,x_i,j} \) being the \((i,j)\)th element of \( R_{x_i,x_i,j} \). For notational convenience, let \( b = a - 1 \), then \( J_n(n) = E[e^2(n)] = \sigma_{\epsilon}^a \). We shall only focus on the NC adaptation mode, as the MSM mode is equivalent to the TDNLMS algorithm with a maximum step-size. The latter can be obtained by assuming \( \mu(n) \) to be a constant and the details can be found in [5].

Based on (9) and expressing the weight error \( v(n) \) as \( V(n) = D_v^{1/2}v(n) \), we get
\[ E[V(n+1)] = (I - E(\mu(n))R_{x_i,x_i})E[V(n)], \tag{12} \]
where \( R_{x_i,x_i} = D_v^{1/2}R_{x_i,x_i}D_v^{-1/2} \) is the correlation matrix of a scaled input vector \( X_n = D_v^{1/2}X_n \). Since it is symmetric, it can be written as the following eigenvalue decomposition (EVD): \( R_{x_i,x_i} = U_{x_i} \Lambda_{x_i} U_{x_i}^T \) and \( \Lambda_{x_i} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_i) \) contains corresponding eigenvalues. Since Eqn. (12) is identical to the difference equation of the LMS algorithm, the classical result of the maximum possible step-size can be obtained as \( \mu_{\text{max}} = 2/\lambda_i \), where \( \lambda_i \) is the maximum eigenvalue of \( R_{x_i,x_i} \).

B. Mean Square Convergence Analysis
To evaluate the mean square behavior, multiplying \( v(n) \) by its transpose and taking expectation on both sides, one gets a difference equation of the weight error covariance matrix:
\[ \mathcal{E}(n+1) = \mathcal{E}(n) - E(\mu(n))D_v R_{x_i,x_i} \mathcal{E}(n) - E(\mu(n))E(V(n)), \tag{13} \]
where \( \mathcal{E}(n) = E(\mu(n))E(v(n)v^T(n)) \), \( x_i = E(\mu(n))E(x_i)x_i^T \). The \((i,j)\)-th element of \( s_i \) is evaluated to be [5]
\[ s_{i,j} = s_{i,j}^{(0)}(r_{x_i,x_i}) \eta_n \eta_n^T \] for \( r_{x_i,x_i} \).

(4) \[ \text{where } s_{(i,j)}^{(0)} = 2\sum p_n \left( -4R_{x_i,x_i,j} \right)^{\alpha_i^{(3/2,2)}} \alpha_i^{(3/2,2)}, \quad s_{(i,j)}^{(1)} = \sum p_n \left( -4R_{x_i,x_i,j} \right)^{\alpha_i^{(3/2,2)}} \alpha_i^{(2,2)}, \quad s_{(i,j)}^{(2)} = \sum p_n \left( -4R_{x_i,x_i,j} \right)^{\alpha_i^{(3/2,2)}} \alpha_i^{(1,2)}, \quad s_{(i,j)}^{(3)} = \sum p_n \left( -4R_{x_i,x_i,j} \right)^{\alpha_i^{(3/2,2)}} \alpha_i^{(0,2)}, \quad s_{(i,j)}^{(4)} = \] and \( \Gamma_{(i,j)} = \sum \exp(-ue_\epsilon g(\tilde{\beta})) \eta_n \eta_n^T \) for \( r_{x_i,x_i,j} \).

Using (14), the last term in (13) can be simplified to
\[ E[V(n)] = \sum (r_{x_i,x_i,j}) \eta_n \eta_n^T \] for \( r_{x_i,x_i,j} \).

(15) \[ \text{where } \mathcal{D}_v = \text{is a diagonal matrix with its } i\text{-th element } \mathcal{D}_v^{(i)} = r_{x_i,x_i,j} \eta_n \eta_n^T, \quad \mathcal{S}(n) = s_{y_0}^T(\Gamma_n), \quad s_{y_0}^T(\Gamma_n) = \sum \mathcal{S}_{(i,j)}^{(0)} \] and \( \sigma_{\epsilon}^{(n)} = \text{Tr}(R_{x_i,x_i}(\mathcal{E}(n))) + \sigma_{\epsilon}^{(n)} \).

It can be shown that a sufficient condition for mean square convergence of (15) is given by
\[ E[\mu^2(n)] \leq \frac{2}{\mu_{\text{max}}(\text{TDNLMS})} \text{Tr}(D_v \mathcal{D}_v + D_v \mathcal{D}_v + D_v \mathcal{D}_v) = \mu_{\text{max}}(\text{TDNLMS}) \tag{16} \]
where \( \mathcal{D}_v = (D_v^{(i)} + D_v^{(i)}) + D_v^{(i)} \) and \( D_v^{(i)} \) are diagonal matrices with the \( i\)-th diagonal element equal to that of \( S^{(i)} \).

To evaluate the steady-state EMSE, \( E[\mu^2(n)] \) is evaluated from (4) and (5) as follows
\[ E[\mu^2(n)] = \mu^2(1 + 2E(\lambda(n)) + E(\lambda(n))) \tag{17} \]
\[ E[\mu^2(n)] = (1 - \beta^2) E(\lambda(n)) + \beta(1 - \beta) E(\lambda(n))J(n) \tag{18} \]
\[ \frac{1}{\text{max}_\epsilon} \] and \( b \). If \( E[\lambda(n)] \) converges, the limiting value of \( E[\mu^2(n)] \) is obtained by using (17)(18).

(19) \[ \frac{1}{\text{max}_\epsilon} \] and \( b \). Here we have dropped the 1st term in (15) since from numerical results it is observed small compared with the 2nd term.

Assuming \( J \) to be small, the terms involving \( J \) and \( J \) can be dropped to obtain a good approximation of \( J \) as
\[ J_n = \frac{1}{\text{max}_\epsilon} \tag{20} \]
\[ \text{where } \delta \text{ is a constant.}

To prevent (22) from being unbound if the denominator is zero, the following gives an approximated condition on the maximum nominal step-size for mean squares convergence
\[ \alpha < \frac{2 - \beta \sigma_{\epsilon}^{(n)}}{(A_i + \text{max}_\sigma) \text{TDNLMS}} \tag{23} \]

C. Switching Threshold and Parameter Search
1) Selection of \( T \): From (22), the steady-state EMSE at a fixed step-size is lower bounded by \( \mu \text{max}^2 \).

Based on (11) and (18), \[ \text{var}(\tilde{\lambda}(\infty)) = E[\tilde{\lambda}(\infty)^2] - E(\tilde{\lambda}(\infty))^2 = \frac{\mu_{\text{max}}^2}{\text{var}^{-1}(\mu_{\text{max}})}((1 - \beta^2)E(\lambda_\text{max})c_r) ] + \sigma_{\epsilon}^{(n)} \] with \( b = 0 \) and \( c_r = \text{TDNLMS} \).

Assuming \( \tilde{\lambda}(\infty) \) is Gaussian distributed, \( T \) can be chosen as \( \text{the upper bound of } \tilde{\lambda}(\infty) \), i.e. \( T = (\text{max}_\epsilon c_r + \mu_{\text{max}} c_r + c_r) \text{TDNLMS} \).

If \( \text{max}_\epsilon \) as in (16) is used, then \( \mu_{\text{max}} c_r = 2 \). On the other hand, \( \epsilon \) can be adjusted
experimentally and appropriate values are around 3 to 5. 2) Choice of $\beta$ and $\beta_1$: Generally, we observe that the parameter $(1 - \beta)$ (or $(1 - \beta_1)$) acts as a forgetting factor and controls the averaging process of the instantaneous MSE. The best value of $\beta$ (or $\beta_1$) depends mildly on the convergence speed. The recommended value for $\beta$ is around 0.01. For $\beta_1$, a larger value around 0.1 can be used because the algorithm is converging at the fastest speed under the MSM mode.

3) Choice of $\alpha$, $\delta$ and $\gamma$: According to (22), the product $\alpha \delta$ is chosen to be 0.1. Since the TD algorithms are usually used, where the validness of the independent assumption in the definition of $\delta$ in (22). If $\sigma^2_\epsilon$ is not exactly known, we recommend to use the upper bound of $\sigma^2_{\epsilon,\text{max}}$ in (22).

IV. SIMULATION RESULTS

In this section, computer simulations are conducted to evaluate the convergence behavior of the proposed algorithm and verify the analytical results obtained in Section III. As a comparison, we also consider the conventional TDNLMS and MVSS-TDNLMS algorithms [6]. These algorithms use a DCT transformation due to its wide usage and efficiency in practice. To simplify the comparison with the other algorithms, the estimated power of input element is chosen as $\epsilon_i(n) = \sigma + \alpha X^2_{\epsilon_i}(n)$, where $\sigma$ is the input power and $\alpha = 0.1$. The results for the estimated power in place of $\sigma$ are similar. The simulations are performed using the system identification model and the unknown system to be estimated is an L-order ($L=8$) FIR filter. Different signal-to-noise ratios (SNRs) are used to examine the performance of the parameter selection scheme proposed in Section IIIIC. The maximum step-size is $\mu_{\text{max}} = 0.13$ and $\kappa = 4$. Since the TD algorithms are usually employed when the input is colored, the first order auto-regressive process is considered: $x(n) = 0.9x(n-1) + g(n)$, where $g(n)$ is a zero-mean white Gaussian noise. For fair comparison, the algorithm parameters are chosen such that all the algorithms achieve the same steady-state EMSE. The step-size for the TDNLMS algorithm is 0.007; $\gamma$ in MVSS-TDNLMS is 0.996. For SNC-TDNLMS $\delta$ is chosen to be 0.1 and $\alpha$ is determined to be 0.0064. Thus, $\gamma$ is calculated from (22) as 0.2, 2 and 20 for SNR=0 dB, 10 dB and 20 dB, respectively. The recommended values $\beta = 0.01$ and $\beta_1 = 0.1$ are used. The learning curves of EMSE are shown in Figs. I(a), (b), (c). It can be seen that the SNC-TDNLMS algorithm generally converges at the fastest speed. The improvement is more significant as the SNR increases. The theoretical and simulation results agree well with each other, especially when the algorithms are near convergence. The deviation at initial convergence at high SNR is caused by the maximum step-size used, where the validness of the independent assumption in (A2) becomes less accurate. Simulations for white Gaussian input can be found in the supplementary document [13]. And results for longer filter length are similar.

REFERENCES


Fig. 1 Learning curves of EMSE for the time-invariant channel identification problem with first-order AR input at SNR= (a) 0 dB (b) 10 dB (c) 20 dB.