ABSTRACT

We propose a new approach to study the dynamical implications of mass models of clusters for the velocity structure of galaxies in the core. Strong and weak lensing data are used to construct the total mass profile of the cluster, which is used in conjunction with the optical galaxy data to solve in detail for the nature of galaxy orbits and the velocity anisotropy in the central regions. We also examine other observationally and physically motivated mass models, specifically those obtained from X-ray observations and N-body simulations. The aim of this analysis is to understand qualitatively the structure of the core, and to test some of the key assumptions of the standard picture of cluster formation regarding relaxation, virialization and equilibrium. This technique is applied to the cluster Abell 2218, where we find evidence for an anisotropic core, which we interpret as indicating the existence of a dynamically disturbed central region. We find that the requirement of physically meaningful solutions for the velocity anisotropy places stringent bounds on the slope of cluster density profiles in the inner regions.

Key words: galaxies: clusters: general – galaxies: clusters: individual: Abell 2218 – gravitational lensing.
dynamically young but also quite disturbed. The nature of orbits is therefore an important indicator of the dynamical state of both the inner and outer regions of the cluster. Principally radial orbits, expected at the outskirts, are the signature of a region dominated by infall, whereas isotropic orbits imply the existence of a well-mixed region.

Previous studies of the velocity dispersion profile and estimates of the degree of anisotropy in clusters have provided ambiguous results, primarily due to the lack of knowledge of the underlying mass distribution. The velocity structure of galaxies in clusters has been studied in detail for Coma and A2670 by Kent & Gunn (1982), Thé & White (1986), Merritt (1987), Sharples, Ellis & Gray (1988), Colless & Dunn (1995) and several other groups. These analyses used the observed galaxy positions and velocities to constrain the distribution of total mass and simultaneously find consistent and physically meaningful solutions for the velocity anisotropy.

Thé & White (1986) examined the uncertainties in the virial mass profiles derived for Coma from observational data. They showed that a wide range of mass models are consistent, consequently permitting a large range of orbital structures. Mass models that were more compact (but had low overall masses) implied circular orbits for the galaxies, whereas the higher mass models implied predominantly radial orbits. Merritt (1987) examined the relative distribution of the dark and luminous matter in Coma, and showed that it was impossible to distinguish between models where the galaxies trace the total mass and were on isotropic orbits and models in which the dark matter was very concentrated and the galaxies were on primarily transverse orbits. Therefore sufficient constraints on the total mass distribution and the velocity anisotropy cannot be obtained simultaneously using only the luminous tracers. In their recent survey of the Coma cluster, Colless & Dunn (1996) found that the velocity distribution is highly non-Gaussian. The dynamics can be better interpreted in terms of an ongoing merger between two subclusters, thus indicating that the system is not in virial equilibrium.

Similar studies of the cluster A2670 have also been inconclusive; in the faint photometric and spectroscopic survey by Sharples, Ellis & Gray (1988), both extremely anisotropic models and nearly isotropic ones were indistinguishable in terms of goodness-of-fit with respect to the available data.

In these clusters and others that have been studied, the principal uncertainty in the determination of the anisotropy arises from ignorance of the distribution of the total mass. In our analysis, independent data from lensing that constrain the distribution of total mass and simultaneously fit the observed number density in a given cluster to the limitations of the available data, we implicitly assume that the E and SO galaxies efficiently trace the overall mass distribution.

While it can be argued that there is insufficient observational evidence for a core in the galaxy distribution in clusters (Merritt & Tremblay 1994; Carlberg et al. 1996), it is nevertheless instructive to examine the modified Hubble law profile, which provides a reasonable fit to sparser samples.

**PROFILE A:**

\[ v_g(r) = \frac{v_0}{(1 + r/r_s)^{1.5}}. \]  

The core radius \( r_s \) for typical clusters ranges from 150 to about 300 \( h_{50}^{-1} \) kpc. A least-squares method is then used to determine the values of \( \rho_0 \) and core radius that simultaneously provide the best fit to the data. For well-sampled clusters (Katgert et al. 1996; Yee et al. 1996), it is found that the number density of galaxies can be fitted by a generic profile of the form

**PROFILE B:**

\[ v_g(r) = \frac{v_0}{(r/s)^{\alpha} (1 + r/s)^{2-\alpha}}. \]

with \( \alpha = 1 \), and \( s \) being a scale-radius ranging from 200 to 400 \( h_{50}^{-1} \) kpc. This profile looks asymptotically like the modified Hubble law, but has a central cusp. We examine both these profiles and their implications for the core dynamics in detail in Section 5.
In principle, non-parametric maximum-likelihood and regression techniques developed by Merritt & Tremblay (1994) and Merritt & Gebhardt (1995) are more accurate in terms of characterizing the surface number density distribution; given the quality of data available at present for lensing clusters (at moderately high redshift) with a secure mass model, good photometry and sufficient velocities, we restrict our current analysis to a parametric approach, primarily on the basis of statistical adequacy, given the sample sizes that we are dealing with and the convenience of working with analytic forms. Besides, the non-parametric methods necessarily involve smoothing the data in order to yield confidence limits; this introduces a bias that increases with the degree of smoothing employed and is hence undesirable for sparse samples. For a more complete data set, however, a non-parametric likelihood method would be more appropriate.

2.1.2 Line-of-sight velocity dispersion profiles

The only measured component of the velocity dispersion of the galaxies is the projection along the line of sight. The interpretation of this measurement might be severely affected by substructure in the cluster and the presence of interacting subgroups. For any distant cluster ($z > 0.1$), the line-of-sight velocity dispersion is not determined accurately enough at present to construct a secure radial profile. Even for nearby clusters, the rich region close to the centre and the sparser outer regions are not sampled adequately to infer conclusively the asymptotic behaviour. For the well-studied cluster Abell 2218, the error bars in the measurements (see Fig. 6) are too large to perform any sensible fit to the available data and extract a radial profile.

2.1.3 X-ray observations

X-ray observations by Einstein, ROSAT and ASCA map the thermal bremsstrahlung emission from the hot intracluster gas at temperatures around $10^7 - 10^8$ K; the typical observed X-ray luminosities range from $10^{42}$ to $10^{45}$ erg s$^{-1}$ (in the 0.5–4.5 keV band). The measured gas mass within the central few Mpc is of the order of $10^{14}$ M$_\odot$, and the inferred cooling times for the gas in some clusters are of the order of $10^9$ yr (Fabian, Nulsen & Canizares 1982). The surface brightness profiles of clusters are sharply peaked at the centre. From X-ray observations, with a few exceptions, most cluster cores appear smooth and uniform with very angular isophotes. Accurate measurements of the temperature profile in the cores are only now becoming feasible (for $z < 0.1$ clusters) with the ASCA satellite.

2.1.4 Mass profiles from X-ray data

Standard deprojection analysis of the X-ray surface brightness profile gives the density profile of the gas, and the cluster potential within which the gas is confined (Sarazin 1988). The main limiting assumptions are spherical symmetry, hydrostatic equilibrium, the inability to take substructure into account, and assumptions regarding the unknown radial temperature profile of the gas (Nulsen & Böhringer 1995). The last assumptions are particularly important when modelling the inner parts of cluster cores with strong cooling flows, where one expects to have a multi-phase ICM (Allen, Fabian & Kneib 1996), or in clusters which have undergone recent mergers, resulting in a complex temperature structure. Preliminary results of measurements of the temperature structure of the core of A2256 (Briel & Henry 1995) seem to indicate that this cluster, which was believed to be one of the smoothest and most uniform from its X-ray image, shows strong evidence for temperature gradients in the core (Markevitch et al. 1996).

2.2 N-body simulations

N-body simulations provide the crucial link in understanding how the observed structure in clusters arises in the context of their evolution from the initial perturbations in the gravitational instability picture. High-resolution simulations that incorporate gas dynamics and some of the important gas physics like shocks and radiative cooling are being used to study the formation, dynamics and evolution of galaxy clusters in the scenario where structure is built up hierarchically in a universe dominated by cold dark matter (CDM) (Evrard 1990; Cen, Gnedin & Ostriker 1993; Navarro & White 1994; Frenk et al. 1995). The evolution of both the dark matter and the baryonic component can be tracked to within the resolution limit. While the core of an individual cluster cannot be resolved in enough detail to understand relaxation processes, an ensemble of clusters can be studied for their ‘average’ properties (Navarro, Frenk & White 1994).

The density profiles of clusters formed in these simulations are sensitive to the underlying cosmological model, the initial conditions, the accuracy of modelling gas dynamic processes, and prescriptions for galaxy and star formation. Efstathiou et al. (1985) found that in simulations with only dark matter particles, the slope of the density profile on cluster scales steepens with increasing $n$, where $n$ is the spectral index of the scale-free, initial perturbations in an Einstein–de Sitter cosmology. It has been suggested by Crane, Evrard & Richstone (1994) and Navarro et al. (1994) that ensemble cluster properties, like abundance, clustering and density profiles, might be a useful discriminant of cosmological parameters. However, properties of currently simulated clusters are not consistent in detail with their observed properties, primarily a reflection of the lack of understanding of the physics of galaxy formation and the role of non-gravitational processes coupled with the lack of knowledge of $\Omega$. Qualitatively though, the morphology of simulated clusters is quite similar to that indicated by ROSAT observations of X-ray clusters, and the physical effects of mass segregation due to dynamical friction and luminosity segregation seem to be borne out in the simulations – evidence to support our naive theoretical picture of the formation of clusters.

In a recent paper, Navarro, Frenk & White (1994) report the results of their $N$-body + Smooth Particle Hydrodynamics (SPH) simulations, wherein a ‘universal density profile’ is found to be a good fit over a large range of scales for dark haloes in standard CDM models. The halo profiles are more or less isothermal, shallower than $r^{-2}$ near the central regions and steeper close to the virial radius. The
density profile has the following form,
\[ \rho(r) = \frac{\rho_0}{r^2 (1 + r/r_s)^2}, \]
where \( r_s \) is a scale radius. The corresponding mass profile is given by
\[ M(r) = M_0 \left[ \ln \left( \frac{1 + r}{r_s} \right) + \frac{1}{1 + r/r_s} \right], \]
We examine this mass model and its consequences for the resulting dynamics of cluster cores in Section 5.

### 2.3 Lensing by clusters

Clusters of galaxies have the optimum cross-section for lensing the nearly isotropically distributed high-redshift faint galaxy population. Lensing by an extended mass concentration can be understood in terms of a mathematical mapping (e.g. Blandford & Narayan 1987; Fort & Mellier 1994) from the source plane on to the image plane, with the properties that it conserves surface brightness and is achromatic. The deflections produced are non-linear with impact parameter, and therefore produce both amplification and distortion of the background sources. There are two important effects: the isotropic magnification and the non-isotropic distortion. The isotropic magnification is caused by mass interior to the beam, and is pronounced in the region of the image plane where the local surface mass density \( \Sigma \) is of the order of the critical surface mass density, \( \Sigma_{\text{crit}} \), which occurs in the dense cores of rich clusters producing multiple images and arcs. \( \Sigma_{\text{crit}} \) depends on the angular distance to the source and lens, and hence on cosmological parameters.) The anisotropic distortion of images is caused by the gradient of the two-dimensional potential and characterizes the 'weak lensing' regime, the signal being arclets (single, weakly sheared images) produced even at large distances from the cluster centre.

A composite mass profile for a cluster can be constructed using a variety of constraints from lensing effects over a range of scales. The strong lensing regime constrains the total mass enclosed within the 'Einstein radius', while weak shear effects (measured statistically from the ellipticities of faint background galaxies) determine the slope of the mass profile at the outskirts.

#### 2.3.1 Constraints from strong lensing

The input from observations for the mass modelling are arc positions, the number of merging images and their parities, and the width, shape and curvature of the arcs. They are used to determine the location of the critical lines in the image plane, which are then mapped back to the source plane in the method developed by Kneib (1993). The difference in parameters implied by each of the multiple images is then minimized in the \( \chi^2 \) sense in the source plane. In order to calibrate the lens model, at least one arc redshift needs to be measured. The usual mass profiles used in modelling the cluster mass distributions are the cored isothermal profile (Blandford & Kochanek 1987), the pseudo-isothermal elliptical mass distribution (PIEMD) (Kassiola & Kovner 1993) or a linear combination of them. The PIEMD model has a two-dimensional surface mass density defined by
\[ \Sigma(r) = \frac{\Sigma_0}{\sqrt{1 + r^2 / r_0^2}}, \]
The corresponding three-dimensional density profile and the mass are
\[ \rho(r) = \frac{\rho_0}{1 + r^2 / r_0^2}, \]
\[ M(r) = M_0 \left( \frac{r}{r_0} - \tan^{-1} \frac{r}{r_0} \right), \]
Modelling the arcs with this profile, the normalization and the core radius (which is a measure of the compactness of the mass distribution) are determined. The core radius of most lensing clusters is observed to be quite small, \( 30 \lesssim r_0 \lesssim 100 \ h_{70}^{-1} \) kpc.

#### 2.3.2 Constraints from weak lensing

The slope of the mass profile at large radii \( r > 200 \ h_{70}^{-1} \) kpc is constrained by the observed weak distortion effects. The weak shear \( \gamma \) induced by the cluster on the background images can be written for the circularly symmetric case as
\[ \gamma < \langle D(z_s) \rangle \left[ \bar{\Sigma}(r) - \Sigma(r) \right], \]
where \( \langle D(z_s) \rangle \) is the mean of the ratio of the angular distances \( D_{\text{lens-source}}/D_{\text{observer-source}} \), and \( \bar{\Sigma} \) is the mean surface density within radius \( r \). The Kaiser–Squires technique (Kaiser & Squires 1993) defines a mapping that relates the image ellipticities to the relative mass map \( \Sigma(r) \) for a cluster. To construct the surface mass density profile, one uses the statistic suggested by Fahlman et al. (1994) and Squires et al. (1995),
\[ \bar{\Sigma}(r) = \frac{2 \Sigma_{\text{crit}}}{1 - r^2 / r_0^2} \int_{\gamma_0}^{\gamma_1} \frac{d\gamma}{r}, \]
where \( \langle \gamma_0 \rangle \) is the mean tangential component of the image ellipticities. For this inversion, deep optical images under exquisite seeing conditions of a wide field over the lensing cluster are required. Details of correction and compensation for the anisotropy of the point-spread function and bad seeing conditions have been demonstrated by Bonnet & Mellier (1995) and Kaiser (1995).

#### 2.3.3 Constraints from the cD galaxy

Dark matter in clusters is sharply peaked about the cluster centre around which the lensed images are seen. In most clusters with spectacular arcs, the centre of the brightest cluster galaxy and the centre of the dark matter distribution, as determined from both X-ray and lensing studies, seem to be coincident to within the errors – of the order of a few arcseconds.

The central bright elliptical galaxies are often cDs with diffuse haloes extending out to beyond the Einstein radius. The orbits of the stars in these haloes trace the overall dark
matter potential. The density profiles of cD galaxies are fairly well determined observationally (Kneib et al. 1995) and are best fitted by a difference of two PIEMD models,

$$\rho(r) = \frac{\rho_0 r^2}{(r_{\text{cut}}^2 - r^2)} \left( 1 - \frac{1}{r_{\text{cut}}^2 + r^2} \right),$$  \hspace{1cm} (10)

where $r_c$ is the core radius, and $r_{\text{cut}}$ is the truncation radius. The velocity dispersion profile of giant ellipticals is also measured in a number of clusters (Fisher, Illingworth & Franx 1995) and is found to be $\sigma_\beta \approx 300-500$ km s$^{-1}$. Close to the cluster centre, the overall mass profile has to be consistent with the measured isotropic velocity dispersion of stars in the cD (Miralda-Escudé 1995).

3 DYNAMICAL EQUATIONS

We model the cluster as a collisionless system in which the individual galaxies move under the influence of the mean gravitational field $\phi$ generated by all the constituents. The system is characterized by its phase-space density $f(x, v, t)$, and a given configuration of the system is specified by $f(x, v, t) \ d^3 x \ d^3 v$ – the number of galaxies having positions in the infinitesimal volume $d^3 x$, with velocities in the range $d^3 v$. It should be noted here that in phase space $x$ and $v$ are independent variables and the potential is not a function of $v$.

The density of points in phase space satisfies the continuity equation,

$$\frac{Df}{Dt} = \partial f/\partial t + v \cdot \nabla f - V \partial f/\partial v = 0,$$  \hspace{1cm} (11)

which is the collisionless Boltzmann equation. Neglecting the explicit time derivative, taking the first velocity moment, and integrating over all possible velocities for a spherical system, we obtain the Jeans equation:

$$\frac{d(\rho \sigma_\beta^2)}{dr} + \rho \left[ 2 \sigma_\beta^2 - (\sigma_\alpha^2 + \sigma_\gamma^2) \right]/r = - \rho \frac{d\phi}{dr},$$  \hspace{1cm} (12)

where $\rho$ is the density profile and $\sigma_\beta^2$ are the components of the velocity dispersion. If, additionally, the velocities and the density are invariant under rotations about the cluster centre, then we have

$$\sigma_{\alpha}^2 = \sigma_{\gamma}^2 = \sigma_\beta^2,$$  \hspace{1cm} (13)

$$\frac{d(\rho \sigma_\beta^2)}{dr} + 2 \frac{\beta \sigma_\beta^2 \rho}{r} - \rho \frac{d\phi}{dr} = 0,$$  \hspace{1cm} (14)

The velocity anisotropy parameter $\beta$ at a given point is

$$\beta(r) = \left( 1 - \frac{\sigma_\alpha^2}{\sigma_\beta^2} \right).$$  \hspace{1cm} (15)

3.1 The isotropic Jeans equation

The isotropic Jeans equation is a special case of the more general equation above, wherein the galaxies are on isotropic orbits. Hence $\beta = 0$, and $\sigma_\beta^2 = \sigma_\gamma^2$,

$$\frac{d(\rho \sigma_\beta^2)}{dr} = - \frac{GM(r) \rho(r)}{r^2} - \rho \frac{v_\star^2}{r},$$  \hspace{1cm} (16)

where $v_\star(r)$ is defined to be the circular velocity. The solutions for the isotropic velocity dispersion are given by

$$\sigma_\beta^2(R) = \frac{GM(R)}{\rho} \int_R^\infty \frac{M(r) \rho(r) \ dr}{r^2} = \frac{1}{\rho} \int_R^\infty \rho(r) v_\star^2(r) \ dr,$$  \hspace{1cm} (17)

The velocity dispersion of both the galaxies and the total mass can therefore be computed, given their respective density profiles and the underlying mass distribution. We plot the solutions obtained for a mass model of the form

$$M(r) = M_0 \left( \frac{r}{r_0} - \frac{1}{r_0} \right),$$  \hspace{1cm} (18)

with $r_0 = 50$ kpc, $\sigma_\beta = 1200$ km s$^{-1}$, and generic density profiles for the tracers (galaxies or isothermal gas), of the form

$$\rho(r) = \frac{\rho_0}{(1 + r^2/r_0^2)^3}.$$  \hspace{1cm} (19)

The solutions for $\alpha = 1.0, 1.2$ and $1.5$ and for various values of $\beta_0/\sigma_\beta = 1.0, 2.5$ and $5.0$ are plotted in Fig. 1. The solutions have the following interesting properties:

(i) the velocity dispersion falls in the centre for small core radii;

(ii) the smaller the core radius of the tracer, the lower the central value of the velocity dispersion, and

(iii) the mean value of the velocity dispersion is a weak function of $r_c$, but depends on the slope $\alpha$.

Therefore one can have different mean values of the velocity dispersion for different components if they do not have the same radial profile.

In Fig. 1, we also plot the ratio of the velocity dispersions of dark matter and galaxies, which provides a qualitative understanding of the velocity bias (as found in the numerical simulations by Carlberg 1994). The difference in the asymptotic slope of the density profiles of dark matter and galaxies, and the ratio of the core radii are found to determine the velocity bias. To first order, the asymptotic behaviour (regardless of core size) is

$$\frac{(\sigma_{\text{DM}})}{\sigma_\beta} \approx \frac{x_{\text{DM}}}{x_\beta}.$$  \hspace{1cm} (20)

3.2 Anisotropic Jeans equation

The Jeans equation is a mathematical statement of detailed pressure balance for an equilibrium stellar system. The dynamical evolution of clusters in N-body simulations has been studied using the distribution function formalism (Natarajan, Hjorth & Van Kampen 1996, in preparation). This analysis indicates that clusters evolve from one quasi-equilibrium state to another. A cluster in a quasi-equilibrium configuration is found to be virialized and has a smooth potential, which is traced by galaxies with one of the following orbital structures:

(i) $\beta = 0$, isotropic orbits;

(ii) $0 < \beta \leq 1$, orbits are mostly radial, or

(iii) $\beta < 0$, when the orbits are primarily transverse.
While $\beta$ has no lower bound, it is strictly required to be less than unity for any physically admissible solutions for the velocity dispersion. In this context, it is instructive to examine and compare with studies of the formation and evolution of elliptical galaxies (Hjorth & Madsen 1991). It has been shown that the observed uniformity in the properties of elliptical galaxies can arise from either of two sets of initial conditions: dissipationless cold collapse or a 'warm collapse' (or merger) with dissipation. The predicted evolution to the final state with a deep potential and significant radial anisotropy arises from the relaxation brought about by global potential fluctuations rather than two-body encounters (Aguilar & Merritt 1990; Londrillo, Messina & Stiavelli 1991). Radial anisotropy can therefore arise naturally in most models as a consequence of relaxation and, as demonstrated by Gerhard (1993), the line-of-sight velocity profiles, being more sensitive to $\beta$ and less so to the potential or to the stellar number density profile, provide a probe of the kinematics of the core. Conversely, for a galaxy cluster, the initial collapse conditions are different and, additionally, many physical processes that can effect energy exchange are active and do occur in the dense core region. For a cluster core that has virialized, we expect the orbits in the core to reflect the efficiency of the energy exchange mechanisms, while outside the core region, we expect and do find that the orbits are largely radial, $0 < \beta \leq 1$ (Natarajan, Hjorth & Van Kampen 1996, in preparation).

4 PROPOSED APPROACH

4.1 The mathematical formalism

In our approach, we solve the full Jeans equation for the velocity anisotropy parameter $\beta$ and for the radial component of the velocity dispersion $\sigma_r^2$, using the projected mass profile for the cluster as constructed independently from gravitational lensing.

From the observed projected galaxy positions, we fit to get a surface number density profile $\Sigma_g(r)$ and use the Abel integral inversion to extract the three-dimensional density profile $\rho_g(r)$. The key assumption made in the analysis below is that of spherical symmetry. Starting with the full Jeans equation,

\[
\frac{d}{dr} \left( v_r \sigma_r^2 \right) + 2 \beta(r) v_r \sigma_v^2 = - \frac{G M_{\text{tot}}(r) v_r}{r^2},
\]

where $v_r(r)$ is three-dimensional galaxy density profile, $\sigma_v^2(r)$ is the radial velocity dispersion of the galaxies, $\beta(r)$ is the velocity anisotropy, and $M_{\text{tot}}(r)$ is the distribution of total mass (most accurately determined from gravitational lensing).

In addition, we have the equation that defines the observed line-of-sight velocity dispersion profile $\sigma_{ls}(R),$
We need to solve these two integro-differential equations numerically for $\sigma^2$ and $\beta(r)$. For an individual galaxy with an assumed mass profile, these coupled equations have been solved by Binney (1982) and Bicknell et al. (1989).

We truncate the integration at a large, finite truncation radius $R_\text{t}$, defined strictly to be the radius at which both $\Sigma(R)$ and $\rho_\text{g}(R)$ tend to zero. Substituting the expression for $\beta$ from equation (22), we have

\[
\frac{1}{2} \left[ \Sigma(R) \sigma_{\text{los}}^2(R) \right] - R^2 = \int_R^\infty \frac{\rho_\text{g}(r) \sigma^2(r)}{r^2} dr.
\]

Integrating the first term on the right-hand side by parts and substituting back, we have

\[
\frac{1}{2} \left[ \Sigma(R) \sigma_{\text{los}}^2(R) \right] - R^2 = \int_R^\infty \frac{\rho_\text{g}(r) \sigma^2(r)}{r^2} dr + \frac{1}{2} \int_R^\infty \frac{d(r \sigma^2)}{r^2} dr.
\]

The equation can be further simplified and reduced after some algebra (for details see Bicknell et al. 1989) to the following integrals,

\[
\sigma^2(r) = I_1(r) - I_2(r) + I_3(r) - I_4(r),
\]

\[
I_1(r) = \frac{1}{3} \int_r^\infty \frac{GM_{\text{tot}}(r) v_g}{r^2} dr,
\]

\[
I_2(r) = -\frac{2}{3r} \int_r^\infty GM_{\text{tot}}(r) v_g dr,
\]

\[
I_3(r) = \frac{1}{r^2} \int_0^r R \Sigma(R) \sigma_{\text{los}}^2(R) dR,
\]

\[
I_4(r) = \frac{2}{\pi r^2} \int_r^\infty R \Sigma(R) \sigma_{\text{los}}^2(R) \left( \frac{r}{\sqrt{r^2 - R^2}} \sin^{-1} \frac{r}{R} \right) dR.
\]

It is to be noted here that the explicit dependence on the mass profile and the observed line-of-sight velocity dispersion profile separate. All the above integrals are well behaved, with the exception of $I_4$, which has an integrable singularity which can be taken care of easily via a simple transformation of variables. Computing these integrals is nevertheless tricky, as the final profile for $\sigma^2(r)$ is sensitive to the precise asymptotic behaviour of all the four terms (see Fig. 2). The numerical solution for $\sigma^2(r)$ is then substituted back into the Jeans equation to obtain $\beta$,

\[
\beta(r) = \frac{r}{2 r \sigma^2} \left[ \frac{GM_{\text{tot}}(r) v_g}{r^2} + \frac{d}{dr} (r \sigma^2) \right].
\]

The variation of $\beta$ with radius can be understood physically in terms of the relative importance of the mass term and the ‘galaxy pressure’ gradient term. Rewriting the above equation as follows,

\[
\nu(r) \sigma^2 = I_1(r) - I_2(r) + I_3(r) - I_4(r),
\]

\[
I_1(r) = \frac{1}{3} \int_r^\infty \frac{GM_{\text{tot}}(r) v_g}{r^2} dr,
\]

\[
I_2(r) = -\frac{2}{3r} \int_r^\infty GM_{\text{tot}}(r) v_g dr,
\]

\[
I_3(r) = \frac{1}{r^2} \int_0^r R \Sigma(R) \sigma_{\text{los}}^2(R) dR,
\]

\[
I_4(r) = \frac{2}{\pi r^2} \int_r^\infty R \Sigma(R) \sigma_{\text{los}}^2(R) \left( \frac{r}{\sqrt{r^2 - R^2}} \sin^{-1} \frac{r}{R} \right) dR.
\]
\[ \beta(r) = \frac{1}{2} \frac{\frac{\sigma^2(r)}{\sigma_0^2} + \frac{1}{\ln \frac{r}{r_0}}}{\sigma_0^2} \]  
\[ \text{eqn (31)} \]

We find that the sign of \( \beta \) depends crucially on the asymptotic behaviour of the mass model at large \( r \), and specifically for \( \rho_{\text{tot}} \) ranges between \( r^{-2} \) and \( r^{-3} \), it is found to be fairly insensitive to the slope of the assumed galaxy density profile. The sensitivity of the sign and magnitude of \( \beta \) to the slope of the mass profile enables its use as a discriminant between the various mass models.

5 RESULTS FOR VARIOUS MASS PROFILES

We consider several physically motivated fiducial density profiles for the total mass, and in what follows we examine both galaxy distribution profiles described in equation (1) (PROFILE A) and equation (2) (PROFILE B). The asymptotic slope of the density profile is defined to be \( \gamma \).

All the mass profiles are normalized to have the same total projected mass enclosed within the Einstein radius \( [M_{\text{Ein}}(r_e=r_{\text{Ein}})=(5 \pm 0.1) \times 10^{13} M_{\odot}] \), as calibrated from strong cluster lensing in A2218. The density profiles and mass models studied are:

**Model I:**

\[ \rho(r) = \frac{\rho_0}{(r^2 + r_0^2)^{\frac{\gamma}{2}}}; \quad \gamma = 2 + 1, \]

\[ \text{eqn (32)} \]

\[ M(r) = 2\pi \rho_0 \ln (r^2 + r_0^2); \quad \alpha = 1. \]

**Model II:**

\[ \rho(r) = \frac{\rho_0}{(r^2 + r_0^2)^{\frac{\gamma}{2}}}; \quad \gamma = 2, \]

\[ \text{eqn (34)} \]

\[ M(r) = 2\pi \rho_0 \ln (r^2 + r_0^2); \quad \alpha = 1. \]

**Model III:**

\[ \rho(r) = \frac{\rho_0}{(r^2 + r_0^2)^{\frac{\gamma}{2}}}; \quad \gamma = 3 + 1, \]

\[ \text{eqn (36)} \]

\[ M(r) = 4\pi \rho_0 r_0 \left( \frac{r}{r_0} - \ln (r + r_0) \right); \quad \alpha = 1. \]

\[ \text{eqn (37)} \]

5.1 Dependence on the slope \( \gamma \)

We analyse here the results for the various mass models with a specified asymptotic slope \( \gamma \), assuming a core radius of \( r_0 = 250 \) kpc for the galaxy distribution of PROFILE A with \( r_0 = 60 \) kpc for the dark matter.

(i) For \( \gamma = -2.0 \) (Fig. 3, top panel), and a range of input values of the line-of-sight velocity dispersion assumed to be constant (\( \sigma_{\text{los}} = 800, 1000 \) and \( 1400 \) km s\(^{-1}\)), we obtain unphysical solutions (\( \rho_1(r) < 0 \)) for the lowest \( \sigma_{\text{los}} \) for all the three models. On increasing \( \sigma_{\text{los}} \) to \( 1000 \) km s\(^{-1}\), the orbits are primarily transverse in the core, progressing to more radial ones in the outer parts, so that \( \beta < 0 \). For the highest \( \sigma_{\text{los}} \), we find evidence for a small core region with mixed orbits, but with primarily radial orbits outside 200 kpc for all three models.

(ii) For \( \gamma = -2.5 \) (Fig. 3, middle panel), unphysical solutions are obtained for the lower \( \sigma_{\text{los}} \) value for all models, but for \( \sigma_{\text{los}} = 1000 \) and \( 1400 \) km s\(^{-1}\) we do find physically admissible solutions. All three mass models have a finite core with mixed orbits leading on to largely radial orbits outside. Model I has the largest mixed region (of the order of 700 kpc), while Models II and III have smaller mixed regions which are of the order of 500 kpc. The highest value of \( \sigma_{\text{los}} \) produces primarily radial orbits from the centre outward.

(iii) For \( \gamma = -3.0 \) (Fig. 3, bottom panel), and the lowest value of \( \sigma_{\text{los}} \), we obtain unphysical solutions, but as \( \sigma_{\text{los}} \) is increased there is evidence for a core with transverse orbits.

Assuming the galaxy distribution to be of the form of PROFILE B with a scale radius \( s = 200 \) kpc and \( r_0 = 60 \) kpc for the dark matter, we find the following trends.

(i) For \( \gamma = -2.0 \) (Fig. 4, top panel), and the same range of input values of the line-of-sight velocity dispersion (\( \sigma_{\text{los}} = 800, 1000 \) and \( 1400 \) km s\(^{-1}\)), we obtain unphysical solutions in the core region for the lowest \( \sigma_{\text{los}} \) for all the three models. On increasing \( \sigma_{\text{los}} \) to \( 1000 \) km s\(^{-1}\), the orbits tend to be transverse, so that \( \beta < 0 \). For the highest \( \sigma_{\text{los}} \) we find primarily radial orbits for all three models.

(ii) For \( \gamma = -2.5 \) (Fig. 4, middle panel), unphysical solutions are obtained for the lower \( \sigma_{\text{los}} \) value for all models, but for \( \sigma_{\text{los}} = 1000 \) we find that all three mass models have a finite core (of the order of 500 kpc) with mixed orbits leading on to largely radial orbits outside. The highest value of \( \sigma_{\text{los}} \) produces primarily radial orbits right from the centre outward.

(iii) For \( \gamma = -3.0 \) (Fig. 4, bottom panel), once again for the lowest value of \( \sigma_{\text{los}} \), we obtain unphysical solutions, but as \( \sigma_{\text{los}} \) is increased there is evidence for a core with tangential anisotropy.

Both PROFILES A and B require high values of the line-of-sight velocity dispersion to produced physically meaningful solutions. The trends above seem to be qualitatively consistent with the physical picture of ongoing isotropization and regularization in the core for all the three fiducial mass models considered.

5.2 Dependence on the circular velocity

An important parameter for the dynamics of the galaxies in the global cluster potential is the circular velocity, \( v_c \) (Fig. 5), which measures the change in slope of the mass profile. For a given mass model with asymptotic slope \( \gamma \), increasing the core radius increases the circular velocity for all three models (where these were normalized to have the same projected mass within the radius of the arc).

Comparing different mass models that have the same asymptotic value of \( v_c \) (but different \( r_0 \) and \( \gamma \)), we find that the velocity structure of the core and anisotropy profiles are fairly similar. The qualitative behaviour of \( \beta \) for fixed asymptotic \( v_c \) depends strongly on \( \sigma_{\text{los}} \), with increasing \( \sigma_{\text{los}} \) we find preferentially radial orbits. For a fixed \( \sigma_{\text{los}} \), increas-
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Figure 3. Top panel: $\beta(r)$ for PROFILE A with $r_g = 250$ kpc, the mass models with asymptotic slope $\gamma = -2.0$; solid curve - Model I, dotted curve - Model II, dashed curve - Model III, for $\sigma_{\text{tot}} = 800, 1000$ and 1400 km s$^{-1}$ respectively. Centre panel: $\beta(r)$ for the mass models with asymptotic slope $\gamma = -2.5$; solid curve - Model I, dotted curve - Model II, dashed curve - Model III, for $\sigma_{\text{tot}} = 800, 1000$ and 1400 km s$^{-1}$ respectively. Bottom panel: $\beta(r)$ for the mass models with asymptotic slope $\gamma = -3.0$; solid curve - Model I, dotted curve - Model II, dashed curve - Model III, for $\sigma_{\text{tot}} = 800, 1000$ and 1400 km s$^{-1}$ respectively.

ing the circular velocity increases the size of the mixed core region, while lowering the value of $\beta$ at large radius. For both high and low circular velocities, and $-3.0 \leq \gamma \leq -2.0$, low line-of-sight velocity dispersion models $\sigma_{\text{tot}} < 1000$ km s$^{-1}$ are ruled out purely from the dynamical point of view.

5.3 Dependence on the central density profile

The best probe of the shape of the density profile at the very centre comes from the observed velocity dispersion of the stars in the cD halo (Miralda-Escude 1995). For the fiducial mass models of Section 5, we solve for the line-of-sight velocity dispersion of the cD halo stars using the isotropic Jeans equation (neglecting the contribution of the mass of the cD galaxy to the total mass of the cluster):

$$\frac{d(\rho_{\text{cD}} \sigma_{\text{cD}}^2)}{dr} = -\frac{GM_{\text{tot}}(r) \rho_{\text{cD}}}{r^2}.$$  \hspace{1cm} (38)

Assuming a scaling of $\rho_{\text{cD}} \propto r^{-4}$, and $\rho_{\text{tot}} \propto r^{-\gamma}$, close to the centre, we obtain

$$\sigma_c = A \frac{r^{2-\gamma}}{(\delta + \gamma - 2)} + \text{constant}.$$  \hspace{1cm} (39)

Therefore, for $0 \leq \gamma < 2$ and $\delta + \gamma > 2$, we expect the velocity dispersion of the stars to rise. The cD profile from Section 2.3.4 is used with a core radius $r_c = 0.05$ kpc and $r_{\text{cut}} = 35.0$ kpc. The three models studied in the previous section predict profiles (Fig. 6) with low central values for $\sigma_{\text{cD}}$, as is steeply with radius. Models I and III have central values $\sim 200-400$ km s$^{-1}$, varying with $\gamma$, such that the steeper the total mass profile the higher the central value. For the stars, $\sigma_{\text{tot}}$ rises to 700 km s$^{-1}$ at $r = 100$ kpc, which is consistent with the measurements of the cD galaxy IC 1011 in A2029 by Dressler (1979) and Fisher, Illingworth & Franx (1995). Model II underpredicts the central value, and is qualitatively incompatible with the data.

5.4 Summary of the important parameters

In the above analysis, there are several parameters to be kept track of in order to interpret the results for the com-

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Figure 4. Top panel: $\beta(r)$ for PROFILE B with $r = 200$ kpc, the mass models with asymptotic slope $\gamma = -2.0$; solid curve - Model I, dotted curve - Model II, dashed curve - Model III, for $\sigma_m = 800, 1000$ and $1400$ km s$^{-1}$ respectively. Centre panel: $\beta(r)$ for the mass models with asymptotic slope $\gamma = -2.5$; solid curve - Model I, dotted curve - Model II, dashed curve - Model III, for $\sigma_m = 800, 1000$ and $1400$ km s$^{-1}$ respectively. Bottom panel: $\beta(r)$ for the mass models with asymptotic slope $\gamma = -3.0$; solid curve - Model I, dotted curve - Model II, dashed curve - Model III, for $\sigma_m = 800, 1000$ and $1400$ km s$^{-1}$ respectively.

6 APPLICATION TO A2218

We apply this technique to the Abell cluster A2218, at a redshift $z = 0.175$, with a mean measured velocity dispersion $\sigma_{\text{mean}} \sim 1370 \pm 160$ km s$^{-1}$ from 56 cluster members. A2218 is a cD cluster with a very peaked mass distribution and a compact core; hence a large number of gravitationally distorted arcs and arclets are observed. The mass model for this cluster was constructed using ground data (Kneib et al. 1995) and refined using HST data by (Kneib et al. 1996). Redshifts of two of the arcs were spectroscopically measured by Pelló et al. (1992), and further redshifts of arclets have been determined by Ebbels et al. (1996, in preparation), hence tightly calibrating the mass model.

6.1 Observational data

The observational input for the galaxies in A2218 for our analysis comes primarily from the photometric and spectroscopic survey by Le Borgne, Pelló & Sanahuja (1992).
Figure 5. Asymptotic behaviour of the fiducial mass models—varying the core radius $r_0 = 30, 60$ and $100$ kpc; solid curves—Model I, dotted curves—Model II, dashed curves—Model III.

Figure 6. Computed line-of-sight velocity dispersion for the cD halo stars: solid curve—Model I, dotted curve—Model II, dashed curve—Model III.
The observed surface density of galaxies in A2218 was fitted to a modified Hubble-law profile (PROFILE A) with a core radius $r_c = 250 h^{-1}_{50} \text{kpc}$,

$$\Sigma_0(r) = \frac{\Sigma_0}{1 + r^2/r_c^2}, \quad (40)$$

and the corresponding three-dimensional density profile from equation (1), as well as by the cuspy profile (PROFILE B) with a scale radius $s = 200 h^{-1}_{50} \text{kpc}$,

$$\Sigma_0(r) = \frac{\Sigma_0}{(rs)^{0.1}(1 + rs)^{0.15}}, \quad (41)$$

and the corresponding density profile from equation (2).

### 6.1.3 Constructing the mass profile

The lensing mass profile for the cluster was constructed from the strong-lensing data (arcs, arclets and resolved multiple images) from ground-based observations and the HST image by (Kneib et al. 1996), and the weak-lensing mass map published by Squires et al. (1995).

A2218 is best fitted by the following functional form of MODEL III with $\alpha = 1.0$ (see Section 5),

$$M(r) = M_0 \left[ \frac{r}{r_0} - \ln \left( 1 + \frac{r}{r_0} \right) \right], \quad (42)$$

where we normalize $M_0$ to the mass enclosed by the arc at $r_{arc} = 78.5 h^{-1}_{50} \text{kpc}$, and $r_o$, the core radius, is $60 h^{-1}_{50} \text{kpc}$. The corresponding three-dimensional density profile is

$$\rho(r) = \frac{\rho_0 r^2}{r(r + r_0)}, \quad (43)$$

with $\rho_0 = 1 \times 10^{-22} \text{ g cm}^{-3}$. The X-ray mass profile was obtained using the standard deprojection technique described by Fabian et al. (1981) to the archival ROSAT HRI map, assuming spherical symmetry and hydrostatic equilibrium for the intracluster gas. The integrated X-ray luminosity (in the 0.1–2.4 keV band) and central temperature of A2218 are measured to be, respectively,

$$L_x = 7 \times 10^{44} \text{ erg s}^{-1}, \quad T = 8 \text{ keV}, \quad (44)$$

in good agreement with the Squires et al. (1995) results. The predicted circular velocity is

$$v^2(r) = \frac{GM(r)}{r} = \frac{kT}{\mu m_p} \left[ \frac{d (\ln \rho_{\text{halo}})}{d \ln r} + \frac{d \ln T}{d \ln r} \right], \quad (45)$$

where $M(r)$ is the total mass as inferred from the X-ray analysis.

The mass model from N-body simulations was also normalized to the mass enclosed within the Einstein radius, and for consistency with the observed arcs is A2218 the scale radius $r_s$ (see equation 3) is required to be of the order of $250 h^{-1}_{50} \text{kpc}$ (Waxman & Miralda-Escudé 1995).

### 6.2 Results

Using the total mass profile constructed from lensing as described above, we solve the equations to obtain solutions (see Fig. 8) for $\sigma_v(r)$, $\sigma_t(r)$ (the radial and transverse velocity dispersion profiles respectively) and the velocity anisotropy parameter $\beta(r)$.

The profile was also checked for consistency with a measured stellar velocity dispersion of the halo stars in the cD galaxy (Fig. 9). (It is to be noted here that the measured line-of-sight velocity dispersion profile for A2218 is inconsistent with an isotropic solution.) We find that the orbits predicted for the best-fitting mass model in the central regions is consistent with the picture of a core not in equilibrium, independent of the assumed form for the galaxy number density distribution. The precise nature of orbits transverse ($\beta < 0$) or radial ($\beta > 1$) depends on the detailed shape of the line-of-sight velocity dispersion profile, which is not measured to adequate precision at present. Both $\sigma_r$ and $\sigma_t$ fall within the inner $600 h^{-1}_{50} \text{kpc}$, with $\sigma_r$ declining more rapidly and then tending to flatten off. From the slope of $\beta$, the trend with increasing $r$ is that the nature of orbits tends to being mainly radial at the outskirts, signaling the existence of a region dominated by infall. The physical picture that emerges for the description of the dynamical state of A2218 is one of a dynamically disturbed cluster core. For lower values of the measured line-of-sight velocity dispersion, we find a tendency for the predominance of transverse orbits in the central 400 kpc (which is precisely of the order of the distance between the two distinct optical clumps seen in the HST image) and could be interpreted as an indication of ongoing energy exchange in the core. Using the mass model from N-body simulations as the input, we
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Figure 8. Projected mass profile: solid triangles - X-ray data, solid circle - weak lensing mass estimates, filled pentagon - mass from cD, filled squares - mass enclosed within giant arc, solid line - HST mass model, dashed line - 'best-fitting' composite lens mass model, dot-dashed line - fitted X-ray mass model, dotted line - 'best-fitting' N-body model.

Figure 9. The best-fitting composite mass model for A2218. The radial and transverse components of the velocity dispersion and the velocity anisotropy parameter for $\sigma_0 = 1100, 1200$ and $1300$ km s$^{-1}$ and the galaxy distribution modelled by PROFILE A with $r_g = 250$ $h_{70}^{-1}$ kpc.
Figure 10. The best-fitting composite mass model for A2218. The radial and transverse components of the velocity dispersion and the velocity anisotropy parameter for $\sigma_{\text{ms}} = 1100, 1200$ and $1300$ km s$^{-1}$, and the galaxy distribution modelled by PROFILE B with $r_e = 200 h_{70}^{-1}$ kpc and $\alpha = 0.1$.

Figure 11. Computed line-of-sight velocity dispersion profile for cD halo stars of A2218.
find that the resultant predictions for $\beta(r)$ agree well with those calculated for the mass profile reconstructed from lensing. For the mass profile from X-ray data for A2218, we obtain qualitative agreement with the predictions from the lensing mass model.

7 CONCLUSIONS

Gravitational lensing provides the ‘cleanest’ way to construct the total mass profile for a cluster independent of the kinematic details; additionally, combining strong and weak lensing removes the scaling ambiguity allowing the calibration of other independent mass models. With ‘good’ data for an individual cluster, the requirements for consistency on the smallest to the largest scales are stringent enough to constrain the slope of generic density profiles for rich clusters. Accurate mass profiles are crucial for settling many important issues, such as the baryon fraction problem, and for understanding the discrepancies and biases arising in the X-ray, lensing and virial mass estimates for clusters.

In this paper, we have demonstrated that the dynamics and velocity structure of the core of galaxy clusters can be probed given an independently inferred total mass profile. The future applications of our method to study cluster cores are promising, given the prospect of collecting more spectrophotometric data of galaxies in cluster lenses (e.g. Yee et al. 1996).

With current data, we find strong evidence for the existence of an anisotropic central region. This is consistent with the picture of ongoing relaxation, wherein anisotropies in the velocity tensor can arise naturally as a consequence of the initial conditions coupled with evolution. Given the range of complex physical processes that operate in cluster cores that could alter galaxy orbits [e.g., dynamical friction (dynamical friction in an aspherical cluster can induce and amplify the velocity anisotropy, as demonstrated by Binney 1977, and is a possible origin for the inferred velocity anisotropy, especially in the case of A2218), potential fluctuations arising due to the presence of substructure, and the frequent presence of a cD galaxy at the centre of the cluster potential], it is not surprising that the core is not isotropic.

Distinguishing between the dynamical effects of the various physical mechanisms in order to model them satisfactorily, in addition to requiring from the observations more accurately determined line-of-sight velocity dispersion profiles for clusters would enable this technique to be applied more effectively. Further extension of this analysis to incorporate the dynamics of the intracluster gas with the lensing model self-consistently is required in order to understand the possible role of baryons in the dynamics of cluster cores.

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REFERENCES

Fort B., Mellier Y., 1994, A&AR, 5, 239